Asymptotic Safety in Quantum Gravity

(Martin Reuter)

The 4 Fundamental Interactions : theoretical status

electromagnetic, weak, strong:
- class.: Yang-Mills theories
- quant.: perturbatively renormalizable QFTh.s

gravity:
- class.: General Relativity, with $S[g_{\mu\nu}] \sim \int d^4x \sqrt{-g}\, R$
- quant.: ???

Quantum field theory of spacetime metric $g_{\mu\nu}(x)$ based upon $\int \sqrt{-g}\, R$ is not renormalizable <u>in perturbation theory</u>:

increasing order in pert. th. $\Rightarrow$
" number of counter terms $\Rightarrow$
" " " undetermined parameters

$\rightsquigarrow$ no/very questionable predictivity at high energies ("effective" rather than "fundamental" theory)

Standard quantization of gravity $\widehat{=}$

degrees of freedom carried by:	$g_{\mu\nu}(x)$
bare action:	$\int d^4x \sqrt{-g}\, R$
calculational method:	perturbation theory in G; infinite cutoff limit at the trivial "Gaussian" fixed point

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity ?

String Theory: d.o.f., action, calc. meth.

Loop Quantum Gravity: d.o.f., calc. meth.

Asymptotic Safety: calc. meth., action

Asymptotic Safety Approach:

- degrees of freedom carried by $g_{\mu\nu}$
- quantization/renormalization is non-perturbative in an essential way
- bare action Γ_* is not an ad hoc assumption, but a <u>prediction</u>:

  $\Gamma_* \sim \int d^4x \sqrt{-g}\, R$ + "more" is a non-Gaussian fixed point of the (∞-dimensional, non-pert.) Wilsonian renormalization group flow

- fixed point "controls" UV divergences

... provided it exists

Weinberg's "asymptotic safety" conjecture (1979):

Perhaps Quantum Einstein Gravity can be defined nonperturbatively at a non-Gaussian fixed point.

$d = 2 + \varepsilon$: FP known to exist

$d = 4$: progress hampered by lack of appropriate calculational scheme

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$\rightsquigarrow$ Use "effective average action" which seems ideally suited. Wetterich 1993

Effective average action for gravity: M.R. 1996

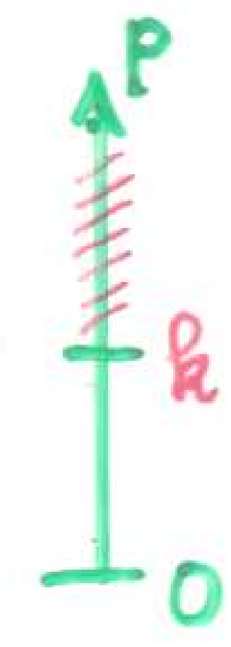
$$\Gamma_k [g_{\mu\nu}, \cdots]$$

C. Wetterich
1993

The Effective Average Action Γ_k

- Wilson-type (coarse grained) free energy functional

- IR cutoff at k : Γ_k contains the effect of all quantum fluctuations with momenta $p > k$, not (yet) of those with $p < k$.



- modes with $p < k$ suppressed in the path integral by $(\text{mass})^2 = R_k(p^2)$

- $\Gamma_{k \to \infty} = S$, classical (bare) action
  $\Gamma_{k \to 0} = \Gamma$, standard effective action

- Γ_k satisfies exact RG equation; symbolically:

$$\text{“} \; k \, \partial_k \Gamma_k = \frac{1}{2} \operatorname{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} k \, \partial_k R_k \right] \; \text{”}$$

- Powerful nonperturbative approximation scheme:
  "truncate" the space of action functionals,
  project RG flow onto finite dimensional subspace

M.R. 1996

Construction of Γ_k for Gravity

- starting point : $\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$

- decompose $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

  $\bar{g}_{\mu\nu}$: fixed backgrd. metric

- add background gauge fixing $S_{gf}[h;\bar{g}]$ + ghost terms

- expand $h_{\mu\nu}$ in $\bar{D}^2$-eigenmodes, and introduce IR cutoff k^2 : only modes with generalized momenta ($\bar{D}^2$-eigenvalues) are integrated out.

- add sources: generating fctl. $W_k[\text{sources}; \bar{g}]$

$$W_k[\text{sources};\bar{g}] \xrightarrow{\text{Legendre transf.}} \Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$$

