UNIVERSAL CANONICAL ENTROPY FOR GRAVITATING SYSTEMS

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AF Black holes exhibit thermal instability: Schw bh has $T \sim 1/M \;\Rightarrow\; C \equiv \partial M/\partial T < 0$! Instability $\Leftarrow \rho(M) \sim \exp M^2 \Rightarrow Z_C(\beta) \nearrow \infty$ Canonical ensemble problematic

- Isolated black holes (isolated horizons) can be consistently described in terms of microcanonical ensemble with fixed $\mathcal{A}_{hor} >> l_{Planck}^2$ (Ashtekar et. al. PRL 1998; Adv. Phys. Math. 2000) having a microcanonical entropy (Kaul, PM PRL 2000; PLB 1998)

  $$S_{MC} = S_{BH} - \frac{3}{2} \log S_{BH} + const. + \mathcal{O}(S_{BH}^{-1})$$

  where $S_{BH} = \mathcal{A}_{hor}/4l_{Planck}^2$

- Canonical ensemble OK for asymptotically adS black holes $\rightarrow S_C \sim \mathcal{A}_{hor}$: Hawking and Page CMP 1983

- Understanding on 'Unified' basis ? **Possible**

- **Logarithmic corrections to canonical entropy due to thermal area fluctuations ? Yes, they are universal**

Based on Chatterjee, PM gr-qc/0309026; gr-qc/0303030.

Canonical partition function : holography ? PM gr-qc/0110198

Recall in canonical GR

$$H = H_{bulk} + H_{bdy}$$

such that

$$H_{bulk} \approx 0$$

In canonical QGR

$$\hat{H} = \hat{H}_{bulk} + \hat{H}_{bdy}$$

Choose as basis eigenstates of the full Hamiltonian $|\psi\rangle_{blk} \otimes |\chi\rangle_{bdy}$ Classical Hamiltonian constraint $\rightarrow$

$$\hat{H}_{bulk}|\psi\rangle_{blk} = 0$$

$\Rightarrow$ (formally)

$$\begin{aligned} Z_{CAN} &\equiv Tr \exp -\beta \hat{H} \\ &= \underbrace{dim\ \mathcal{H}_{bulk}}_{indep.\ of\ \beta.} \ \underbrace{Tr_{bdy} \exp -\beta \hat{H}_{bdy}}_{boundary} \end{aligned}$$

$\Rightarrow$ $dim \mathcal{H}_{blk} \rightarrow$ additive const. in S_{CAN} !

$\Rightarrow$ For GR systems with (internal) boundary,

$$Z_C(\beta) = \sum_{n \in \mathcal{Z}} \underbrace{g(E_{bdy}(n))}_{degeneracy} \exp -\beta E_{bdy}(n) .$$

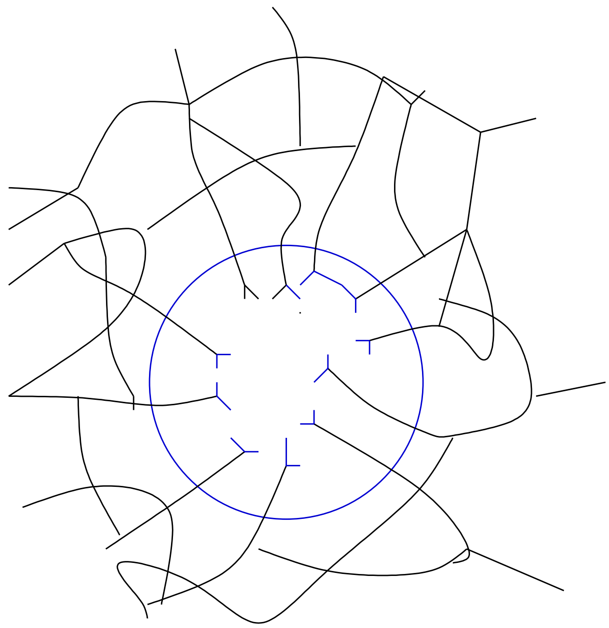
Spin network basis of NCQGR Rovelli, Smolin NPB 1992; Ashtekar et. al. JMP 1995, CQG 1996

- Network with links $l_1, \ldots, l_n$ carrying spins $j_1, \ldots j_n$

- Spacetime curvature has support only on network

- Area operator $\rightarrow$ diagonal with discrete spectrum

- Internal boundary : punctured $\mathcal{S}^2$ with each puncture having a deficit angle $\theta = \theta(j_i), i = 1, \ldots, p$



For $\mathcal{A}_{bdy} >> l^2_{Planck}$, area spectrum dominated by $j_i = 1/2, \forall i = 1, \ldots, p, p >> 1$ $\Rightarrow$

$$\mathcal{A}_{bdy}(p) \ \sim \ p \ l^2_{Planck} \ , \ p >> 1$$

Back to $Z_C(\beta) \rightarrow$ energy spectrum ?

