Black Hole Thermodynamics: Entropy, Information and Beyond

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Plan:

- Black Hole Entropy and Temperature
- Statistical Mechanical origins - CFT, LQG, ST
- Leading corrections, agreements and disagreements
- Information loss & Attempts at resolution
- Potential experiments, observations
- Discussions

Black Hole Thermodynamics: $M, Q, \ldots$

$$ds^2_{RN} = -\left(1 - \frac{16\pi G_d M}{(d-2)c^2\Omega_{d-2}r^{d-3}} + \frac{16\pi G_d Q^2}{(d-2)(d-3)c^4 r^{2(d-3)}}\right) dt^2 + (\cdots)^{-1}\, dr^2 + r^2 d\Omega^2_{d-2}$$

$$r_+^{d-3} = \frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}} + \sqrt{\left(\frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}}\right)^2 - \frac{2G_d\ Q^2}{(d-2)(d-3)\ c^4}}\ \ , \ \Phi = \sqrt{\frac{2(d-3)}{d-2}}\,\frac{Q}{r^{d-3}}$$

$$T_H = \frac{(d-3)\hbar c}{2\pi r_+^{d-2}}\sqrt{\left(\frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}}\right)^2 - \frac{2G_d Q^2}{(d-2)(d-3)c^4}}$$

$$S_{BH} = \frac{\Omega_{d-2} r_+^{d-2}}{4\lambda_{Pl}^{d-2}}\ , \quad \lambda_{Pl} = \text{Planck length}$$

$$\boxed{d(Mc^2) = T_H dS_{BH} + \Phi dQ}$$

$$\boxed{\Delta S_{BH} \geq 0}$$

Laws of Black Hole Thermodynamics

Bekenstein '73, Hawking '76

Statistical Mechanical Origins:

I. Horizon Conformal Field Theory:

(Carlip, hep-th/0203001)

Start with: $I_d = \frac{c^3}{16\pi G_d} \int \mathcal{R}_d \sqrt{-g} d^d x$

Most General Spherically Symmetric Metric: $\phi(x) = r^2(x)/\lambda_{Pl}^2$

$$ds_d^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + \phi(x) d\Omega_{d-2}^2 \quad , \ \mu, \nu = t, x$$

Then Action: (Gegenberg, Kunstatter, Louis-Martinez, gr-qc/9408015)

$$I = \int \mathcal{L} \sqrt{-\gamma} \ d^2 x = \frac{c^3}{2G_2} \int \left(\phi R_2 + \frac{1}{\ell^2} V[\phi] \right) \sqrt{-\gamma} \ d^2 x$$

$\Theta = \frac{1}{\phi} \ell^a \nabla_a \phi = 0 \leftarrow$ Horizon

Under $\mathcal{T}$:

$$\begin{aligned} \delta g_{ab} &= \nabla_c (f \ell^c) g_{ab} \\ \delta \phi &= (\ell^c \nabla_c h + \kappa h) \end{aligned}$$

$\delta \mathcal{L} \sim \Theta \to 0$ at horizon, i.e. 'Asymptotic Symmetry'

Now, $\mathcal{T}$ generated by 'Hamiltonian'

$$L[f] = -\tfrac{c^3}{2G_2} \textstyle\int_{\tilde{\Delta}} \left(2\ell^a \nabla_a s - \kappa s\right) f \sqrt{\gamma} d^2x \quad , \; s = \ell^a \nabla_a \phi$$

Basis: $(f = \Sigma\, c_n f_n)$

$$f_n = \tfrac{\phi_+}{2\pi s} z^n \; , \; z = \exp(2\pi i \phi/\phi_+) \; , \; \{f_m, f_n\} = i(m-n) f_{m+n}$$

$$L[f_n] = \underbrace{-\frac{\kappa \phi_+^2}{4\pi G_2 s}}_{\Delta} \delta_{n0} \leftarrow \text{Ground State}$$

$$\{L[f_m], L[f_n]\} = \underbrace{-\frac{24\pi s}{G_2 \kappa}}_{\mathcal{C}} \frac{n^3}{12} \delta_{m+n,0} \leftarrow \text{Virasoro Alg.}$$

