Cosmology with Clusters in the CMB

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ICGC-2007, IUCAA-Pune, December 17, 2007

Overview of the talk

Clusters, CMB and Cosmlogy
The basic idea

Cosmology with Clusters - 1
(When the clusters are unresolved)
Example1 - Constraining mass fluctuation

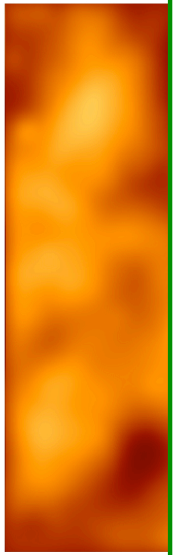
Cosmology with Clusters - 2
(When we just about resolve/detect the clusters)
Example 2 - Constraining cosmo parameters, say DM & DE

Cosmology with Clusters - 3
(When we can adequately resolve the clusters)
Example 3 - A geometric handle on the expansion history of the Universe

From Large Scales (WMAP etc) to Small Scales (Upcoming)
From Primary CMB to Secondary CMB Anisotropies …

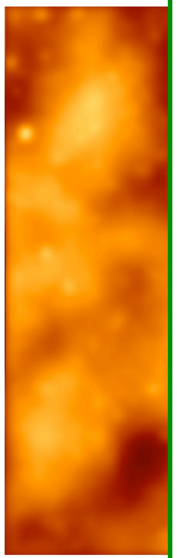
4’

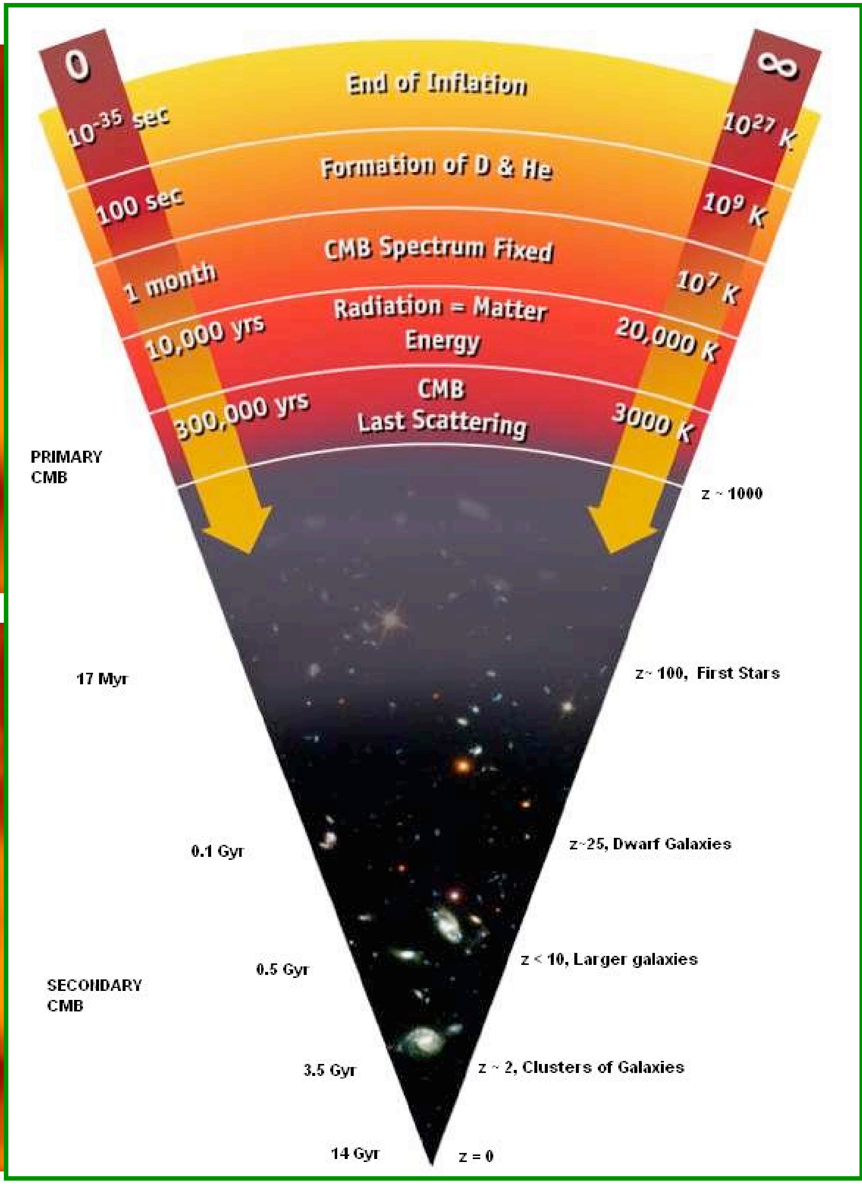
CMB
+SZ



2’

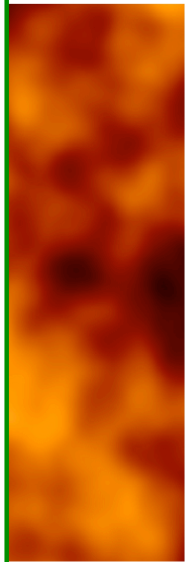
CMB
+SZ





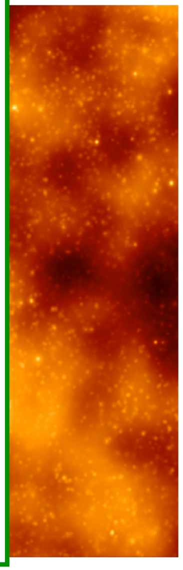
CMB
ONLY

Un-filtered



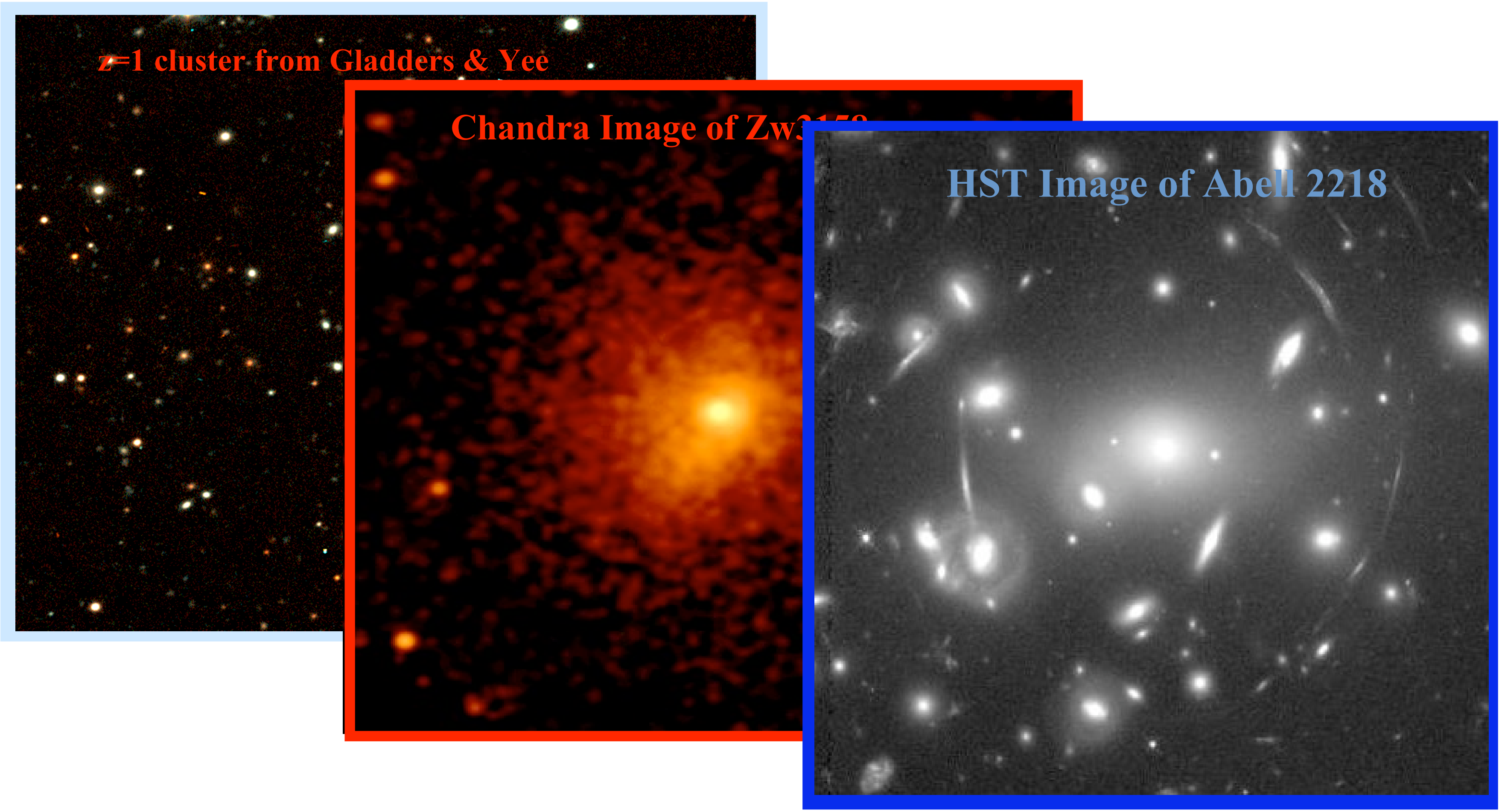
10”

CMB
+SZ



Galaxy clusters - as we have known them…

Galaxy clusters are the most massive (> 10^{14} M_{sun}), collapsed structures in the universe.
They contain galaxies, hot ionized gas and dark matter.
Clusters are good probes, because they are huge (> 1 Mpc) and “easy” to detect.