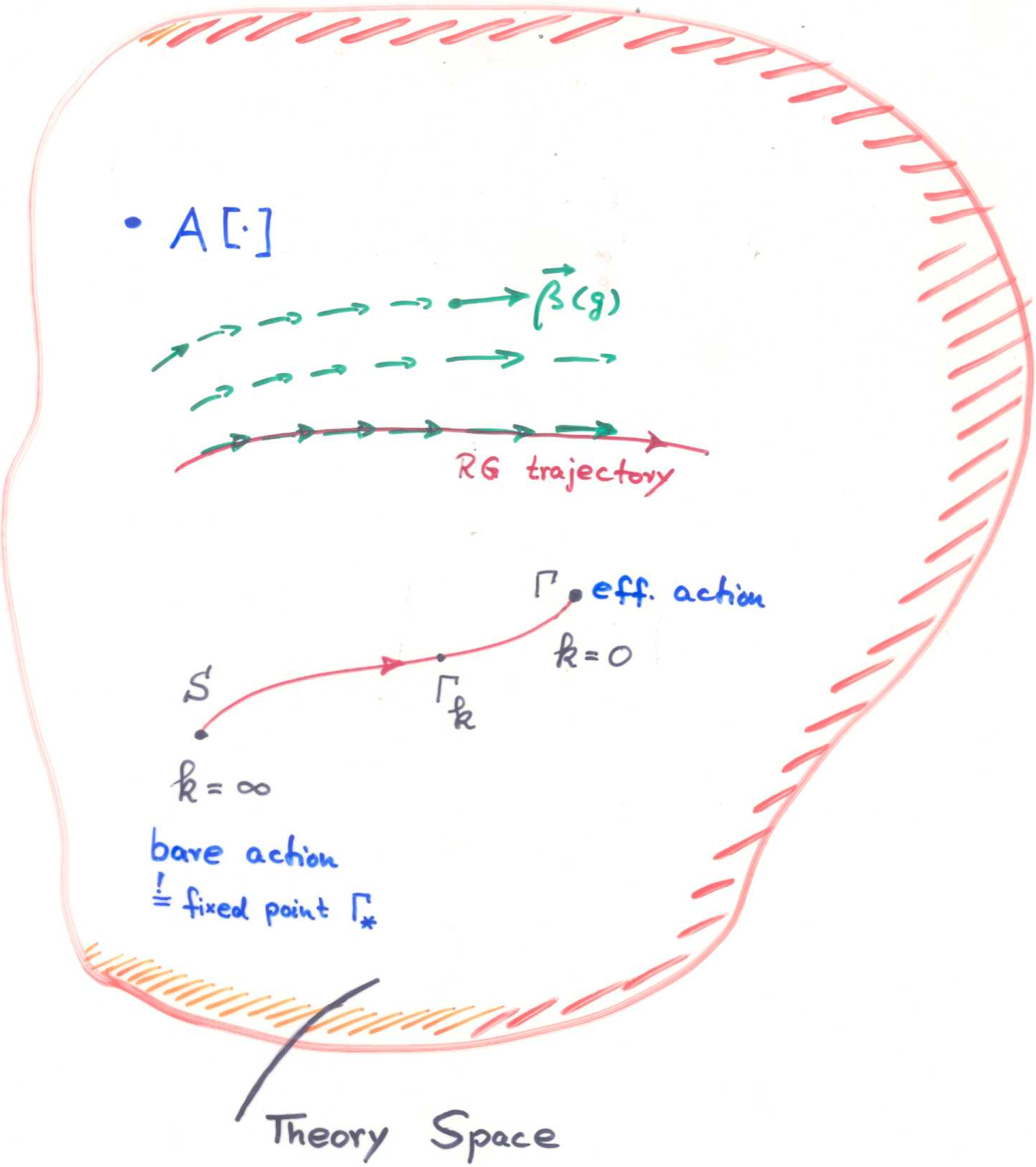
$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$

- derive exact RG equation from path integral:

$$k\frac{\partial}{\partial k}\Gamma_k[g,\bar{g},\cdots] = \mathrm{Tr}(\cdots) \qquad \begin{aligned}\Gamma_\infty &= S\\ \Gamma_0 &= \Gamma\end{aligned}$$

- "Ordinary" diffeomorphism invariant action:

$$\Gamma_k[g] = \Gamma_k[g, \bar{g}=g, \text{ghosts}=0]$$



• A[·]
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
k = 0
S
Γ_k
k = ∞
bare action
$\stackrel{!}{=}$ fixed point Γ_*
Theory Space

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$\Lambda_k \equiv \bar{\lambda}_k$

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k , dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k , dimensionless: $\lambda(k) = \Lambda_k / k^2$

