What do black holes tell us about the early Universe ?

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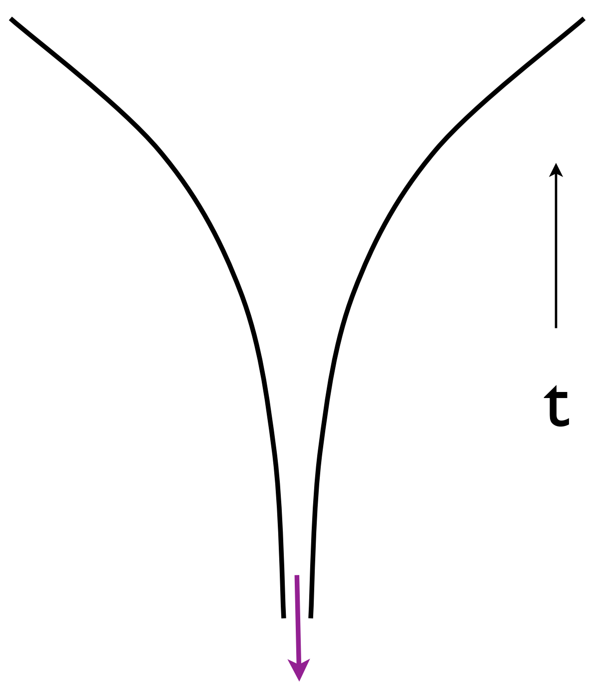
Consider an expanding Universe



High energy density,
Quantum gravity effects ...

Where else do we encounter high energy densities ?

When matter collapses to form a black hole ....

String theory has had remarkable success in understanding the quantum structure of the black hole interior...

What do the results suggest about the early Universe?

Plan of the talk :

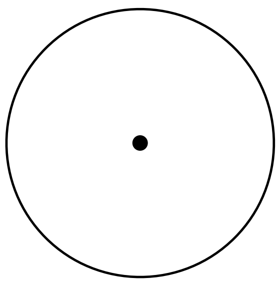
(A) Review how string theory resolves the paradoxes arising with black holes, and extract the relevant properties of quantum gravity at high density

(B) Extend the ideas here to get an expression for the entropy S and pressures P as functions of the energy E, and then find the evolution of the Universe with this equation of state

(A) Black holes in string theory

Puzzles with black holes :

The entropy problem : Black holes behave as if they have an entropy given by their surface area



$$S_{bek} = \frac{A}{4G}$$

(Bekenstein, 72)

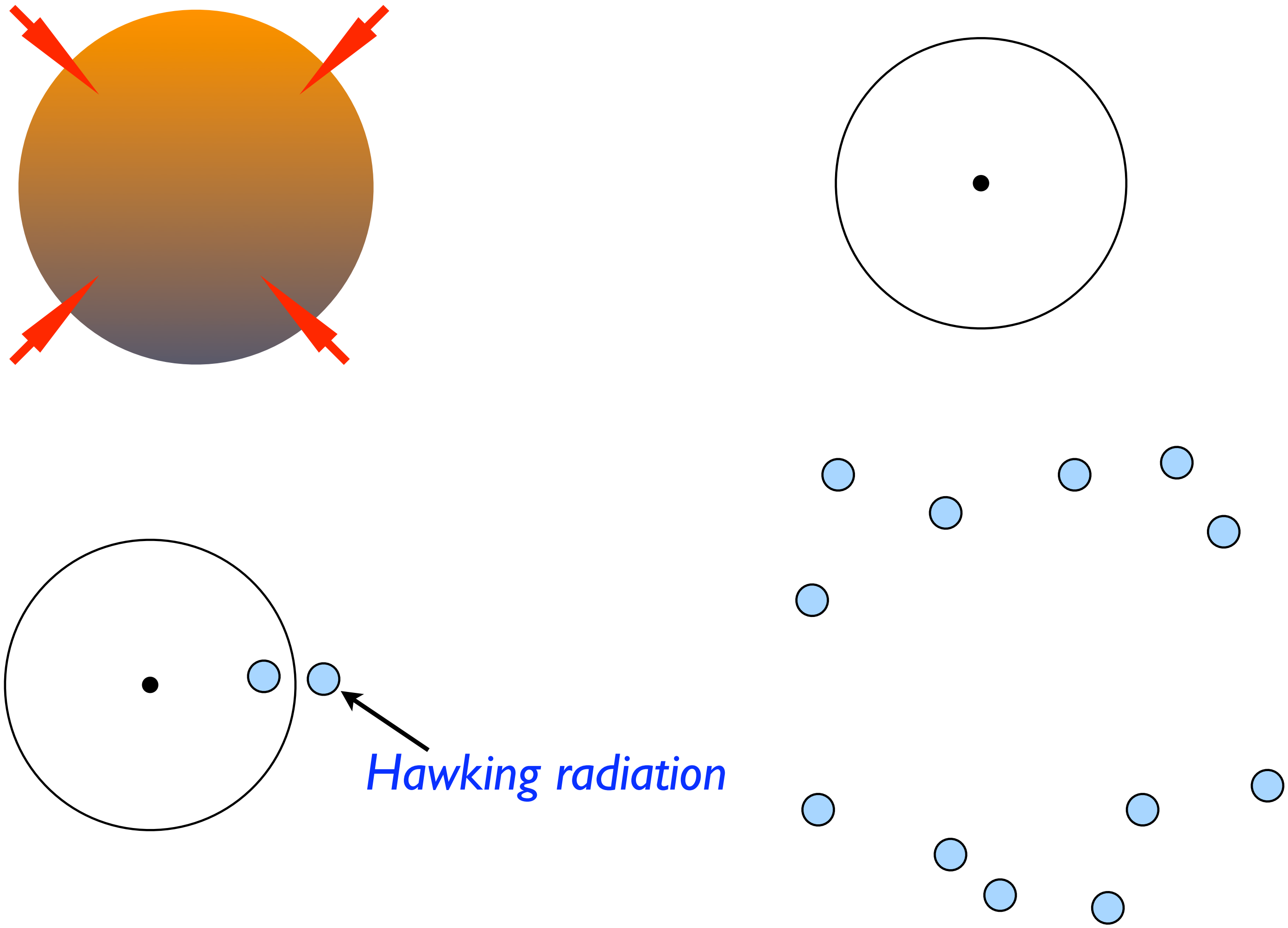
But statistical mechanics then says that there should be

$e^{S_{bek}}$ states of the hole for the same mass and charge

Can we show that there are $e^{S_{bek}}$ states of the hole ?

(Classical relativity finds that black holes have no hair, so there is only *one* state)

The information problem



How can the Hawking radiation carry the information of the initial matter ?

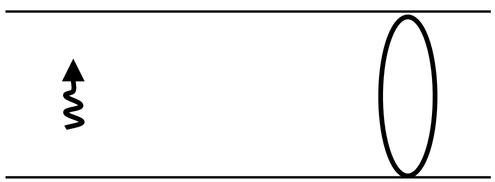
(Hawking '74)

A simple example: 2-charge holes

(Susskind, Sen, Vafa '94-'95)

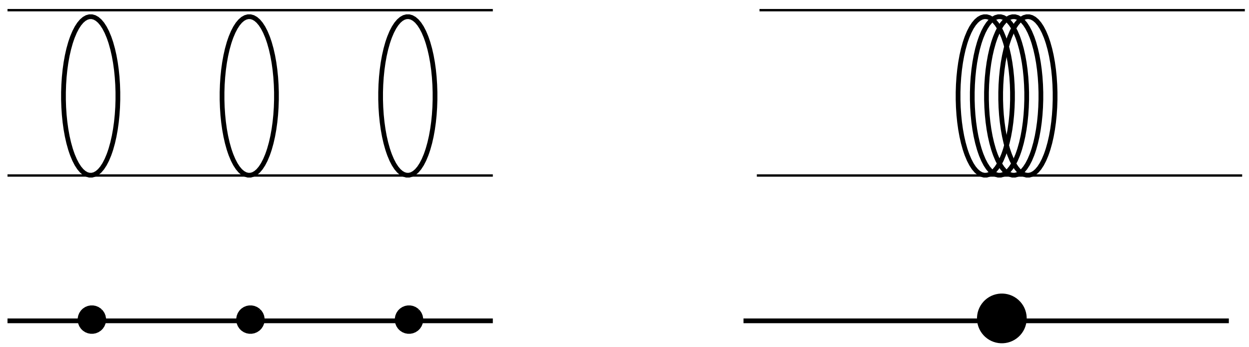
In string theory, we must make black holes from the objects present in string theory.