Scattering of photons by hot electrons…

(Sunyaev-Zeldovich 1969, Rephaeli 1984, Bernstein & Dodelson 1990, Birkinshaw 1999, SM 2001)

$$n_\alpha = (e^{h\nu/k_B T_{rad}} - 1)^{-1} \quad \text{for } \alpha = 1, 2 \qquad f_\alpha(\boldsymbol{r}, \boldsymbol{p}, t) = h^{-3} n_\alpha(\boldsymbol{r}, \boldsymbol{p}, t)$$

$$I_\nu(\hat{\boldsymbol{k}}, \boldsymbol{r}, t) = \sum_{\alpha=1}^{2} \left(\frac{h^4\nu^3}{c^2}\right) f_\alpha(\boldsymbol{r}, \boldsymbol{p}, t)$$

Kompaneets eqn : distortion of the photon occupation number distribution due to scattering by electrons (in the non-relativistic limit).

$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e} x_e^4 \left(\frac{\partial n}{\partial x_e} + n + n^2\right) \qquad \boxed{y = \frac{k_B T_e}{m_e c^2}\frac{ct}{\lambda_e}}\boxed{= \int n_e \sigma_T \, dl \frac{k_B T_e}{m_e c^2}}$$

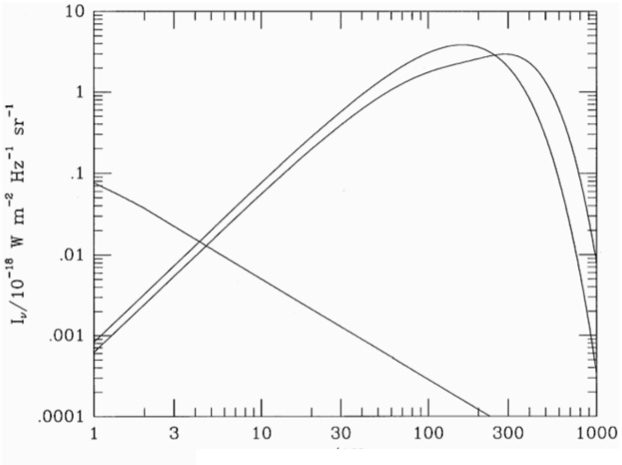
$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e} x_e^4 \frac{\partial n}{\partial x_e} \xrightarrow[\xi = 3y + \ln x_e]{\text{substitute}} \left(\frac{\partial n}{\partial y}\right) = \frac{\partial^2 n}{\partial \xi^2}$$

Soln:

$$I(\nu) = \int_{-\infty}^{\infty} P_K(s) I_0(\nu_0)\, ds \quad \text{with} \quad P_K(s) = \frac{1}{\sqrt{4\pi y}} \exp\left(-\frac{(s+3y)^2}{4y}\right)$$

$$\boxed{\Delta n = x\, y \frac{e^x}{(e^x - 1)^2}(x \coth(x/2) - 4)}$$

The one-of-a-kind spectrum…



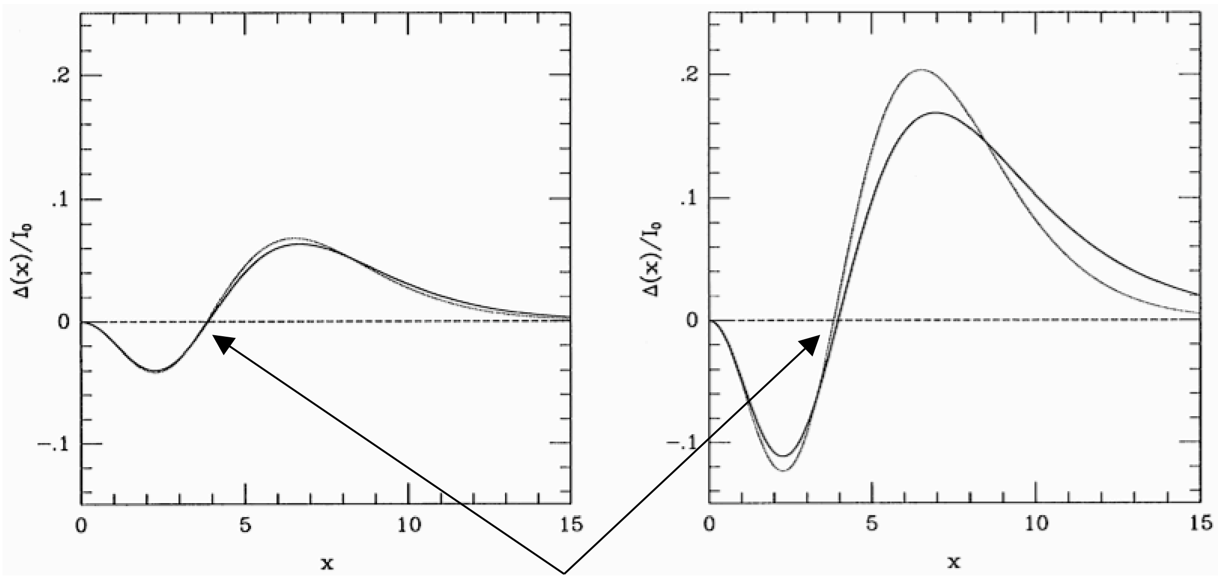
Photons from lower energies are kicked To higher energies --> blackbody spectrum is distorted

$x = h\nu/k_B T_e$

x (null) = 3.83 (non-relativistic)

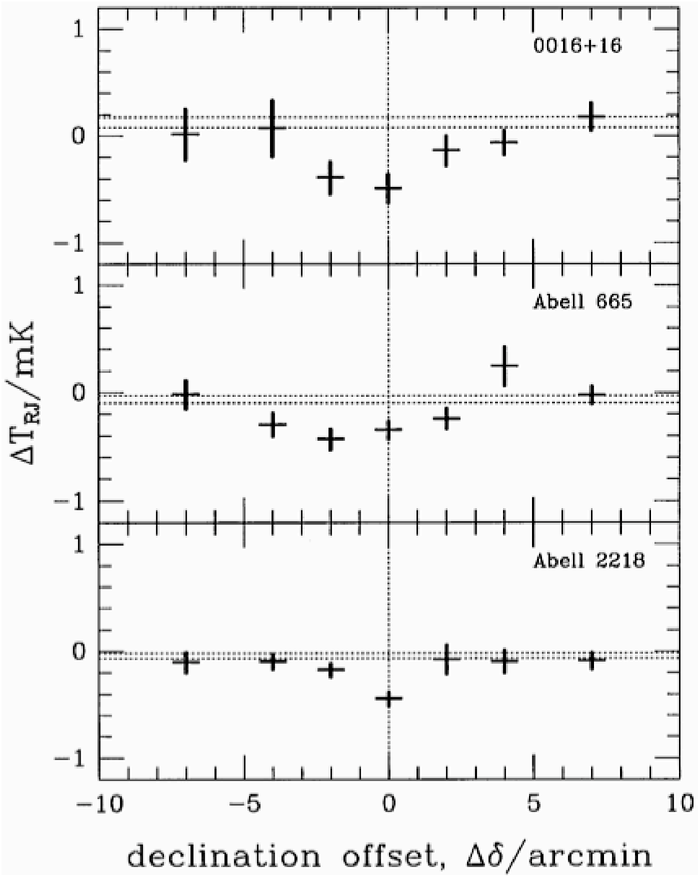
x (min) = 2.26

x (max) = 6.51

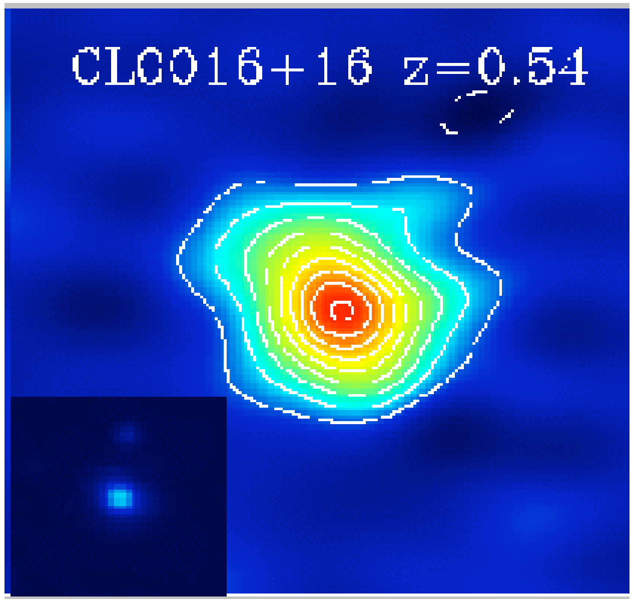


Null : No distortion at 227 GHz

Cluster in the CMB…



Birkinshaw etal 1999, OVRO



Reese etal 2001, SuZIE

The cluster appears as a hole in the sky

Notice the decrement ($\sim 10^{-4}$)- Coincides or close to X-ray cluster centre.

