Status of Numerical Relativity: From my personal point of view

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1 Introduction

2 General issues in numerical relativity

3 Current status of implementation

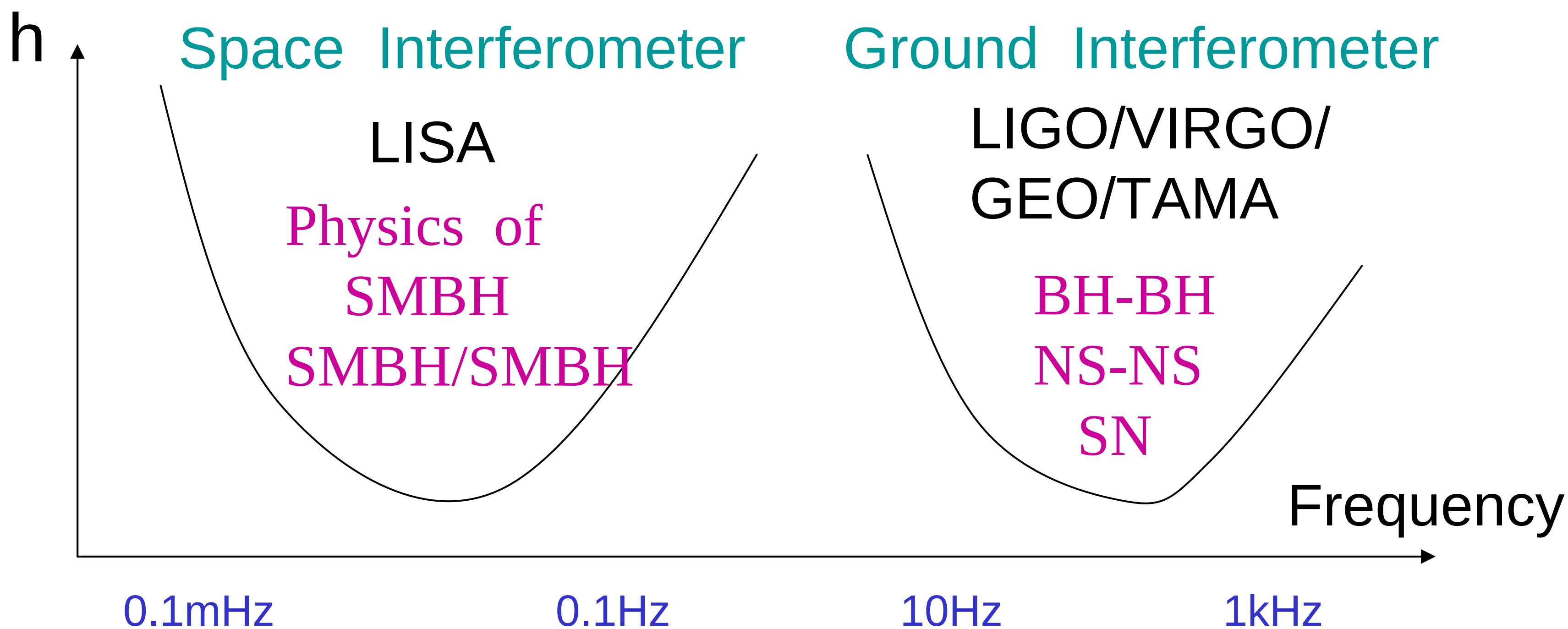
4 Some of our latest numerical results:
NS-NS merger & Stellar core collapse

5 Summary & perspective

1: Introduction: Roles in NR

A To predict gravitational waveforms:

Two types of gravitational-wave detectors work now or soon.

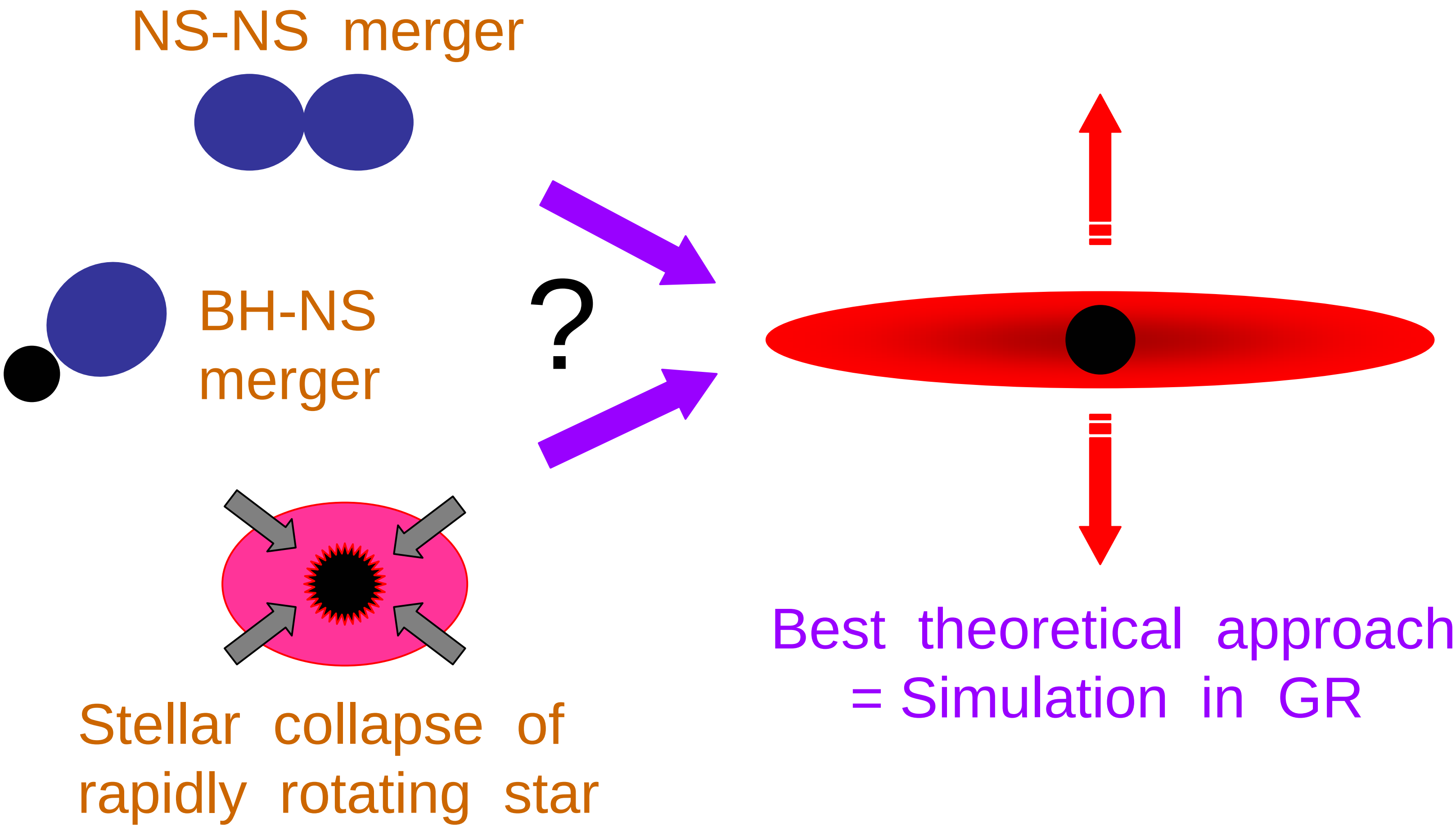


Templates (for compact binaries, core collapse, etc) should be prepared

B　To simulate Astrophysical Phenomena

e.g. Central engine of GRBs

= Stellar-mass black hole + disks (Probably)



Best theoretical approach
= Simulation in GR

C To discover new phenomena in GR

In the past 20 years, community has discovered e.g.,

1: Critical phenomena (Choptuik)

2: Toroidal black hole (Shapiro-Teukolsky)

3: Naked singularity formation (Nakamura, S-T)

GR phenomena to be simulated ASAP

- NS-NS / BH-NS /BH-BH mergers

  (Promising GW sources/GRB)
- Stellar collapse of massive star to a NS/BH

  (Promising GW sources/GRB)
- Nonaxisymmetric dynamical instabilities of rotating NSs

  (Promising GW sources)
- ….

In general, 3D simulations are necessary

2 Issues: Necessary elements for GR simulations

- Einstein's evolution equations solver
- GR Hydrodynamic equations solver
- Appropriate gauge conditions (coordinate conditions)
- Realistic initial conditions in GR
- Gravitational wave extraction techniques
- Apparent horizon (hopefully Event horizon) finder
- Special techniques for handling BHs / BH excision
- Micro physics (EOS, neutrino processes, B-field …)
- Powerful supercomputers

RED = Indispensable elements

3: Current Status: Achievements in the past decade

Here, focus on progress in main elements:

- Einstein evolution equation solver in 3D
- GR Hydro equation solver
- Appropriate gauge conditions in 3D
- Supercomputers

Progress I

- Formulations for Einstein's evolution equation

Many people 10 yrs ago believed the standard ADM formalism works well.

BUT:

Unconstrained free evolution

Standard ADM

Variables in standard ADM formalism:

$$\gamma_{ij}, \quad K_{ij}$$

12 components

→

Numerical simulation becomes unstable even in the evolution of linear GW

(Nakamura 87, Shibata 95, Baumgarte-Shapiro (99)

Due to constraint violation instabilities

- New formulations for Einstein's evolution eqs :

(i) BSSN formalism

Nakamura (87), Shibata-Nakamura (95), Baumgarte-Shapiro (99).....

Choose variables:

$$\phi \equiv \frac{1}{12}\ln\left(\det(\gamma)\right)$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}$$

$$K \equiv K_k^k$$

$$\tilde{A}_{ij} \equiv e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right)$$

$$F_i \equiv \delta^{jk}\partial_j\tilde{\gamma}_{ik}$$

17 components

→

An Important step

Rewrite ADM equations using

$$\left\{\begin{array}{l} constraint\ \ equations \\ \det\left(\tilde{\gamma}_{ij}\right) = 1 \end{array}\right\}$$

↓ Unconstrained free evolution

Stable numerical simulation

(So far no problem in the absence of black holes)

- New formulations for Einstein's evolution eqs. :

(ii) Hyperbolic formulations

Bona-Masso (92) ……… many references ……..

Kidder-Scheel-Teukolsky (KST) (01)

$$\partial_t g^{ij} + \partial_k Q^{kij} = \underline{F^{ij}(g, Q, \ldots)}$$

No derivatives

30~40 variables are defined

Advantage for imposing boundary conds. at BH
→ Perhaps, robust for BH spacetimes
But, no success in 2BH merger so far.
(Something is short of. Need additional ideas.)

Progress II

- GR Hydro scheme

Trend until the middle of 1990

$\Rightarrow$ Add artificial viscosity to capture shocks

(Wilson 1980, Centrella 1983, Hawley et al. 1984,

Stark-Piran 1985, Evans 1986, Nakamura 1993, Shibata 1999)

Schematically,

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial(\rho v_i v^j + P\gamma_i^{\ j})}{\partial x^j} = \underline{[Viscous\ \ term]_i} + ...$$

Very phenomenological;
Not very physical

Drawback : Strong shocks cannot be captured accurately.

Concern : We do not know if it always gives the correct answer for any problems ???

- Hydro scheme: Current trend

High-resolution shock-capturing scheme
= Solve equations using characteristics
(+ Piecewise-Parabolic interpolation
+ Approximate Riemann solver) : very physical !

