# Gravitational Collapse and Naked Singularities

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# 1. Introduction

# 1-1 Gravitational Collapse

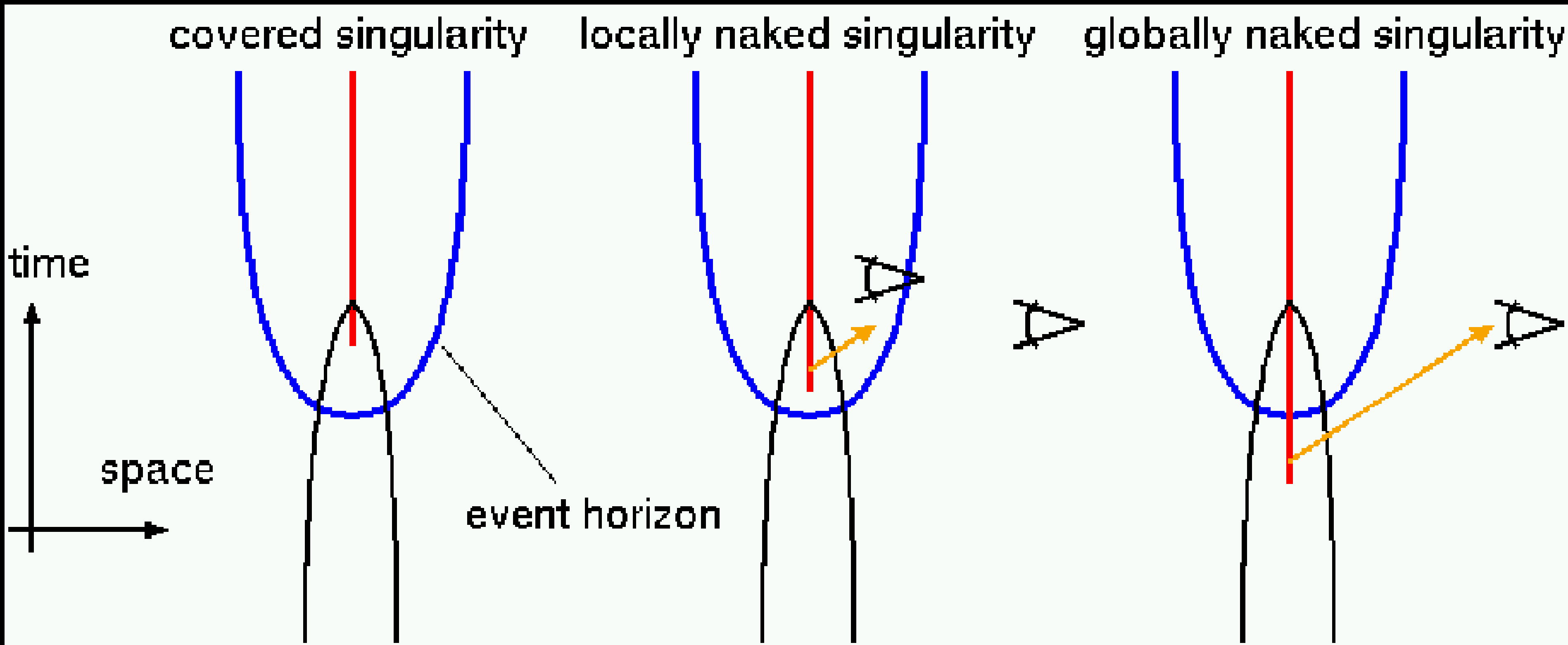
- A body may continue collapsing due to its self-gravity.
  - Upper limit to the maximum possible mass of a spherical body of cold nuclear matter
  - Cosmological initial density perturbations may have collapsed to BHs.
  - Observational evidence for the existence of massive/supermassive BHs
- Singularity theorem
  - There exist spacetime singularities in generic gravitational collapse.
- Spacetime singularity
  - The smoothness of spacetime is lost.
  - Classical physics cannot be applicable.

# 1-2 Cosmic Censorship

- Cosmic Censorship Conjecture (Penrose 1969, 1979)
  - Weak censorship
    - A system which evolves, according to classical general relativity with reasonable equations of state, from generic non-singular initial data on a suitable Cauchy hypersurface, does not develop any spacetime singularity which is visible from infinity.
  - Strong censorship
    - A physically reasonable classical spacetime is globally hyperbolic.
- Physical reasonableness
  - Matter fields
    - Energy conditions
    - Fluids with reasonable EOS's
    - Fundamental fields
  - Initial data
    - Nonsingular/sufficiently smooth initial data
    - Initial data generic in some appropriate topology

