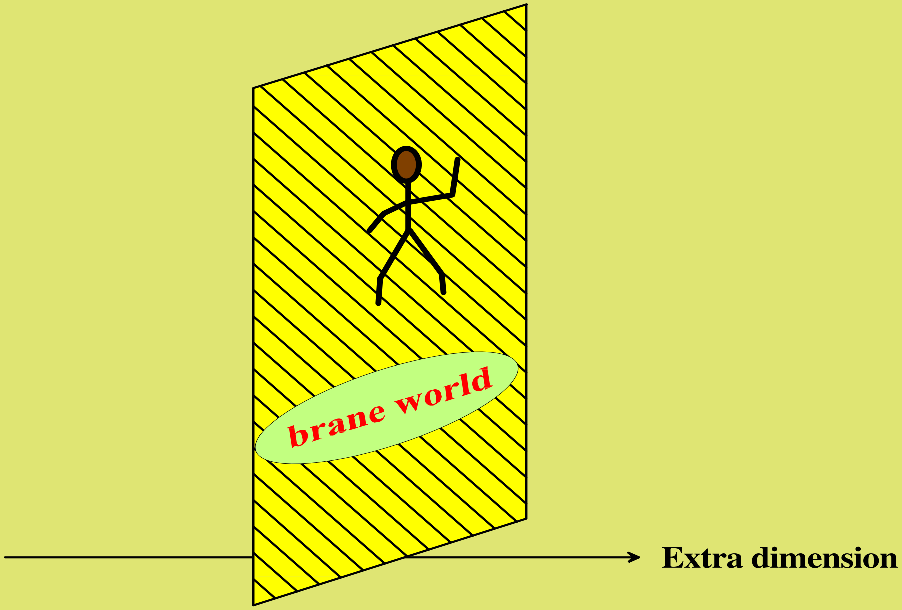
Brane-World Cosmology and Inflation



$$\boxed{G_{\mu\nu} = \kappa T_{\mu\nu} \ ?}$$

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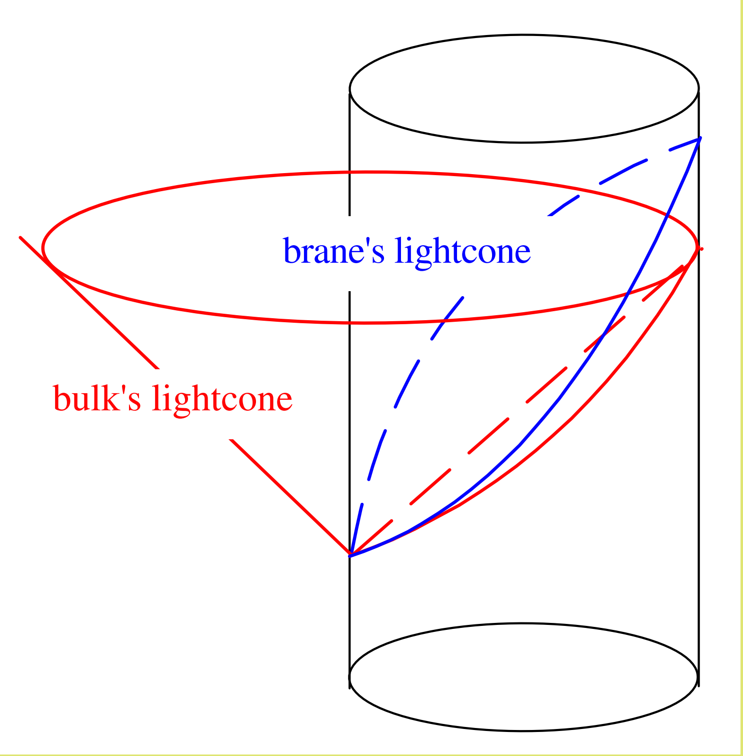
§1. Introduction

- **Braneworld $\approx$ domain wall**

  $\star$ $(n-1)$-brane = singular (time-like) hypersurface embedded in $(n+1)$-dim spacetime

  $\star$ brane tension (σ) = vacuum energy ($\sigma > 0$? or $\sigma < 0$?)
  vacuum energy $\neq$ cosmological constant on the brane

  $\star$ causality on the brane $\neq$ causality in the bulk
  H. Ishihara, PRL86 (2001)