No artificial viscosity

Developed by Valencia (Ibanez, Marti, Font, …)
& Munich (Mueller …) groups in 1990s.
Now used by many groups (including myself)

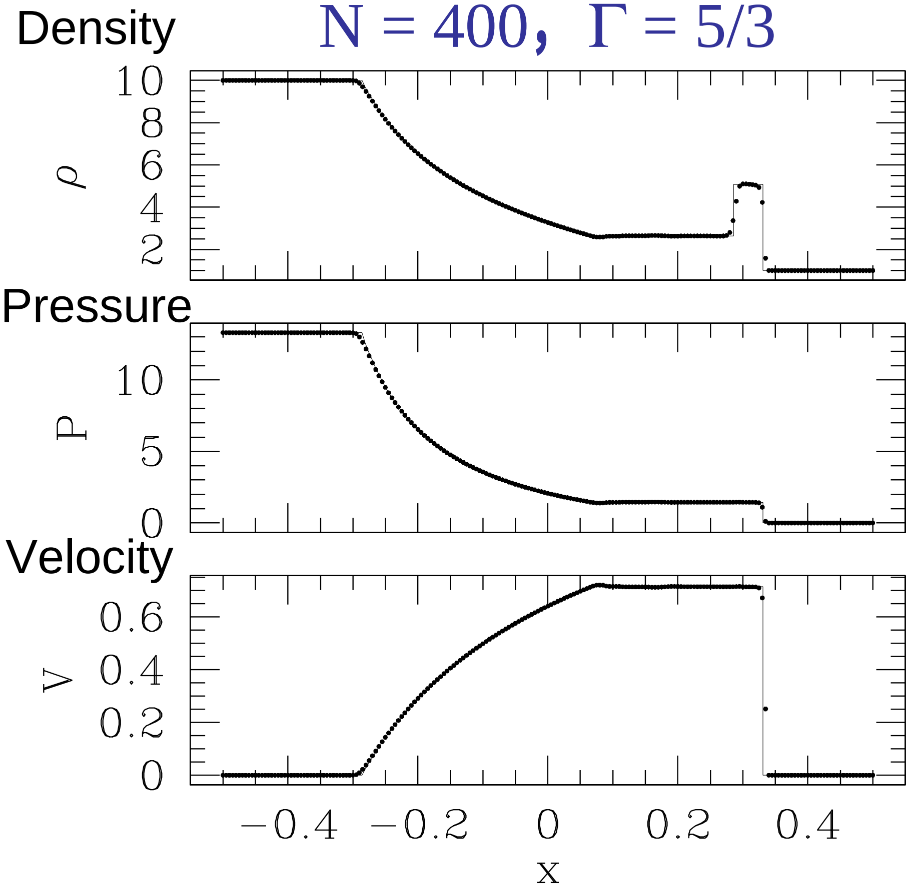
- Strong shocks & oscillations of stars are computed accurately
- Physical Scheme → No concern on the outputs

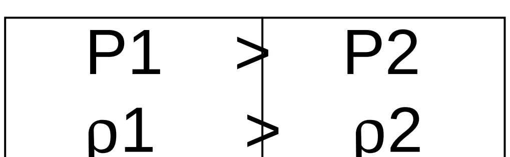
⇒ This is currently the best choice for simulations of
-- Stellar core collapse
-- NS-NS merger

Standard tests for hydro code in special relativity

Riemann Shock Tube

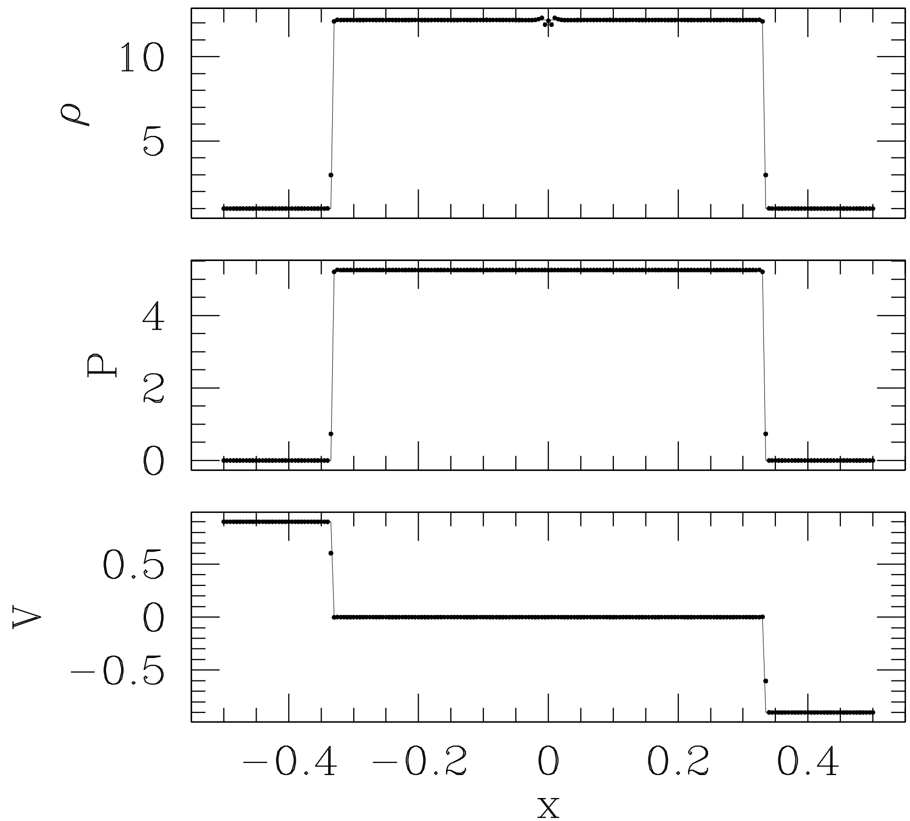
$N = 400, \ \Gamma = 5/3$

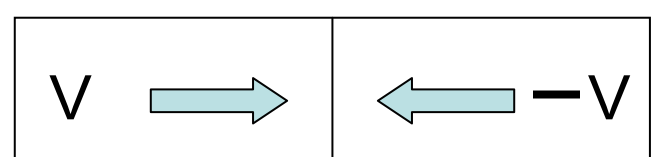




V = 0.9c. Wall Shock

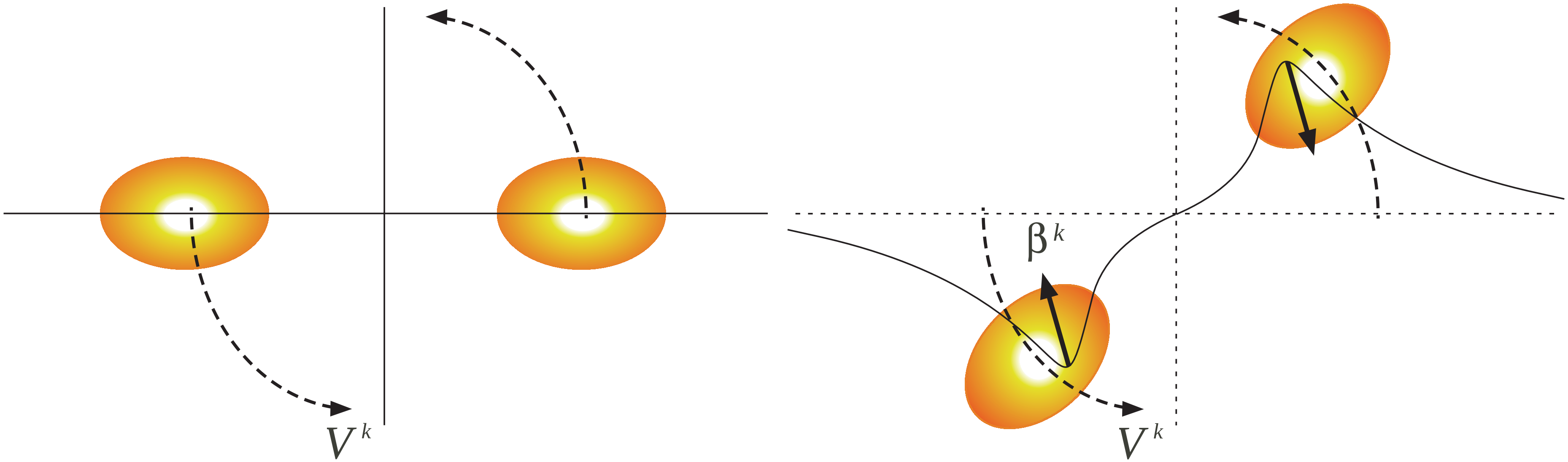
$N = 400, \ \Gamma = 4/3$





Progress III

- Choice of appropriate spatial gauge condition :



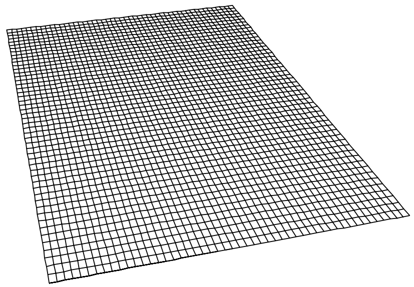
Frame dragging → Coordinate distortion

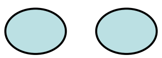
Could increase the magnitude of unphysical parts of metric

We need to suppress it for a long-term evolution.

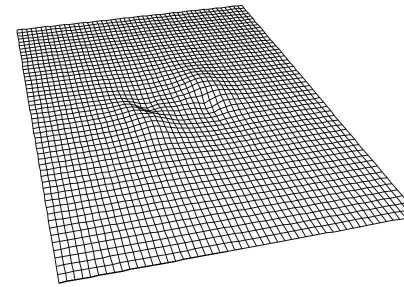
γ_{xx} on the equatorial plane with zero shift vector

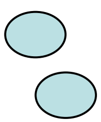
T=0



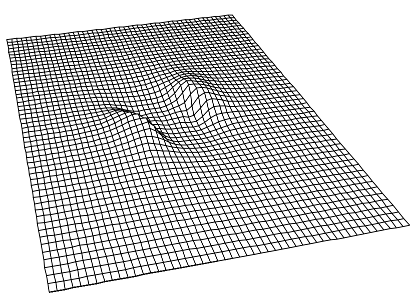


T~P/6

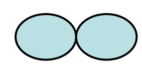


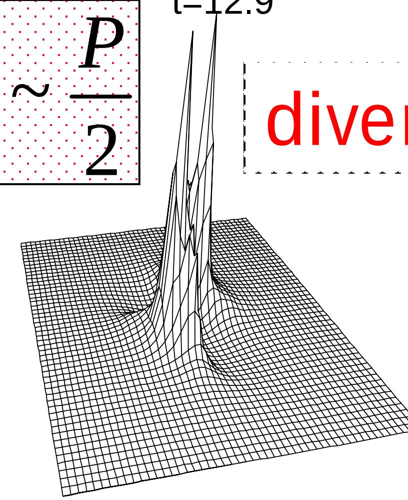


T~P/3



$$\gamma_{xx} - 1 \approx 2 \quad \text{at} \quad t \sim \frac{P}{2}$$





diverge

Distortion monotonically increases to crash

Previous belief: Minimal distortion gauge

(Smarr & York 1978)

Require that an action which denotes the global magnitude of the coordinate distortion is minimized.

$$\text{MD gauge}: \Delta\beta^k + \frac{1}{3} D^k D_j \beta^j = S^k$$

Physically good.
But, computationally time-consuming

New Trend: Dynamical gauge (Alcubierre et al 2000, Lindblom & Scheel 2003, Shibata 2003 …..)