Cardy formula:

$$S = \ln \rho(\Delta) = 2\pi \sqrt{\tfrac{\mathcal{C}\Delta}{6}} = \tfrac{2\pi \phi_+}{G_2} = \tfrac{A_H}{4\lambda_{Pl}^2} = S_{BH}$$

Loop Quantum Gravity:

(Ashtekar, Baez, Corichi, Krasnov, gr-qc/9710007)

Conditions on Δ:

1. Null
2. No radiation falling in
3. No rotation

Equivalent to:

$\frac{A_H}{2\pi\gamma} F_{ab}^{AB} + \Sigma_{ab}^{AB} = 0$

Action:

$$I = -\tfrac{i}{8\pi G} \int Tr\,(\Sigma \wedge F) - \tfrac{i}{8\pi G_4} \tfrac{A_H}{4\pi} \underbrace{\int_\Delta Tr\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)}_{Chern-Simons}$$

Quantum Mechanically:

$$\left(I \otimes \tfrac{A_H}{2\pi\gamma} F_{ab} \cdot r + \Sigma_{ab}^{AB} \cdot r \otimes I\right) \underbrace{\Psi_V}_{Vol} \otimes \underbrace{\Psi_S}_{Sur} = 0$$

.

Physical Hilbert Space:

$$\mathcal{P} = \{(p_1, j_{p_1}), \ldots, (p_n, j_{p_n})\}$$

$$\mathcal{H}_{Phy} = \frac{\oplus_{\mathcal{P}} \left[\mathcal{H}^V_{\mathcal{P}} \otimes \mathcal{H}^S_{\mathcal{P}}\right]}{Gauge}$$

Trace over Vol d.o.f. and count Surface d.o.f.:

$$A_H = 8\pi\gamma\lambda^2_{Pl} \sum_p \sqrt{j_p(j_p+1)} \approx 4\sqrt{3}\,\gamma\lambda^2_{Pl}\,P$$

$$dim\left(\mathcal{H}^S_{\mathcal{P}}\right) \sim \prod_{j_p \in \mathcal{P}} (2j_p+1) \approx 2^P$$

$$\begin{aligned} S_{BH} &= \ln dim\left(\mathcal{H}^S_{\mathcal{P}}\right) \\ &\approx P \ln 2 \\ &= \left(\frac{\gamma_0}{\gamma}\right) \frac{A_H}{4\lambda^2_{Pl}} \quad , \; \gamma_0 = \frac{\ln 2}{\pi\sqrt{3}} \end{aligned}$$

Different spectrum, but area-law for entropy: Alekseev, Polychronakos, Smedbäck, hep-th/0004036; Dasgupta, hep-th/0310069

String Theory: (Strominger, Vafa, hep-th/9601029)

$\underline{\underline{d = 5 \text{ Extremal RN Black Holes}}}$

'Strong Coupling' (Large G_d)

$$I_{LEEA} = \frac{c^3}{16\pi G_{10}} \int d^{10}x\sqrt{-g_{10}} \left[R + \frac{1}{2}(\nabla \underbrace{\phi}_{Dil})^2 - \frac{1}{12} e^{\phi} \underbrace{\overset{2}{H}_{(3)}}_{RR\ 3-form} \right]$$

Solution on $M^{10} = T^4 \otimes S^1 \otimes M^5$ (AF, 2 Charges, KK Mom)

$$ds^2_{10} = \underbrace{e^{2\chi}}_{Sca} \underbrace{dx_i dx^i}_{\substack{4-Torus \\ \text{(Small)}}} + \underbrace{e^{2\psi}}_{Sca}(\underbrace{dx_5}_{\substack{Circle \\ \text{(Large)}}} + A_\mu dx^\mu)^2 + \underbrace{e^{-2(4\chi+\psi)/3}}_{Sca} \underbrace{ds^2_5}_{Extr\ \ RN}$$