# 1-3 Naked Singularities

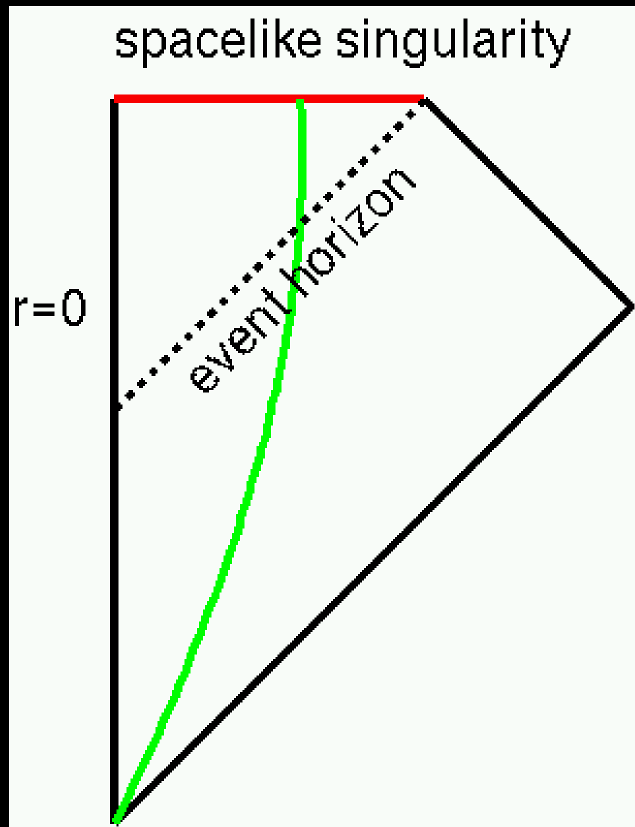
- Naked singularity (NS)
  - Singularity visible to some observer
- Globally NS
  - Singularity visible to an observer at infinity
- Locally NS
  - NS which is not globally naked