Schematic form:

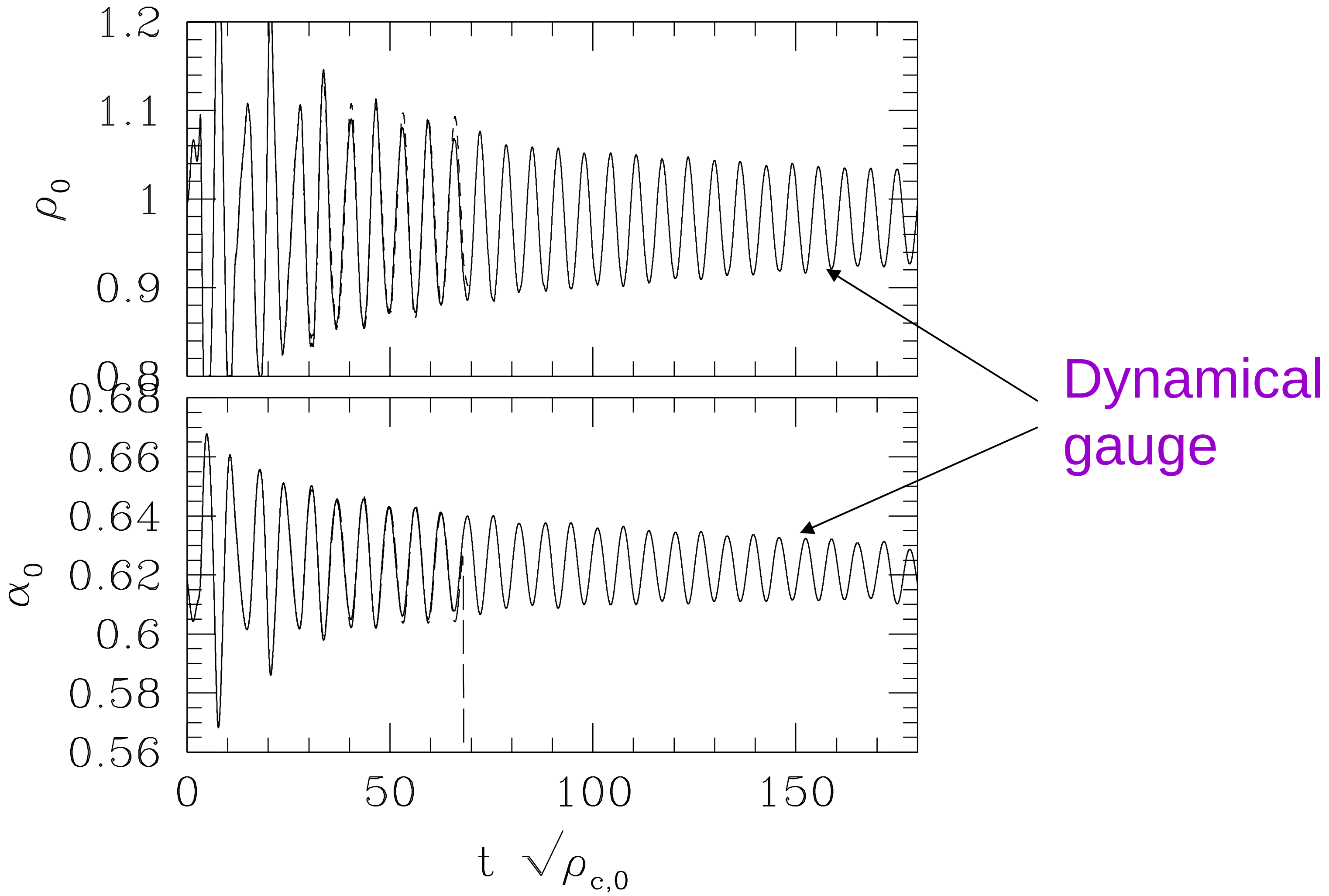
$$\ddot{\beta}^l \approx \Delta\beta^l + \frac{1}{3} D^l D_j \beta^j - S^l$$

Save CPU time significantly !!
Recent numerical experiments show it works well !!

Evolution of compact, rapidly rotating & oscillating NS in a dynamical gauge

Central density

Lapse function at r=0



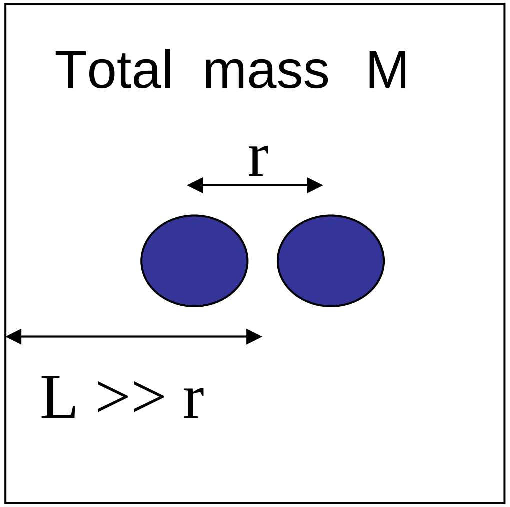
Stable evolution for > 30 oscillation (~ rotation) periods.

Progress IV

Computational resources

Minimum required grid number for extraction of gravitational waveforms

$$\lambda_{GW} \le \lambda_{ISCO} \approx 58 \left(\frac{GM}{c^2} \right) \left(\frac{rc^2}{7GM} \right)^{3/2}$$



Require $\quad L \ge \lambda_{GW} \quad \& \quad \Delta x \le 0.2 \left(\frac{GM}{c^2} \right)$

$$\Rightarrow \quad \frac{L}{\Delta x} \ge 290 \left(\frac{rc^2}{7GM} \right)^{3/2} \quad \& \quad N \ge 580 \left(\frac{rc^2}{7GM} \right)^{3/2}$$

Minimum grid number required (in uniform grid):
~ 600 * 600 * 300 (equatorial symmetry is assumed)
⇒ Memory required ~ 200 GBytes (~200 variables)

An example of current supercomputer

FUJITSU FACOM VPP5000 at NAOJ

- Vector-Parallel Machine (60 vector PEs)
- Maximum memory → 0.96TBytes
- Maximum speed → 0.58TFlops
- Our typical run with 32PEs

Typical current memory & speed

633 * 633 * 317 grid points = 240 Gbytes memory (in my code)

About 20,000 time steps ~ 100 CPU hours /model

Minimum grid number can be taken

But, hopefully, we need hypercomputers for well-resolved simulations.
(e.g. Earth simulator ~ 10TBytes, ~ 40TFlops)

Summary of current status

- Einstein evolution equations solver OK
- Gauge conditions (coordinate conditions) OK
- GR Hydrodynamic equations solver OK
- Powerful supercomputer ~OK

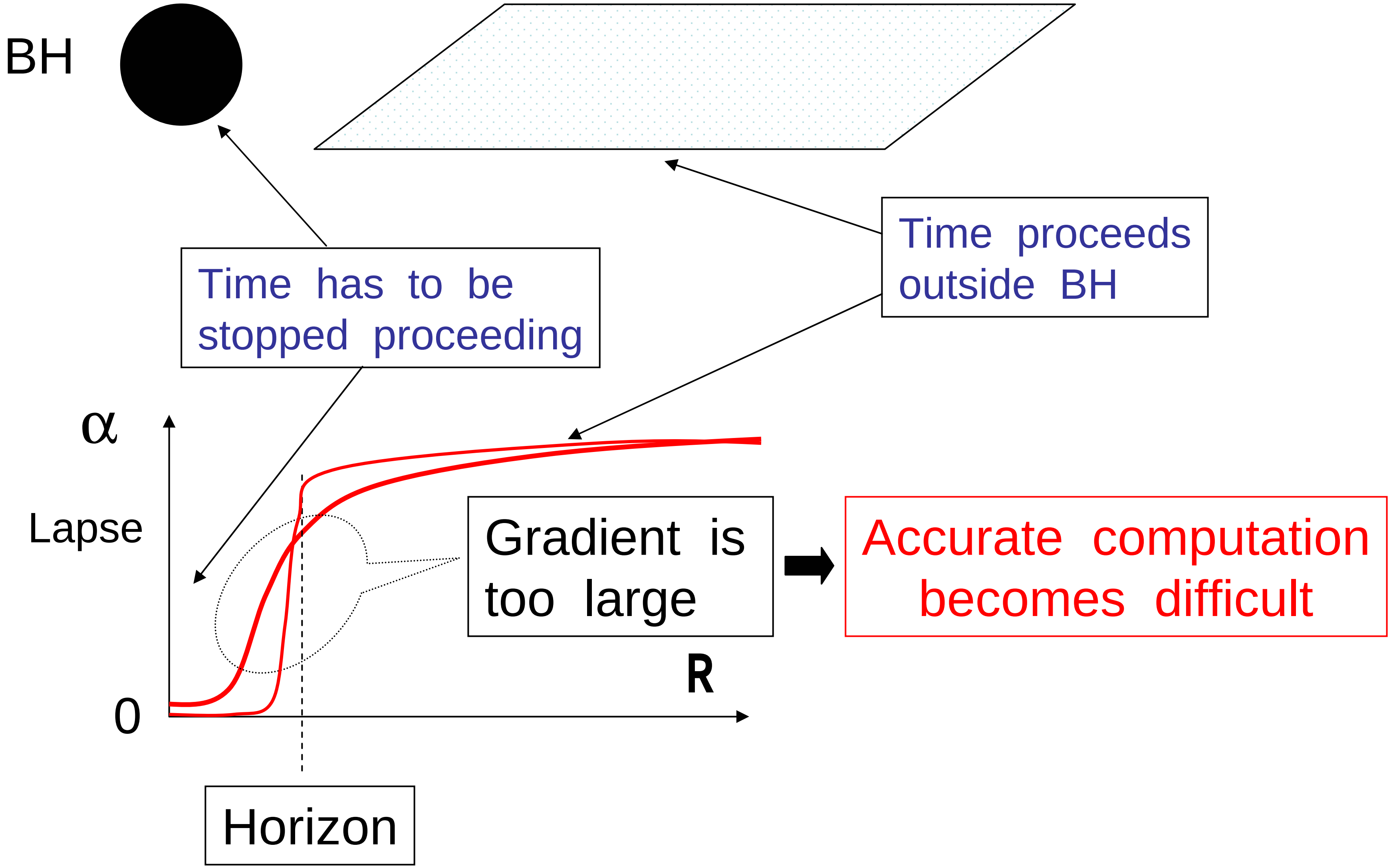
but hopefully need hypercomputers

Long-term GR simulations are feasible
(*in the absence of BHs*)