'Weak Coupling' (Small G_d)

N_1 $D{-}1{-}branes$ $\leftarrow$ Couple to $H_{(3)} \sim \frac{Q}{\sqrt{c\hbar\lambda_{Pl}}} \sim \left(\frac{r_+}{\lambda_{Pl}}\right)^2$ $(d = 5\ RN)$

N_5 $D - 5 - branes$ $\leftarrow$ Couple to ${}^\star H_{(7)} \sim \frac{Q}{\sqrt{c\hbar\lambda_{Pl}}} \sim \left(\frac{r_+}{\lambda_{Pl}}\right)^2$

N Kaluza-Klein Momenta on $S^1 \sim \frac{Q}{\sqrt{c\hbar\lambda_{Pl}}} \sim \left(\frac{r_+}{\lambda_{Pl}}\right)^2$

Equivalent to:
$4N_1N_5$ oriented strings from 1-branes to 5-branes in length L with energy $E = \frac{N\hbar c}{L}$ & $n_{B,F} = 4N_1N_5$ d.o.f.

$T^4 \ll S^1 \;\Rightarrow\;$ *Effectively* - 1-dimensional gas:

$$\begin{aligned}
S &= \sqrt{\frac{\pi(2n_B + n_F)LE}{6\hbar c}} \sim E^{(d-1)/d}\ , \text{ in general} \\
&= 2\pi\sqrt{N_1N_5N} \qquad , \ (n_{B,F} = 4N_1N_5,\ E = N/L) \\
&= 2\pi\left[\left(\frac{Q}{\sqrt{\hbar c\lambda_{Pl}}}\right)^3\right]^{1/2} \\
&= 2\pi\left[\left(\left\{\frac{r_+}{\lambda_{Pl}}\right\}^2\right)^3\right]^{1/2} \\
&= \frac{\Omega_3 r_+^3}{4\lambda_{Pl}^3} \\
&= \frac{A_5}{4\lambda_{Pl}^3} \\
&= S_{BH}
\end{aligned}$$

$\boxed{AdS/CFT \text{ Correspondence:}}$ $\leftarrow$ Holography?

(Maldacena, hep-th/9711200)

Asymptotically AdS_d Spacetimes $\Leftrightarrow$ Dual (Conformal Field) Theory on 'boundary'

$AdS-$Schwarzschild Black Hole

$$ds_d^2 = -\left(1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2} c^2 r^{d-3}} + \frac{r^2}{\ell^2}\right) dt^2 + (\cdots)^{-1} dr^2 + r^2 d\Omega_{d-2}^2$$

$$S_{BH} = \frac{\Omega_{d-2} r_+^{d-3}}{4\lambda_{Pl}^{d-2}} \approx c_1' T_H^{d-2} \quad , \quad \left[c_1' = \frac{\Omega_{d-2}}{4\lambda_{Pl}^{d-2}} \left(\frac{4\pi\ell^2}{\hbar c(d-1)}\right)^{d-2} \right]$$

$$T_H = \hbar c \frac{(d-1) r_+^2 + (d-3)\ell^2}{4\pi \ell^2 r_+} \approx \hbar c \frac{(d-1) r_+}{4\pi \ell^2}, \;\; r_+ \gg \ell \quad \text{['High temp limit']}$$

Dual Gas in Δ spacetime dimensions (Bose/Fermi Perfect Fluid):