$$\boxed{\text{A new picture of the universe!}}$$

- **Randall-Sundrum (RS) Brane-World**

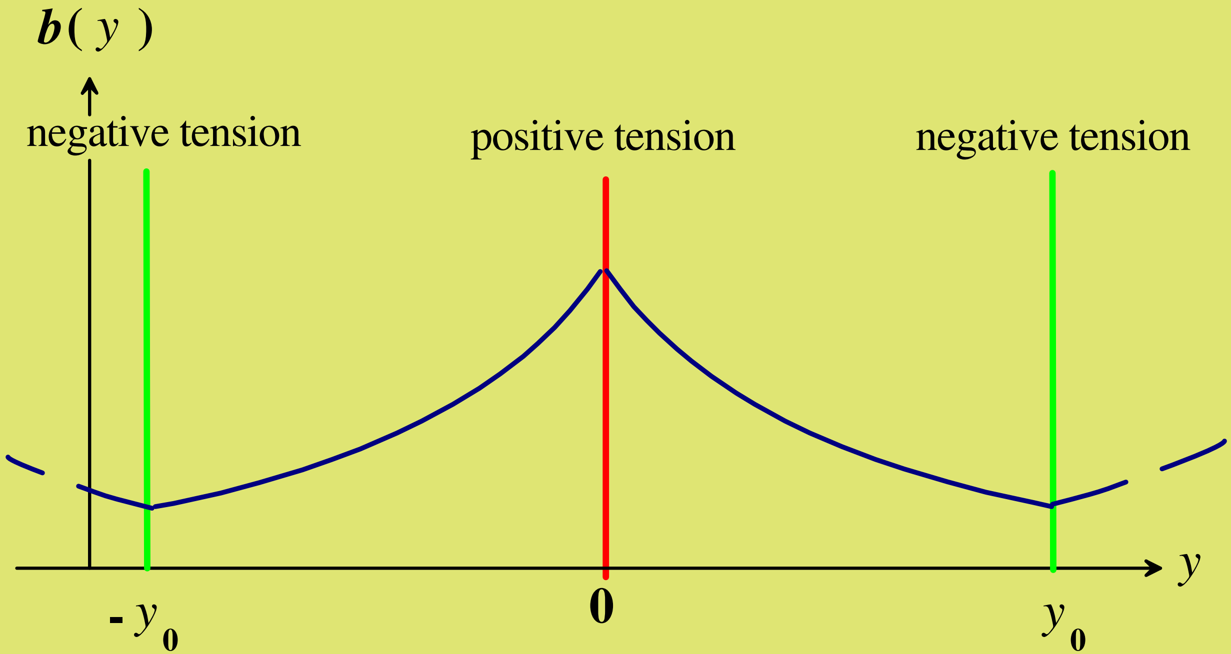
PRL 83, 3370 (1999); 83, 4690 (1999)

- **5D-AdS bounded by 2 branes:** $R^4 \times (S^1/Z_2)$

$$ds^2 = dy^2 + e^{-2|y|/\ell}\eta_{\mu\nu}dx^\mu dx^\nu \quad (-r_c \leq y \leq r_c)$$

$$\ell^2 = -\frac{6}{\Lambda_5}, \qquad \text{brane tension: } \sigma_\pm = \pm\frac{3}{4\pi G_5 \ell}$$

  - $b(y) = e^{-|y|/\ell}$ **is called the warp factor**



⋆ **vacuum energy (brane tension) $\neq$ cosmological constant.**

$$\Lambda_4 = \frac{1}{2}\Lambda_5 + (8\pi G_5 \sigma)^2/12$$

⋆ r_c **is arbitrary** $\Rightarrow$ **Existence of Radion mode.**

$$\boxed{\text{Brans-Dicke type gravity on the branes}}$$

unless $^\exists$stabilization mechanism. (Garriga & Tanaka, '99)

- **In the limit $r_c \to \infty$, the negative tension brane disappears.**

single-brane model $\cdots$ 5th dimension is non-compact

⋆ **Gravity confined within $\ell \sim |\Lambda_5|^{-1/2}$ from the brane**

⋆ **No "radion" modes (no relative motion)**

$$\boxed{\text{Einstein gravity is recovered on scales} \gg \ell}$$

$$\Phi_{\text{Newton}} = -\frac{G_5 M}{r^2} \underset{r \gg \ell}{\Rightarrow} -\frac{G_5 M}{\ell\, r} = -\frac{G_4 M}{r}$$

The single-brane model is cosmologically more attractive.

§2. Brane Cosmology in AdS$_5$-Schwarzschild Bulk

Kraus ('99); Ida ('00); $\cdots$

- **5D AdS-Schwarzschild in Static Chart:**

$$ds^2 = -A(R)dT^2 + \frac{dR^2}{A(R)} + R^2 d\Omega_K^2$$

$$A(R) = K + \frac{R^2}{\ell^2} - \frac{\alpha^2}{R^2} \quad (K = \pm 1, 0, \quad \alpha^2 = 2G_5 M)$$

$\star$ For $\alpha^2 = 0$ and $K = 1$, $(T, R) \;\to\; (t, r)$;

$$R(t, r) = \ell \sinh(r/\ell) \cosh(Ht)$$
$$T(t, r) = \ell \arctan(\tanh(r/\ell) \sinh(Ht))$$

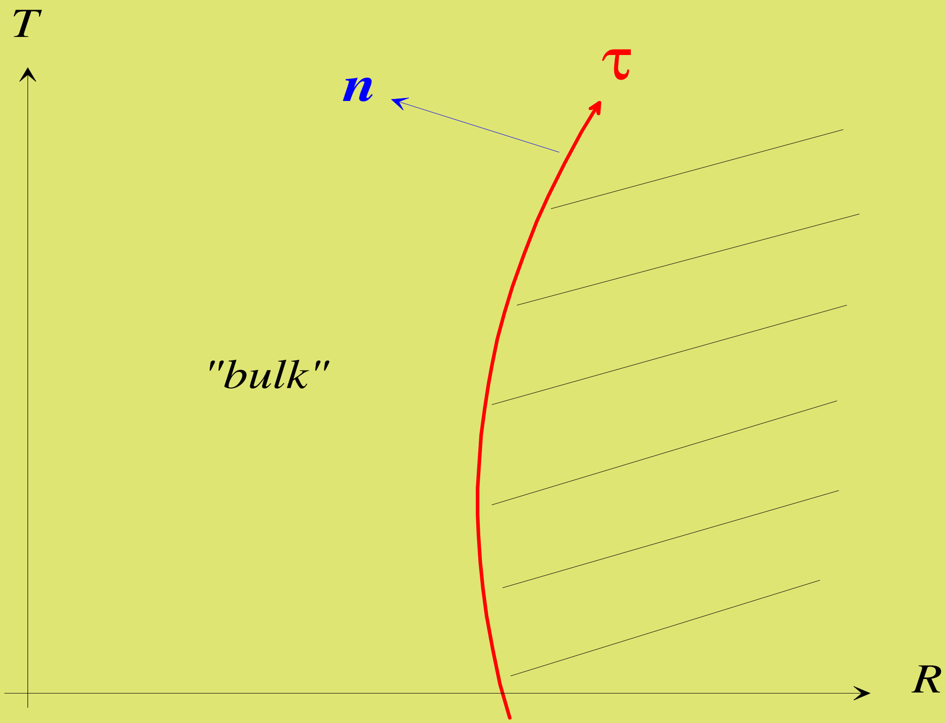
$$ds^2 = dr^2 + \ell^2 \sinh^2(r/\ell)(-H^2 dt^2 + \cosh^2(Ht) d\Omega_{(3)}^2)$$