insert ansatz into flow equation , expand

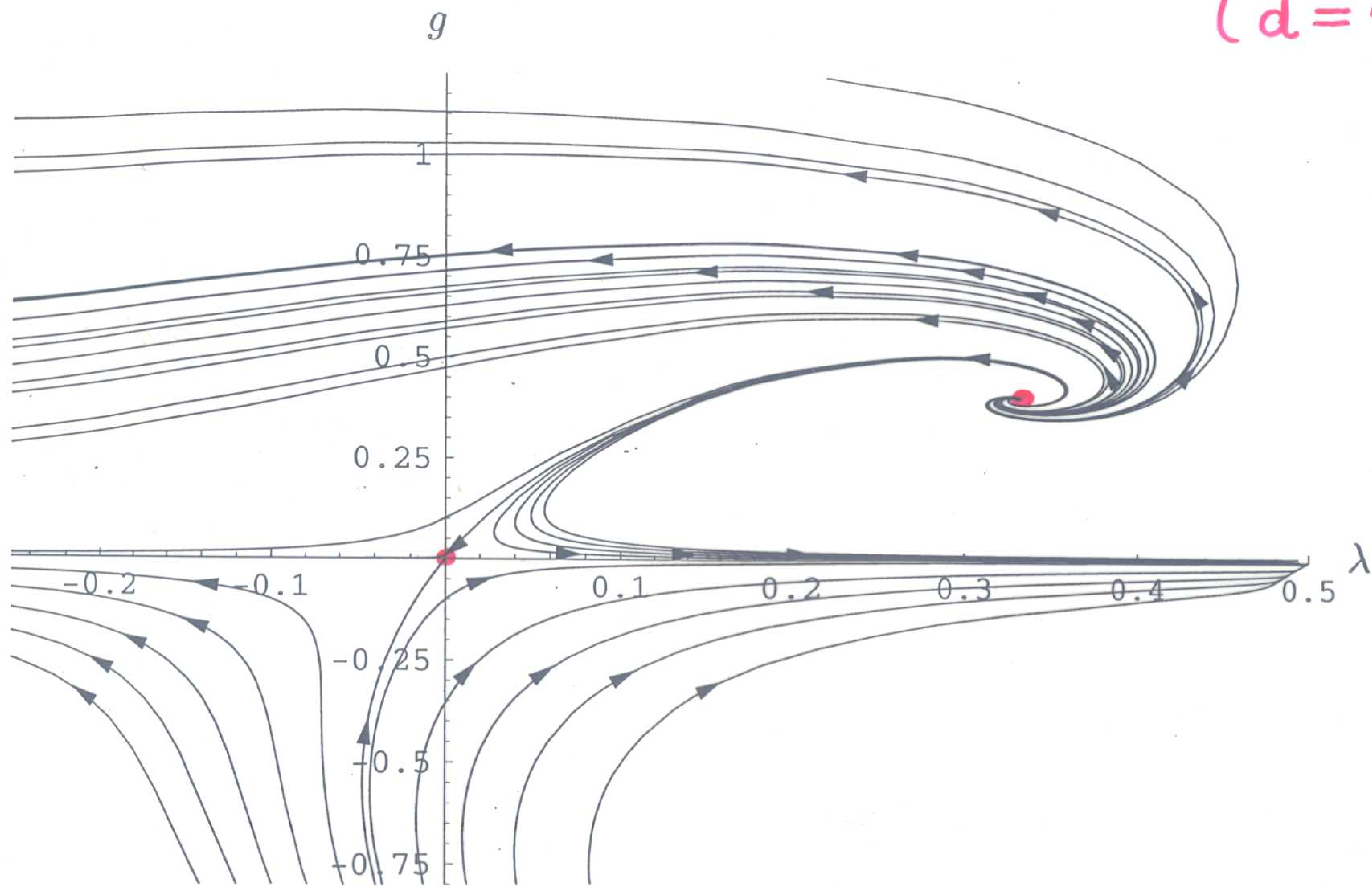
$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots$$

$\rightsquigarrow$

$$\boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

RG - Flow in the Einstein - Hilbert Truncation

(d = 4)



M.R., F. Saueressig, hep-th/0110054

O. Lauscher, M.R.
F. Saueressig, M.R.
R. Percacci, ...
D. Litim, ...

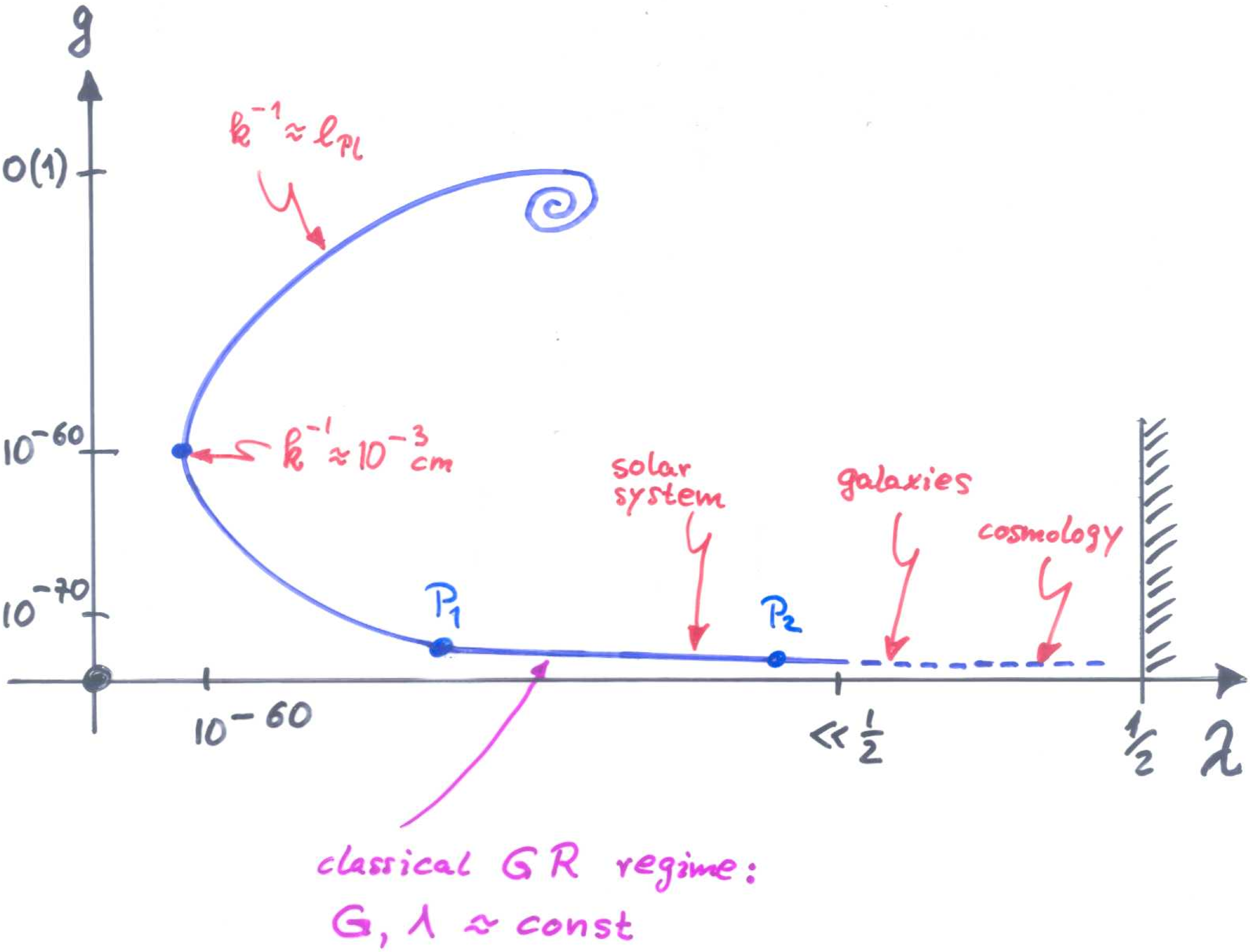
Reliability checks (universality, ...),
more general truncations $\Longrightarrow$