# 2. Spherically Symmetric Dust Collapse

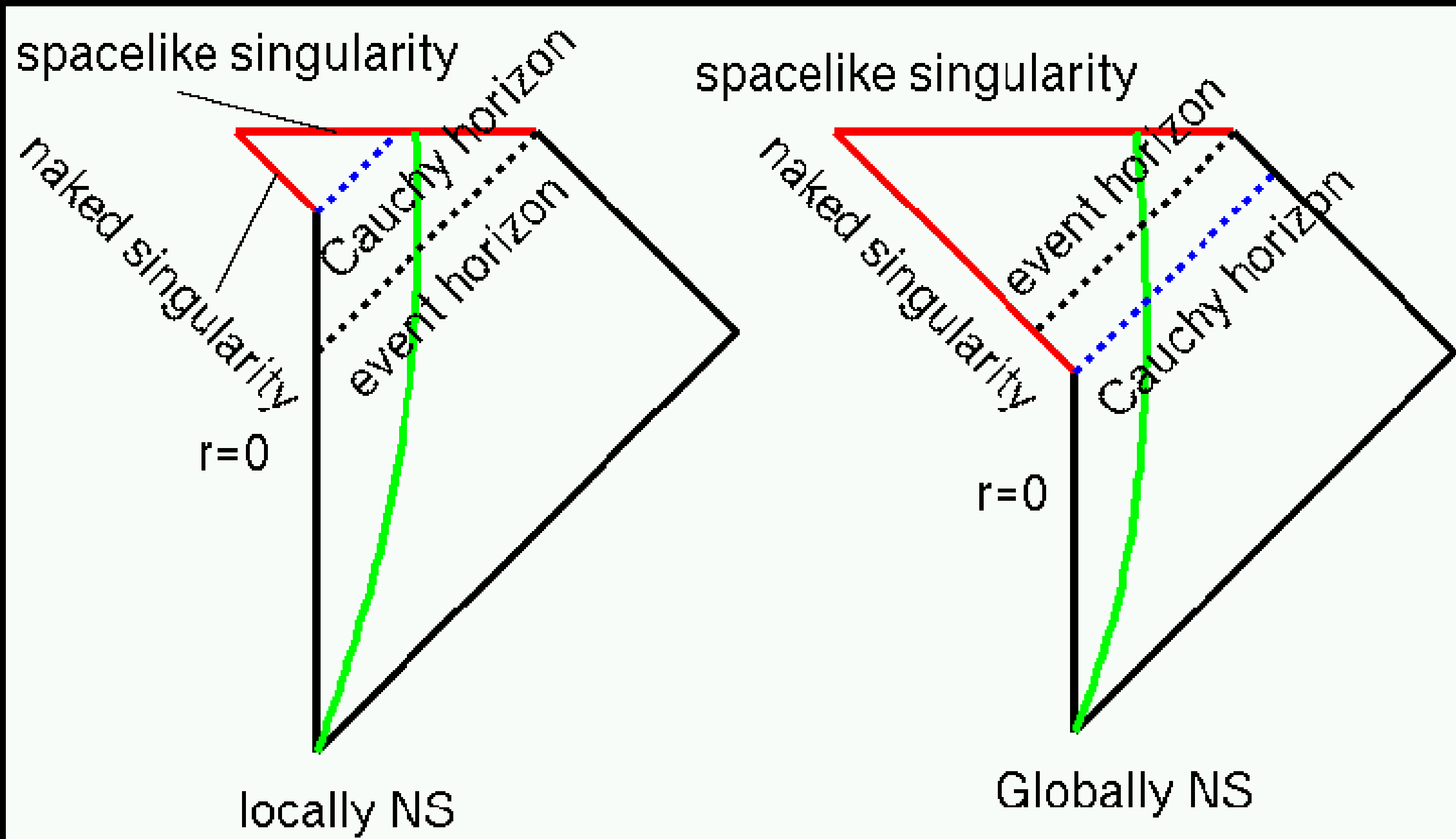
# 2-1 Homogeneous dust ball

- Oppenheimer-Snyder solution
  - The interior and exterior are described by the Friedmann solution and the Schwarzschild solution, respectively.
- Causal Structure
  - An event horizon is formed. A spacelike singularity appears inside the event horizon.



# 2-2 Inhomogeneous dust ball

- Lemaitre-Tolman-Bondi solution
  - A general exact solution
- Causal Structure
  - A central shell-focusing singularity can be naked. (Eardley and Smarr 1979)



# 2-3 NS in LTB solution

- Appearance of shell-focusing NS
  - Mathematical proof (Christodoulou 1984)
  - Condition and genericity for very general cases(Joshi and Dwivedi 1993, Singh and Joshi 1996, Jhingan and Joshi 1997)
    - The appearance of NS is generic for regular/sufficiently smooth initial data.
- Structure of the shell-focusing NS
  - The redshift is finite for the first null geodesic but becomes infinite for the subsequent null geodesics.
  - Curvature strong (Newman 1986, Deshingkar et al. 1999)
  - Not only radial geodesics but also nonradial null geodesics come out from the NS.(Mena and Nolan 2001, Deshingkar et al. 2002)
- The shell-focusing NS's are genuine singularities. However, a dust fluid would not be physically reasonable.

# 3. Recent Examples of NS's

# 3-1 BH threshold (1)

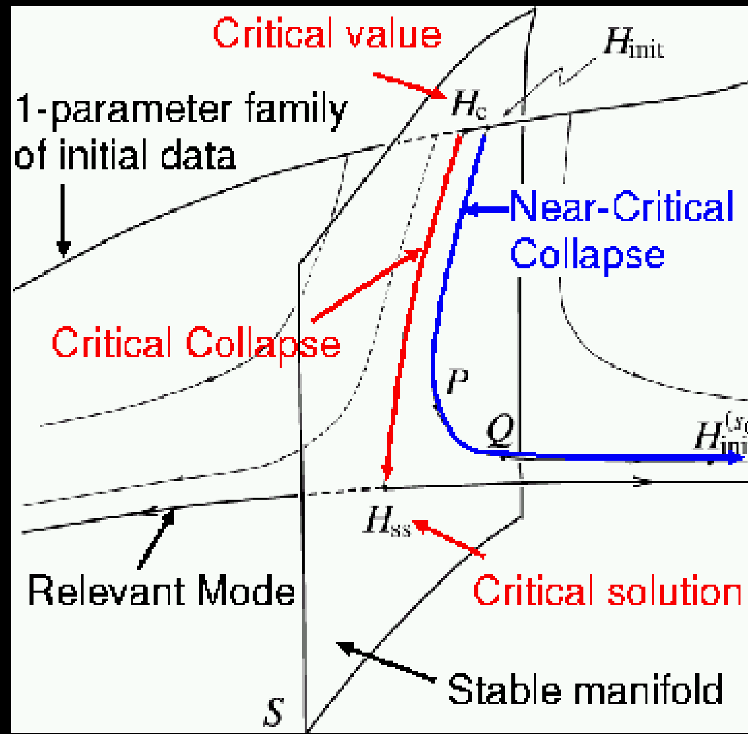
- Critical behaviour (Choptuik 1993)
  - For a one-parameter (p) family of initial data sets, there exists a critical value p* for the BH formation.
  - The mass of the formed BH obeys the scaling law for supercritical collapse.

$$M_{\rm BH} \propto |p - p^*|^\gamma \quad \text{for} \quad p \approx p^*$$

  - For a near-critical case, the collapse first approaches a self-similar solution, which is called a critical solution. And then it deviates from the critical solution.
  - A variety of matter fields
    - Massless scalar field
    - Axisymmetric GWs
    - Radiation fluid
    - Perfect fluid with EOS $P=k\rho$