$$k_B T_e \approx \frac{GMm_p}{2R_{\rm eff}} \approx 7\,(M/3\times10^{14}\,M_\odot)(R_{\rm eff}/\mathrm{Mpc})^{-1}\,\mathrm{keV}$$

$$(\Delta\nu/\nu) \approx (k_B T_e/m_e c^2) \sim 10^{-2}$$

$$\tau_e \approx n_e \sigma_T R_{\rm eff} \sim 10^{-2}$$

PART ONE:

WHEN THE CLUSTERS ARE FULLY RESOLVED…

Preamble - connecting clusters to cosmology…

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2(a) = \frac{8\pi G_N \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\dot{\rho} = -3H(\rho + P)$$

In GR, **scale factor a(t)** describes growth of Universe. We know the time evolution of a(t) → H(a)

$$\boxed{H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_X a^{-3(1+w)}\right]}$$

Expansion history H(z) can be written in terms of energy density of each component

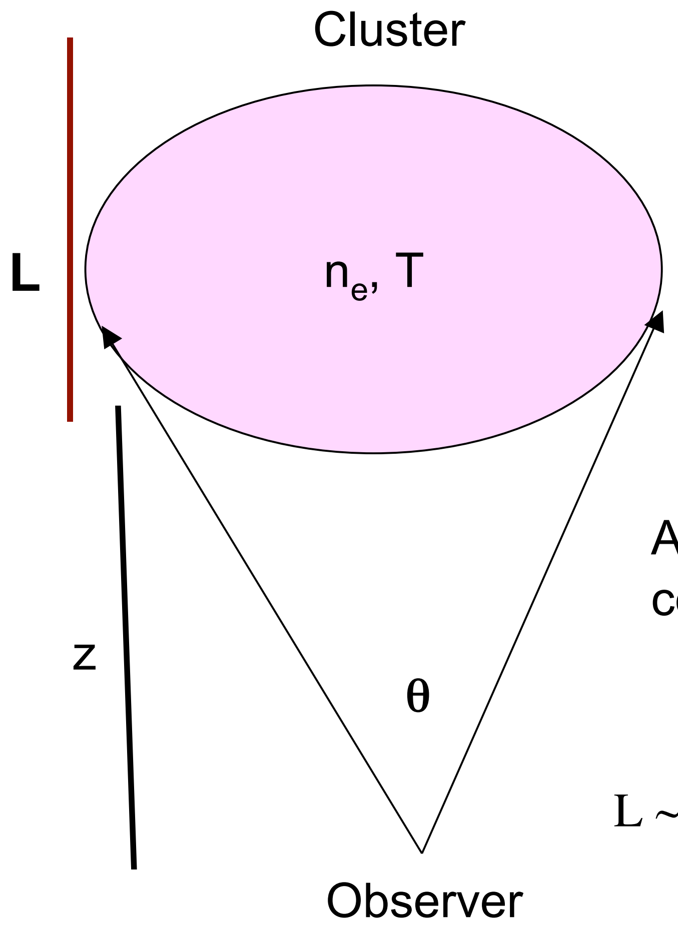
$$D(z) = \int_0^r \frac{dr'}{\sqrt{1 - kr'^2}} = \int_t^{t_0} \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{H(z')}.$$

Distance (or co-ordinate) to source at z

Standard measure $d_L \sim D(z)$ and $d_A \sim D(z)$

where $a = 1/(1+z)$

Hubble const from clusters as standard rulers:



Simply:

$$y_{SZ} \sim nk_BTL$$
$$S_X \sim n^2T^{0.5}\,L$$

Estimate

$$L \sim Y_{SZ}^2 / S_X\,T^{1.5}$$
$$= d_A\,\theta$$

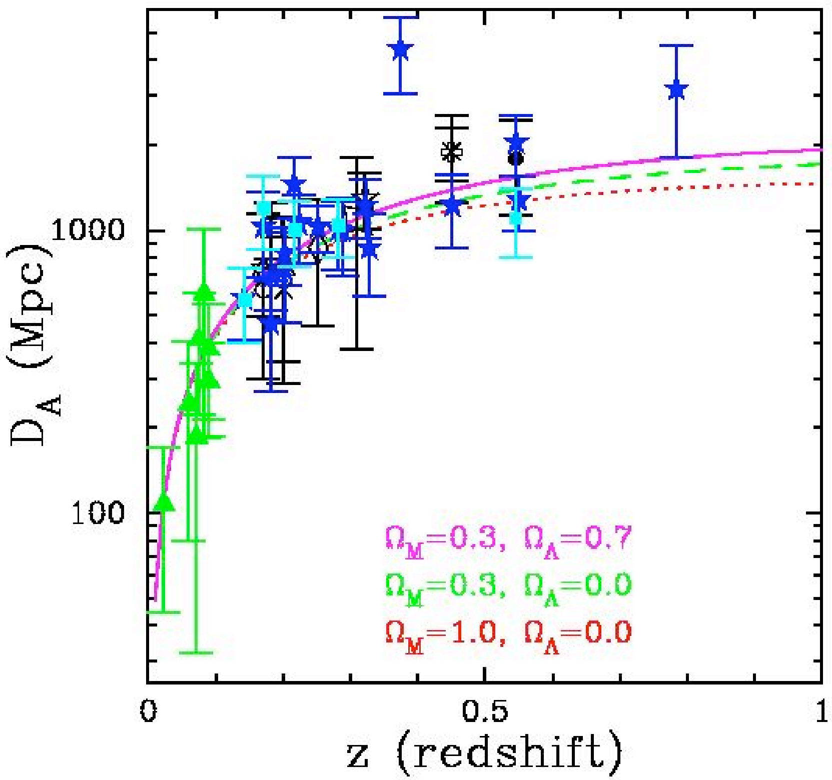
Compare with observed θ to get $\mathbf{d_A(H(z))}$

Assume a cluster model, say β-model (estimate the core radius as the standard ruler) and do the math

$$L \sim r_{c,est} = \frac{\left[\frac{\Delta T(\theta)}{T}\right]^2_{obs}}{S_X(\theta)_{obs}} \frac{\Gamma(3\beta-1/2)\Gamma(3\beta/2)^2}{\Gamma(3\beta/2-1/2)^2\Gamma(3\beta)} \frac{m_e^2c^4\alpha}{16\pi^{3/2}(1+z)^4\sigma_T k_B T_{e,fit}^2}\left[1+\left(\frac{\theta}{\theta_{X,core}}\right)^2\right]^{-1/2}$$

Note: This result does NOT depend on β-model

Observational result…



Reese, PhD Thesis

Assume cosmology, then H_0 is the overall scaling.

This results will improve consderably with more clusters and better SZ measurements.