The NGFP seems to exist in the full un-truncated theory beyond any reasonable doubt.

Cosmological Applications / Implications

A. Bonanno, M.R. (2002)
M.R., H. Weyer (2005)
M.R., F. Saueressig (2005)
A. Bonanno, M.R. (2007)

The RG trajectory "realized in Nature"

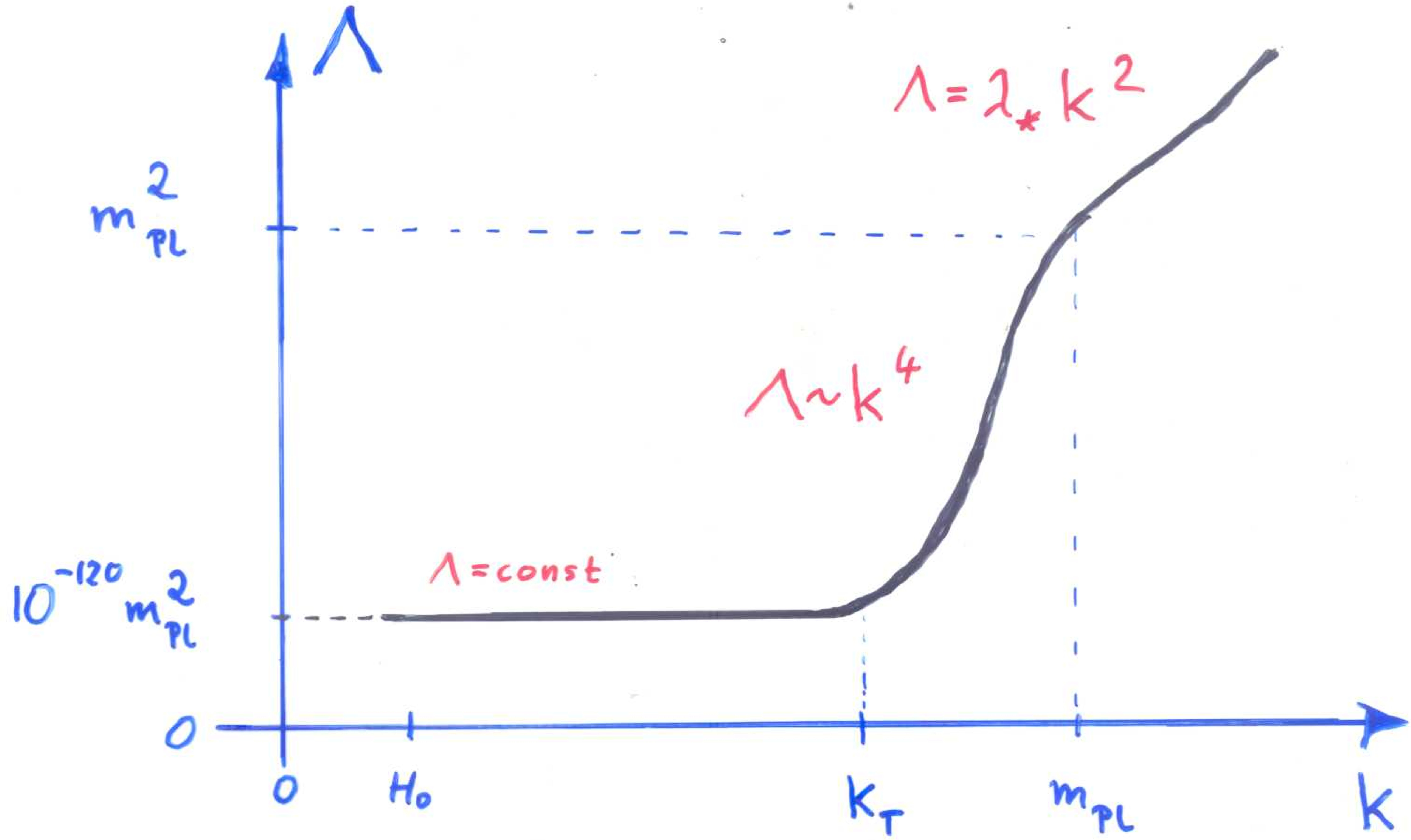


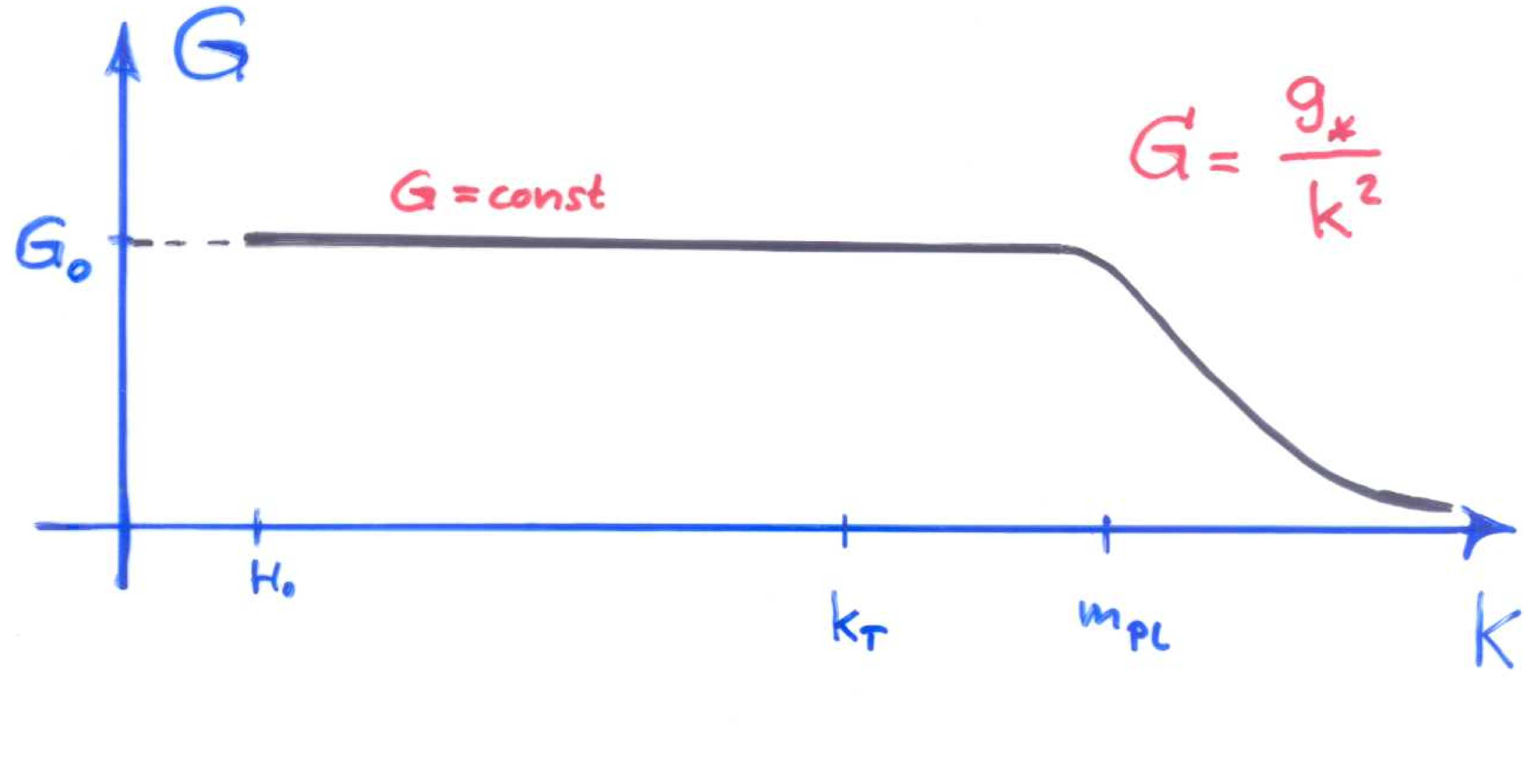
"Today" in cosmology:

$$\lambda_{cosmo} \equiv \frac{\Lambda_{cosmo}}{k^2_{cosmo}} = O(1) \; !!!$$

$\Lambda_{cosmo} \approx H_0^2$

$k^2_{cosmo} \approx H_0^2$





Definition of Planck scale: $m_{Pl} \equiv \ell_{Pl}^{-1} \equiv G_0^{-\frac{1}{2}}$

Are there observable / observed physical phenomena related to the RG-running implied by QEG ?

Candidates in cosmology:

a) Entropy carried by cosmological matter (CMBR photons, ...)

$$\left[S_{\mathrm{CMBR}} / \text{Hubble volume} \right]_{\text{today}} \approx 10^{88} \gg 1$$

most plausible initial value $= O(1)$!