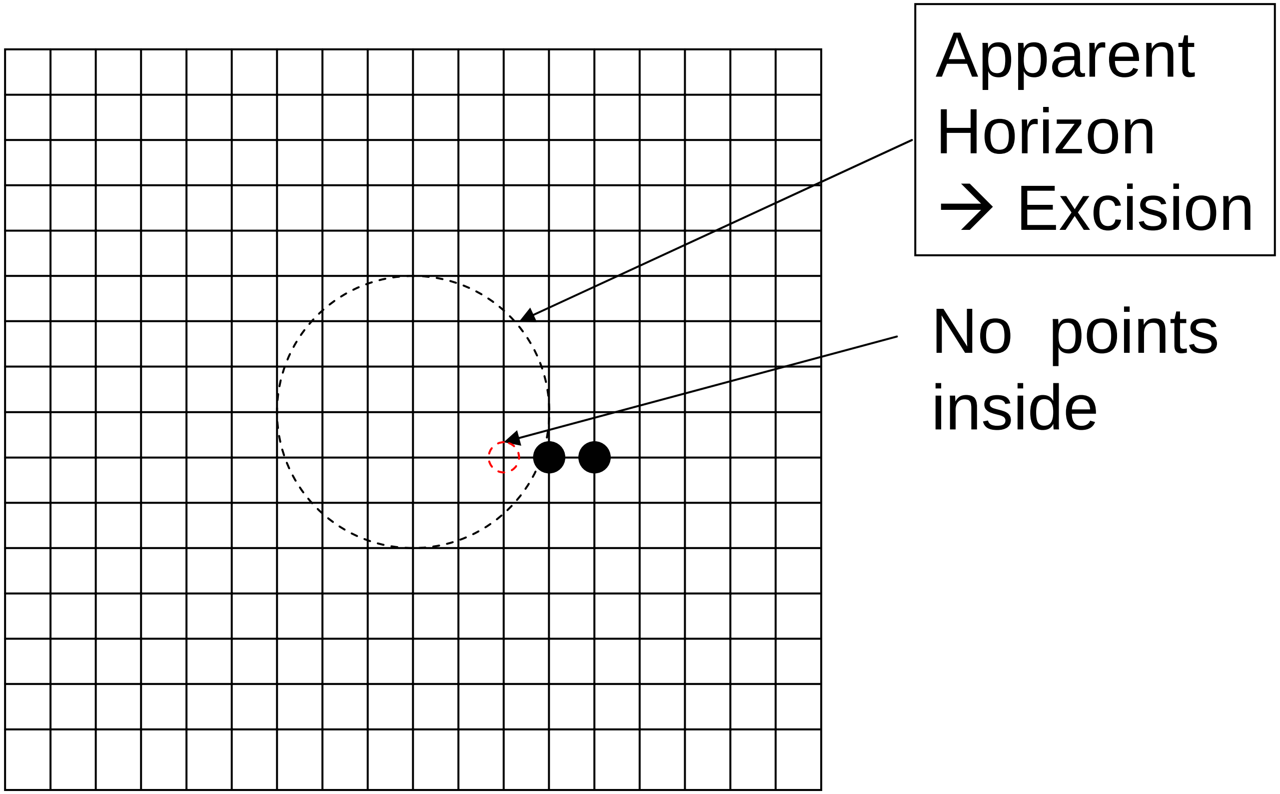
In the past 5 yrs, computations have been done for

- NS-NS merger (Shibata-Uryu, Miller, …)
- Stellar core collapse (Font, Papadopoulas, Mueller, Shibata)
- Collapse of supermassive star (Shibata-Shapiro)
- Bar-instabilities of NSs (Shibata-Baumgarte-Shapiro)
- Oscillation of NSs (Shibata, Font-Stergioulas, ….)

Unsolved Issue : Handling BHs



A solution = Excision (Unruh)



What are appropriate formulation, gauge, boundary conditions .... ?

-- 1BH → OK (Cornell, Potsdam, Illinois...)

-- 2BH → No success for a longterm simulation

(But see gr-qc/0312112, Bruegmann et al. for one orbit)

4. Our latest numerical results:

Current implementation in our group

1. GR : BSSN (or Nakamura-Shibata). But modified year by year; e.g., latest version = Shibata et al. 2003 has improved accuracy significantly

2. Gauge : Maximal slicing (K=0) + Dynamical gauge

3. Hydro : High-resolution shock-capturing scheme (Roe-type method with 3rd-order PPM interpolation)

Latest results for merger of 2NS

EOS: Initial; $P = K \rho^\Gamma$, $\Gamma = 2$; K = 1.535e5 cgs

M = 1.40 M_solar → R = 14.8 km

1.60 M_solar → R = 13.3 km

(Maximum mass for the spherical case = 1.68 M_solar)

During the evolution: $P = (\Gamma-1)\rho\varepsilon$

I here show animations for merger of 2NS

(a) 1.40 – 1.40 M_solar,

(b) 1.33 – 1.46 M_solar,

(c) 1.52 – 1.52 M_solar,

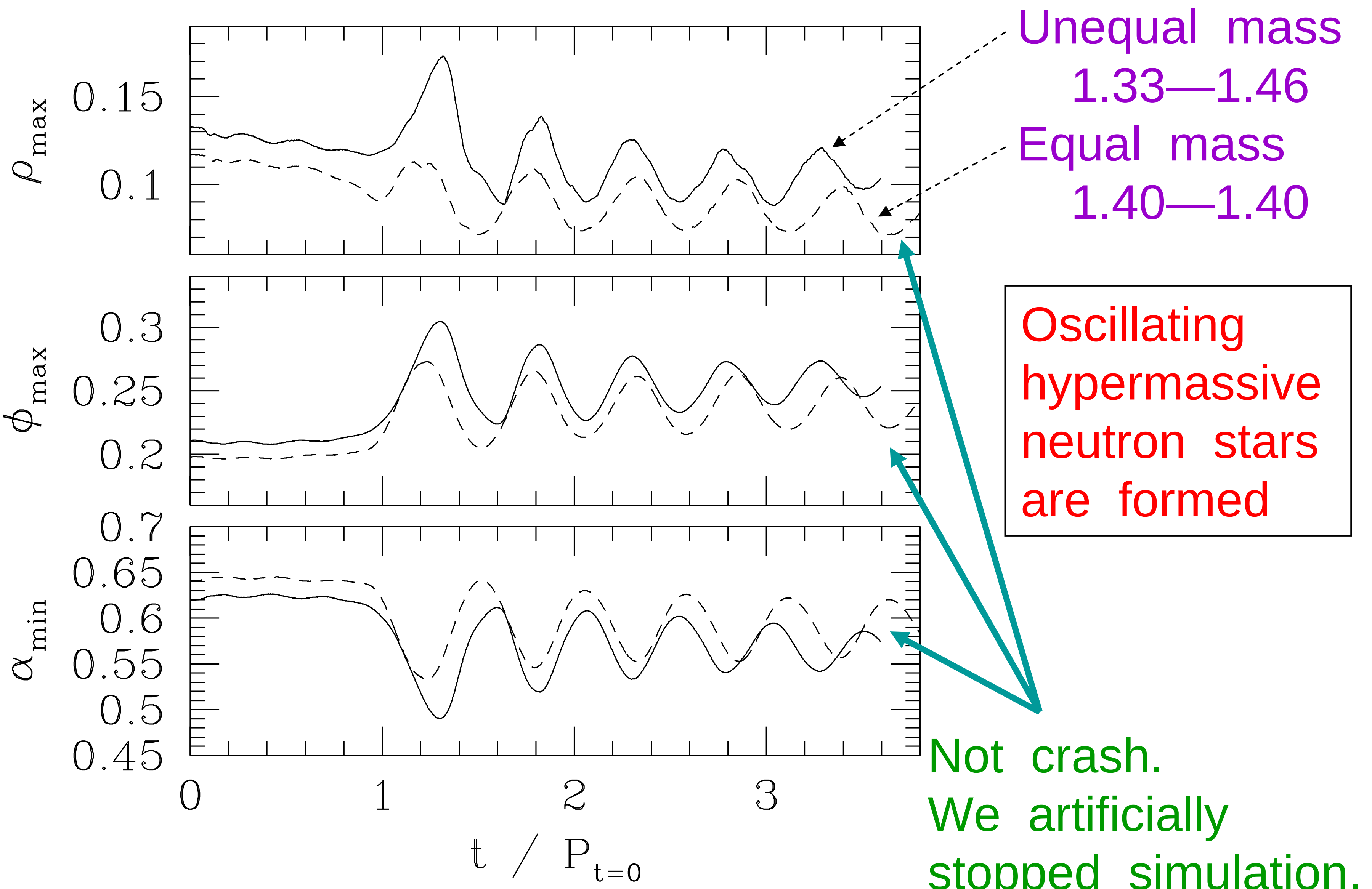
(d) 1.40 – 1.60 M_solar

(See, Shibata et al. PRD 68, 084020, 2003)

Typical grid size : 633 * 633 * 317

(max we have taken, 761 * 761 * 381)

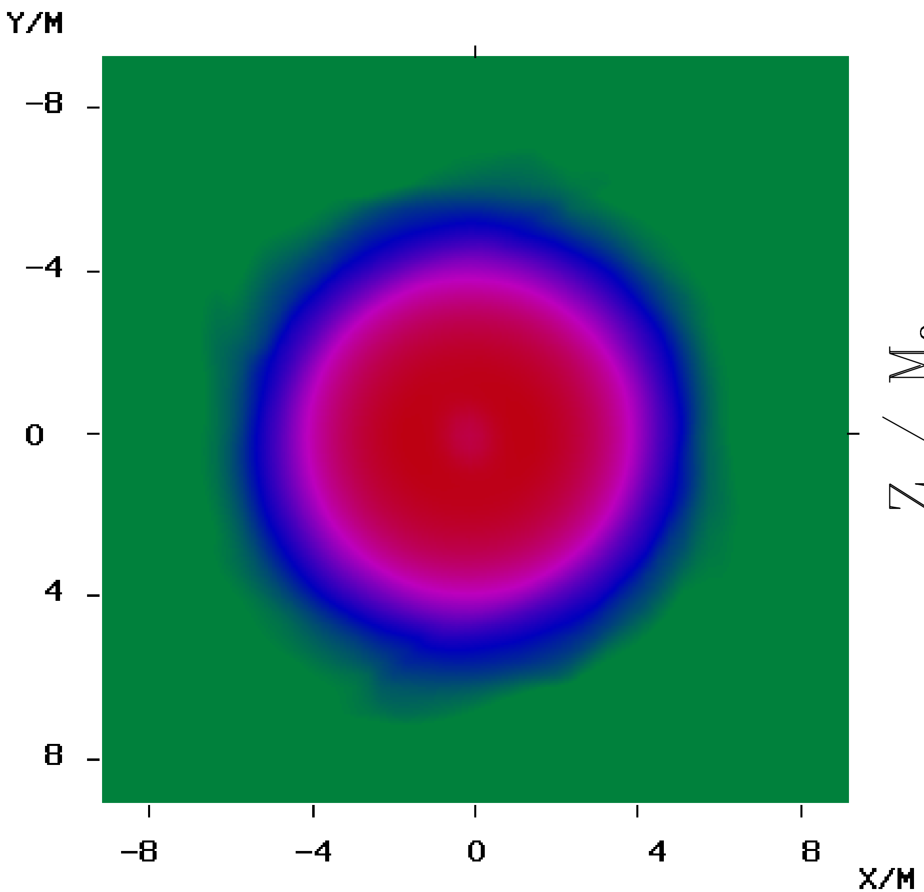
Evolution of maximum density in NS formation