One can think of using it as a geometrical probe in cosmology.
(more later)

Just like SNe, from future cluster surveys, one may construct H(z) (from D_A(z)) and use it to break cosmo parameter degeneracies when combined with other probes

Satej + SM

One parameter cluster structure…

Mass $M_{\Delta c}$ is enclosed in radius $R_{\Delta c} \rightarrow$ $M_{\Delta c} \sim \rho_c(z)\Delta_c R^3_{\Delta c}$

$\rho_c(z) \sim E^2(z)$

$R \sim M^{1/3}E^{-2/3}(z)$

Assuming, hydrostatic equilibrium,

1) M-T reln: $$M_{\Delta_c} \propto T^{3/2}E^{-1}(z)$$

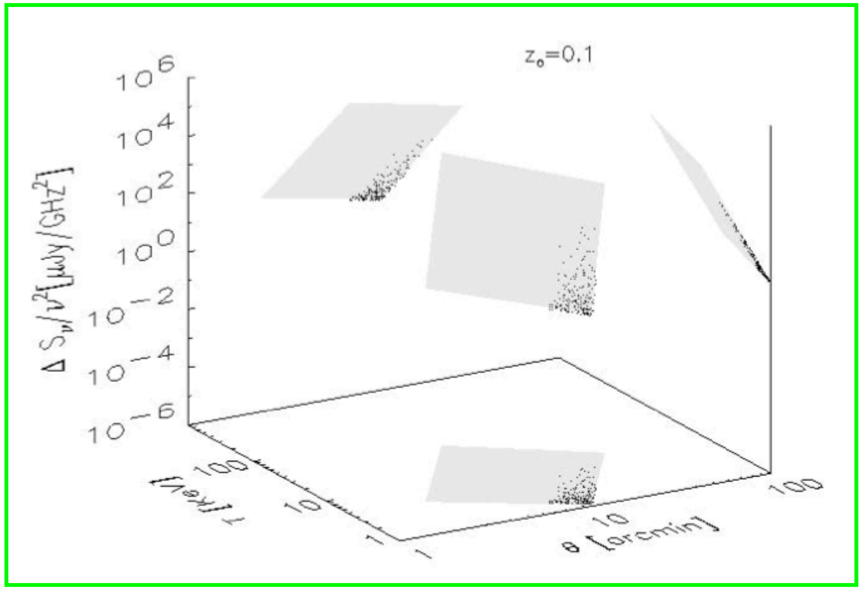
2) SZ relns: $$\Delta S \propto \int y(\theta)\mathrm{d}\Omega \propto d_A^{-2} \int T n_e \mathrm{d}^3 r \propto d_A^{-2} T^{5/2} E^{-1}(z)$$

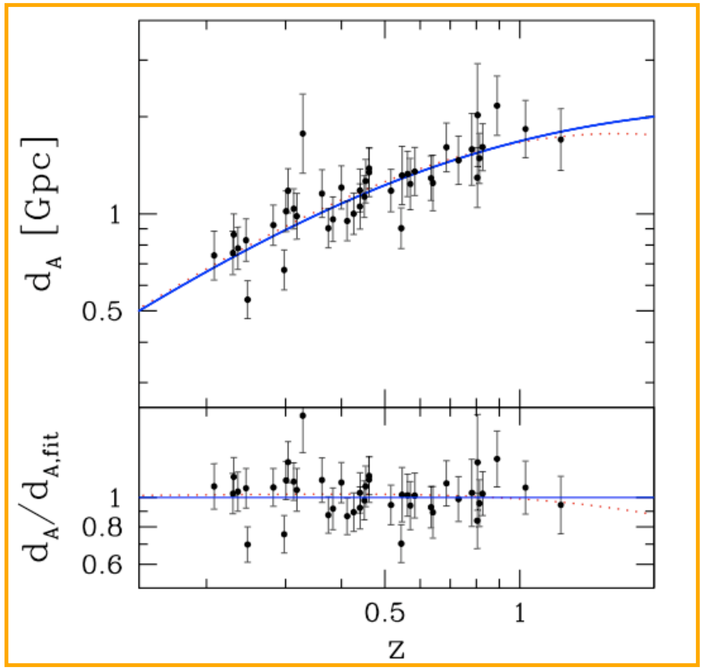
$$y_0 \propto T^{3/2}E(z) \propto L_X^{3/4}E^{1/4}(z)$$

Assumption: Gravity ONLY decides the thermodynamic state of the ICM. No preferred scale→ hence self-similar (Kaiser)

Real Life clusters are NOT self-similar.
However, **they still have mean scaling relation with some scatter.**

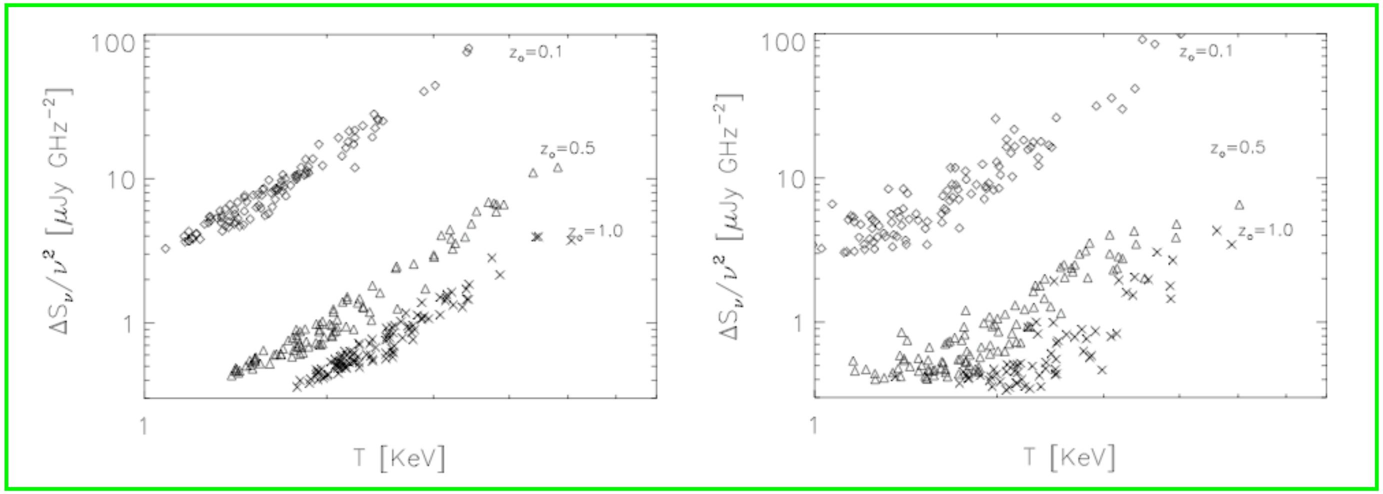
Clusters- cosmo rulers at every location…





70 clusters

O'hara etal 07



Verde et al 04

Before we proceed further.
Why study clusters in the CMB?

There are many advantages. Most importantly, the total integrated SZ flux is amazingly robust to cluster gas physics and maps strongly only with the cluster total mass.

i.e. Total SZ flux towards a cluster ⟷ Mass of the cluster

The total integrated flux (Y or S_ν)

$$S_\nu = j(x)\int d\Omega \int \frac{k_B T_e}{m_e c^2}\sigma_T n_e \, dl$$

$j(x)$ = freq dependence

Integrating of the volume (angular diameter distance comes in)

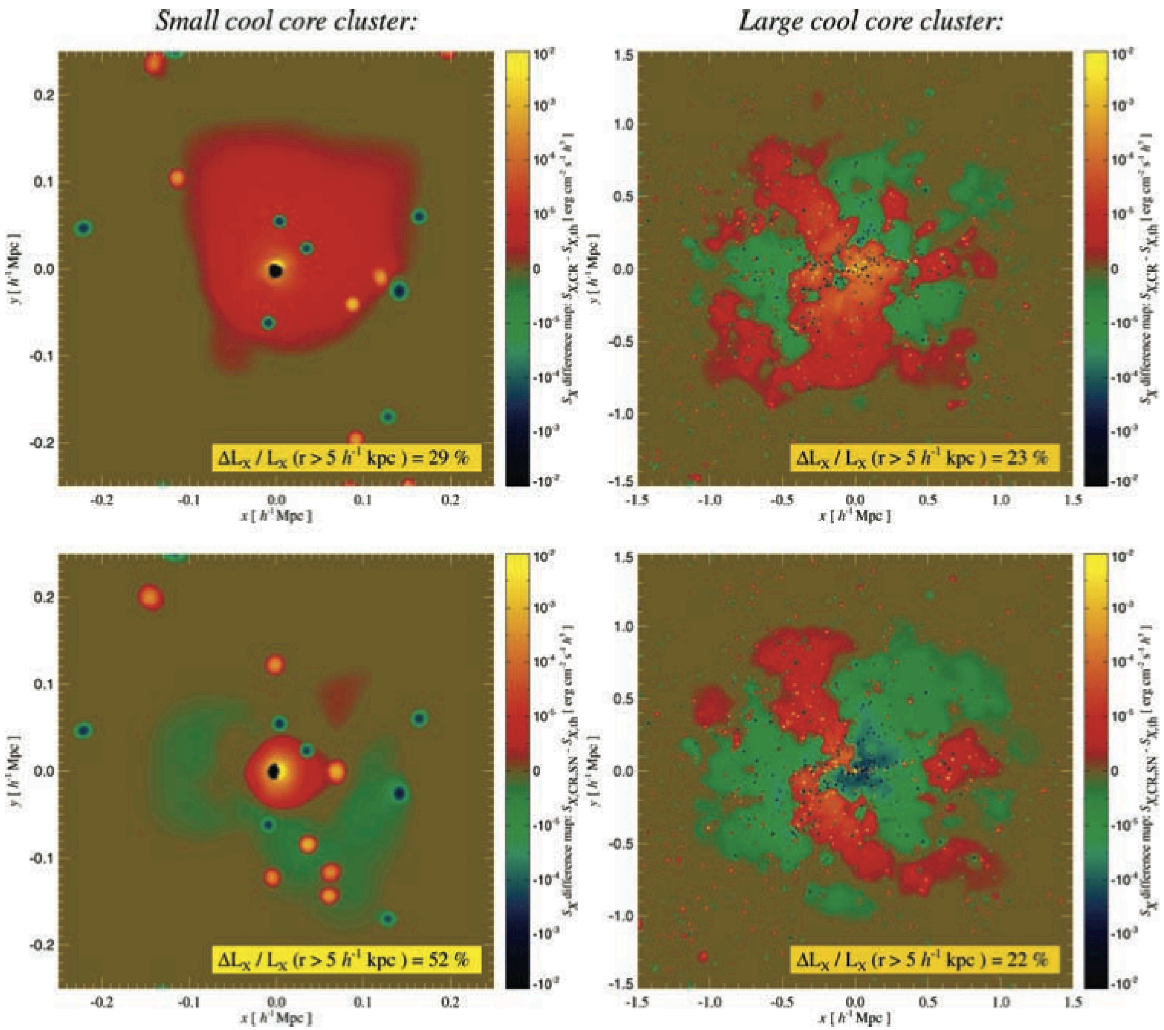
$$S_\nu = j(x)\int d^3x \frac{k_B T_e}{m_e c^2}\frac{\sigma_T n_e}{d_A^2}$$