b) Automatic inflation in the NGFP regime

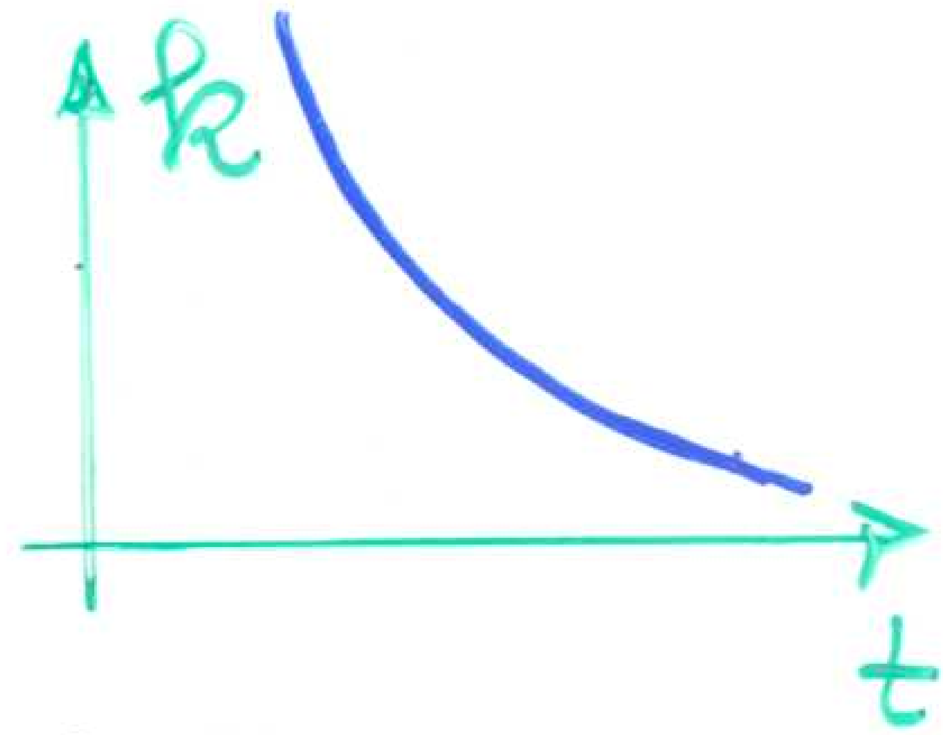
no inflaton needed,
no reheating necessary;
$\Lambda(k)$ large: cosmolog. const. drives inflation
$\Lambda(k)$ small: inflation stops automatically

c) Generation of primordial density perturbations

spectrum scale free:

big bang = "critical phenomenon" governed by the NGFP

For monotonic cosmological cutoff identification $k = k(t)$ and below the NGFP regime:



$$\boxed{\Lambda(t) \equiv \Lambda(k = k(t)) \text{ is a positive and decreasing function of time.}}$$