Energy $\rightarrow$ $\underline{\epsilon = \kappa p^\alpha}$ $\leftarrow$ Momentum

$$S_{gas} = c_2' V_{\Delta-1} T^{\frac{\Delta-1}{\alpha}}$$

$$T = \frac{T_H}{\sqrt{-g_{00}}} = \frac{\ell \; T_H}{r_0} \rightarrow \quad r_0 = \text{'Location' of gas}$$

$$S_{gas} = c_2' V_{\Delta-1} \left(\frac{\ell}{r_0}\right)^{\frac{\Delta-1}{\alpha}} \; T_H^{\frac{\Delta-1}{\alpha}}$$

$$c_2' = \frac{\Omega_{\Delta-2} \; ((\Delta-1)/\alpha+1) \; \zeta((\Delta-1)/\alpha+1) \; \Gamma((\Delta-1)/\alpha+1) \left(n_B + n_F - \frac{n_F}{2^{(\Delta-1)/\alpha}}\right)}{(\Delta-1)\kappa^{(\Delta-1)/\alpha} (2\pi\hbar)^{\Delta-1}}$$

Matching powers of T_H in S_{BH} and S_{gas}:

- $\Delta = \alpha(d-2)+1 \quad \Rightarrow \Delta = d-1$, iff $\alpha = 1$
- $c'_1 = c'_2 \, \frac{(d-1)\Omega_{\Delta-1}\ell^{d-2} r_0^{(\alpha-1)(d-2)}}{(\Delta-1)/\alpha+1}$

Observations:

- if $\alpha = 1$ (Relativistic Dispersion), r_0 disappears
- For $AdS_5 - CFT_4$,
  $d = 5, \ \alpha = 1 \ , \Delta = 4$

  $N_B = N_F = 8N^2 \ (of \ \mathcal{N} = 4 \ , \ SU(N) \ SYM)$

  $\left(\frac{\ell}{\lambda_{Pl}}\right)^3 = \frac{2N^2}{\pi}$

$$c'_1 = \frac{3}{4} \, c'_2 \ \ , \ S_{BH} = \frac{3}{4} S_{gas}$$

Almost Exact Agreement !!

Holography at work?

Bulk Information = Boundary Information $\leq$ 1 bit per λ_{Pl}^{d-2}

Leading Order Corrections to Entropy

(Kaul,Majumdar, gr-qc/0002040;Das, Majumdar, Bhaduri, hep-th/0111001)

$$Partition\ Function:\quad Z(\beta) = \int_0^\infty \rho(E)\ e^{-\beta E} dE$$

$$\text{Density of States:}\quad \rho(E) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \underbrace{Z(\beta)e^{\beta E}}_{S=\ln Z+\beta E} d\beta = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{S(\beta)} d\beta$$

$$\beta_0^{-1} = Equilibrium\ Temperature$$

Near $\beta = \beta_0$,

$$S(\beta) = S_0 + \frac{1}{2}(\beta - \beta_0)^2 S_0'' + \cdots$$

$S_0 := S(\beta_0)$ and $S_0'' := (\partial^2 S(\beta)/\partial\beta^2)_{\beta=\beta_0}$.

$$\rho(E) = \frac{e^{S_0}}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{1/2(\beta-\beta_0)^2 S_0''} d\beta \qquad (\beta - \beta_0 = ix)$$

$$\rho(E) = \frac{e^{S_0}}{\sqrt{2\pi S_0''}} \quad .$$

$$\mathcal{S} := \ln\rho(E) = S_0 - \frac{1}{2}\ln S_0'' + \text{ (smaller terms)}.$$

FIND S_0''

.

$$S(\beta) = \ln Z(\beta) + \beta E \quad . \qquad \Rightarrow \quad S''(\beta) = \frac{1}{Z}\left(\frac{\partial^2 Z(\beta)}{\partial \beta^2}\right) - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial \beta}\right)^2$$

$$\Rightarrow S''_0 =< E^2 > - < E >^2$$

$$\left[\quad E \equiv < E > = -\left(\frac{\partial \ln Z}{\partial \beta}\right)_{\beta=\beta_0} = -\frac{1}{Z}\left(\frac{\partial Z}{\partial \beta}\right)_{\beta=\beta_0} \quad . \quad < E^2 > = \frac{1}{Z}\left(\frac{\partial^2 Z}{\partial \beta^2}\right)_{\beta=\beta_0} \quad \right]$$