Assuming isothermality, the effect is proportional to the total # of electrons

$$S_\nu = j(x)\frac{k_B T_e}{m_e c^2}\frac{\sigma_T N_e}{D_A^2}$$

~ Mass (since N_e ~ Mass)

Robustness of SZ flux is verified in simulations with different gas physics…



Difference map:
Hydro simulations with different gas physics
Pfrommer etal 07

SZflux-Mass scaling is minimally affected

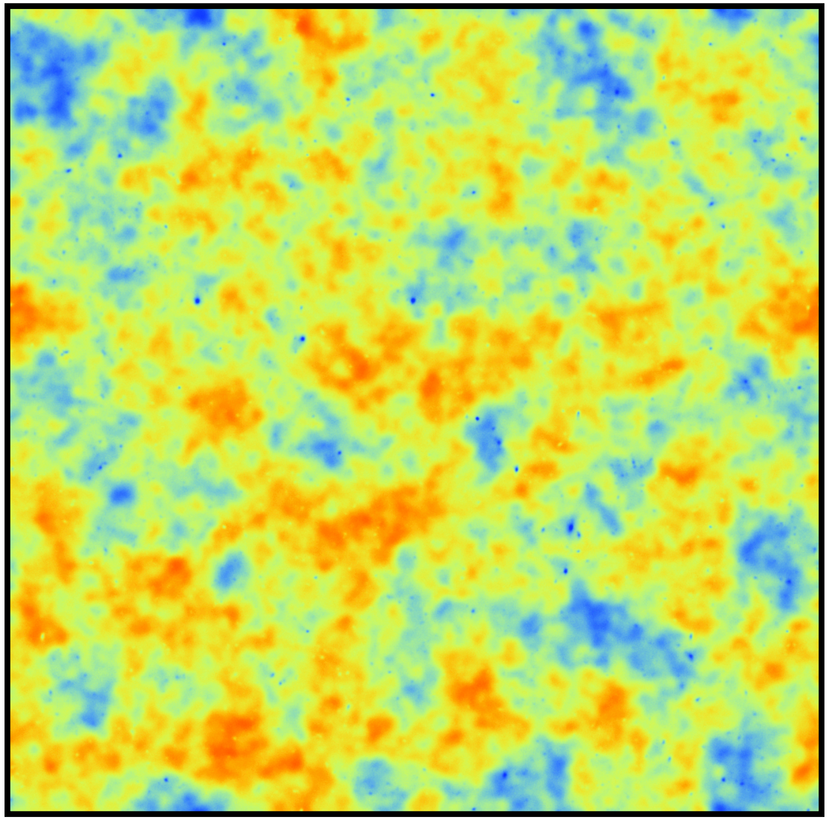
… and hence is good as cosmological probes --> less bias

Pfrommer & SM 2007

PART TWO:
WHEN THE CLUSTERS ARE JUST RESOLVED/DETECTED …

Simulating a *very simplified* SZ observation

Let's suppose we've *subtracted* all the *astrophysical foregrounds*.
We are left with only CMB signal and instrumental noise



10^o x 10^o map
145 GHz

ACT survey simulations:

1. **freq maps 145, 225, 265 GHz**
2. **Resolution of 1.7', 1.1', 0.93'**
3. **Sensitivity 300, 500, 700 μk $s^{1/2}$**
   or ~ few 10 μK for 100 deg2 /month

4. **Gaussian CMB with input Cl**
5. **Hydro simulation of SZ clusters (from Martin White)**
6. **Expected ACT noise in 3 bands (6, 10, 15 μK/pixel of 35 arc-sec)**
7. **Convolution with antenna beam**

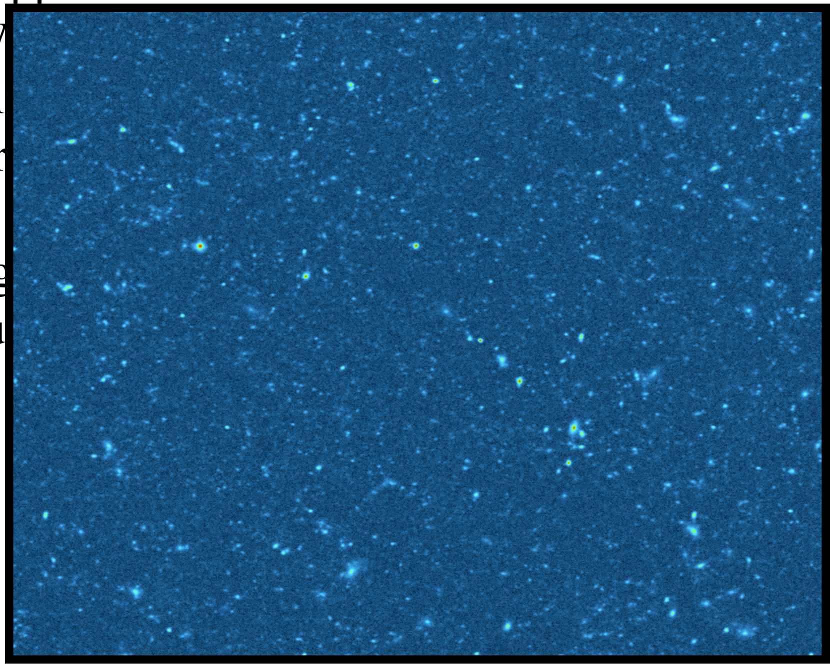
Diego & SM 04

Getting the clusters in the ACT survey…

Remember that SZ has a unique freq dependence (decrement, zero at 217 GHz and then increment).
Typically we need 3 frequencies to bring out the SZ (we just use 2) .
Simplest approach:

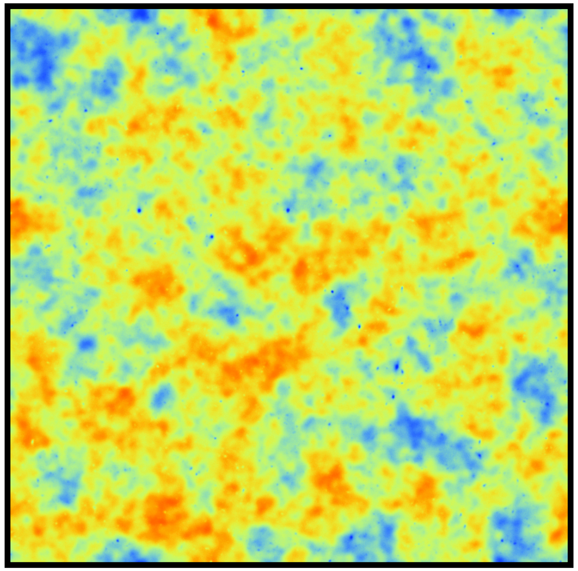
1. W … Estimator of SZ at 145 GHz
2. Fi … map to 145 GHz map
3. Fr … t in temperaure units.