Let us compactify spacetime as $M_{9,1} \to M_{4,1} \times T^4 \times S^1$



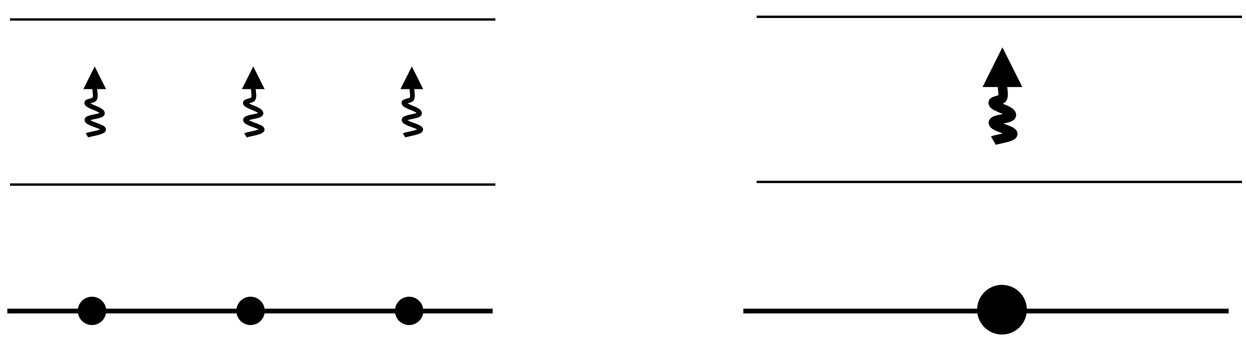
momentum mode P

winding mode NS1



Winding charge n_1

Mass = Charge



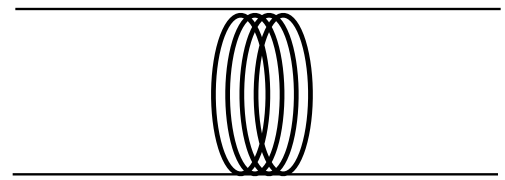
Momentum charge n_p

Mass = Charge

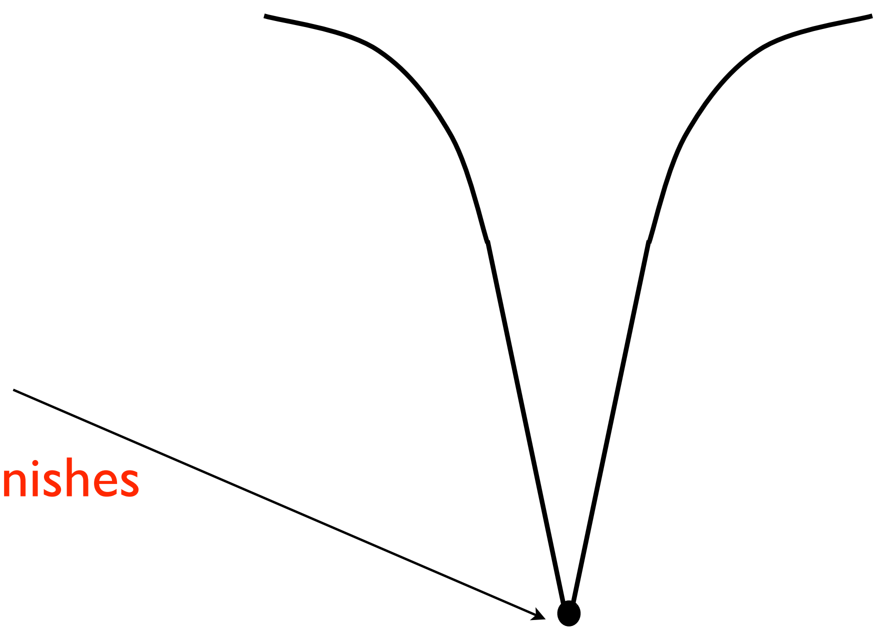
A black hole with winding charge only

$$S_{micro} = \ln[256] \sim 0$$

(Does not grow with n_1)



n_1



Horizon is singular

$$A \;=\; 0$$

Bekenstein entropy vanishes

$$S_{micro} = S_{bek} = 0$$

A black hole with momentum charge only

$$S_{micro} = \ln[256] \sim 0$$

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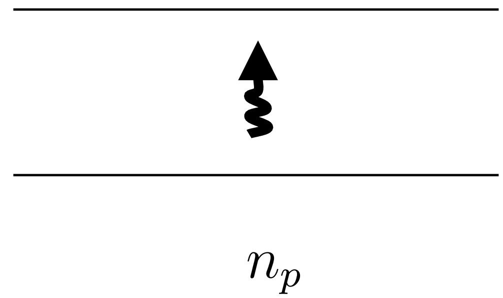

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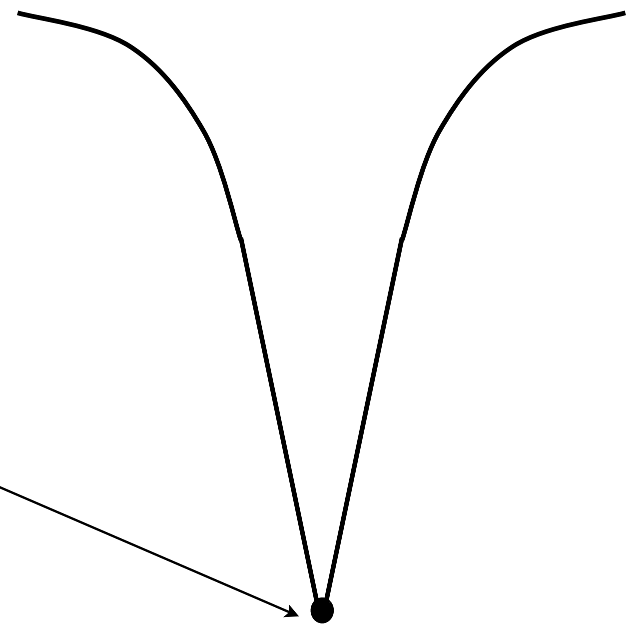
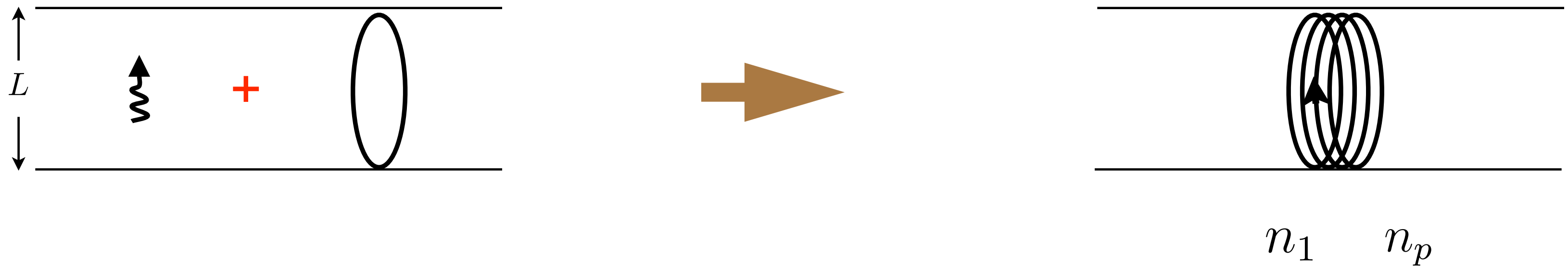


$$S_{micro} = S_{bek} = 0$$

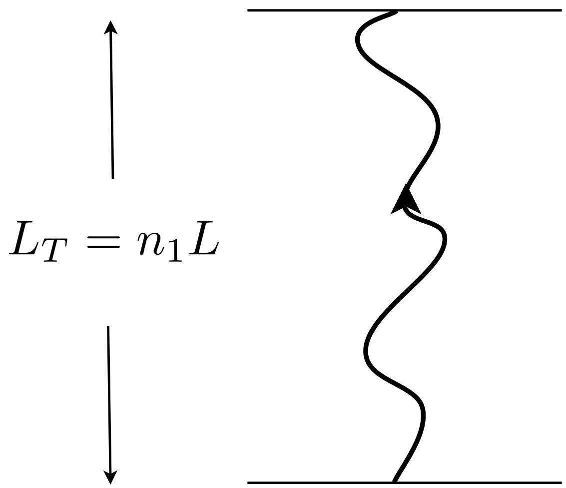
A black hole with winding AND momentum charge