Also, specific Heat $= C \equiv \left(\frac{\partial E}{\partial T}\right)_{T_0} = \frac{1}{T^2}\left[\frac{1}{Z}\left(\frac{\partial^2 Z}{\partial \beta^2}\right)_{\beta=\beta_0} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial \beta}\right)^2_{\beta=\beta_0}\right] = \frac{S''_0}{T^2}$

$$\Rightarrow S''_0 = CT^2$$

$$S''_0 \; = \; -\left(\frac{\partial E}{\partial \beta}\right)_{T_0} \; = \; C\,T^2$$

$$\mathcal{S} = S_0 - \tfrac{1}{2}\ln\left(CT^2\right) + \cdots$$

$\leftarrow$ Leading log corrections

$\uparrow$

Master Equation, valid for <u>any</u> system

For BHs use $T = T_H$ & $S_0 = S_{BH} = \frac{A}{4\lambda_{Pl}^{d-2}}$ & calculate C

<u>For example - BTZ:</u>

$$C = T_H = S_{BH} \quad \Rightarrow \quad \mathcal{S} = S_{BH} - \tfrac{3}{2}\ln S_{BH}$$

Back to Schwarzschild anti-de Sitter

$$
\begin{aligned}
T_H &= \hbar c \, \frac{(d-1)r_+^2 + (d-3)\ell^2}{4\pi\ell^2 r_+} = \#\underline{S_0}^{1/(d-2)} \\
C &= (d-2)\left[\frac{(d-1)r_+^2/\ell^2 + (d-3)}{(d-1)r_+^2/\ell^2 - (d-3)}\right] S_0 \\
&= (d-2)\ \underline{S_0}\ > 0 \qquad [\ \ell \ll r_+\ ]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S} &= S_{BH} - \frac{1}{2}\ln\left(S_{BH} S_{BH}^{2/(d-2)}\right) \\
&= S_{BH} - \frac{d}{2(d-2)}\ln\left(S_{BH}\right)
\end{aligned}
$$

Dual Gas

$$
\mathcal{S} = S_{gas} - \tfrac{1}{2}\ln(CT^2) = S_{gas} - \tfrac{1}{2}\ln\left(\tfrac{V_{\Delta-1}c_2'(d-1)}{(d-2)}\ \tfrac{dT^{d-1}}{dT}T^2\right)
$$

$$
= S_{gas} - \tfrac{1}{2}\ln\left(c_2\ V_{\Delta-1}\ T^d\right) \qquad \leftarrow \quad S_{gas} = c_2 V_{\Delta-1} T^{d-2}
$$

$$
= S_{gas} - \tfrac{1}{2}\ln\left[\left(S_{gas}^{1/(d-2)}\right)^d \left(c_2 V_{\Delta-1}\right)^{-2/(d-2)}\right]
$$

$$
= \underbrace{S_{gas}}_{S_{BH}} \quad - \underbrace{\frac{d}{2(d-2)}\ln S_{gas}}_{\text{Same as BH corr incl. coeffs.}} \quad + \ \tfrac{1}{d-2}\ \underbrace{\ln\left[(n_B+n_F)V_{\Delta-1}\right]}_{\text{Subleading. Depends on details of theory}}
$$

Entropy matching *does not* guarantee corrections matching !