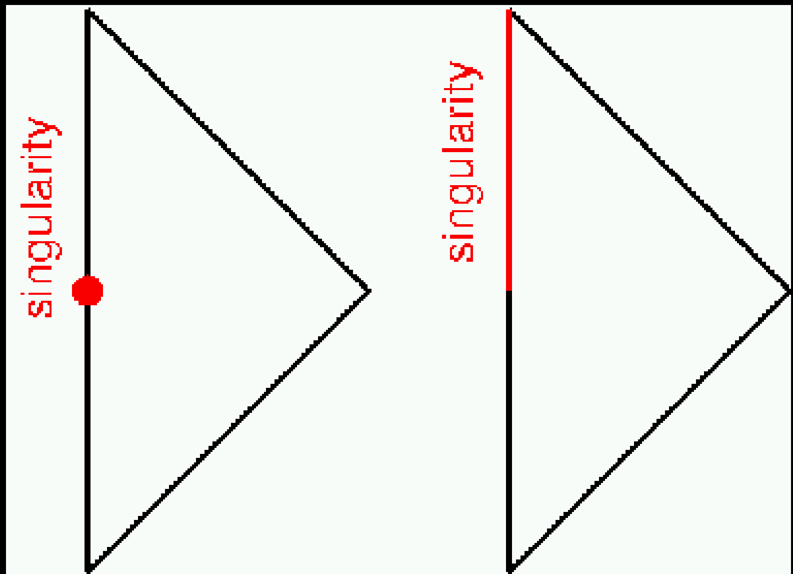
# 3-1 BH threshold (2)

- Intermediate attractor (Koike et al. 1995)
  - The critical solution is identified with a self-similar solution with a single unstable mode.
  - The critical phenomena are well described by the intermediate behaviour around the critical solution.
  - The critical exponent $\gamma$ is given by the eigenvalue of the single unstable mode.



# 3-1 BH threshold (3)

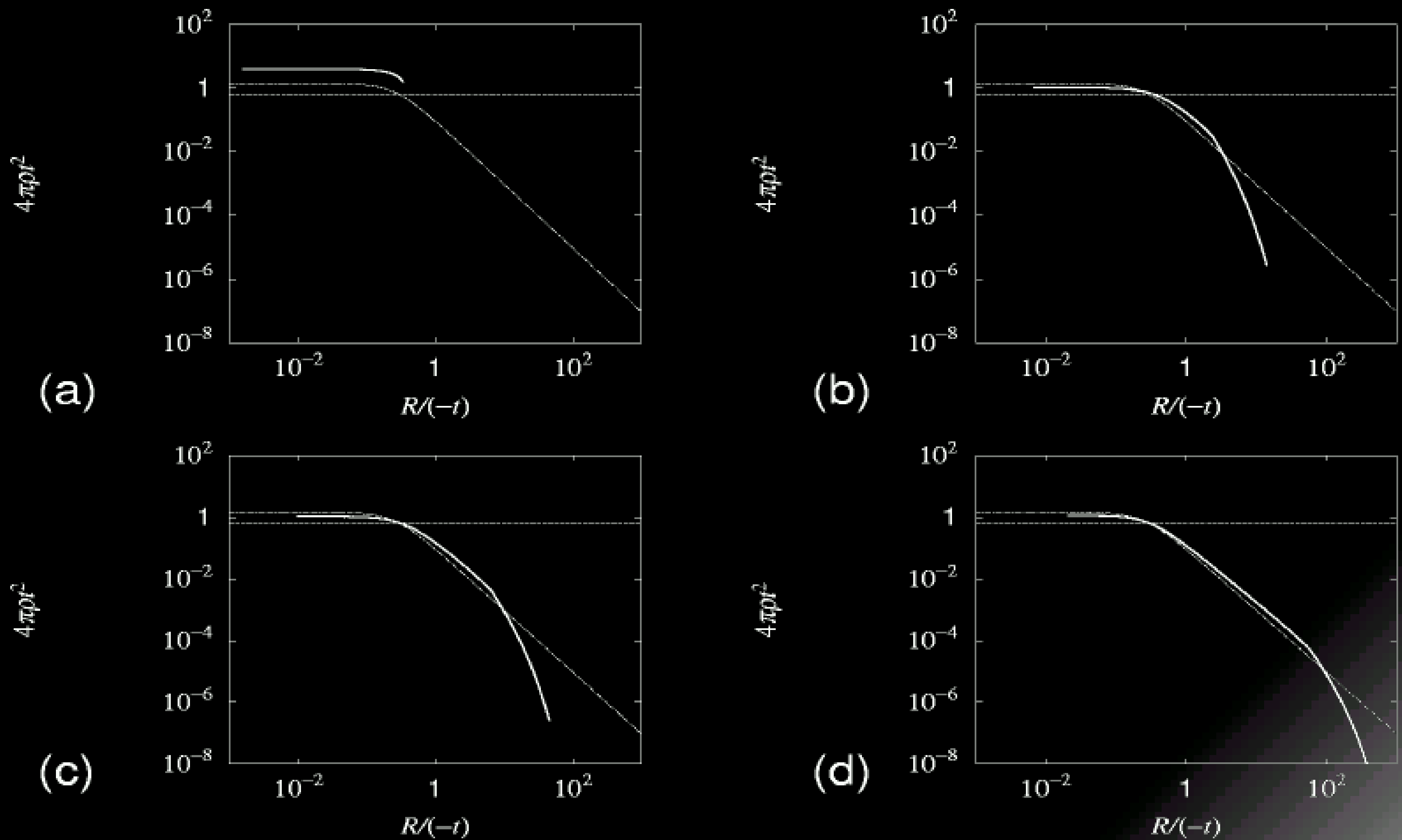
- BH threshold as a NS
  - Intuitively, a "zero-mass" BH is a NS. The curvature just outside the horizon scales as 1/M^2 for the formed BH. Take the limit M→0.
  - The Choptuik critical solution actually has a NS. (Gundlach and Martin-Garcia 2003)