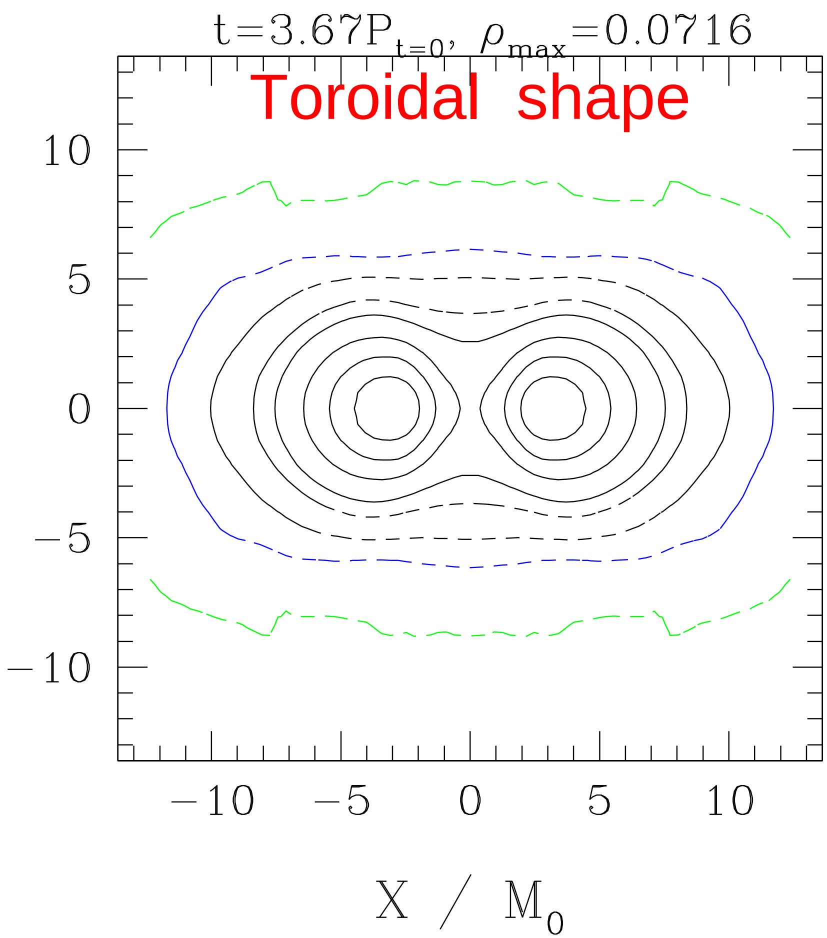
1.40 – 1.40 M_solar case : final snapshot

Massive toroidal neutron star is formed

(slightly elliptical)

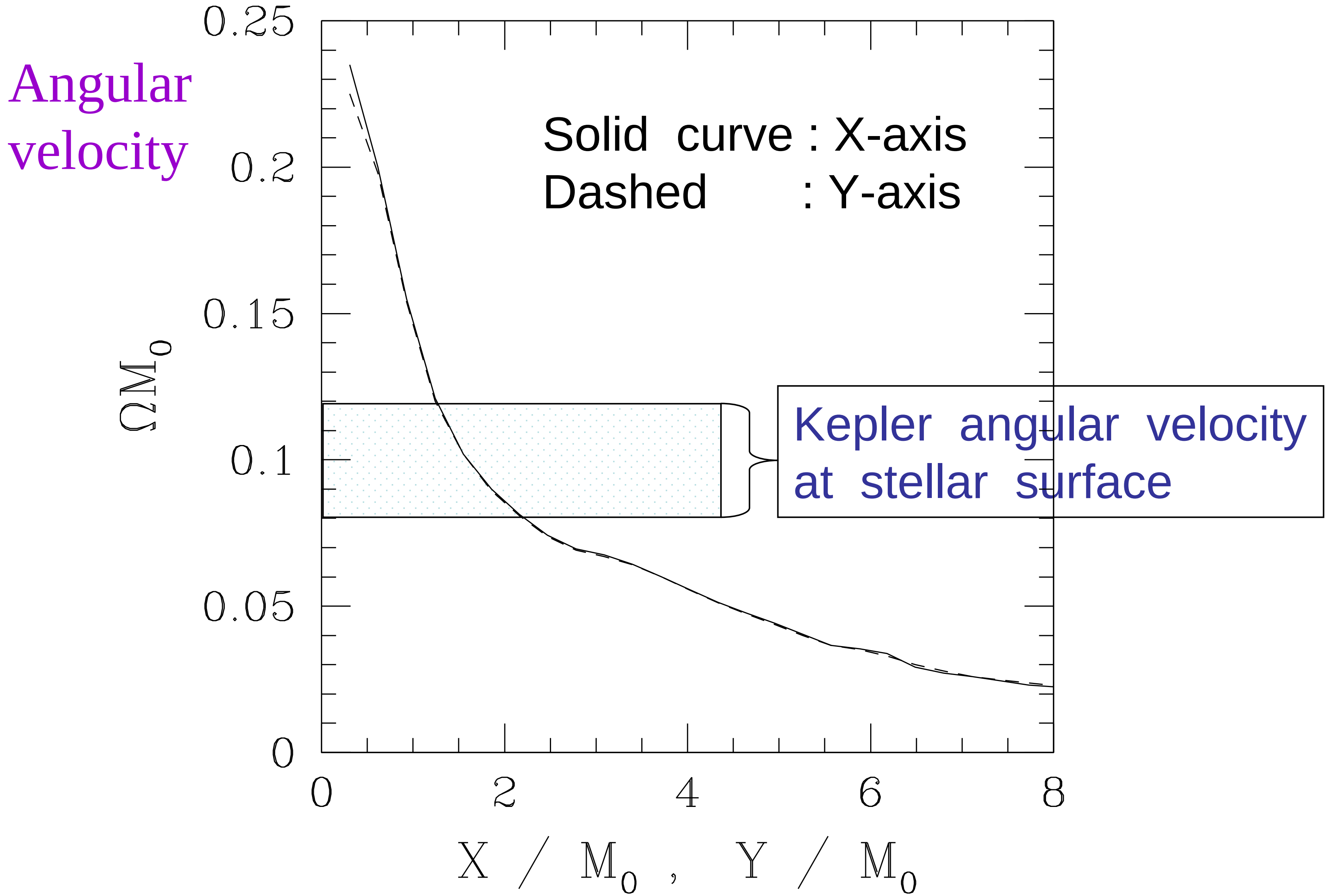


X – Y contour plot



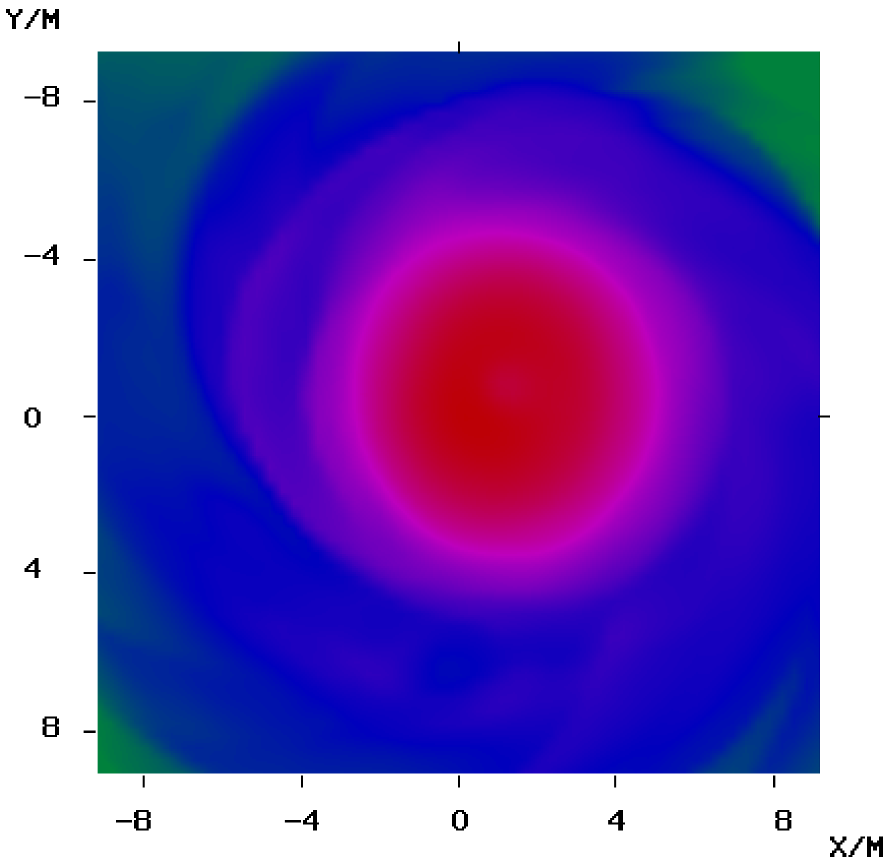
X – Z contour plot

Formed Massive toroidal NS is differentially and rapidly rotating



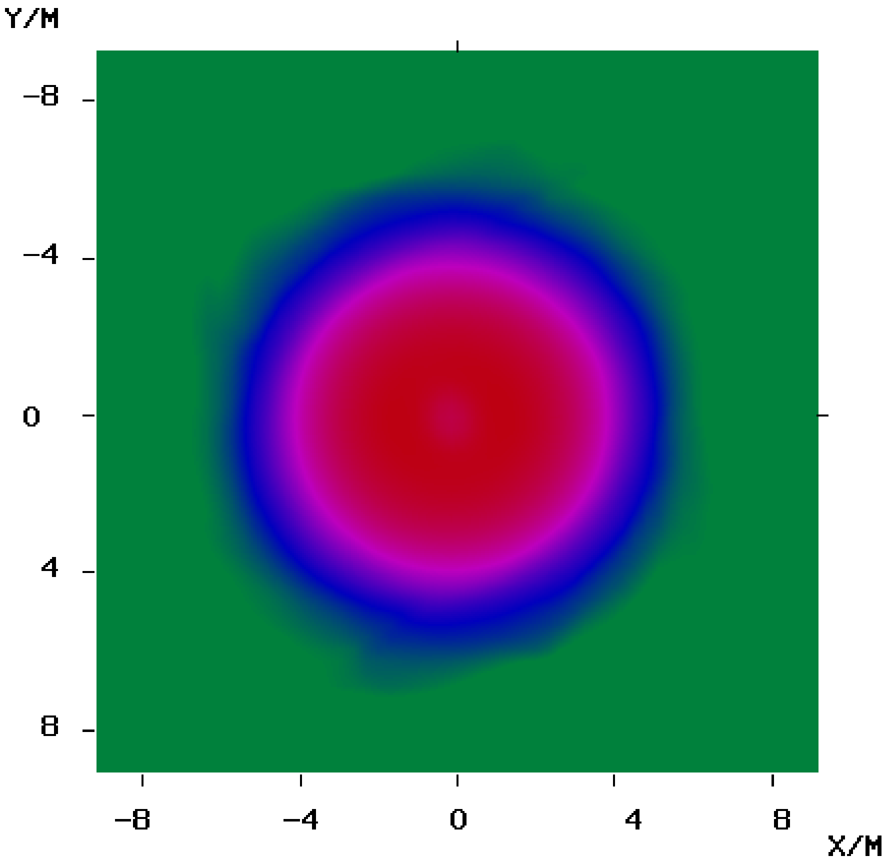
Comparison between equal and unequal mass mergers