(Das, Husain, hep-th/0303089)

Hawking Radiation

$$\frac{dE}{d\omega dt} = \frac{\sigma\ \omega^{d-1} d\omega}{e^{\omega/T_H} \mp 1}$$

$$\frac{dE}{dt} = \sigma\ T_H^d$$

- LQG: (Krasnov, gr-qc/9710006)

- ST: (S R Das, Mathur, hep-th/9606185)

Theory $\rightarrow$	CFT	LQG	S T	AdS|CFT
$d = 4$ SC	$\surd$	$\surd$	$\times$	$\times$
$d = 4$ Extr RN	$\surd$	$\surd$	$\surd$	$\times$
$d = 4$ SC-AdS	$\surd$	?	$\times$	$\times$
$d = 5$ SC-AdS	$\surd$	$\times$	$\times$	$\surd$
Location of dof	$\surd$	$\surd$	$\times$	$\surd$
Hawking Radn	$\times$	$\surd$	$\surd$	$\times$

Comparison

Information loss from a Black Hole

Unitary Cloning?

CFT/LQG/ST ... are Unitary

Theorem

$$\mathcal{U}|\Psi\rangle \otimes \underbrace{|T\rangle}_{Target} = |\Psi\rangle \otimes |\Psi\rangle \quad , \quad \mathcal{U}^\dagger = \mathcal{U}^{-1}$$

$$\mathcal{U}|\Phi\rangle \otimes |T\rangle = |\Phi\rangle \otimes |\Phi\rangle$$

Taking inner product

$$\langle\Psi| \otimes \underbrace{\langle T|\underbrace{\mathcal{U}^\dagger\,\mathcal{U}}_{I}|T\rangle}_{1} \otimes |\Phi\rangle = \left(\langle\Psi| \otimes \langle\Psi|\right) \cdot \left(|\Phi\rangle \otimes |\Phi\rangle\right)$$

$$\langle\Psi|\Phi\rangle = \left(\langle\Psi|\Phi\rangle\right)^2$$

$$\Rightarrow \langle\Psi|\Phi\rangle = 1 \;\; or \;\; 0 \quad \longleftarrow \text{ Not a general QM State}$$

There is no Quantum XeroX Machine !

Possible Resolutions

•$\mathcal{U}$ is Not-Unitary ← Hawking. **Theories suggest otherwise**

• Planck-Size Remnant ← Retains all information

But, can there be such large information concentrated in such a small volume?

• Black Hole Complementarity ('t Hooft, Susskind)

- Construct pure state: $|\mathcal{R}\rangle = \Sigma_{n=0}^{\infty} p^n \ |n\rangle_o |n\rangle_I$, $p = \frac{1}{e^{\hbar\omega/T_H} \mp 1}$

- Density matrix: $\rho = |\mathcal{R}\rangle\langle\mathcal{R}| = \Sigma\Sigma\, p^{k+m} |kk\rangle\langle mm|$ ← Pure

-Trace over **Inside:** $\rho_o = \Sigma\, p^{2n} |n\rangle_o \langle n|_o$ ← Mixed ($\rho_o^2 \neq \rho_o$)

- Outside observer should wait for at least half the BH to evaporate to get 1 bit of information (Page, hep-th/9306083)

- Bulk information appears at very late times, when BH is Planck sized ←

- After getting the info, outside observer dives inside BH to find out if Xeroxing has taken place. But semi-classical physics fails, including Hawking evaporation etc

At best, an 'effective theory'

• Unique BH Final State (Horowitz/Maldacena, hep-th/0310281)

- Final state collapses to unique state $|\psi_0\rangle$, all info transported to outgoing Hawking radiation à la EPR

- Final BH entropy à la Shannon $= -\Sigma_n p_n \ln p_n = -1 \ln 1 = 0$
$\Rightarrow$ No info is absorbed by singularity

Ad-hoc $\oplus$ weak violations of unitarity due to body, Hawking particle interactions (Gottesmann, Preskill, hep-th/0311269)

Observations, Experiments, Data, ...

• Compelling evidence for event horizons in tens of X-ray binaries (Narayan, astro-ph/0310692). BH thermodynamics next?