  - The critical collapse is realised as a consequence of the exact fine-tuning. In other words, the critical collapse is NOT generic.

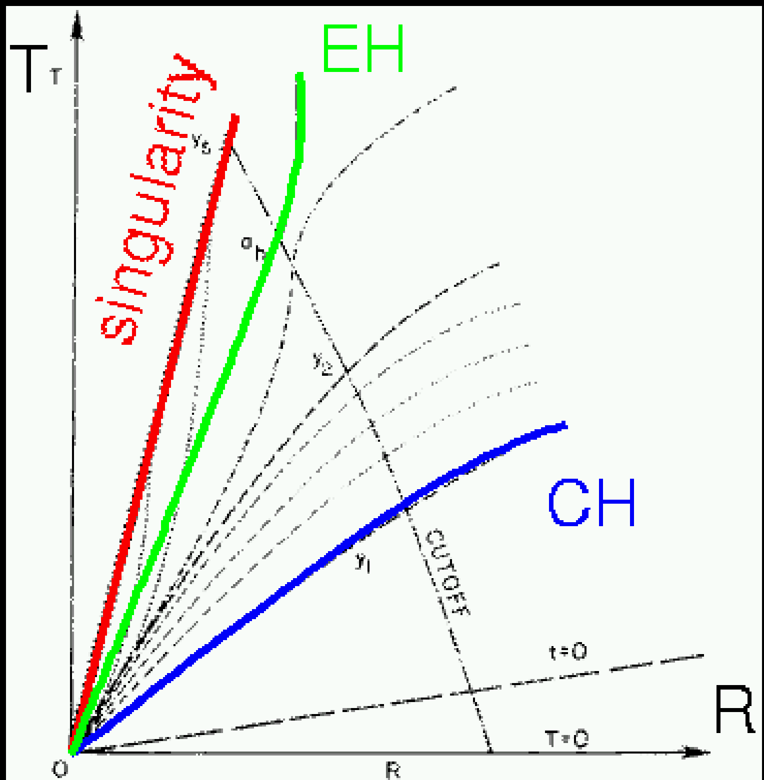
# 3-2 Self-similar attractor (1)

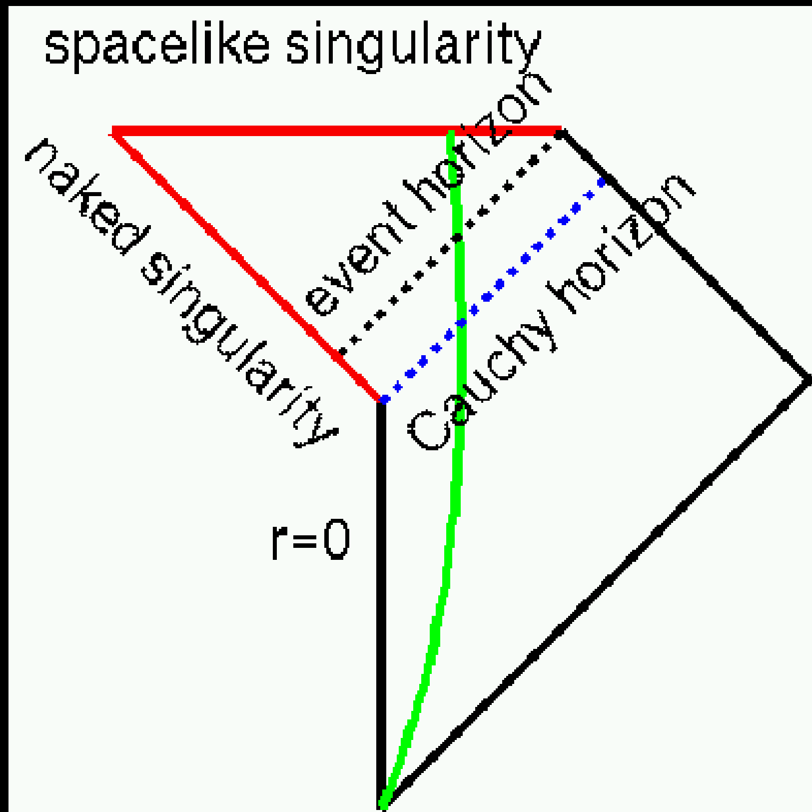
- Attractor self-similar solution
  - A spherically collapsing perfect fluid dynamically approaches a self-similar solution WITHOUT anyfine-tuning for EOS $P=k\rho$ $(0<k\leqq 0.03)$. (Harada and Maeda 2001)