$\Longrightarrow$

$$\boxed{\text{Energy transfer into the matter system ("heating up").}}$$

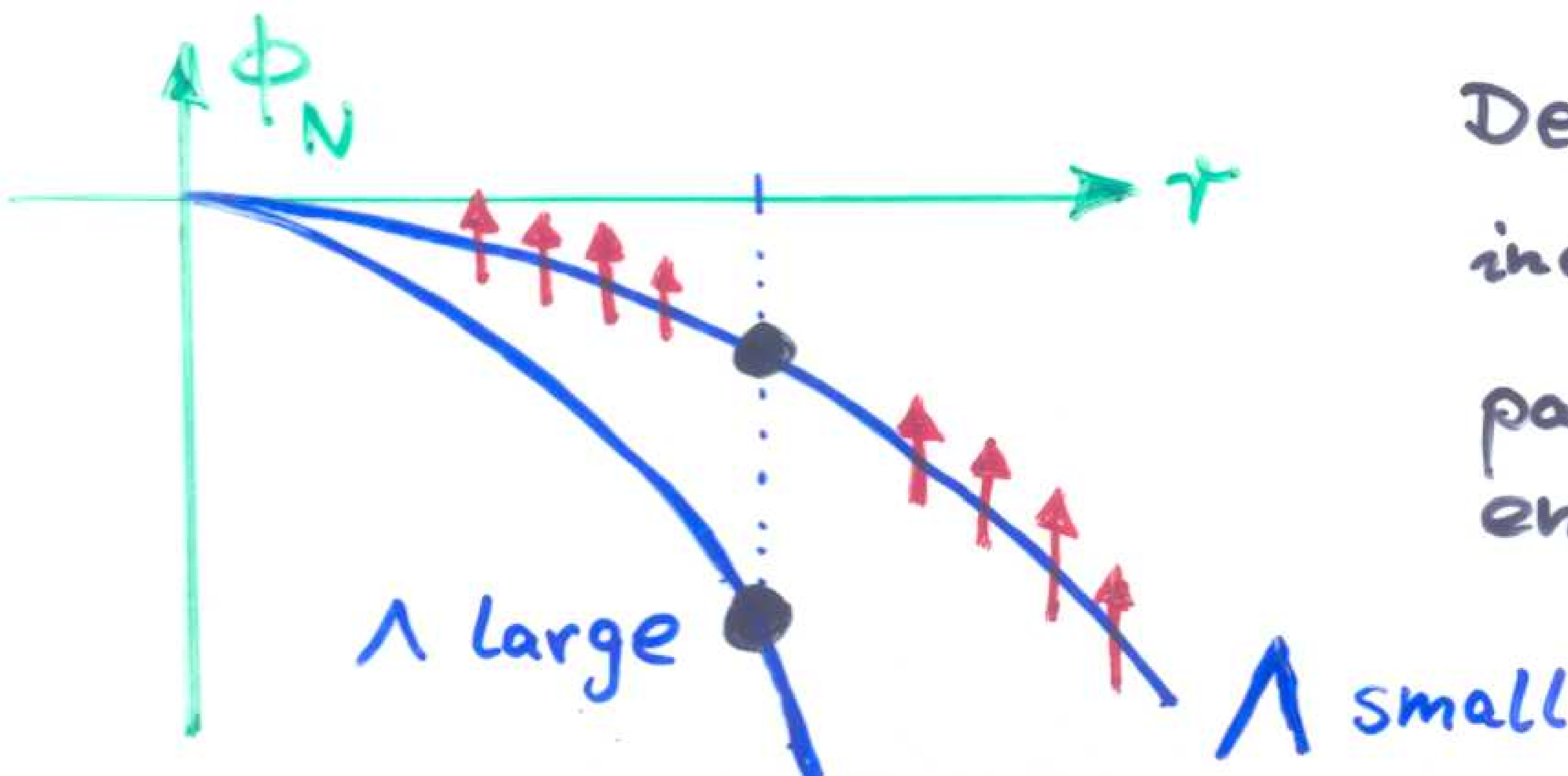
Example: de Sitter plus test particle

$$ds^2 = -\left[1 + 2\phi_N(r)\right] dt^2 + \left[1 + 2\phi_N(r)\right]^{-1} dr^2 + r^2 d\Omega^2$$

$$\phi_N(r) = -\frac{1}{6} \Lambda \, r^2$$

Newtonian interpretation:

$\Lambda > 0$:



Decreasing Λ increases the test particle's potential energy!

RG-Improved Cosmology

Spatially flat RW geometry:

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)\right]$$

$$T_\mu{}^\nu = \mathrm{diag}\left[-\rho(t), p(t), p(t), p(t)\right]$$

The scale factor is determined by