• Brane World Black Holes in detectors? (ADD:$M^4 \times T^{d-4}$ & RS)

$$\underbrace{\overset{d-2}{M_{Pl(d)}}}_{1-TeV/c^2} = \frac{\hbar^{d-3}}{c^{d-5} G_d} = \frac{\hbar^{d-3}}{c^{d-5} \underbrace{V_{d-4}}_{L^{d-4}} G_4} = \left(\frac{\hbar}{c \underbrace{L}_{mm}} \right)^{d-4} \underbrace{M^2_{Pl(4)}}_{10^{19}-TeV/c^2}$$

Low $(d = 6)$ High

$$\underbrace{r_{+(d)}}_{10^{-4}Fm} = \left(\frac{G_d M}{c^2} \right)^{\frac{1}{(d-3)}} = \left(\frac{V_{d-4} G_4 M}{c^2} \right)^{\frac{1}{(d-3)}} = (V_{d-4} \underbrace{r_{+(4)}}_{10^{-29}Fm})^{\frac{1}{(d-3)}}$$

Large $(G_d = \hbar^{\frac{1}{d-3}} c^{5-d} / M_{Pl}^{\frac{1}{d-2}})$ Small

At Planck Scale, GUP replaces HUP (ST, NCQM, ...)

$$\Delta x \geq \frac{\hbar}{\Delta p} + (A\lambda_{Pl})^2 \frac{\Delta p}{\hbar} \quad \leftarrow \text{GUP } (A = \mathcal{O}(1), \text{ theory dependen}$$

$$\Delta p = \frac{\hbar \Delta x}{2(A\lambda_{Pl})^2} \left[1 - \sqrt{1 - \frac{4(A\lambda_{Pl})^2}{\Delta x^2}}\right]$$

$$\left(\frac{\Delta x}{\lambda_{Pl}} \gg 1 \,, \; \frac{\Delta p}{M_{Pl(d)} c} \ll 1 \;\Rightarrow\; \Delta p \, \Delta x \geq \hbar \quad \text{(HUP)} \right)$$

For a Hawking particle, $\Delta x \approx 2r_+ \,, \; \Delta p \, c \approx T_H$

Using: $r_+ \approx m^{1/(d-3)}$ $\qquad (m \equiv \frac{M}{M_{Pl(d)}})$

$$T_H = \underbrace{m^{\frac{1}{d-3}} \left[1 - \sqrt{1 - \frac{4A^2}{m^{\frac{2}{d-3}}}}\right] M_{Pl(d)}}_{\text{GUP corrected temp}} \approx \left[\underbrace{\frac{1}{m^{1/(d-3)}}}_{\text{Usual temp}} + \underbrace{\frac{1}{m^{3/(d-3)}}}_{\text{Corrections}} + \cdots \right] M_{Pl(d)}$$

$\frac{dm}{dt} \propto (\text{Area}) \times T_H^d \quad \leftarrow$ (Stefan-Boltzmann Law)

Observe:

- $T_H(\text{GUP}) > T_H(\text{Uncorrected}) \quad \leftarrow$ Radiates faster
- Radiation stops at $M_{Min} = (4A^2)^{\frac{d-3}{2}} \, M_{Pl} \quad \leftarrow$ Missing energies!

Cavaglia, Das, Maartens, hep-ph/0305223

• Analog Black Holes (Unruh '81, Visser gr-qc/9901047)

Start with Continuity and Navier-Stokes for irrotational, inviscid fluid

$$\begin{aligned}\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho v) &= 0\\ \rho\left(\frac{\partial v}{\partial t} + (v\cdot\nabla)v\right) &= -\nabla p\\ \nabla\times v = 0 \quad \Rightarrow v &= \nabla\psi\end{aligned}$$