Any r =const. timelike hypersurface is 4D de Sitter space.

$\cdot$ de Sitter brane at $r = r_0$ (with Z_2 symmetry): $\quad (T_{\mu\nu} = -\sigma\, g_{\mu\nu})$

$$\sigma = \frac{3}{4\pi G_5 \ell} \coth(r_0/\ell), \qquad H^2 \ell^2 = \frac{1}{\sinh^2(r_0/\ell)}$$

⋆ Deviates from de Sitter if $T_{\mu\nu}$ is non-trivial:



brane trajectory :

$$\begin{cases} R = R(\tau) \\ T = T(\tau) \end{cases}$$

$$ds^2|_{\text{brane}} = \left(-A(R)\dot{T}^2 + \frac{\dot{R}^2}{A(R)} \right) d\tau^2 + R^2(\tau) d\Omega_K^2$$

- Choose τ to be Proper time on the Brane:

$$-A(R)\dot{T}^2 + \frac{\dot{R}^2}{A(R)} = -1$$

$$\Rightarrow \quad \boxed{A^2(R)\dot{T}^2 = \dot{R}^2 + A(R)}$$

· **Junction condition under Z_2-symmetry:**

$$[K_{\mu\nu}]^+_- = 2K_{\mu\nu}(+0) = -8\pi G_5[T_{\mu\nu} - (1/3)Tg_{\mu\nu}]$$

$$K_{\mu\nu} = \frac{1}{2}\pounds_n q_{\mu\nu}\,; \quad n_a = (\dot{R}, -\dot{T}, 0, 0, 0)$$

$q_{\mu\nu} \cdots$ **induced metric on the brane**

$$T^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p) - \sigma_c \delta^\mu{}_\nu\,; \quad \sigma_c = \frac{3}{4\pi G_5 \ell}$$

$$\Rightarrow \quad \boxed{\frac{A(R)}{R}\dot{T} = \frac{4\pi G_5}{3}(\rho + \sigma_c) = \frac{4\pi G_5}{3}\rho + \frac{1}{\ell}}$$

$G_4 = G_5/\ell \cdots$ **4D Newton const.**

$$\Downarrow \quad A(R) = K + R^2\ell^2 - \frac{\alpha^2}{R^2}$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G_4}{3}\rho + \ell^2\left(\frac{4\pi G_4}{3}\rho\right)^2 + \frac{\alpha^2}{R^4}$$

- **Friedmann equation on the brane:** Binetruy et al. ('99)

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{8\pi G_4}{3}\rho + \ell^2\left(\frac{4\pi G_4}{3}\rho\right)^2 + \frac{\alpha^2}{R^4}$$

⋆ **presence of $\propto \rho^2$ and α^2/R^4 terms.**

· **ρ^2-term dominates in the early universe: $H \propto \rho$.**

For $\rho \propto R^{-4}$, $K = \alpha^2 = 0$; $\quad R \propto \left(t + \frac{2t^2}{\ell}\right)^{1/4}$

· **reduces to standard Friedmann equation for $\ell^2 G_4 \rho \ll 1$.**

$(\ell^2 G_4 \rho \ll 1 \;\Leftrightarrow\; \rho \ll \sigma_c)$

· **α^2/R^4-term: "dark radiation" (or Weyl fluid)**

$$\alpha^2 = 2G_5 M \sim \text{5D (bulk) BH mass}$$

$$\frac{\alpha^2}{R^4} = -\frac{E_{tt}}{3}; \quad E_{\mu\nu} = \overset{(5)}{C}{}_{\mu a \nu b} n^a n^b \quad \text{(5D Weyl contribution)}$$