The momentum charge is carried as traveling waves on the 'multiwound string'

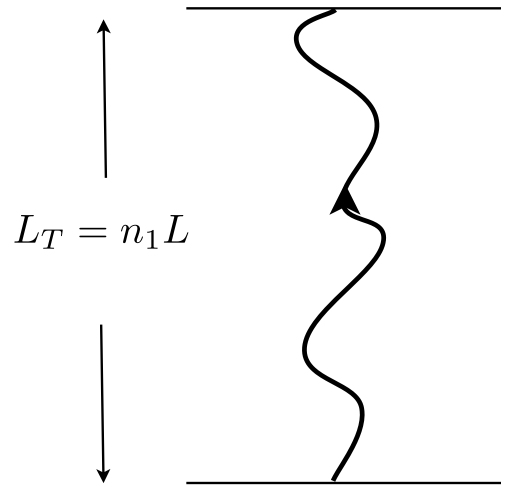
But there are many ways to do this ...

For example, we can put all the energy in the lowest harmonic, or some in the first and some in the second harmonic etc ....



Computing the entropy

Each quantum of harmonic k carries momentum $\dfrac{2\pi k}{L_T}$



Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi(n_1 n_p)}{L_T}$$

So we have to count 'partitions' of $n_1 n_p$

$$\sum k\ n_k = n_1 n_p$$

8 bosonic + 8 fermionic degrees of freedom

$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}}$ *states* →

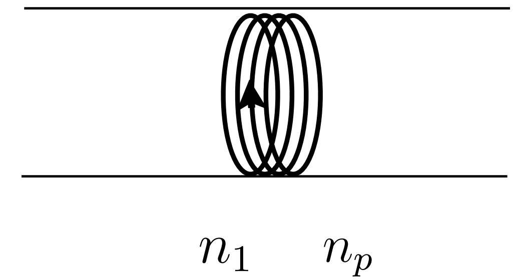
$$S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p} \qquad T^4 \times S^1$$

$$S_{micro} = 4\pi\sqrt{n_1 n_p} \qquad K3 \times S^1$$

(Susskind '93, Sen '94)

Now let us make a black hole with these charges ...

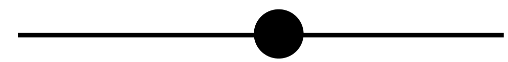
(Extremal black hole, Mass = Charge)



Make metric for this mass and charges ...

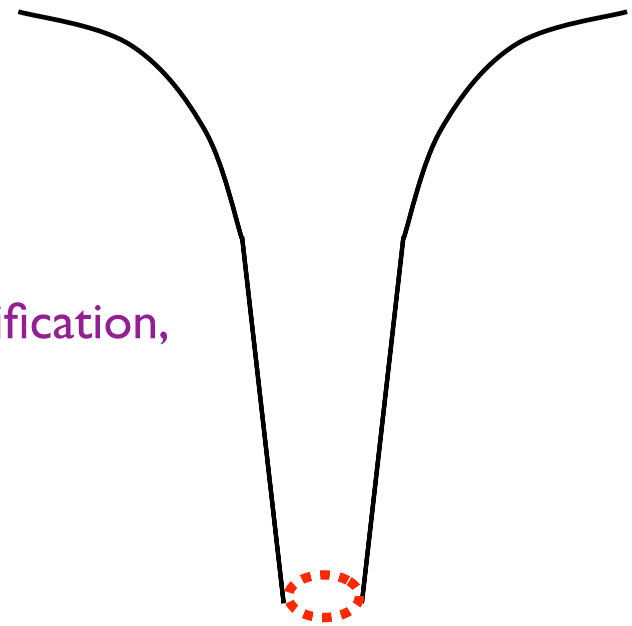
look at the horizon ...

Bekenstein - Wald entropy is found to be



$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$

($K3 \times S^1$ conmpactification, Dabholkar '04)

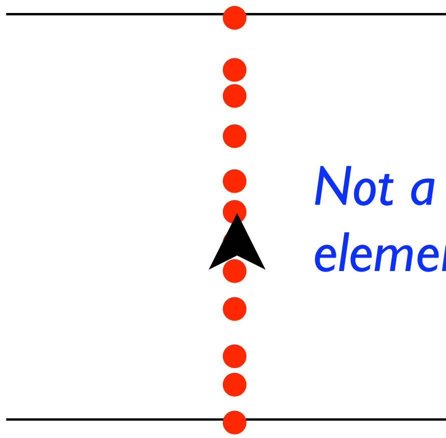


Thus we can understand the entropy of black holes from a microscopic count of states in string theory

What about the information problem ?

The elementary string (NS1) does not have any LONGITUDINAL vibration modes

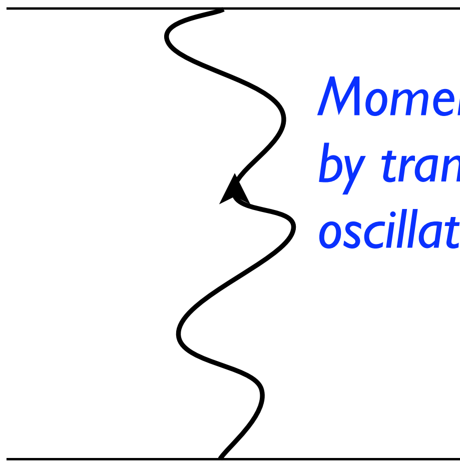
This is because it is not made up of 'more elementary particles'



Not a mode for the elementary string

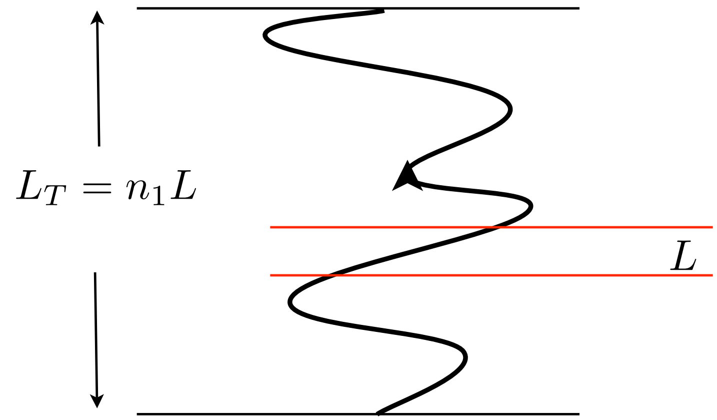
Thus only transverse oscillations are permitted

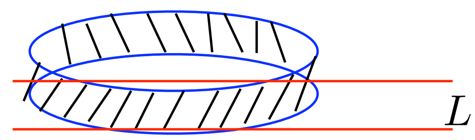
This causes the string to spread over a nonzero transverse area