$$\boxed{R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\Lambda(t)\, g_{\mu\nu} + 8\pi G(t)\, T_{\mu\nu}}$$

where $\Lambda(t) \equiv \Lambda(k = k(t))$, $G(t) \equiv G(k = k(t))$

"Improved" Einstein's eq. $\Longleftrightarrow$

modified Friedmann equation:

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G(t)\, \rho + \frac{1}{3}\Lambda(t)$$

modified conservation law $D^\mu \left[-\Lambda(t) g_{\mu\nu} + 8\pi G(t) T_{\mu\nu}\right] = 0$:

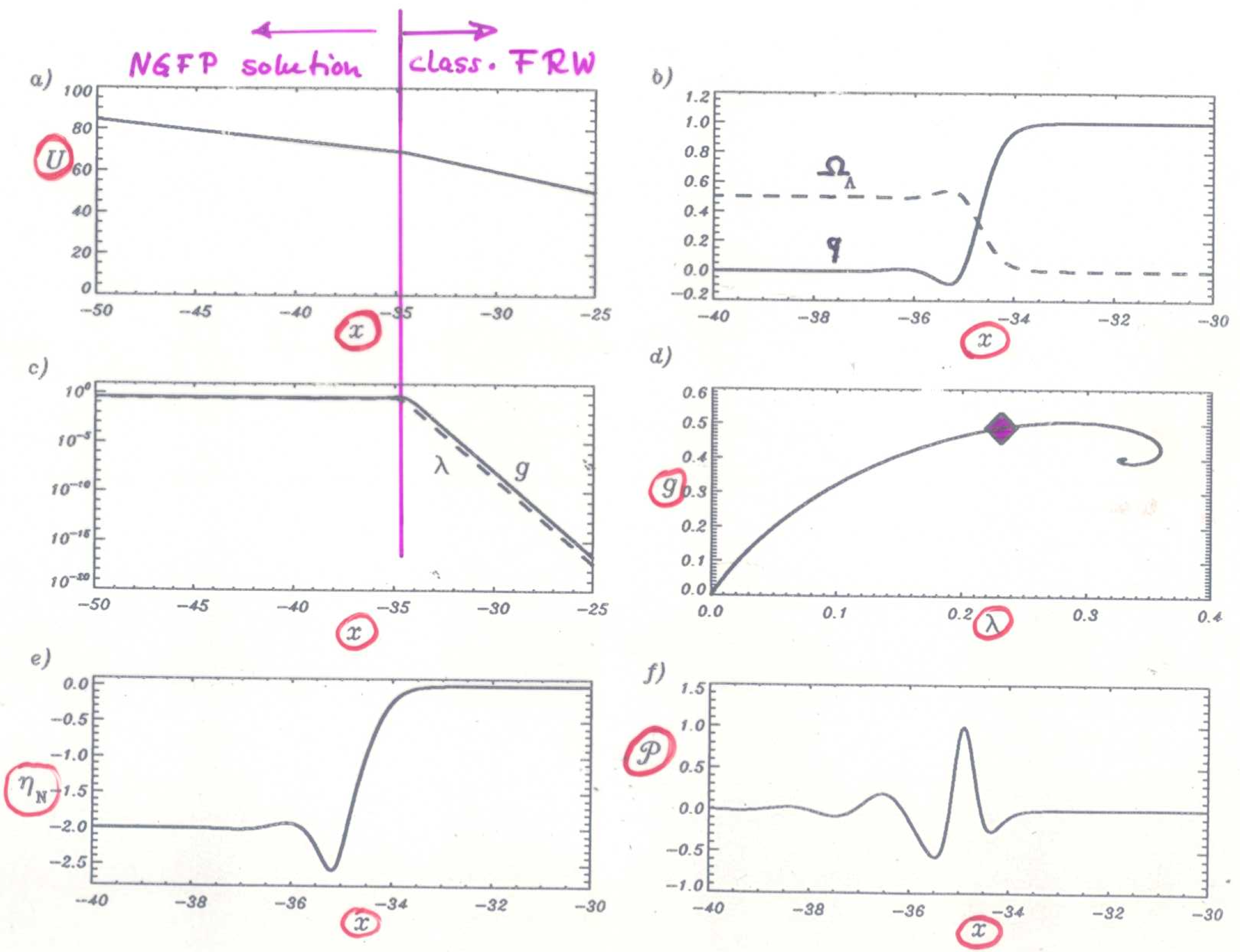
$$\underbrace{\dot\rho + 3H(\rho + p)}_{\sim D^\mu T_{\mu 0}} = -\frac{\dot\Lambda + 8\pi\rho\, \dot G}{8\pi G}$$

Describes energy exchange between matter and fields Λ, G ("vacuum").