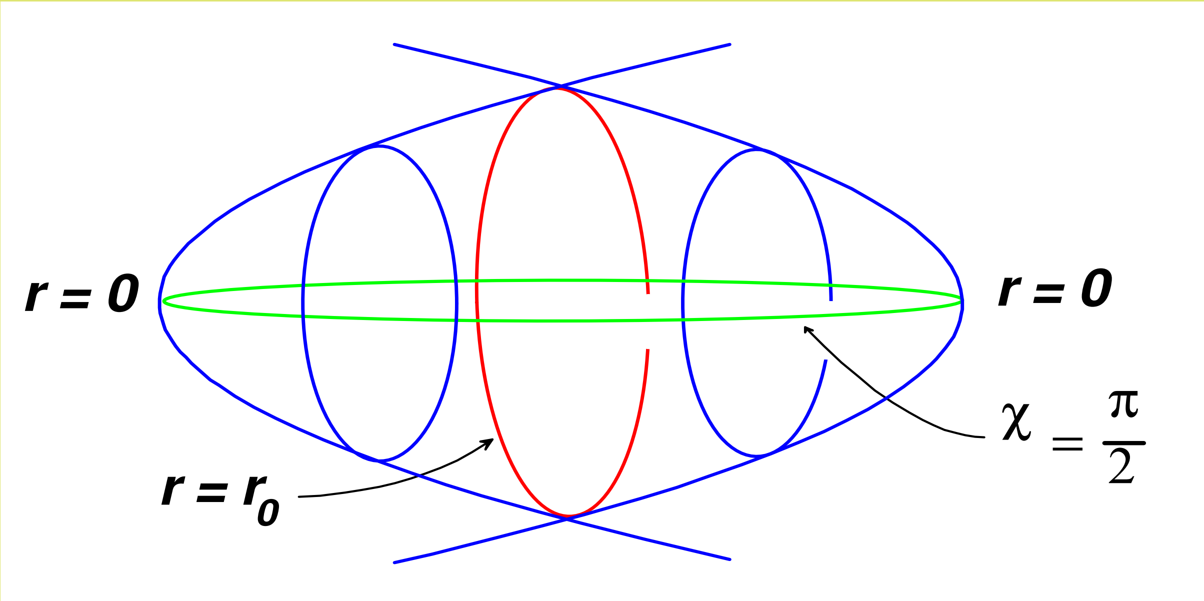
Shiromizu, Maeda & MS ('99), Maartens ('00), ...

§3. Quantum Creation of Inflationary Brane-World

- **Euclidean AdS: H^5 ($O(5,1)$-symmetric)**

$$ds^2 = dr^2 + \ell^2 \sinh^2(r/\ell)(d\chi^2 + \sin^2\chi \, d\Omega_{(3)}^2)$$

★ **Brane at $r = r_0$ (with Z_2-symmetry) = deS brane instanton**



topology $\sim$ S^5

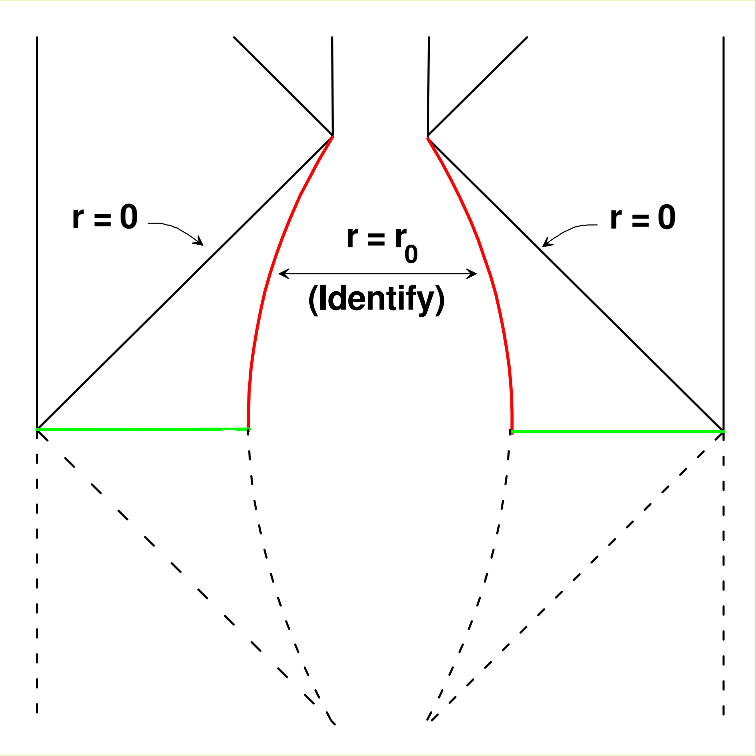
$\chi = \pi/2$: totally geodesic 4-surface

- **Creation of inflationary brane-world**

Garriga & MS ('99); Koyama & Soda ('00)

Analytic continuation: $\chi \to iHt + \pi/2$

$$ds^2 = dr^2 + (H\ell)^2 \sinh^2(r/\ell)(-dt^2 + H^{-2}\cosh^2 Ht\, d\Omega_{(3)}^2) \quad (r \le r_0)$$



Spatially Compact 5D Universe

Universe is naturally born with inflation.

Initial value (Cauchy) problem is well-posed.

(RS flat brane is recovered in the limit $r_0 \to \infty$)

§4. 4D Graviton and Kaluza-Klein Excitations

- **5D gravitational perturbation of de Sitter brane universe:**

$$\begin{aligned} ds^2 &= dr^2 + (H\ell)^2 \sinh^2(r/\ell) ds^2_{\mathrm{dS}_4} + h_{ab} dx^a dx^b \\ &= b^2(z) \left(dz^2 + H^2 ds^2_{\mathrm{dS}_4} \right) + h_{ab} dx^a dx^b \end{aligned}$$

· $dz = \dfrac{dr}{\ell \sinh(r/\ell)}$ **(conformal radial coordinate):**

$$b(z) = \frac{\ell}{\sinh(|z| + z_0)} \qquad (\sinh z_0 = H\ell\,; \quad -\infty < z < \infty)$$