Interesting … good results.
We land u…

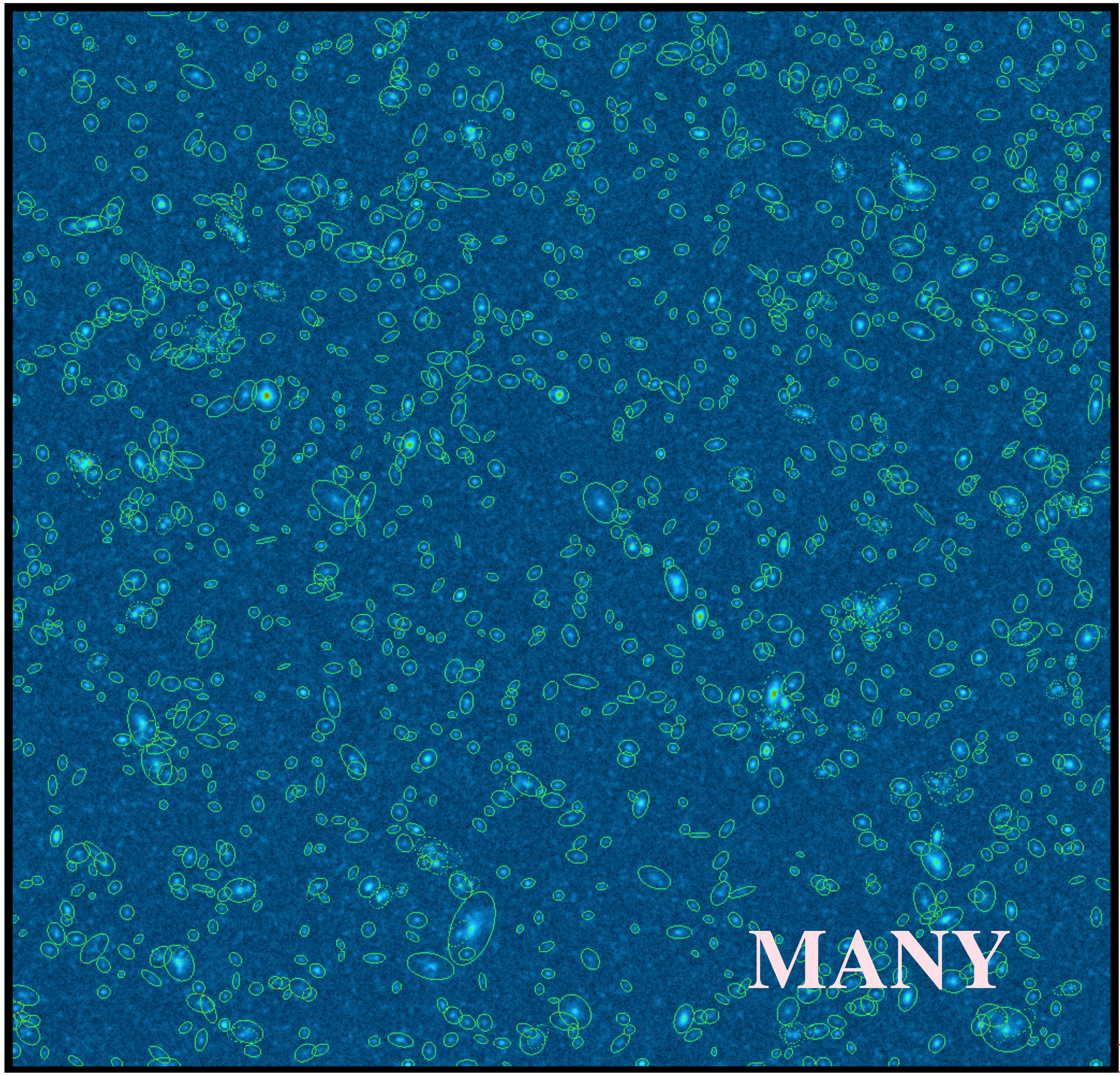


Diego & SM 04

There are distinct SZ sources, some amount of diffuse SZ and some leftover noise



Clusters (at last)...



Diego & SM 04

Doing some Angular or matched filtering.

Just for demonstration, we SExtractor to get the sources.

Surveys yielding SZ (and related) clusters …

On last count, there are ~ 30 surveys dealing fully/partially with clusters (DETF report) !

Planck >12 mK, 10' for LFI; >5mk , 5' for HFI , whole sky #5000-8000
(All the rest, typically few μK)

SPT : 1' resolution, 150, 220, (>220 ?) GHz, 4000 sq-deg
1000 bolometer array in a 8 m dish # 15000-25000

ACT 0.9'-1.7', 145, 225, 267 GHz, 100 sq-deg
Cerro-Toco, Chile # 1000-2000

APEX 0.6'-0.75', 150, 220, 270(?) GHz, 150-200 sq-deg
330 bolometers, Atacama # 1000-2000

SZA : interferrometer, array of 8, 3.5 m telescopes, 12 sq-deg # 100

AMI, AMiBA interferrometric # 100-300, ?

Mosaic Survey: optical followup of 100 deg of SPT field, Blanco 4m, g r i bands #500

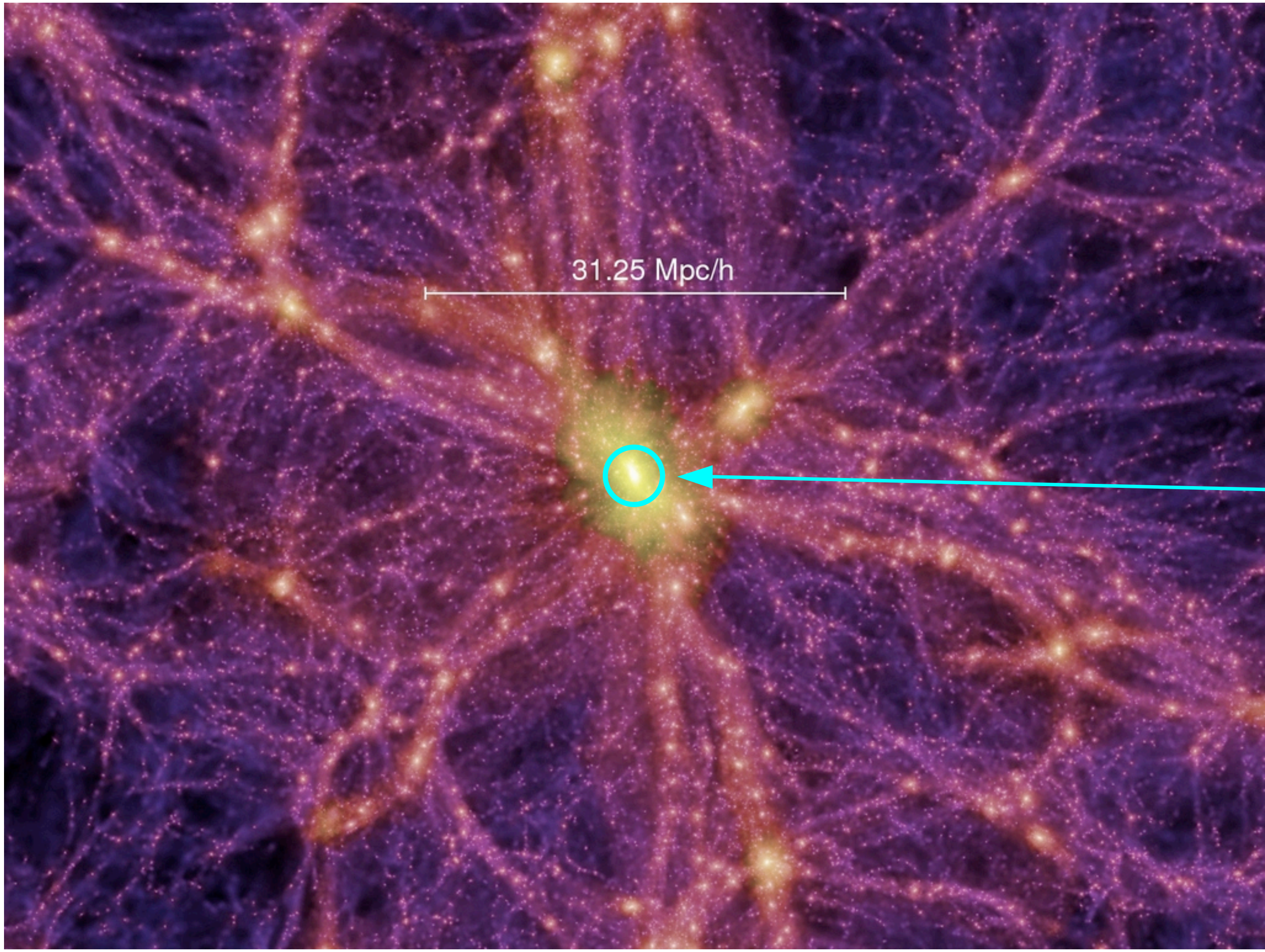
SMI optical followup of 500 deg of SPT field, Magellan 4.5m, g r i z bands #3000