Assume $E_p \equiv E(\mathcal{A}_p)$ $\Rightarrow$

$$Z_C \ = \ \sum_p g(E(\mathcal{A}_p)) \ \exp -\beta E(\mathcal{A}_p)$$

Poisson resummation

$$\sum_{n=-\infty}^{\infty} f(n) \;=\; \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dx\;\; \mathrm{exp}(-2\pi i m x)\; f(x)$$

Restrict to $p >> 1 \rightarrow \mathcal{A}_{hor} >> l^2_{Planck} \Rightarrow$

$$Z_C \;\simeq\; \int_{-\infty}^{\infty} dx\; g(E(A(x)))\;\; \mathrm{exp} -\beta E(A(x))$$

$$= \;\int dE\;\; \mathrm{exp}[S_{MC}(E)\; - \;\log|\frac{dE}{dx}|\; - \;\beta E]$$

where $S_{MC} \equiv \log g(E)$

If $\tilde{S}_{MC} \equiv \log \rho(E)$

$$\tilde{S}_{MC} \;=\; S_{MC} \; - \; \log|\frac{dE}{dx}|$$

Ambiguity inherent in definition of S_{MC}; irrelevant for BHAL, but relevant for log corrections

Saddle point approximation $(E = M)$

$$Z_C \simeq e^{\{S_{MC}(M) - \beta M - \log|\frac{dE}{dx}|_{E=M}]\}} \left[\frac{\pi}{-S''_{MC}(M)}\right]^{1/2}$$

Using $S_C = \log Z_C + \beta M$

$$S_C = S_{MC}(M) \underbrace{- \frac{1}{2}\log(-\Delta)}_{\delta_{th} S_C}$$

where,

$$\Delta \equiv \frac{d^2 S_{MC}}{dE^2} \left(\frac{dE}{dx}\right)^2 |_{E=M}$$

Chain rule $\Rightarrow$

$$\Delta = \left[\frac{d^2 S_{MC}}{d\mathcal{A}^2} - \left(\frac{dS_{MC}}{d\mathcal{A}}\right) \underbrace{\frac{d^2E/d\mathcal{A}^2}{dE/d\mathcal{A}}}_{non-univ.}\right] \left(\frac{d\mathcal{A}}{dx}\right)^2 |_{E=M}$$

Upto corrections due to thermal fluctuations of area, $S_C = S_{MC}$.

- Recall $\mathcal{A} \sim x \; for \; x >> 1 (large \; area)$

- Assume $E(\mathcal{A}) = const. \; \mathcal{A}^r$

- Assume $S_{MC}(\mathcal{A}) \sim \mathcal{A}$

$\Rightarrow$

$$\delta_{th} S_C = \frac{1}{2} \log S_{BH} - \frac{1}{2} \log(r-1) + const.$$

Role of 'Jacobian' term $|dE/dx|$: Gour,Medved CQG 2003, although adhoc; physical interpretation absent. Similar calculations also done earlier Das, Bhaduri, PM CQG 2002, but dE/dx missed.

- The thermal fluctuation correction to the canonical entropy of a spacetime with an inner boundary is **universal**, independent of r; it is also insensitive (for large areas) to the log(area) corrections in the microcanonical entropy due to quantum spacetime fluctuations

- Substitute for S_{MC} ; 'Universal' Carlip CQG 2000; Govindarajan et. al. CQG 2001; Sen et. al. PLB 2002; ...

$$S_{MC} = S_{BH} - \frac{3}{2} \log S_{BH} + const. + \mathcal{O}(S_{BH}^{-1})$$

$\Rightarrow$

$$S_C = S_{BH} - \log S_{BH} - \frac{1}{2} \log(r-1) + const. + \dots$$

- (If gauge group on Cauchy surface : $U(1) \rightarrow$ $S_{MC} = S_{BH} - \frac{1}{2} \log S_{BH} + \dots \Rightarrow$

$$S_C = S_{BH} - 0. \log S_{BH} + \dots \ !$$

$\Rightarrow$ Non-renormalization of BHAL ?)

- Role of r : SPA OK for $r > 1$
  - AF Schw : $r = 1/2 \rightarrow S_C$ becomes complex $\Rightarrow$ thermal instability

– NR BTZ : $r = 2$ for $r_H >> \ell \equiv (-\Lambda)^{-1/2}$

– AdS Schw : $r = 3/2$ for $r_H >> \ell$

– AdS : $r = 1$ Hawking-Page ‘phase transition’

Conclusions

- **Universal** thermal fluc. corr. $(1/2) \log S_{BH}$ in S_C independent of r

- If S_{MC} also universal, then S_C with both fixed-area quantum spacetime fluctuations $+$ thermal area fluctuations has universal $-\log S_{BH}$ correction to S_{BH} (net log corr. $= 0$ if gauge group on Cauchy surface is $U(1)$)

- Index r in energy-area relation signifies domain of validity of SPA ($r > 1$)

- Top priority : obtain $E(\mathcal{A})$ from NCQGR