· **(generalized) Randall-Sundrum gauge:**

$$h_{55} = h_{5\mu} = h^\mu{}_\mu = D_\mu h^{\mu\nu} = 0\,;$$

D_μ : **4D covariant derivative**

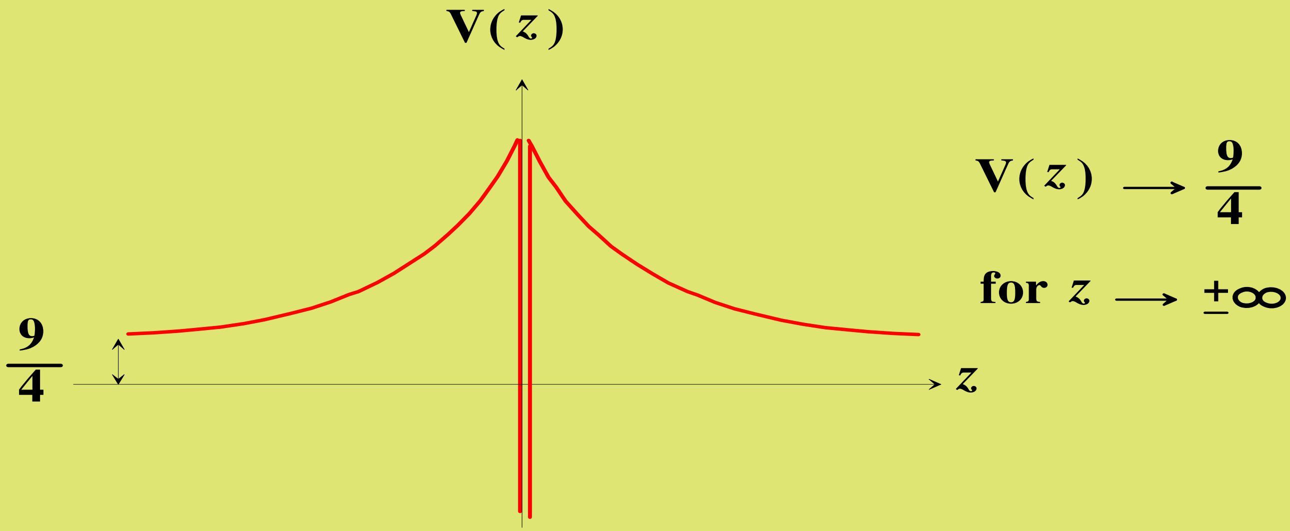
$h_{\mu\nu}$ ··· **5 degrees of freedom (= 1 scalar + 2 vector + 2 tenser)**

- **Perturbation equations:**

$$h_{\mu\nu} = b^{1/2}\,\varphi(z) H_{\mu\nu}(x) \quad \Rightarrow \quad \begin{cases} -\varphi'' + \dfrac{(b^{3/2})''}{b^{3/2}}\varphi = \dfrac{m^2}{H^2}\varphi \\ (-\overset{(4)}{\Box} + 2H^2 + m^2) H_{\mu\nu} = 0 \end{cases}$$

⋆ **"Volcano" potential for φ:**

$$\boxed{V(z) \equiv \frac{(b^{3/2})''}{b^{3/2}} \quad = \frac{15}{4\sinh^2(|z| + z_0)} + \frac{9}{4} - 3\coth\eta_0\,\delta(z)}$$

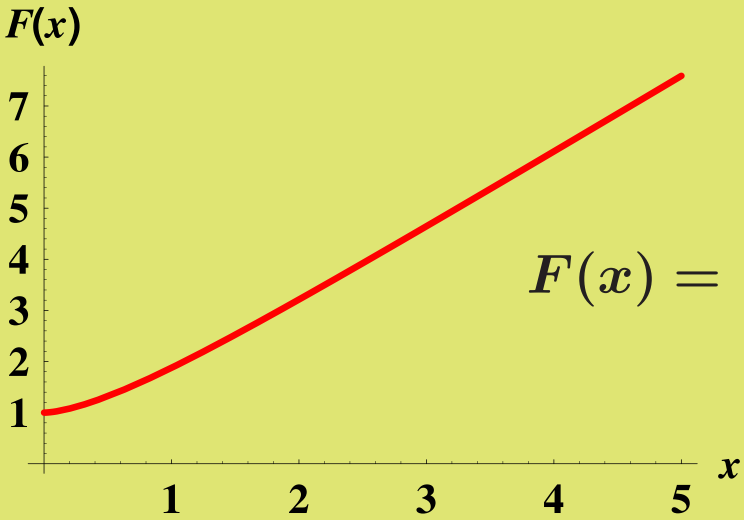


⋆ **4D graviton (zero mode $m = 0$):** $\varphi \propto b^{3/2} \ \Rightarrow \ h_{\mu\nu} \propto b^2$

⋆ **KK excitations ($m > 0$):** $V \to (9/4) \ \Rightarrow \ m > (3/2)H$

- **Zero mode is confined just like the case of the RS flat brane. When quantized, however, the normalization (amplitude) is non-trivial:** Langlois, Maartens & Wands ('00)

$$\frac{P_{gw}(k)k^3}{G_4} \sim \left(\frac{H}{2\pi}\right)^2 F(H\ell)$$



$$F(x) = \left(\sqrt{1+x^2} - x^2 \ln\left[\frac{1+\sqrt{1+x^2}}{x}\right]\right)^{-1}$$

– **large enhancement at $H\ell \gg 1$: $F \sim \frac{3}{2}H\ell$.**

- **Mass gap in KK spectrum: $m_{KK} \geq (3/2)H$**
  - **The same is true for any bulk scalar field with $M \lesssim H$.**
  - **No 'zero-mode' (bound-state mode) for $M \gg H$.**

§5. Bulk Inflaton Model

(Brane Inflation without Inflaton on the Brane)

⋆ **Randall-Sundrum's "default" parameters:**

$$\text{brane tension:} \quad \sigma_c = \frac{3}{4\pi G_5 \ell}; \quad \ell = \left|\frac{6}{\Lambda_5}\right|^{1/2}.$$

If $|\sigma| > \sigma_c$, then inflation occurs on the brane:

$$H^2 = \frac{1}{\ell_\sigma^2} - \frac{1}{\ell^2} = \frac{1}{\ell_\sigma^2} - \frac{|\Lambda_5|}{6}; \quad \sigma \equiv \frac{3}{4\pi G_5 \ell_\sigma}$$

If $|\sigma| = \sigma_c$ but $|\Lambda_{5,eff}| < |\Lambda_5|$, inflation also occurs on the brane:

$$H^2 = \frac{|\Lambda_5 - \Lambda_{5,eff}|}{6}$$

$$\Downarrow$$

Brane-world inflation can be driven solely by bulk (gravitational) dynamics.