# 3-2 Self-similar attractor (2)

- NS in the attractor solution
  - The approached solution is a self-similar solution already discovered, which describes NS formation for $0<k<0.0105$. (Ori and Piran 1987)

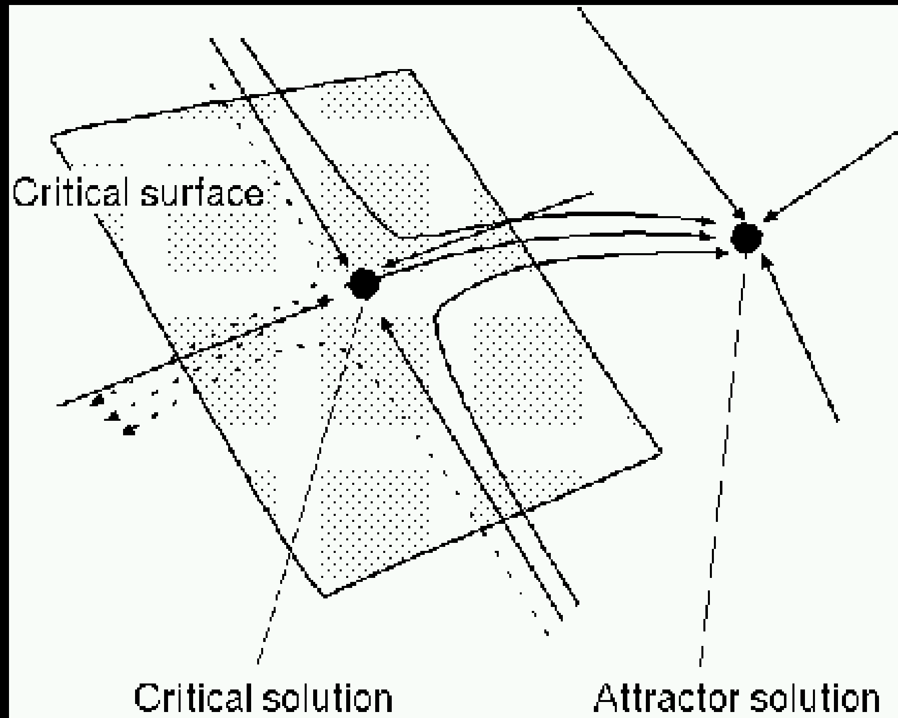


# 3-2 Self-similar attractor (3)

- Generic spherical collapse results in NS formation for a perfect fluid with nonvanishing pressure.
  - Fine tuning is unnecessary.
  - Generic violation of censorship for spherical case
- Self-similarity hypothesis (Carr 1993, Carr and Coley 1999)

- This is the first example of an attractor self-similar solution in generic spherical collapse in GR.
- It is expected that self-similar solutions can describe the asymptotic behaviour of more general solutions in a variety of systems.

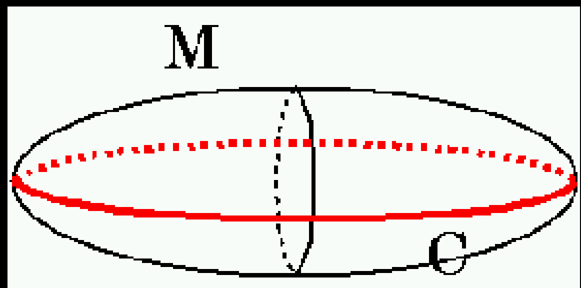


# 3-3 Other recent examples

- Spherical system of a massless scalar field (Christodoulou 1987, 1991, 1993, 1997, 1999)
- Spherical cluster of counterrotating particles (Harada et al. 1998, Jhingan and Magli 2000, Kudoh et al. 2000)
- Spherical collapse of null dust (Joshi and Dwivedi 1992)
- Spherical collapse of type II matter (Harko and Cheng 2000)
- Quasi-spherical dust collapse (Joshi and Krolak 1996, Deshingkar et al. 1998)
- Spherical dust collapse in higher dimensions (Patil 2003, Banerjee et al. 2003)
- Spherical collapse with unspecified matter fields

# 3-4 Highly nonspherical collapse

- **Cylindrically symmetric collapse**
  - Nonexistence of horizons (Thorne 1972, Hayward 2000)
  - Collapse with (counter)rotation (Apostolatos and Thorne 1992, Pereira and Wang 2000, Nolan 2002)
  - Strong GW emission?(Echeveria 1993, Chiba 1996)
- **Axisymmetric collapse**
  - Spindle singularity?(Shapiro and Teukolsky 1991, 1992)
  - Very little is known.
- **General case???**
  - Hoop conjecture (Thorne 1972)
    - BHs form when and only when a mass M gets compacted into a region whose circumference C in every direction is $C \lesssim 4\pi M$.