Complete Cosmology: $\boxed{\Omega_\Lambda^* = \frac{1}{2}}$ $(\alpha = 1)$

$U \equiv \ln \frac{H}{H_T}$

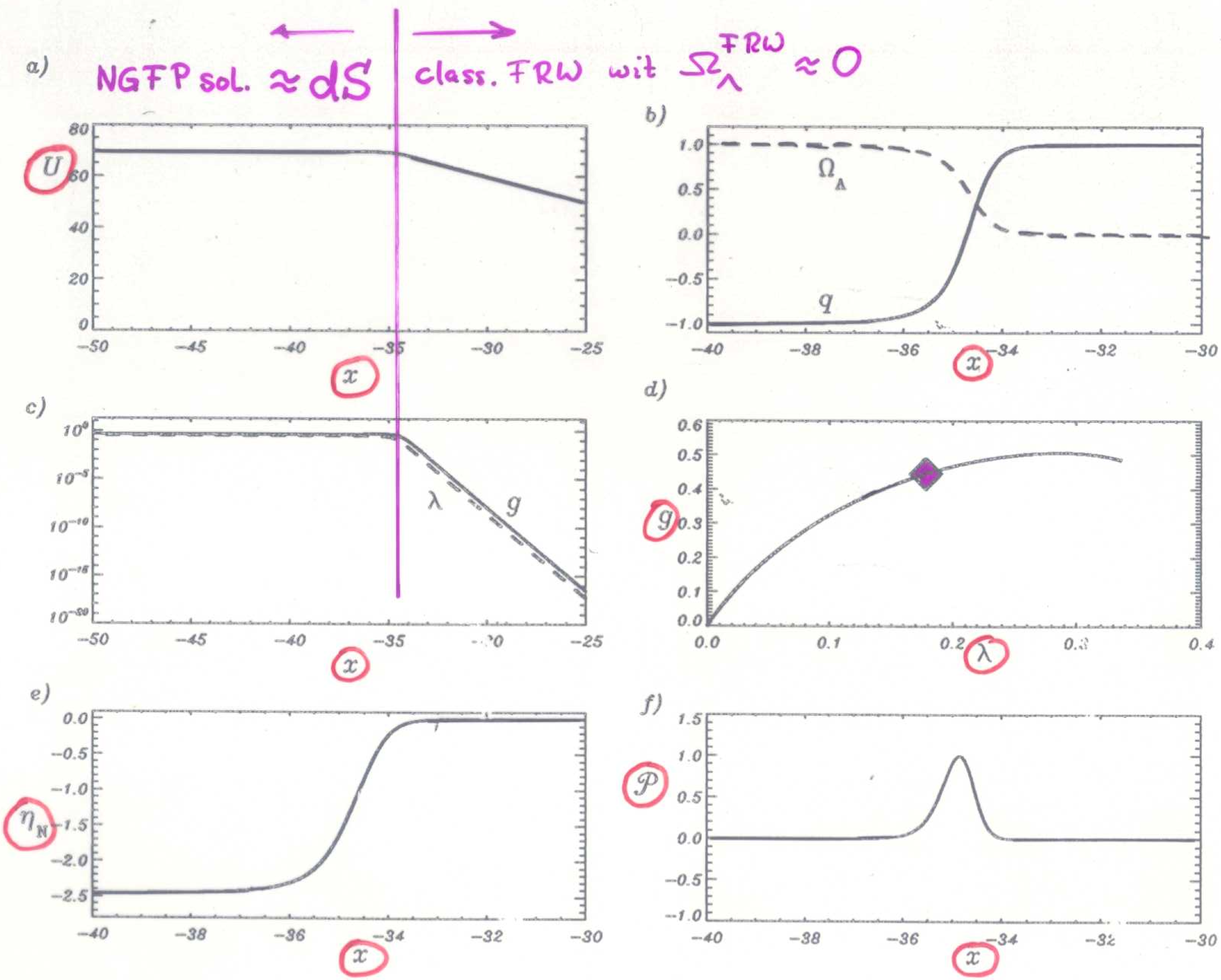
$x = \ln \frac{a}{a_T}$



Complete Cosmology : $\boxed{\Omega_\Lambda^* = 0.98}$ $(\alpha = 25)$

$U \equiv \ln \frac{H}{H_T}$

$x \equiv \ln \frac{a}{a_T}$



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Conclusion

Cosmology is a natural laboratory for confronting the predictions of asymptotically safe QEG with observations.

Candidates for observable / observed phenomena possibly explained by QEG :

- entropy of matter
- primordial density perturbations
  - NGFP regime is a "critical phenomenon"
  - Λ-driven inflation
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