Kobayashi & Soda ('00), Himemoto & MS ('00)
Himemoto, Sago & MS ('01), Himemoto, MS & Tanaka ('02)

• 5D Einstein-scalar system with a brane

$$G_{ab} + \Lambda_5\, g_{ab} = \kappa_5^2\, T_{ab}\,; \qquad \kappa_5^2 = 8\pi G_5$$

∗ (4 + 1)-decomposition (Gaussian Normal Coordinates)

$$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & q_{\mu\nu} \end{pmatrix}\ ; \quad q_{\mu\nu} \cdots \text{4D metric}$$

$$\Updownarrow$$

$$ds^2 = dr^2 + q_{\mu\nu} dx^\mu dx^\nu\ ; \quad r \cdots \text{5th dimension}$$

∗ Energy-momentum tensor

$$T_{ab} = \phi_{,a}\phi_{,b} - g_{ab}\left(\frac{1}{2} g^{cd}\phi_{,c}\phi_{,d} + V(\phi)\right) + S_{ab}\delta(r - r_0)\,,$$

$$S_{ab} = -\sigma q_{ab}.$$

(Standard matter is assumed to be in the vacuum on the brane.)

⋆ Z_2-symmetry and RS brane tension

$$q_{ab}(r_0+y)=q_{ab}(r_0-y)\,,\quad \phi_{,r}(r_0)=0\,,$$

$$\sigma=\sigma_0=\frac{6}{\kappa_5^2\,\ell_0}\,,\quad \ell_0^2=\frac{6}{|\Lambda_5|}\,.$$

- 4D "Einstein-scalar" equations on the brane

$$G_{\mu\nu}=\;\kappa_5^2\,T^{(\mathrm{bulk})}_{\mu\nu}-E_{\mu\nu}\,,$$

$$T^{(bulk)}_{\mu\nu}=\frac{1}{6}\left(4\phi_{,\mu}\phi_{,\nu}-\left(\frac{5}{2}q^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}+3V(\phi)\right)q_{\mu\nu}\right),$$

$$E_{\mu\nu}=\;{}^{(5)}C_{rbrd}\,q^b_\mu\,q^d_\nu\,.$$

· Unconventional form of $T^{(\mathrm{bulk})}_{\mu\nu}$

· $E_{\mu\nu}$ is determined by the bulk scalar dynamics

§6. Quadratic Potential Model

$$L_5 = \frac{1}{16\pi G_5} R - \frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - U(\phi)$$

$$\boxed{\sim \text{ a conformally transformed scalar-tensor gravity}}$$

If ϕ varies very slowly (near and on the brane),

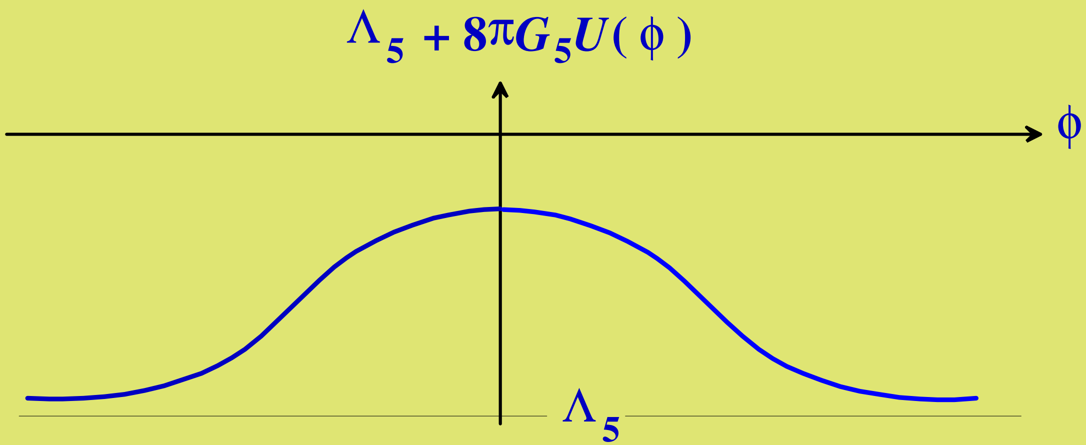
$$|\Lambda_{5,eff}| = |\Lambda_5 + 8\pi G_5 U(\phi)| \; < |\Lambda_5| \,,$$

$$H^2 = \frac{4\pi G_5 U(\phi)}{3} = \frac{8\pi G_4}{3} U_4 \,; \quad G_4 \equiv G_5/\ell_* \,, \quad U_4 = \frac{\ell_*}{2} U(\phi) \,.$$

(ℓ_* here is arbitrary; $G_4 U_4 = \dfrac{1}{2} G_5 U$.)

- Quadratic potential:

$$U = U_0 + \frac{1}{2} M^2 \phi^2$$



When $|M^2|\phi^2 \ll U_0$, one can solve the field equations iteratively.