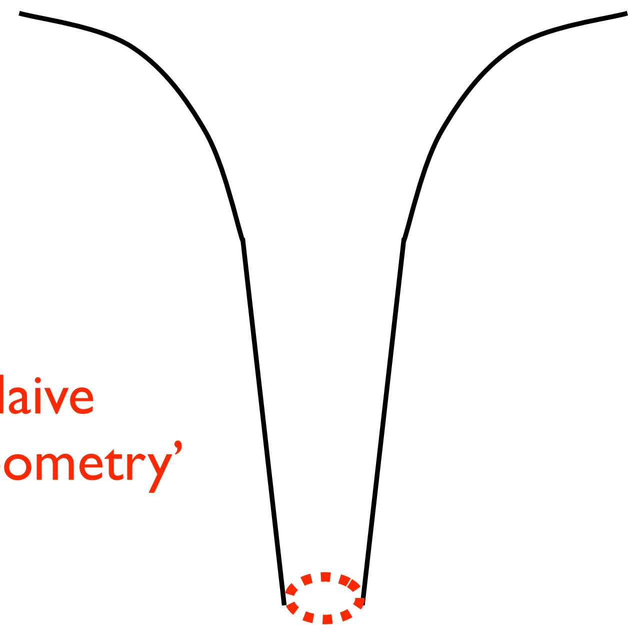


Momentum is carried by transverse oscillations

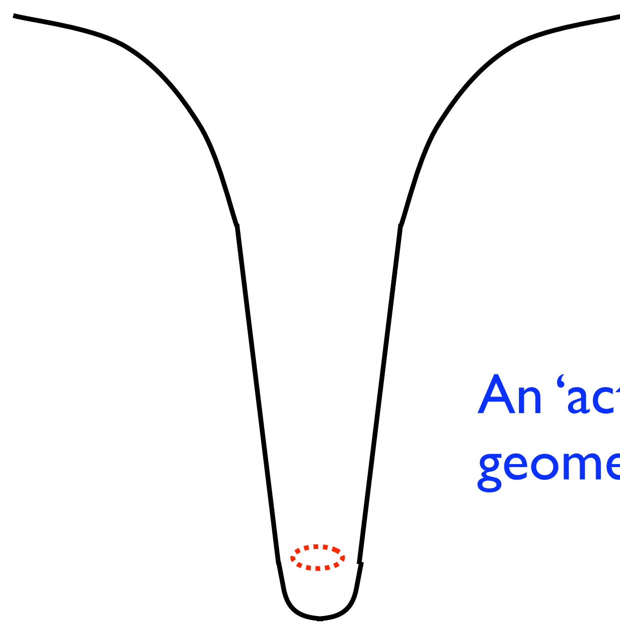
The transverse vibrations of the string break spherical symmetry, so there is no state corresponding to the usual spherically symmetric ansatz







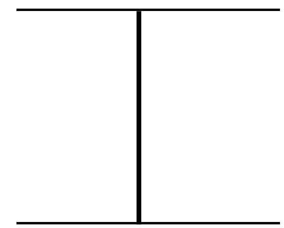
'Naive geometry'



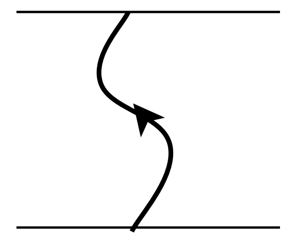
An 'actual geometry'

Making the geometry

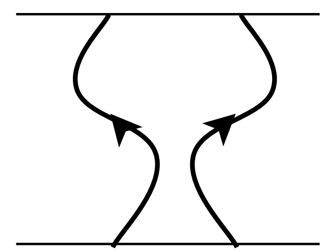
We know the metric of one straight strand of string



We know the metric of a string carrying a wave -- ‘Vachaspati transform’

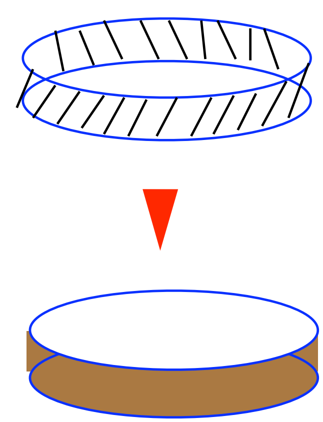


We get the metric for many strands by superposing harmonic functions from each strand

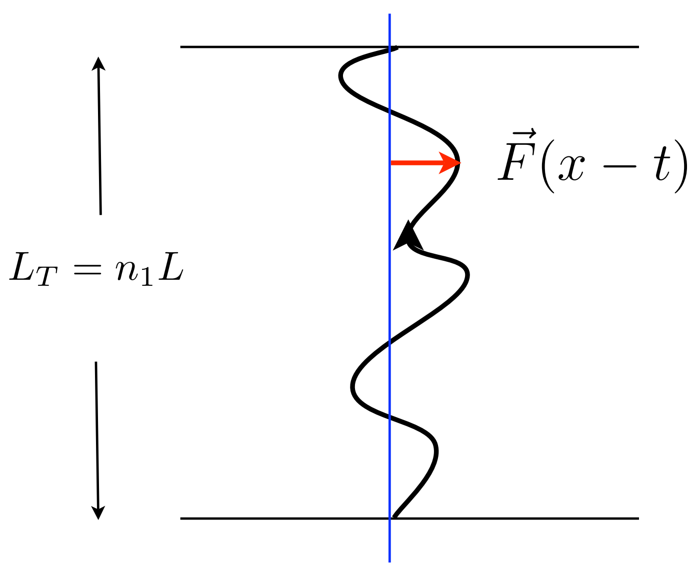


(Dabholkar, Gauntlett,Harvey, Waldram ’95, Callan,Maldacena,Peet ’95)

In our present case, we have a large number of strands, so we ‘smear over them to make a continuous ‘strip’ (Lunin+SDM ’01)

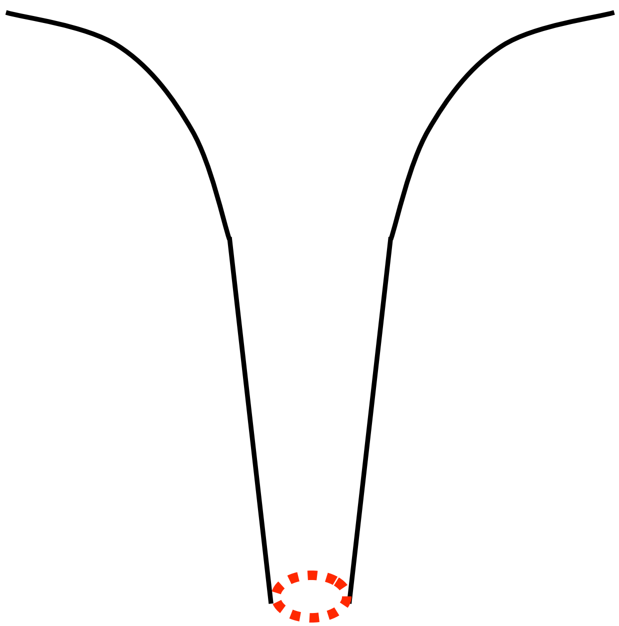


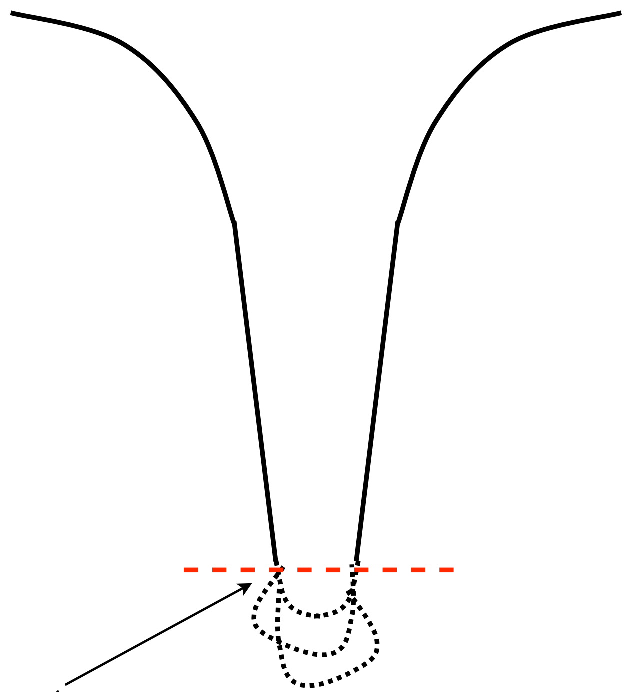
$$
\begin{aligned}
ds^2_{string} &= H[-dudv + Kdv^2 + 2A_i dx_i dv] + \sum_{i=1}^{4} dx_i dx_i + \sum_{a=1}^{4} dz_a dz_a \\
B_{uv} &= \frac{1}{2}[H-1], \qquad B_{vi} = HA_i \\
e^{2\phi} &= H
\end{aligned}
$$