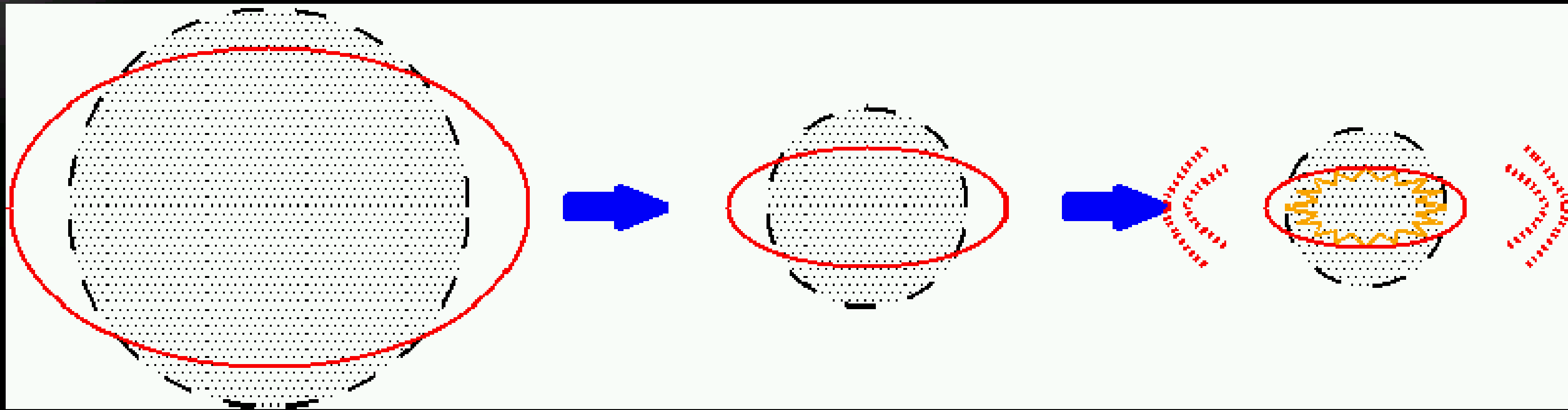



    - Condition in higher-dimensional gravity (Eardley and Giddings 2002, Yoshino and Nambu 2002, Ida and Nakao 2002, Nakao et al. 2003)

# 4. Physics around NS's

# 4-1 Perturbation and GWs

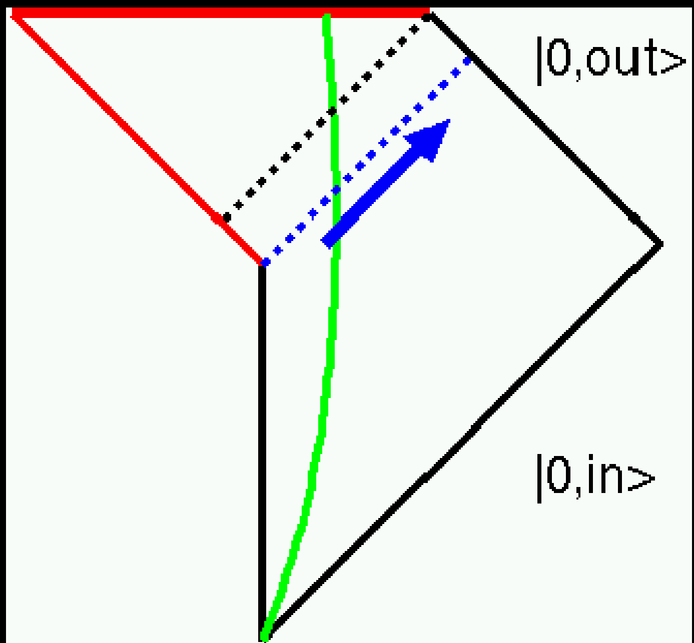
- Nonspherical perturbation of LTB spacetime
  (Iguchi et al. 1998, 1999, 2000)