1.33—1.46:
Massive NS + disk



Unequal-mass case
Mass ratio ~ 0.90

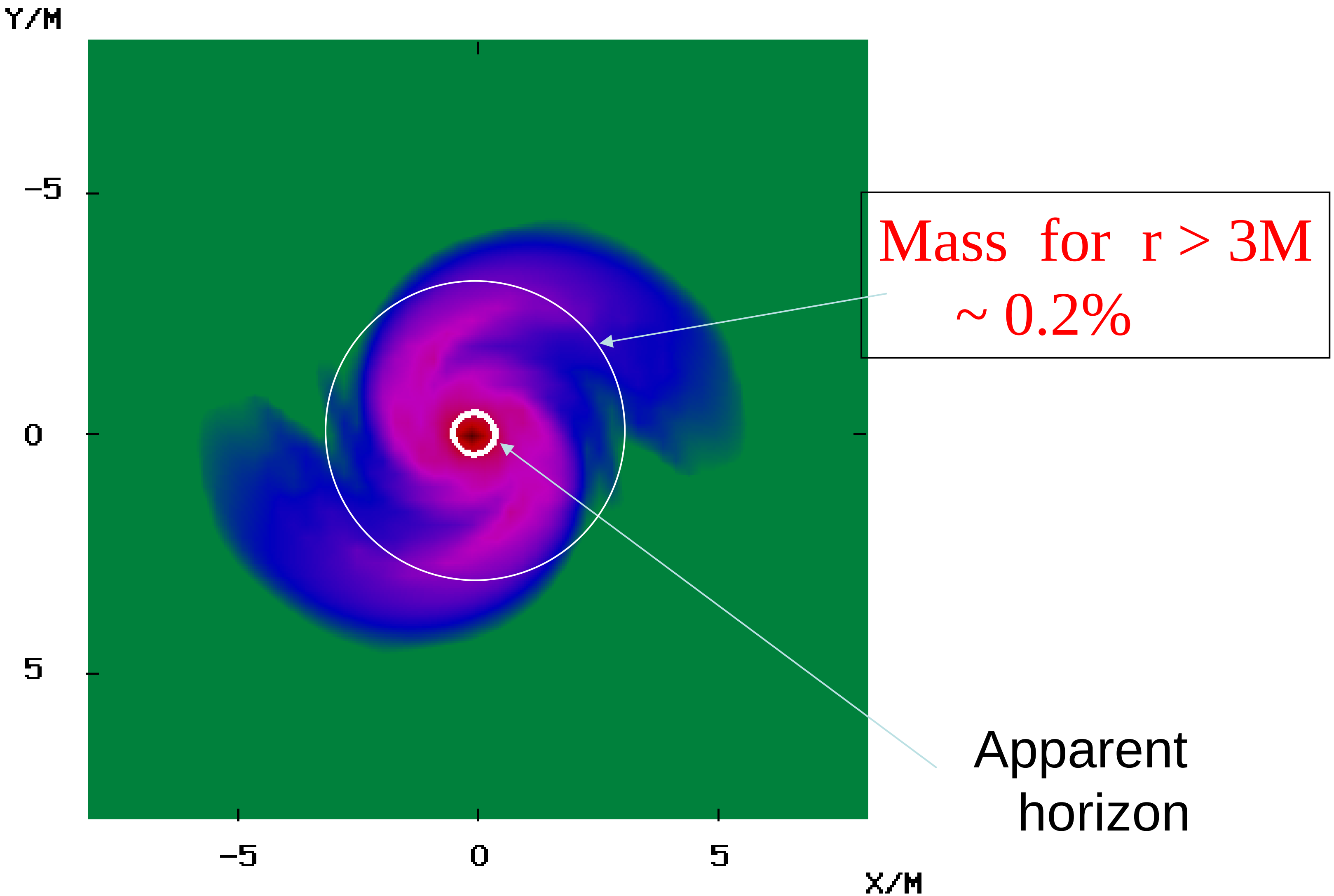
1.40—1.40:
Massive NS



Equal-mass case

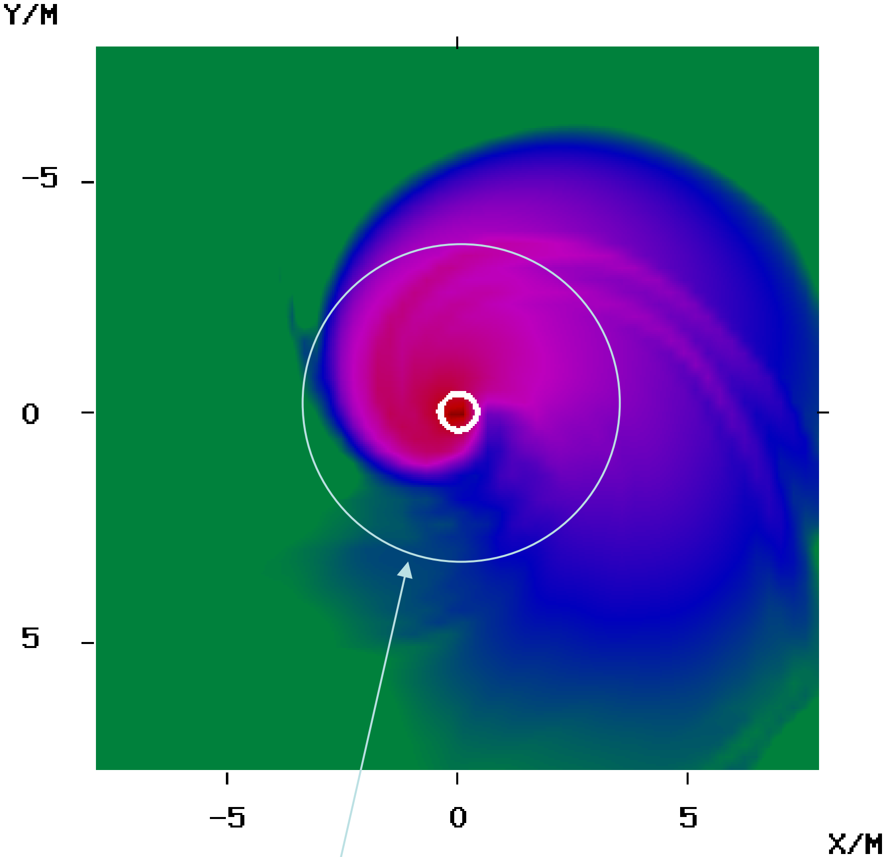
Black hole formation case: 1.52-1.52

Equal-mass case

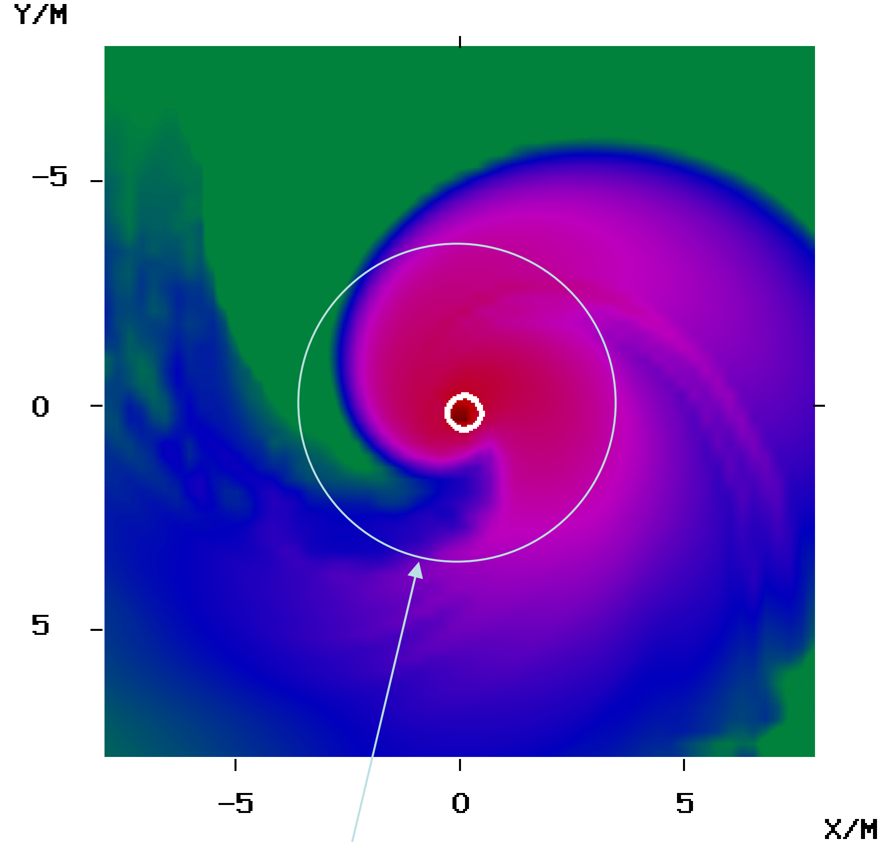


Disk mass for unequal-mass merger

1.45—1.55, Mass ratio 0.925



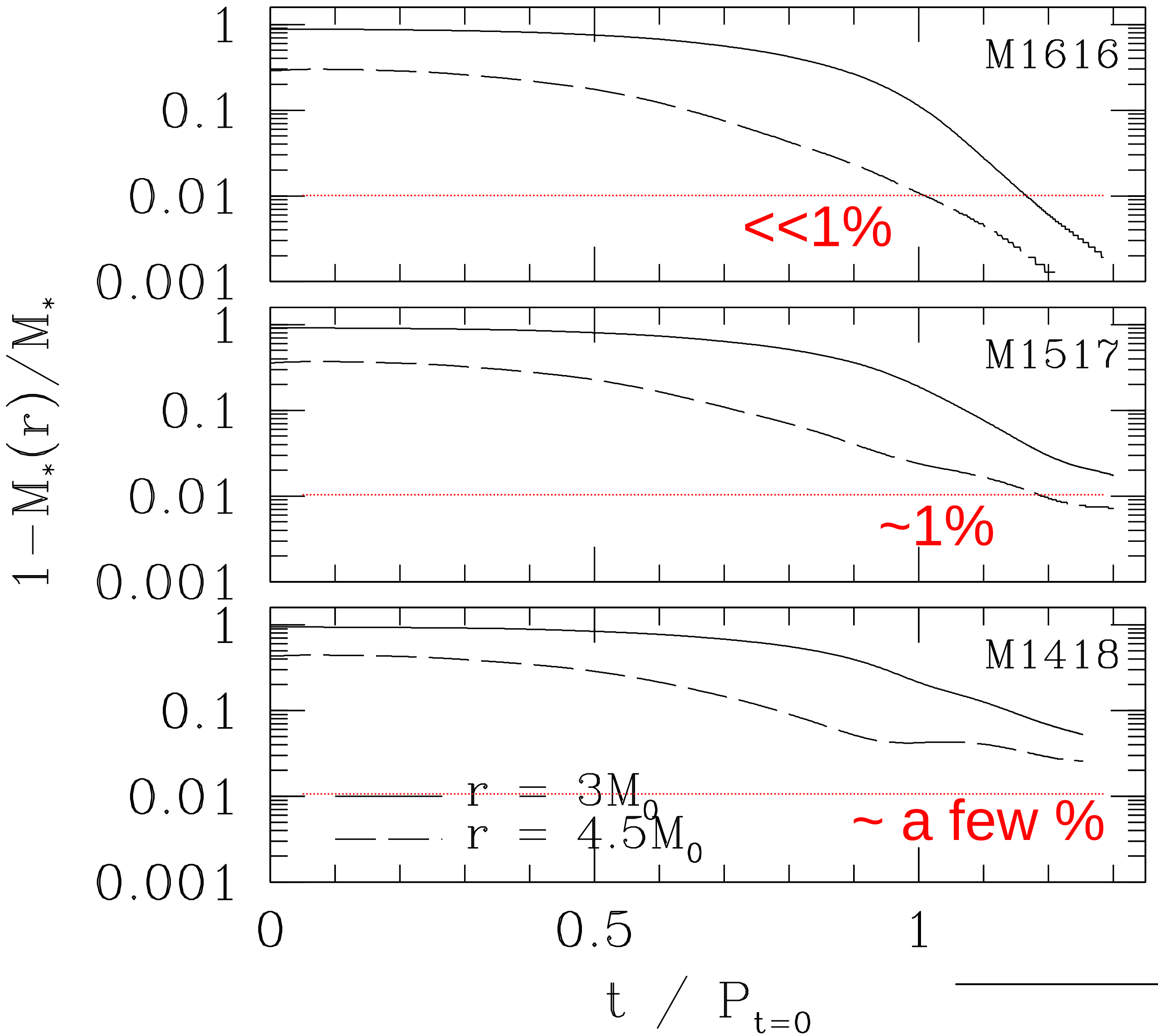
1.40—1.60, Mass ratio 0.855



Mass for r > 3M ~ 2%

Mass for r > 3M ~ 4%

Mass fraction outside a sphere for BH formation case



Equal mass
1.52-1.52 M_s

Mass-ratio
=0.925
1.45-1.55 M_s

Mass-ratio
=0.855
1.40-1.60 M_s

Time

Products of mergers for $\Gamma = 2$

Equal – mass cases

- Low mass cases

  Hypermassive neutron stars

  of nonaxisymmetric & quasiradial oscillations.