$$
\begin{aligned}
H^{-1} &= 1 + \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \\
K &= \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv(\dot{\vec{F}}(v))^2}{|\vec{x} - \vec{F}(v)|^2} \\
A_i &= -\frac{Q_1}{L_T}\int_0^{L_T} \frac{dv\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}
\end{aligned}
$$

(Lunin + SDM 2001)



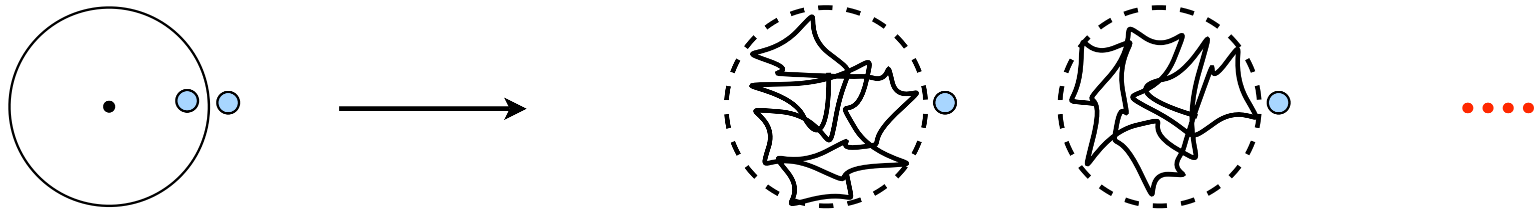


(Lunin+SDM '02)

$$\frac{A}{G} \sim \sqrt{n_1 n_p} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p}$$

Naive geometry, has horizon,
with singularity inside

Actual states,
throat ends in 'fuzzball cap', no
horizon or singularity
Information of infalling matter
eventually leaks out ..

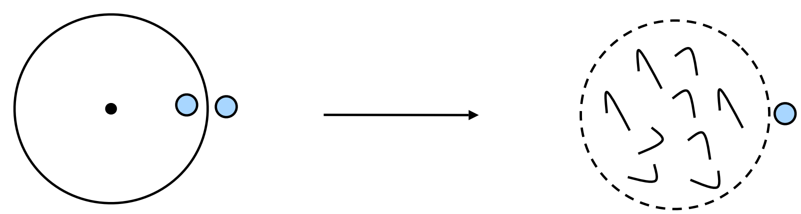


Usually we think that quantum gravity effects are relevant at distances of order planck length l_p

But a black hole is made of a large number of particles N, so we have to ask if the relevant length is l_p or $N^\alpha \, l_p$ .....

With string theory we find

$$l_p \longrightarrow N^\alpha \; l_p$$

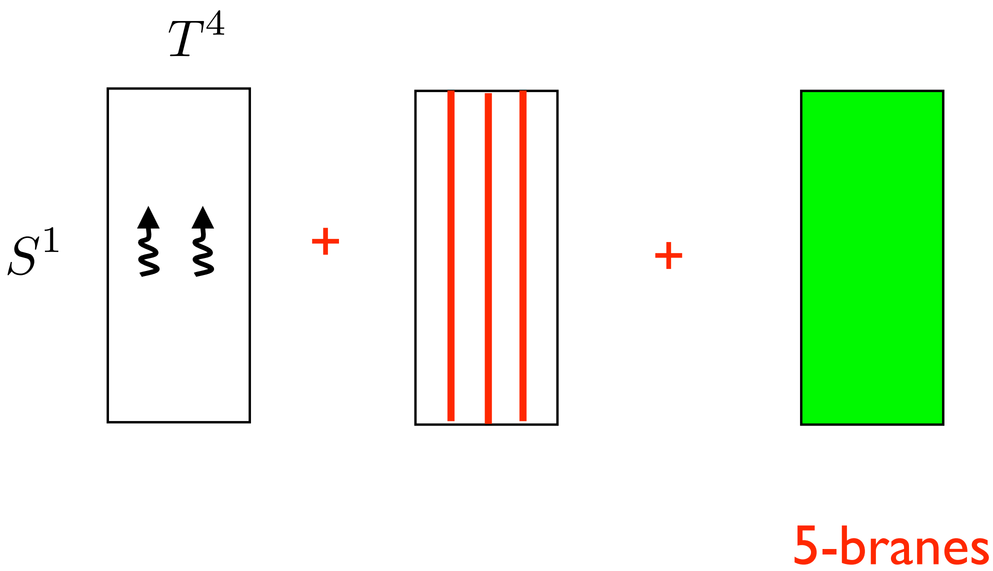


This resolves the information paradox

All this works with much more general black holes ....

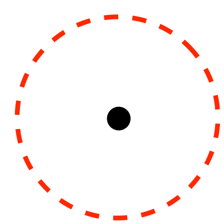
Make an extremal black hole with 3 kinds of charges :

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$



Make a bound state of 3 kinds of charges

$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$



Make the metric of a black hole carrying this mass and charges :

$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1 n_5 n_p}$$

Microscopic entropy formulae

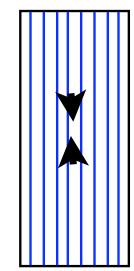
2-charges $S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$

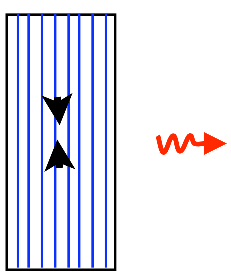
3-charges $S = 2\pi\sqrt{n_1 n_2 n_3}$

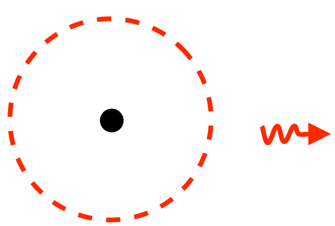
4-charges $S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$

2 charges
+ nonextremality $S = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$

3-charges
+ nonextremality $S = 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$







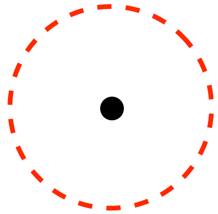
$$\Gamma_{micro} = \Gamma_{hawking}$$

Callan - Maldacena '96, Dhar, Mandal, Wadia, '96

Das+SDM '96, Strominger+Maldacena '96

General case

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$



Maximize S_{micro} *subject to the constraints*

$$n_5 - \bar{n}_5 = \hat{n}_5$$

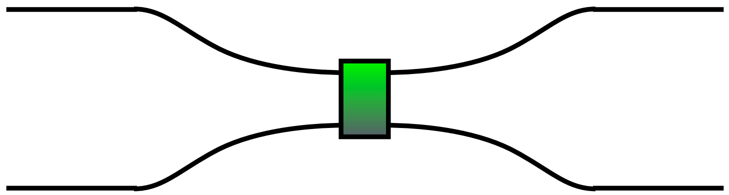
$$n_1 - \bar{n}_1 = \hat{n}_1$$

$$n_p - \bar{n}_p = \hat{n}_p$$

$$E = m_5(n_5 + \bar{n}_5) + m_1(n_1 + \bar{n}_1) + m_p(n_p + \bar{n}_p)$$

$$S_{micro} = S_{bek}$$

(Horowitz, Maldacena, Strominger ’96)



The behavior of compact directions implies a ‘pressure’ due to the black hole.