Perturb around equilibrium → to $\mathcal{O}(\epsilon)$:

$$\rho = \rho_0 + \epsilon\rho_1\ ,\ p = p_0 + \epsilon p_1\ ,\ \psi = \psi_0 + \epsilon\psi_1\ ,\ \vec{v} = \vec{v}_0 + \epsilon\vec{v}_1$$

$$\begin{aligned}\frac{\partial\rho_1}{\partial t} + \nabla\cdot(\rho_1\nabla\psi_0 + \rho_0\nabla\psi_1) &= 0\\ \rho_0\left(\frac{\partial\psi_1}{\partial t} + \nabla\psi_0\cdot\nabla\psi_1\right) &= p_1\end{aligned}$$

Combine: $\left(c^2 = \frac{\partial p}{\partial\rho} = \text{ Speed of sound }\right)$

$$\frac{\partial}{\partial t}\left(c^{-2}\rho_0\left(\frac{\partial\psi_1}{\partial t} + \vec{v}_o\cdot\nabla\psi_1\right)\right) = \nabla\cdot\left(\rho_0\nabla\psi_1 - c^{-2}\rho_0 v_0\left(\frac{\partial\psi_1}{\partial t} + \vec{v}_0\cdot\nabla\psi_1\right)\right)$$

Define:

$$g^{\mu\nu} = \frac{1}{\rho_0\ c}\left(\begin{array}{c|c} -1 & -v_0^j \\ \hline -v_0^i & (c^2\delta^{ij} - v_0^i v_0^j)\end{array}\right)$$

Then:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi_1\right) = 0$$

Scalar field (phonon) in curved background

$$g_{\mu\nu} = \frac{\rho_0}{c} \left(\begin{array}{c|c} -(c^2 - v_0^2) & -v_0^j \\ \hline & \\ -v_0^i & \delta^{ij} \end{array} \right)$$

$$\Rightarrow \quad ds^2 = \tfrac{\rho_0}{c} \; \left[-(c^2 - v_0^2)\; dt^2 + d\vec{r}^2 - 2\vec{v} \cdot d\vec{r}\; dt\right]$$

<u>Choose following velocity and density profiles:</u>

$$v_0 = \sqrt{\tfrac{2G_4M}{r}}\ , \;\; \rho_0 = k r^{-3/2}$$

<u>Then:</u>

$$ds^2 = \tfrac{k}{c}\, r^{-3/2} \left[-c^2 \left(1 - \tfrac{2G_4M}{c^2 r}\right) dt^2 - 2\sqrt{\tfrac{2G_4M}{r}}\; dr dt + dr^2 + r^2 d\Omega_2^2\right]$$

<u>Define:</u> $\quad t' = t + \left[\tfrac{4G_4M}{c^3} \arctan\left(\sqrt{\tfrac{2G_4M}{c^2 r}}\right) - 2\sqrt{\tfrac{2G_4Mr}{c^4}}\right]$

<u>In (t', r) coords:</u>

$$ds^2 = \tfrac{k}{c}\; r^{-3/2} \; \underbrace{\left[-c^2 \left(1 - \frac{2G_4M}{c^2 r}\right) dt'^2 + \left(1 - \frac{2G_4M}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega_2^2\right]}_{\text{-Schwarzschild-}}$$

Acoustic metric conformal to Schwarzschild

Acoustic BH Hawking radiates with $T_H = \frac{\hbar c^3}{8\pi G_4 M} \approx 10^{-4}\ K$

(Similarly, Superradiance: Basak, Majumdar, gr-qc/0303012)

<u>Measurable??</u>

Discussions

- Black Holes are Thermodynamic Systems

- Variety of microscopic dofs account for entropy, Hawking radiation: CFT, LQG, ST, AdS/CFT
  - How are they related ?

- Microscopic theories may not account for fluctuation corrections. More careful analysis required.

- Info lost inside BH: Fundamentally evolution non-unitary? Although QG theories seem to be unitary. Various proposals, none complete.

- Observations, Experiments, ... ?