- High mass cases

  Direct formation of Black holes

  with very small disk mass

Unequal – mass cases (mass ratio ~ 0.9)

- Likely to form disks of mass

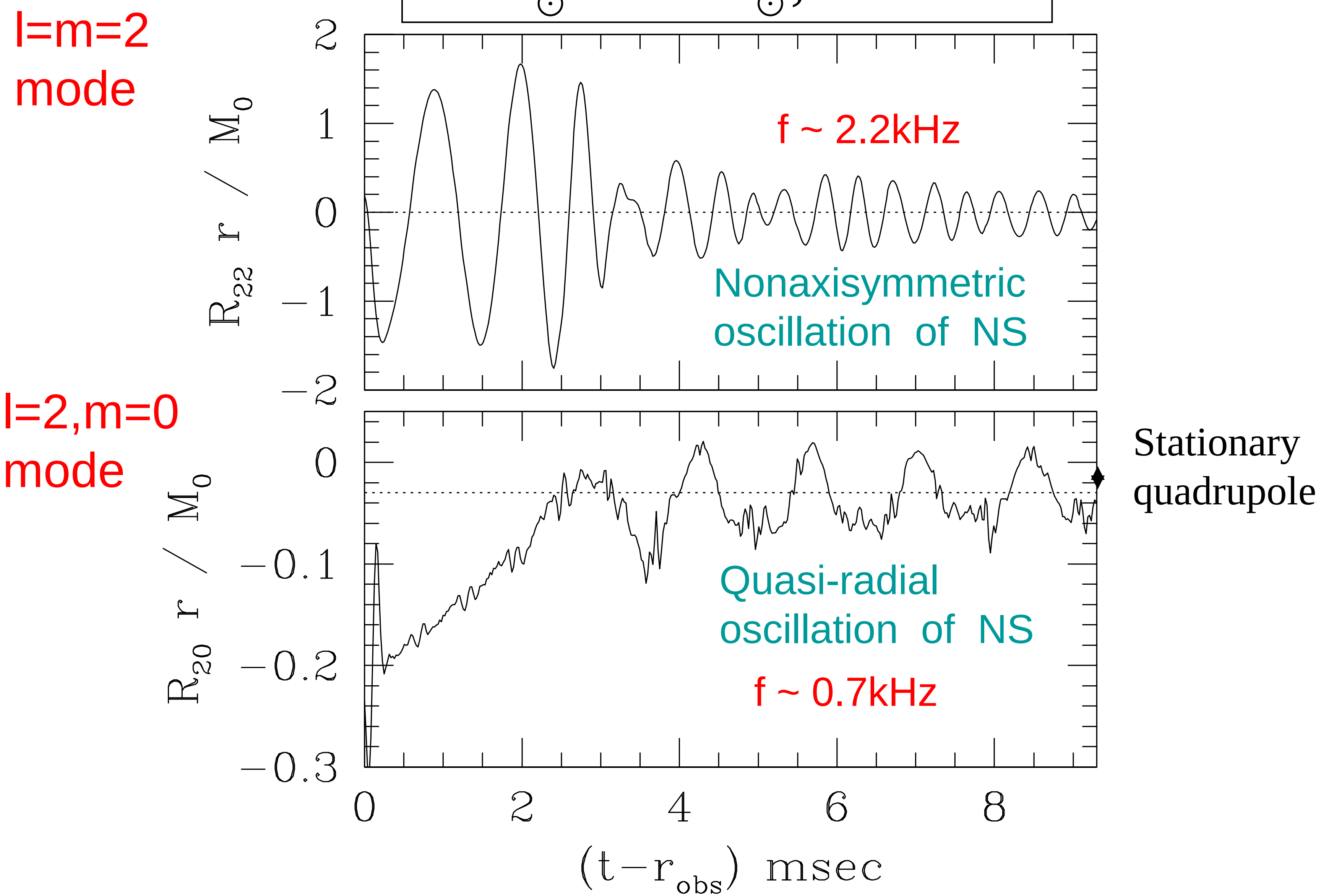
  ~ several percents of total mass

  → BH(NS) + Disk (~0.1M_solar)