This pressure is given by summing the tensions of all the branes and anti-branes (no intercations)

Summary

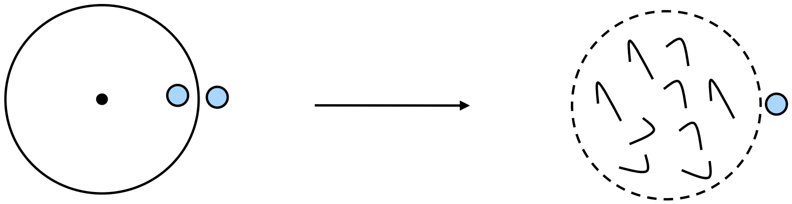
(A) Even if we have a neutral black hole, the energy of the hole goes to creating branes and anti-branes.

These branes/anti-branes are of several different types, and the entropy is of the form

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

(B) Quantum gravity effects extend not over planck length l_p but over lengths $N^\alpha\, l_p$, which turns out to be order horizon size.

Thus the information of the state of the black hole is not at the singularity but instead is distributed all over interior of the horizon, making the black hole a 'quantum fuzzball'.



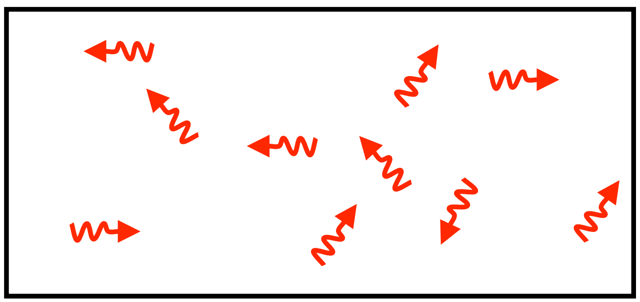
(B) Applying these facts to Cosmology

Cosmology

Start with a box of volume V

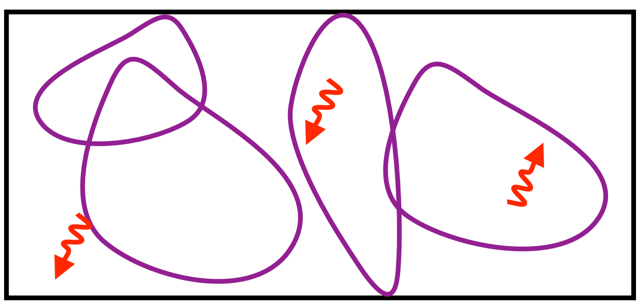
In the box put energy E

Question: What is the state of maximal entropy S, and how much is S(E) ?



Radiation

$$S \sim E^{\frac{D-1}{D}}$$

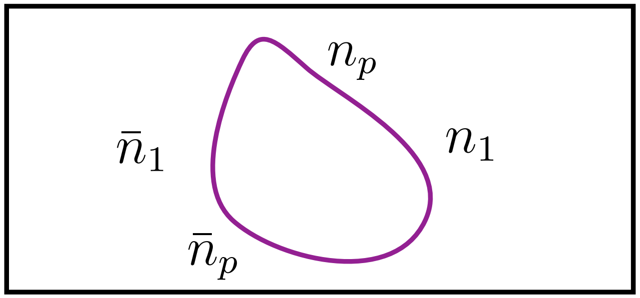


String gas
'Hagedorn phase'

$$S \sim E \sim \sqrt{E}\sqrt{E}$$

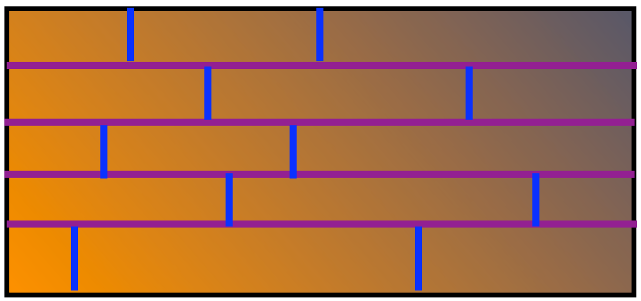
(Brandenberger+Vafa)

Can we do better than this ?



Two charges

$$S = 2\pi\sqrt{2}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim \sqrt{E}\sqrt{E} \sim E$$



Three charges (4+1 d black hole)

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim E^{\frac{3}{2}}$$

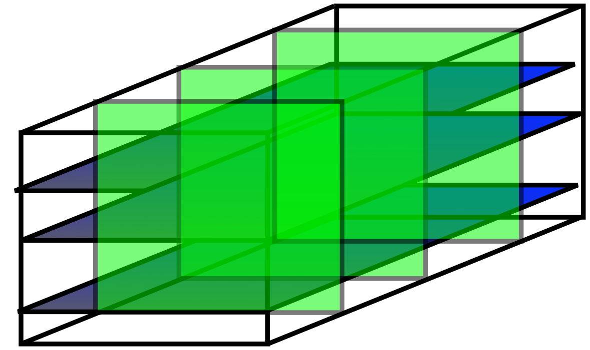
Four charges (3+1 d black hole)

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_2} + \sqrt{\bar{n}_2})(\sqrt{n_3} + \sqrt{\bar{n}_3})(\sqrt{n_4} + \sqrt{\bar{n}_4}) \sim E^2$$

N charges, postulate .....

$$S = A_N \prod_{i=1}^{N}(\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}$$

Plan of the computation :



Assume Universe is a torus ...

Branes can wrap all directions of space ...

Assume entropy relation like the one for black holes ..

$$S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}$$

We will not choose N, but work for general N
(though in principle string theory should fix this
(N=9?)

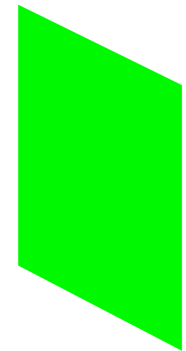
Just as for black holes, the energy and pressure are taken to be a simple sum over the energies and pressures of the branes/antibranes.

Solve for the evolution !

Step A : Find the number of branes and anti-branes by maximizing entropy S for given total mass of branes/anti-branes

Mass of a brane is given by its tension times its area

$$m_i = T_p \prod_j L_j$$



The Universe is neutral, so

$$n_i = \bar{n}_i$$

Maximize S for given total energy E

$$\tilde{S} = S - \lambda(E_{branes} - E) = A \prod_{i=1}^{N} \sqrt{n_i} - \lambda(2 \sum_i m_i n_i - E)$$

We find

$$n_k = \bar{n}_k = \frac{E}{2 N m_k}$$

Energy is equi-partitioned among different types of branes

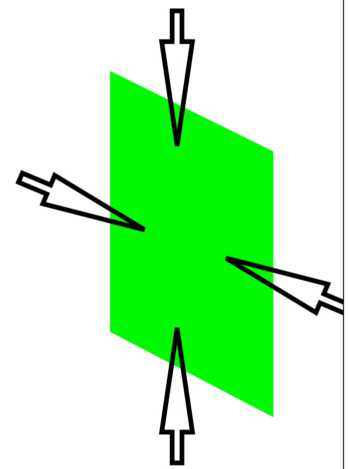
$$E_k = n_k m_k = \frac{E}{2N}$$

Step B : Find the stress tensor due to the branes/anti-branes

A brane has tension (negative pressure) along the directions where it extends, and zero pressure in the remaining directions

$$T^{(p)k}{}_k = -T_p \prod_{i=p+1}^{D-1} \hat{\delta}(x_i - \bar{x}_i), \qquad k = 1, \dots, p$$

$$T^{(p)k}{}_k = 0, \qquad k = p+1, \dots, (D-1)$$

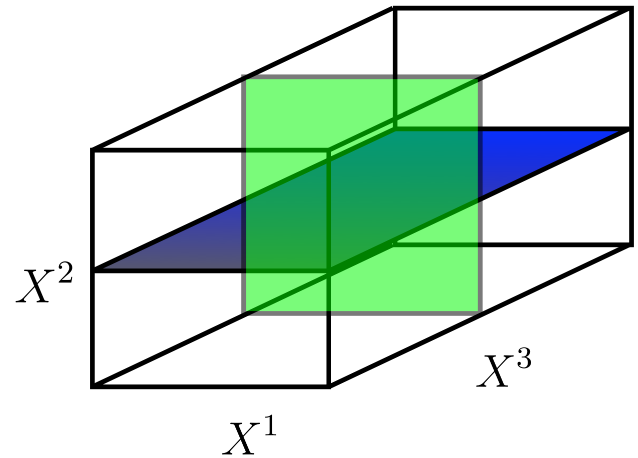


Following what we learnt from black holes, we will simply add the stress tensors from all the branes and anti-branes