⋆ **0-th order:**

$$U = U_0\,, \quad \phi = 0\,,$$

$$ds^2 = dr^2 + \ell^2 \sinh^2(r/\ell)(-H^2 dt^2 + \cosh^2 Ht\, d\Omega^2_{(3)})$$

$$(r \le r_0)$$

$$\ell^2 = \frac{6}{|\Lambda_5 + \kappa_5^2 U_0|}\,, \quad H^2 = \frac{\kappa_5^2}{6} U_0$$

This is just an AdS$_5$-dS brane system with
a modified AdS curvature: $\ell > \ell_0$

⋆ **1st order:** $O(\phi)$

For de Sitter brane at $r = r_0$,

$$\phi(r,t) = u_0(r)\phi_0(t) + \int_{3/2}^{\infty} d\lambda\, u_\lambda(r)\phi_\lambda(t)$$

ϕ_0 : "zero mode" (bound-state mode)

ϕ_λ : Kaluza-Klein modes $M_\lambda^2 = \lambda^2 H^2 \quad (\lambda > 3/2)$

Effective 4d mass of bound-state mode when $|M^2| \lesssim H^2$:

$$M_0^2 = \begin{cases} M^2/2 & \text{for } H^2\ell^2 \ll 1 \\ 3M^2/5 & \text{for } H^2\ell^2 \gg 1 \end{cases}$$

⋆ No bound state when $M^2 > H^2$.

(But there is a quasi-normal mode with $M_0 = M/\sqrt{2} - i\Gamma$)

$\Gamma/M = O(M^2\ell^2)$ for $H^2\ell^2 \ll 1$

⋆ **For $|M^2| \lesssim H^2$, slow-roll inflation is realized on the brane,**
irrespective of the value of $H\ell$.

⋆ **The bound-state mode dominates at late times if $H^2\ell^2 \ll 1$.**

$\star$ **2nd order:** $O(\phi^2)$ (for $H^2\ell^2 \ll 1$)

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right] = \kappa_4^2 \rho_{\rm eff} = \frac{\kappa_5^2}{2}\left(\frac{\dot{\phi}^2}{2} + U(\phi)\right) + E^t{}_t\,. \quad \left(\kappa_4^2 = \frac{\kappa_5^2}{\ell_0}\right)$$

From Bianchi Ids. on the brane,

$$E^t{}_t = -\frac{\kappa_5^2}{2a^4}\int^t a^4\dot{\phi}(\partial_r^2\phi + \frac{\dot{a}}{a}\dot{\phi})\,dt = \frac{\kappa_5^2}{4}\dot{\phi}^2 + X\,,$$

where $\ddot{\phi} + 3H\dot{\phi} + \frac{1}{2}M^2\phi = -\Gamma\dot{\phi}$ ($\Gamma \neq 0$ **only when** $M \gtrsim H$)

$\dot{X} + 4HX = \Gamma\dot{\phi}$ ($X \sim$ **dark radiation**)

$$\Rightarrow \quad \rho_{\rm eff} = \rho_\phi + X = \ell_0\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}U(\phi)\right) + X\,.$$

Consistent with 1st order (bound-state) solution when $H^2\ell^2 \ll 1$.

$$\Rightarrow \quad \rho_\phi = \frac{\dot{\varphi}^2}{2} + V(\varphi)\,; \quad \varphi \equiv \sqrt{\ell_0}\,\phi\,,$$

$$V(\varphi) \equiv \frac{\ell_0}{2}U(\varphi/\sqrt{\ell_0}) = \frac{\ell_0}{2}U_0 + \frac{1}{2}m_{\rm eff}^2\varphi^2, \quad m_{\rm eff}^2 = \frac{M^2}{2}\,.$$

$\star$ **5d scalar behaves like a 4d scalar on the brane!**

§7. Cosmological Perturbations in the Bulk Inflaton Model

- Essentially a 5-dimensional, PDE problem.
- However, some simplifications on super-horizon scales.

Langlois, Maartens, MS & Wands ('01)

- **Basic equations:** Kanno & Soda, hep-th/0303203

$$G_{\mu\nu} = \kappa_4^2 T^{\rm eff}_{\mu\nu}(\varphi) + X_{\mu\nu}\,;$$

$$T^{\rm eff}_{\mu\nu} = \nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{2}\left(\nabla^\alpha\varphi\nabla_\alpha\varphi + 2V(\varphi)\right) \quad \cdots \text{ effective 4d field}$$

$$X_{\mu\nu} = -E_{\mu\nu} - \frac{\kappa_4^2}{3}\left(\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{4}g_{\mu\nu}\nabla^\alpha\varphi\nabla_\alpha\varphi\right) \quad \cdots \text{ dark radiation}$$

where $X^\mu{}_\mu = 0$, and $T^{\rm eff}_{\mu\nu}$ and $X_{\mu\nu}$ are conserved separately

at $O(H^2\ell^2)$: $\quad \nabla^\mu T^{\rm eff}_{\mu\nu} = O\left(H^4\ell^4\right)\,, \quad \nabla^\mu X_{\mu\nu} = O\left(H^4\ell^4\right)\,.$

(during inflation when $\Gamma = 0 \quad \because\ |M^2| \lesssim H^2$)

$\boxed{\text{standard 4d theory is applicable}}$

⋆ Where is the effect of 5D bulk?