DES optical followup of a total SPT field, Blanco 4m, g r i z bands #20000

RCS-2 optical, 1000 sq-deg, mass calibrators, CFHT, g r z bands #10000

Here, we are looking *only* at SZ related surveys

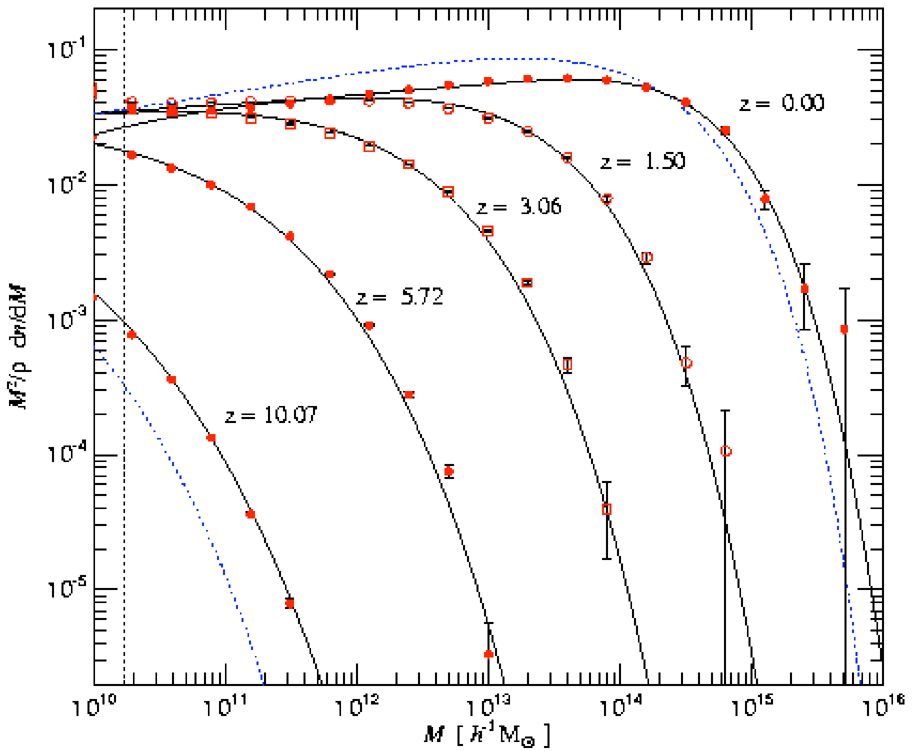
Connecting clusters to cosmology…



Volume of
fixed overdensity
-> mass within
Is cluster mass!

Millenium simulation

The space density of haloes…



Two assumptions:

1. Underlying density field is Gaussian

2. Objects (local patches) collapse if the density contrast is over a threshold δ_c

3) Density field has cosmology Dependent variance σ_M when filtered on scale corresponding to M.

Universality when written in terms σ_M

# of haloes/mass

$$f(\sigma_M, z) = \frac{M}{\bar{\rho}} \frac{dn(M, z)}{d \ln \sigma_M^{-1}}$$

$$f(\sigma_M, z) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma_M} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

Press-Schecter form, theoretical (since 1976)

$$f(\sigma_M, z) = \sqrt{\frac{2a}{\pi}} C \left[1 + \left(\frac{\sigma_M^2}{a\delta_c^2}\right)^q\right] \frac{\delta_c}{\sigma_M} \exp\left(-\frac{a\delta_c^2}{2\sigma_M^2}\right)$$

Seth-Tormen: ellipsoidal collapse & Nbody

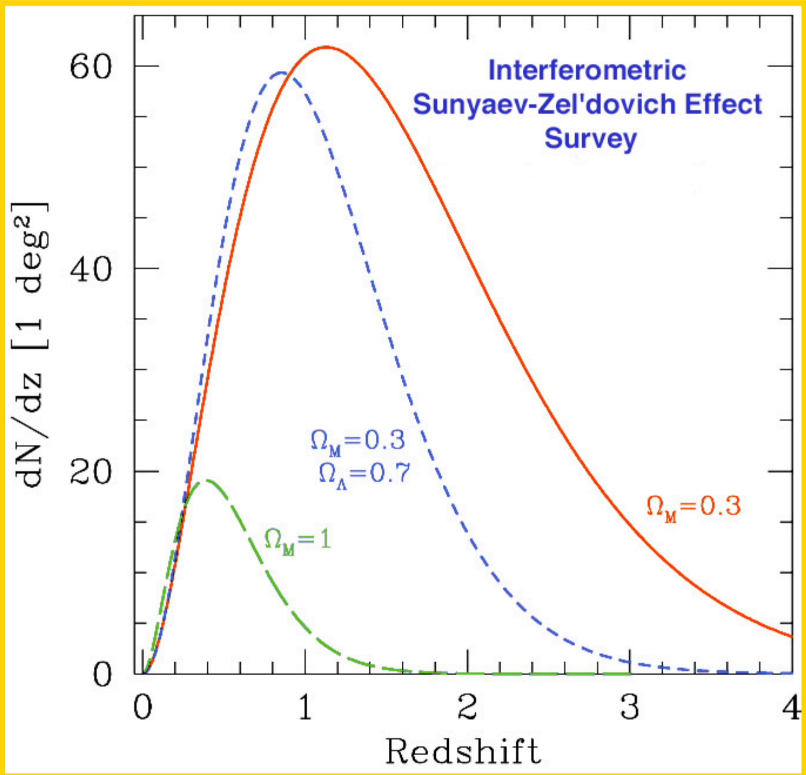
The Galaxy Cluster Redshift Distribution …

$$\frac{dn}{dz\,d\Omega} = \frac{c}{H(z)} d_A^2 (1+z)^2 \int_{M_{\lim}}^{\infty} dM \frac{dn(M,z)}{dM}$$

Cosmo dependent volume

detection limit

Space density of haloes



Holder etal, 2000

Cluster redshift distribution probes:

1) **volume-redshift relation**
2) **abundance evolution**
   (both depend explicitly on cosmological parameters)

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3) **cluster structure and evolution.**
   (depends implicity on cosmology)

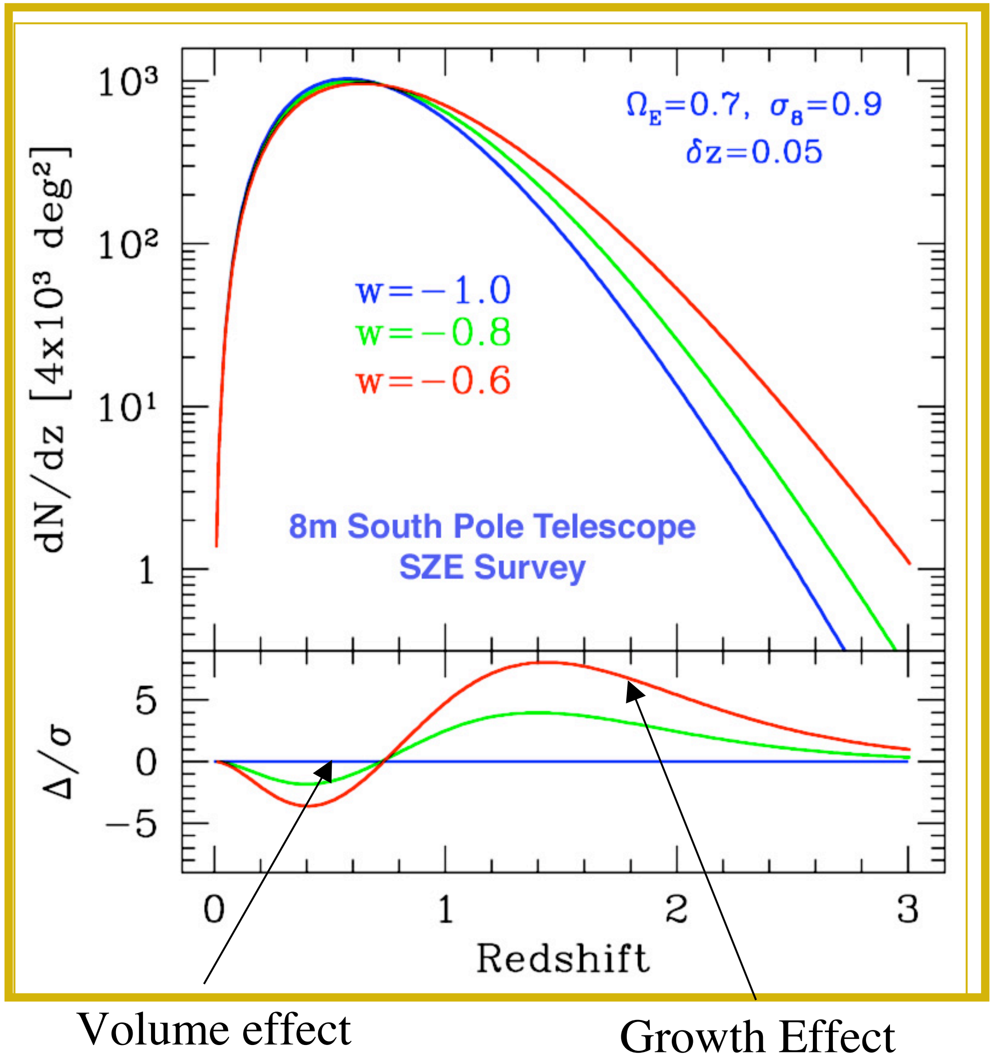
Constraining the different energy densities in cosmology.