Let there be N different types of branes/antibranes

Let N_i of these types extend along the direction X^i

Define $$w_i \equiv -\frac{N_i}{N}$$



$N = 2$

$w = \{1, .5, .5\}$

Then we find that when the entropy is maximized, the pressure in the direction X^i is given by

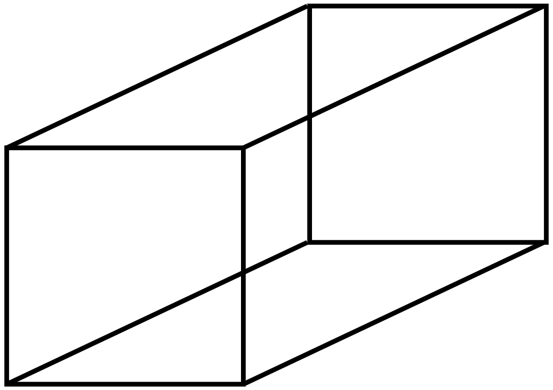
$$p_i = w_i \rho$$

where ρ is the energy density of the Universe

Step C : Solving Einstein's equations

We take a 'Kasner-type' metric ansatz

$$ds^2 = -dt^2 + \sum_{i=1}^{D-1} a_i^2(t) dx_i^2$$



And solve the Einstein equations with $p_i \;=\; w_i\;\rho$

Interestingly, the problem can be solved in closed form

(B. Chowdhury + SDM, 2006)

(some earlier work with similar equations had found numerical solutions)

The solution

Define the constants

$$W \equiv \sum_i w_i, \qquad U \equiv \sum_i w_i^2$$

(Recall that $w_i \equiv -\frac{N_i}{N}$)

Compute the constants

$$\begin{aligned} K_1 &= \frac{(D-1-W)}{2(D-2)} \\ K_2 &= -\frac{1}{2}[\frac{1-W}{D-2} W + U] \end{aligned}$$

$$\delta_k = \frac{1}{2}[\frac{1-W}{D-2} + w_k]$$

Then

$$a_k = C_k \, (\tau - r_1)^{\frac{2(\delta_k r_1 + f_k)}{(K_1+K_2)(r_1-r_2)}} (\tau - r_2)^{-\frac{2(\delta_k r_2 + f_k)}{(K_1+K_2)(r_1-r_2)}}$$

where τ is an auxiliary time parameter given by

$$(t - t_0) = \frac{1}{A_4} \int_0^{\tau} (\tau' - r_1)^{\frac{2(-r_1 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} (\tau' - r_2)^{-\frac{2(-r_2 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} d\tau'$$

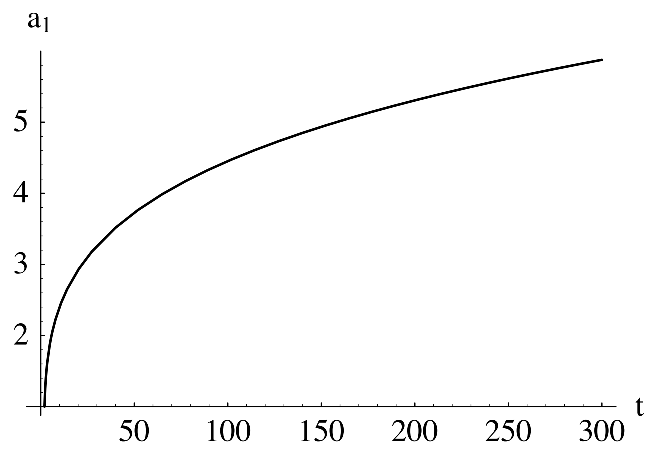
Recall that this integral is just the incomplete Beta function

$$B_x(p,q) = \int_0^x s^{p-1}(1-s)^{q-1} ds$$

At late times the evolution becomes power law ...