  → Maybe a candidate for short GRB

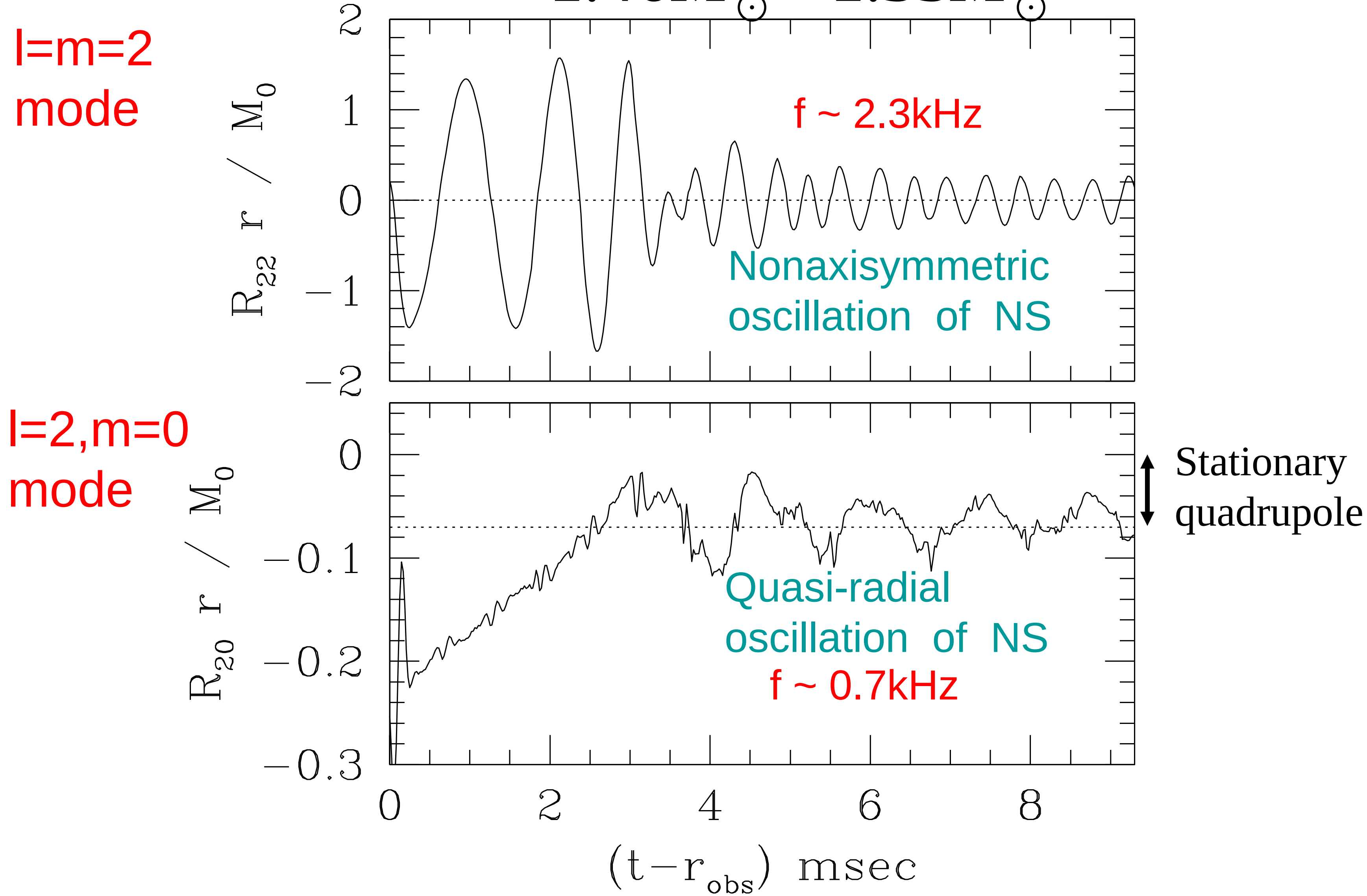
Gravitational waves for NS formation

$1.4M_\odot - 1.4M_\odot, R = 15\text{km}$



Gravitational waves from unequal-mass merger to NS formation

$$1.46M_{\odot} - 1.33M_{\odot}$$



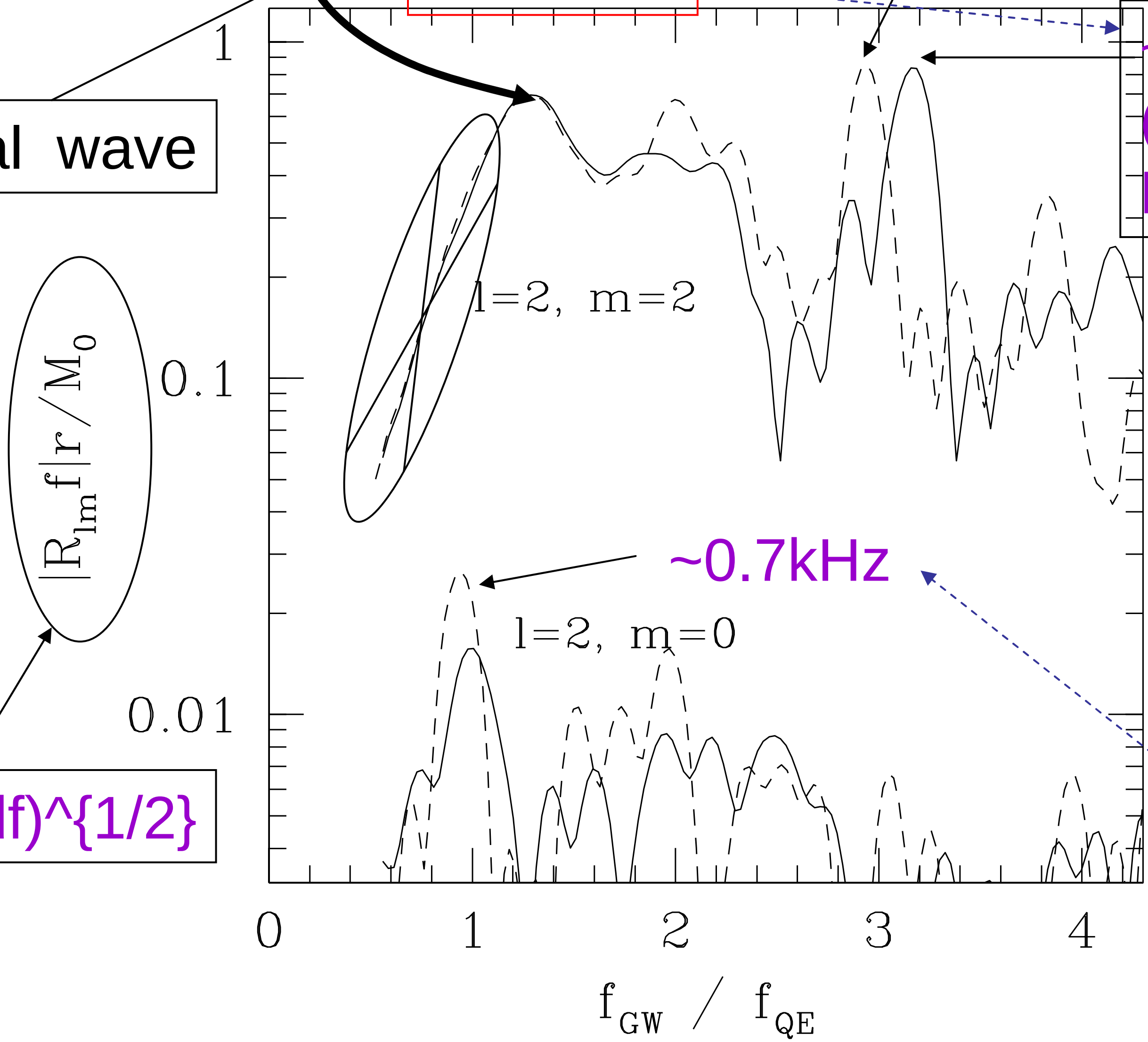
Fourier spectrum in NS formation

Non-axisym. oscillation

~2.2kHz (equal mass)

~2.3kHz (unequal mass Mass ratio=0.9)

Inspiral wave



~0.7kHz

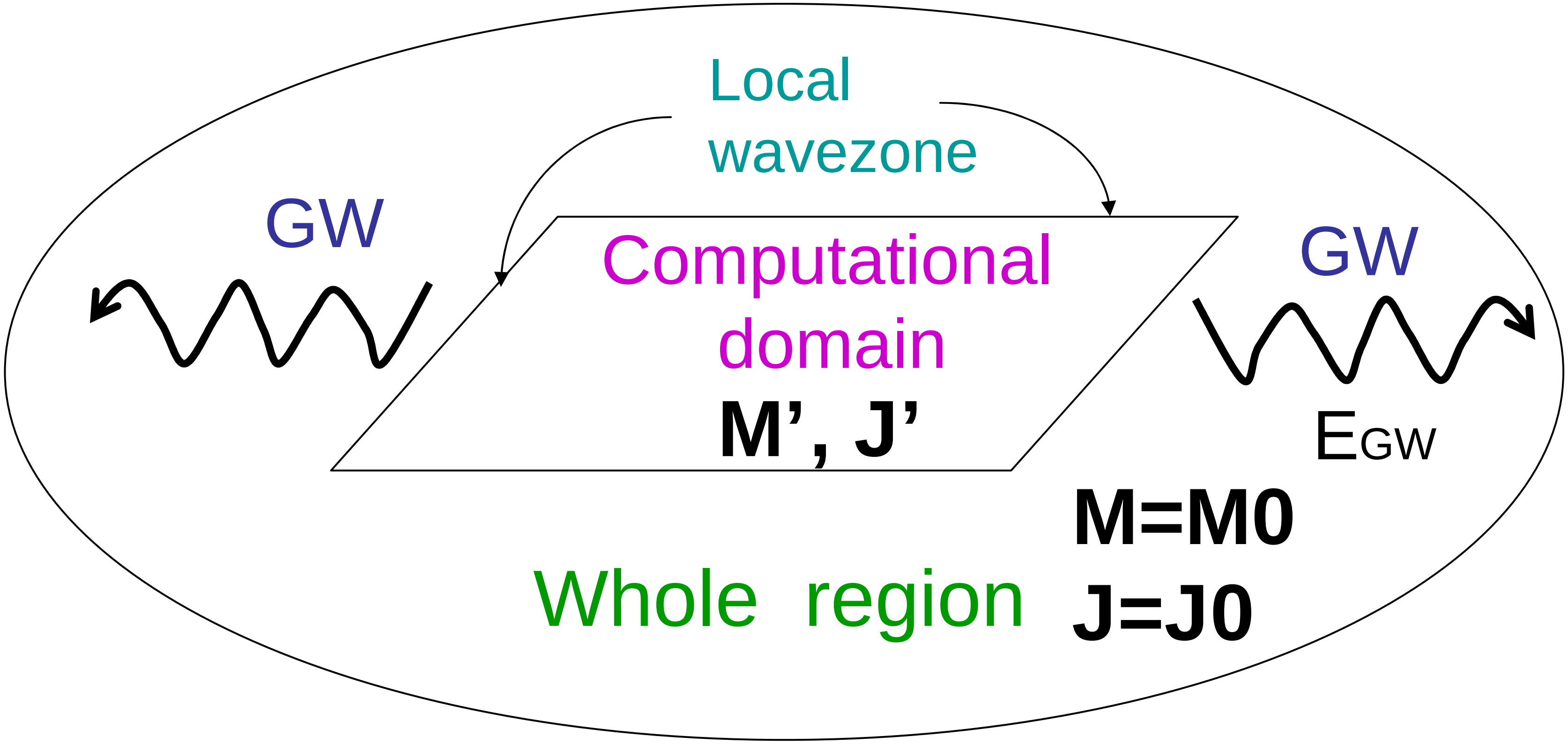
~(dE/df)^{1/2}

Frequency also depends on EOS.

Quasi-radial oscillation

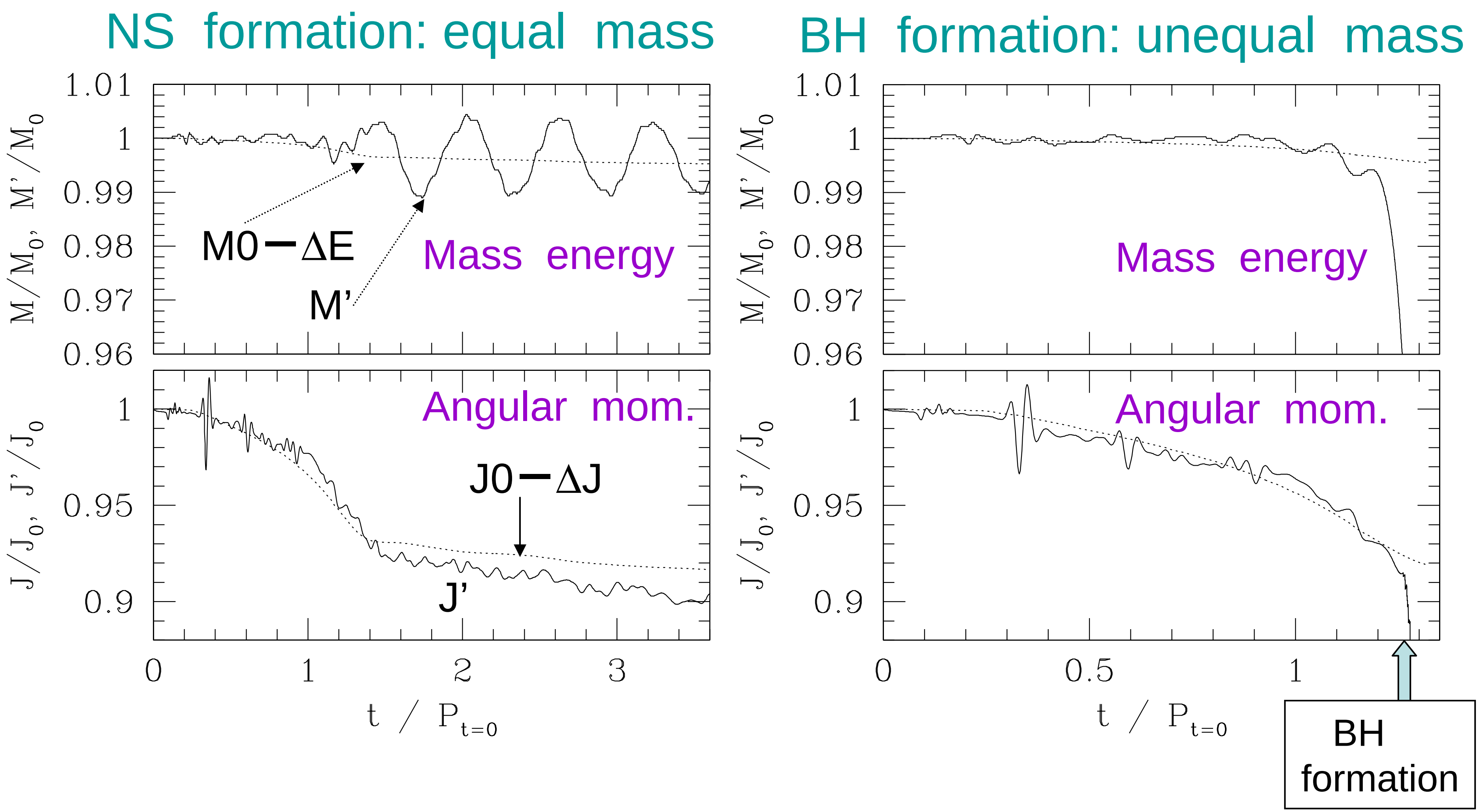
Computation of mass and angular momentum

-- Check of the conservation --



M0 − E_{GW} = M' & J0 − J_{GW} = J'
should be satisfied

Radiation reaction : OK within ~ 1%



Solid curves : computed from data sets in finite domain.

Dotted curves: computed from fluxes of gravitational waves

5 Summary

1 Rapid progress in particular in the past 5 yrs

2 Scientific (quantitative) runs are feasible now.

3 (Astrophysically) Accurate and longterm simulations are feasible for many phenomena in the absence of BHs : NS-NS merger, Stellar collapse, Bar-instabilities of NSs ....

4 (I think) numerical implementations for fundamental parts have been almost established (for the BH-absent spacetimes)

Issues for the near future

1 Several (technical) Issues still remain :

- Grid numbers are still not large enough in 3D
  → We would need hypercomputer (~10TBytes, ~10TFlops)
  Probably becomes available in a couple of yrs.
- Computation crashed due to grid stretching around BH horizon
  → We need to develop excision techniques.
- How to achieve a very high accuracy for making GW templates ?

2 Incorporate more realistic physics in hydro simualtion

More realistic EOS, Neutrino cooling, Magnetic fields

Where are we ?

- 1: Make a code which runs anyhow stably
  (do not care accuracy)
- 2: Improve the code which can provide a qualitatively correct result; care accuracy somewhat (say we admit an error of ~10%)
- 3: Improve the code gradually getting qualitatively new results which can be obtained only by an improved code

★ 4: Goal: Make a code which provides a quantitatively accurate result.

We are here.

Similar to construction of detectors in some sense

Animations

- http://esa.c.u-tokyo.ac.jp/~shibata/anim.html