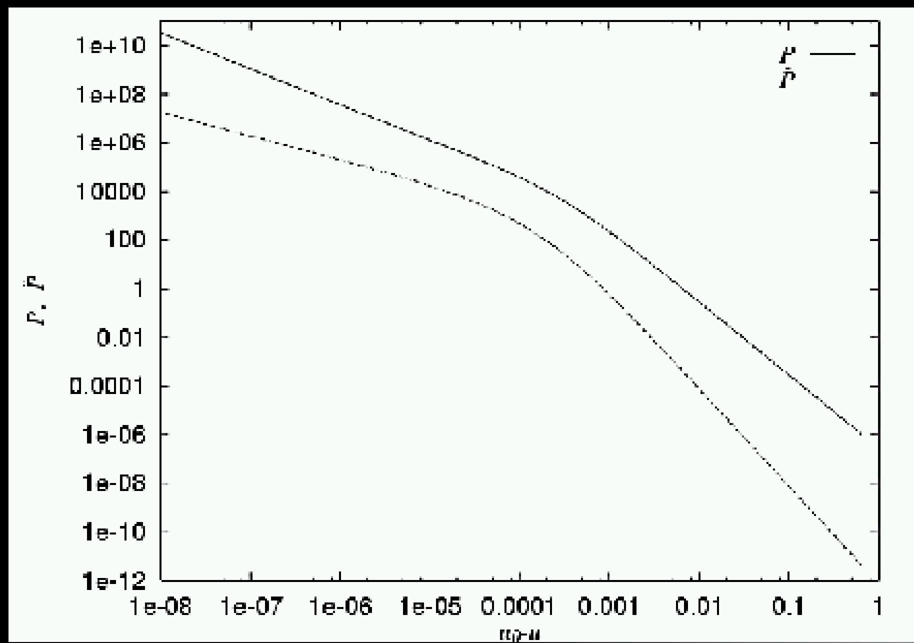


  - Both metric and matter perturbations are included linearly. The numerical simulations calculate the evolution of perturbations up to near the Cauchy horizon.
  - Some tetrad components of the Weyl curvature are diverging in an approach to the Cauchy horizon. Nevertheless the energy flux of GW remains finite.
  - Nonlinear analysis is needed for further investigation.
- GW emission for other examples?

# 4-2 QFT in curved spacetime

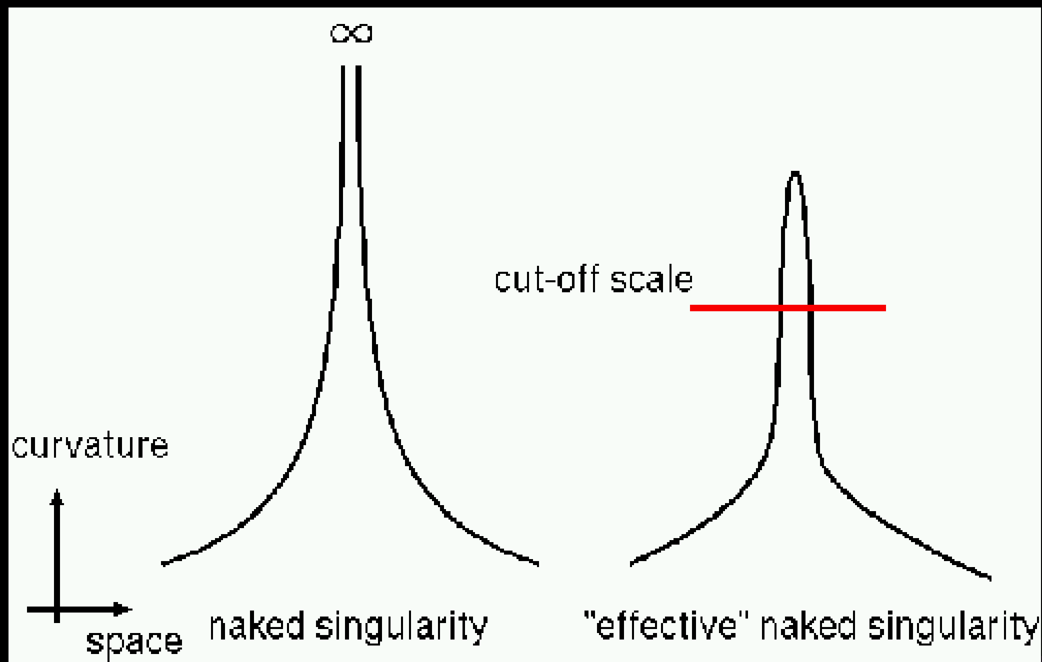
- Diverging energy flux for a shell-focusing NS in LTB solution (Barve et al. 1998, Vaz and Witten 1998, Harada et al. 2000, Iguchi and Harada 2001, Tanaka and Singh 2001)
  - The diverging flux suggest the Cauchy horizon instability due to quantum effects.
  - Objections
    - The calculation relies on the geometrical optics approximation, which has not been justified yet.
    - The amount of energy flux is subject to a cut-off scale. (Harada et al. 2001)



# 4-3 "Effective" NS

- NS
  - The curvature strength outside horizons is diverging.
  - The censorship claims vanishing probability of its appearance, in other words, its instability.
- "Effective" NS
  - The curvature strength outside horizons is beyond some cut-off scale.
  - It appears with nonzero probability.



# 5. Summary

- It is still uncertain whether or not naked singularities are censored in physically reasonable gravitational collapse.
- However, recent examples of naked singularities suggest that "effective" naked singularities appear with nonzero (and maybe not too small) probability for physically reasonable matter fields.
- Naked singularities are worth studying as the appearance of extremely strong curvature in principle observable, in classical physics and also in some form of quantum gravity.