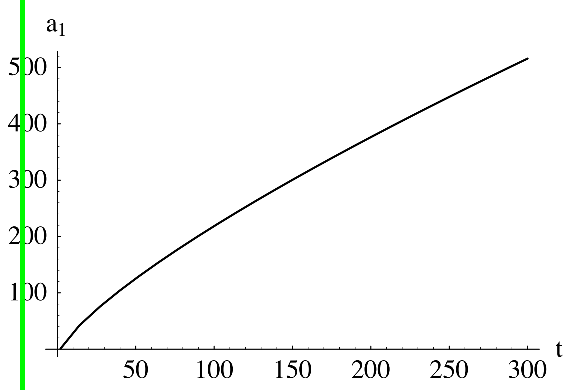
$$w_i = -.2 \text{ for all } i.$$

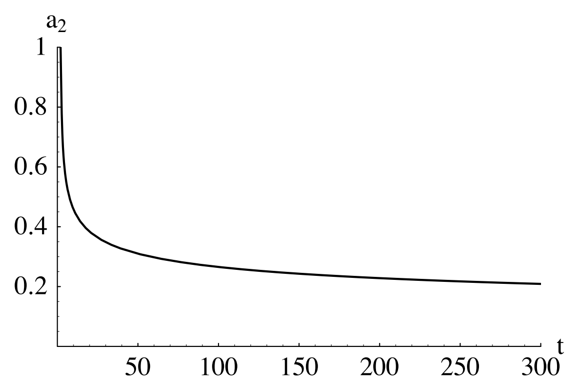
All a_i equal

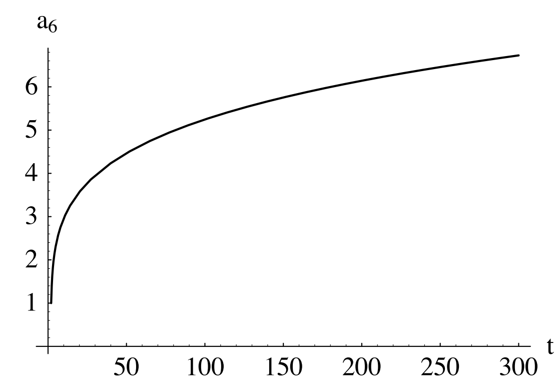


$$w_i = \{.9, -.9, -.9, -.9, -.9, -.1, -.1, -.1, -.1, -.1\}$$

All a_i start off with same value and same time derivative





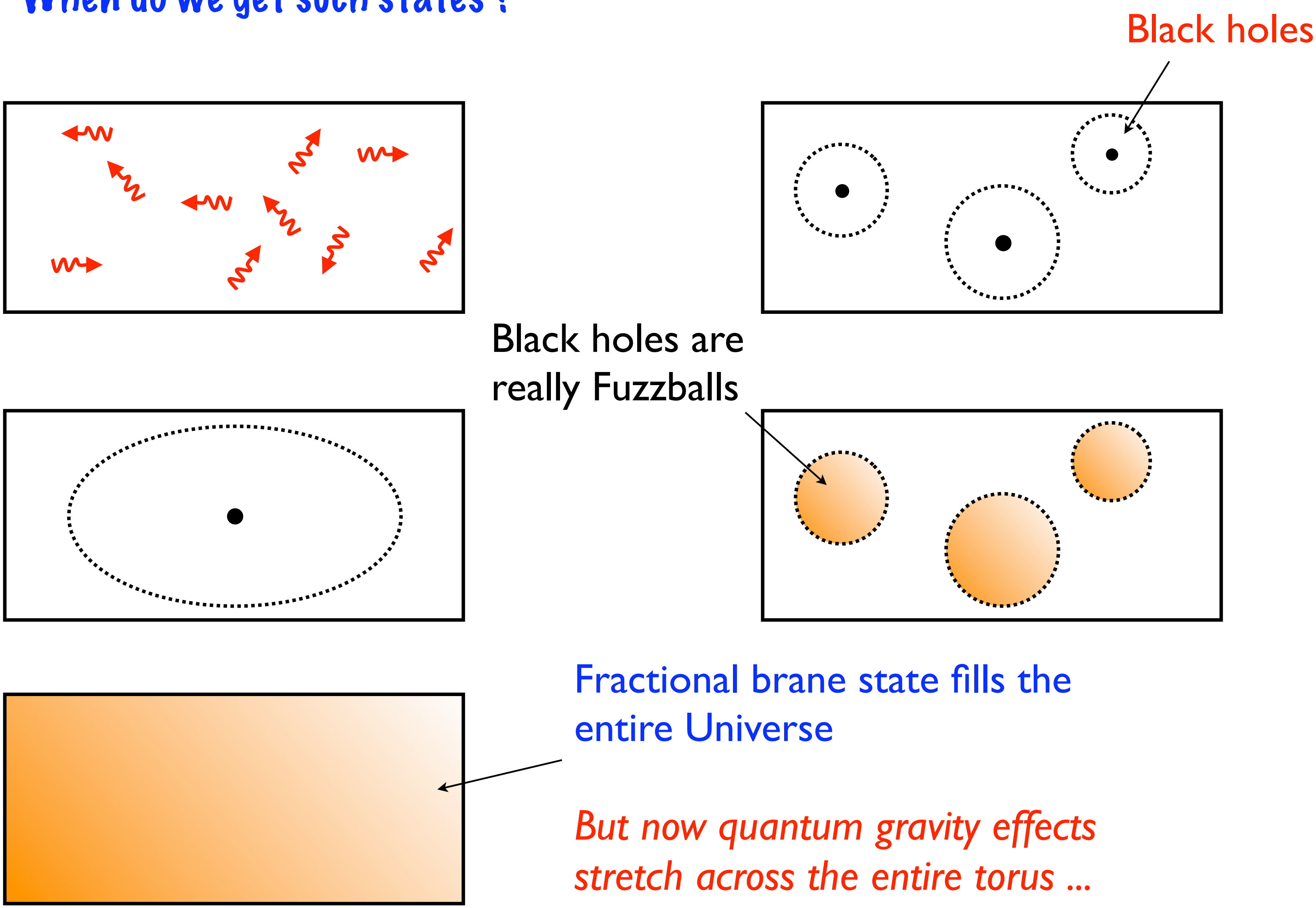


(Several other cases and related ideas studied by Kalyanrama 2007)

We do not seem to get an inflationary evolution ...

What is the physics of this Universe ? When do we get into a phase like the one that we are studying ?

When do we get such states ?



Summary

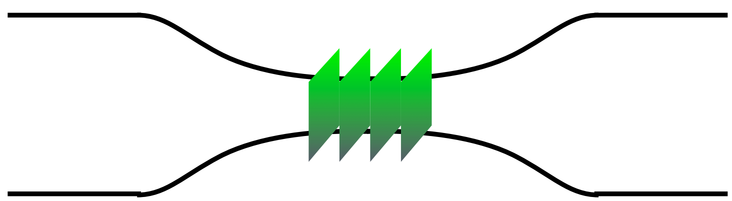
String theory explains the interior structure of black holes ...

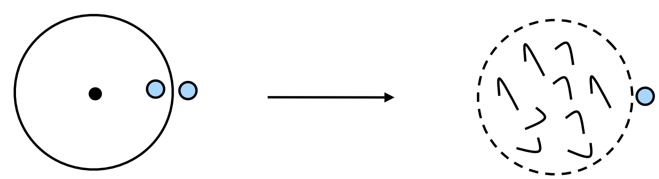
The energy E goes to making several different kinds of branes and anti-branes

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

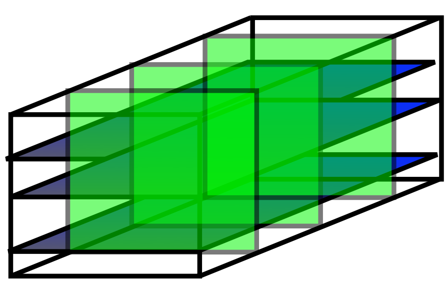
Energies, pressures of these branes/antibranes simply add up
to give the total energy, pressures ...

Quantum nonlocality effects extend all over the interior of the horizon





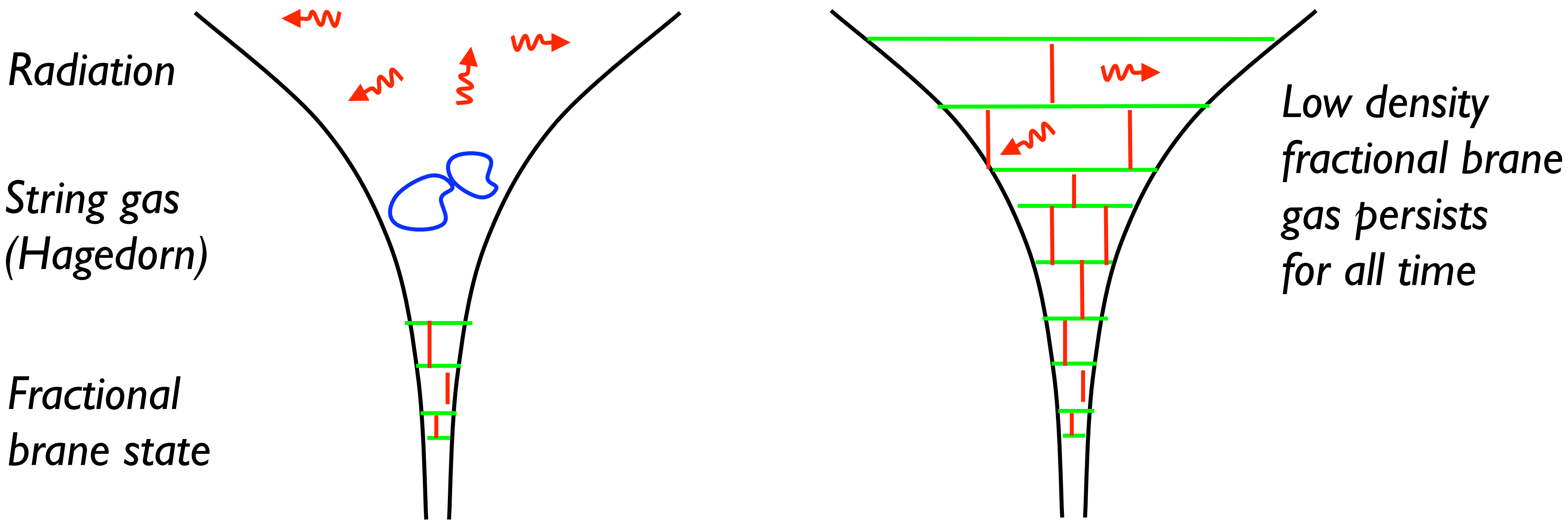
We speculated that a similar description might hold for the early Universe ...



Quantum nonlocal effects across the box ... perhaps can we bypass the 'horizon problem' without inflation ?

Questions

Do the fractional branes persist as a low density fluid for all time?
(Dark matter/dark energy?)



Should we assume that the matter in the early Universe is in a high entropy state ?

Should all the matter be bound up in this ‘fractional brane soup’ ?