"$X_{\mu\nu}$"

- Isotropic part of $X_{\mu\nu}$ ($X^0{}_0$ or $X^i{}_i$) can be determined from $\nabla^\mu X_{\mu\nu} = 0$ (on superhorizon scales).
(Only energy conservation law is essential on superhorizon)

- Anisotropic stress part of $X_{\mu\nu}$ cannot be determined from 4d equations.

$$X_{ij}^{\text{aniso}} \equiv X_{ij} - \frac{1}{3} g_{ij} X^k{}_k$$

$$\left(X_{\mu\nu}^{\text{aniso}} = -E_{\mu\nu}^{\text{aniso}} \quad \because \quad T_{\mu\nu}^{\text{eff, aniso}}(\varphi) \equiv 0 \text{ at linear order} \right)$$

$$\underline{\underline{\text{Need to solve 5D equations for } E_{\mu\nu}}}$$

• Evaluation of $E_{\mu\nu}$ in the bulk inflaton model

Minamitsuji, Himemoto & MS, PRD68, 024016 (2003)

· **Full background spacetime:**

$$ds^2 = dr^2 + b^2(r,t)\left(-dt^2 + a^2(r,t)d\Omega^2_{(3)}\right), \quad \phi = \phi(r,t).$$

where

$$b(r,t) = b(r) + O(\phi^2), \quad a(r,t) = a(t) + O(\phi^2);$$
$$b(r) = H\ell \sinh(r/\ell), \quad a(t) = H^{-1}\cosh Ht$$

$\Rightarrow$ **perturbations to $O(\phi^2)$ as well as to $O(\delta\phi)$.**

· **Lowest order background approximated by AdS$_5$:**

$$ds^2 = dr^2 + b^2(r)(-dt^2 + a^2(t)d\Omega^2_{(3)})$$

Then we have

$$E_{\mu\nu} = O(\phi^2), \quad \delta E_{\mu\nu} = O(\phi\delta\phi) \quad (\Leftrightarrow \quad \delta g_{\mu\nu} = O(\phi\delta\phi))$$

- To $O(\phi^2)$, $\delta\phi$ satisfies the linearized field equation on AdS_5 :

$$\left(-\Box_{\mathrm{AdS}_5}+M^2\right)\delta\phi=0\,.$$

- $\delta E_{\mu\nu}$ also satisfies a similar equation on AdS_5:

$\mathcal{L}\,[\delta E_{\mu\nu}]=S_{\mu\nu};$ $\mathcal{L}\cdots$ Lichnerowicz ($\Box_{\mathrm{AdS}_5}$-like) operator
$S_{\mu\nu}\cdots$ source term of $O(\phi\delta\phi)$

with the boundary condition at the brane,

$$\partial_r(b^2\delta E_{\mu\nu})=\delta\sigma_{\mu\nu}\,;\quad \sigma_{\mu\nu}\ \sim\ \partial(\text{energy momentum of }\phi\,)$$

Strategy:

1. Solve $\delta\phi$ in the AdS bulk.
2. Solve $\mathcal{L}\,[\delta E_{\mu\nu}]=S_{\mu\nu}[\delta\phi]$ by the Green function method:

$$\delta E(x)\sim\int_{\text{bulk}}d^5x'\,G(x,x')\delta S(x')+\int_{\text{brane}}d^4x'\,\partial_r(b^2\delta E(x'))G(x,x')$$

3. Analyze the late time behavior of $\delta E_{\mu\nu}$ on the brane on superhorizon scales.

After a long and tedious calculation, we find

$$\delta X_{\mu\nu} \equiv -\delta E_{\mu\nu} - \frac{8\pi G_4}{3}\delta\left(\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{4}g_{\mu\nu}\nabla^\alpha\varphi\nabla_\alpha\varphi\right)$$
$$= O(H^4\ell^4)\ !$$

⋆ $\delta X^0{}_0 = -\delta X^i{}_i$ **part decays like radiation** ($\propto a^{-4}$).

⋆ **No anisotropic stress** ($\delta X^i{}_j - \frac{1}{3}\delta^i_j\,\delta X^k{}_k$) **remains at late times.**

$$\Updownarrow$$

$$\boxed{\text{Up through } O(H^2\ell^2)\text{, Bulk Inflaton Model is completely equivalent to standard 4d inflation}}$$

Difference appears only at $O(H^4\ell^4)$ **or when** $H^2\ell^2 \gtrsim 1$.

§8. Summary

Brane-world gives a new picture of the universe

Can we find cosmological evidence?

- **Quantum brane cosmology**

⋆ Spatially compact 5D Universe created from nothing
- Well-posed initial value problem

⋆ 4D Universe created in de Sitter (inflationary) phase
- Non-trivial quantum fluctuations if $H\ell \gg 1$
- Effects of KK modes need to be investigated.

⋆ Inflation without inflaton on the brane (bulk inflaton model)
- Inflation as a result of 5D gravitational dynamics

⋆ Mass gap ($\Delta m = (3/2)H$) in the KK spectrum
- Isolation of the zero mode

$\Leftrightarrow$ Decoupling of 5D effects at low energy scales

- **Evolution of a brane universe**

⋆ **Presence of ρ^2 term in Friedmann equation**

· **Modified evolution when $\ell^2 G_4 \rho \gtrsim 1$**

⋆ **Dark radiation (Weyl fluid) term in Friedmann equation**

· **Effect of 5D bulk gravity**

⋆ **Bulk inflaton model is equivalent to standard 4d model up through $O(H^2\ell^2)$**

· **5D effect is encoded in Weyl anisotropy, but absent at $O(H^2\ell^2)$.**

$$\boxed{\text{No trace of braneworld if } H^2\ell^2 \ll 1}$$

⋆ **However, for time-varying H, 5d mode mixing will occur.**

⇒ generation of KK modes from zero (bound-state) mode.

Hiramatsu, Koyama & Taruya, hep-th/0308072,
Easther, Langlois, Maartens & Wands, hep-th/0308078
(for tensor KK mode generation)

⋆ **Generation of scalar KK modes needs to be studied.**