Sensitivity of Cluster Redshift Distribution to Dark Energy Equation of State

w constraints:

Increasing w keeping Ω_E fixed has The following effects:

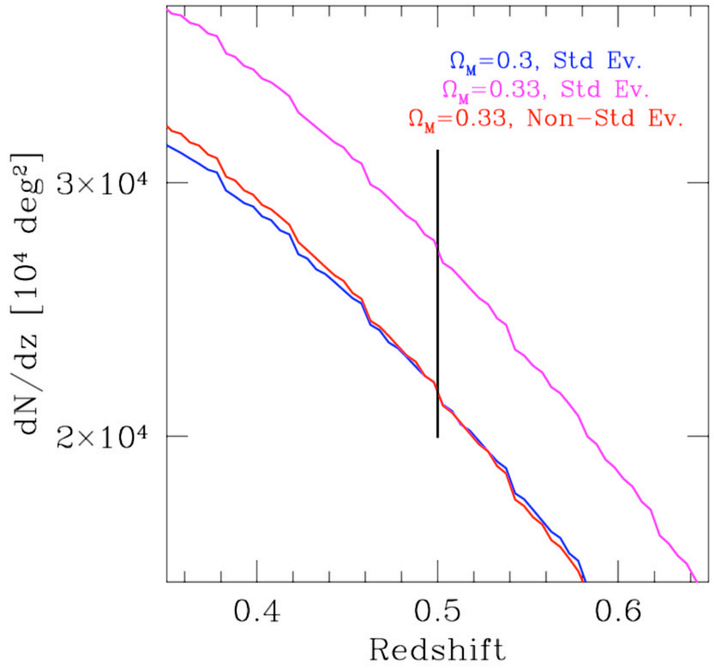
- **it decreases volume surveyed**
- **It decreases growth rate of density perturbations**

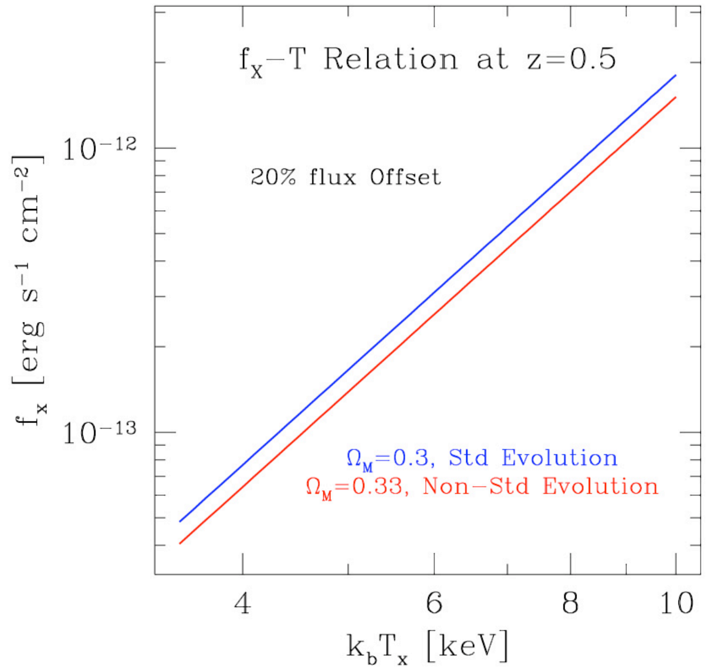


Perils of an uncertain evolution…

Introducing a non-standard evolution model to offset a change of $\delta\Omega_m$=0.03 leads to a 20% offset in the X-ray flux- temperature (fx-Tx) relationship for the clusters in this z=0.5 redshift bin.

Assuming scatter in Lx-T of 50%, the 200 clusters with measured Tx in this redshift bin would provide enough information to discern this shift with great confidence (~6σ significance).





A similar prescription to include non-standard evolution: Add $(1+z)^\gamma$ to cluster scaling

Mass Calibration is needed!
(Direct and Indirect Calibration/Self Calibration) …

1.Limited mass follow-up (using full hydro equilibrium/weak lensing)

(Majumdar & Mohr 2003,2004, Majumdar 2005)

2.Using shape of mass-function in redshift slices

(Hu 2003)

3.Using the cluster power spectrum and P(k) oscillations

(Majumdar & Mohr 2004, Hu & Haiman 2004, Huetsi 2005)

4.Adding information from counts-in-cell

(Lima & Hu 2004, 2005)

5.Time or flux slicing of survey: using shape of dndz

(Majumdar 2007, in prep)

6. For SZ surveys, adding SZ rms distortions to number counts

(Diego & Majumdar, 2004)

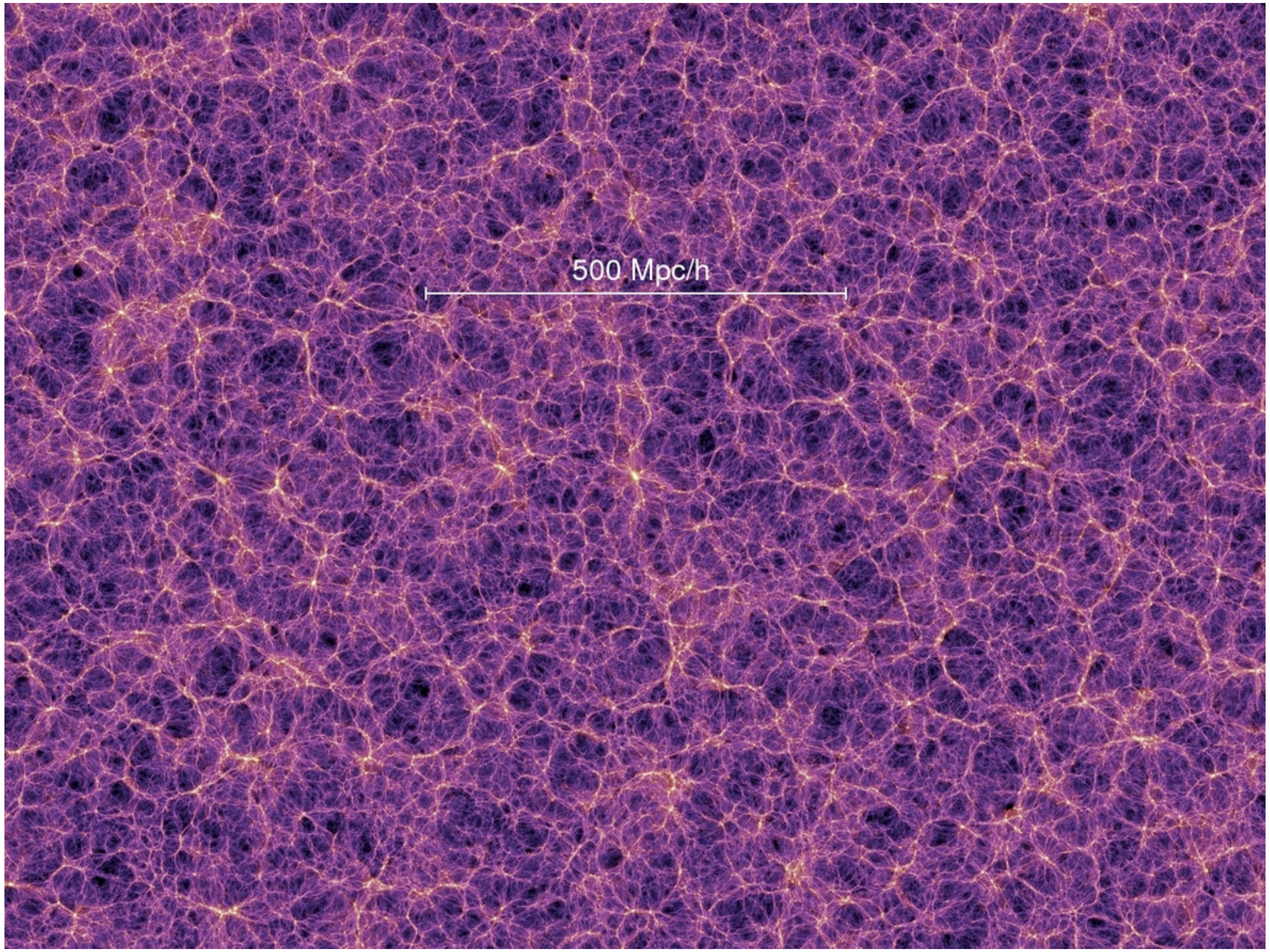
7. Scatter is self-calibrated using both dndz and mass (flux) binning

(Lima & Hu, 2005)

8. Having a subset of clusters observed in both SZ & Xray (assumption on cluster structure crucial)

(Molnar etal, 2004, Khedekar & SM in prep)

Correlations - the distribution is not random

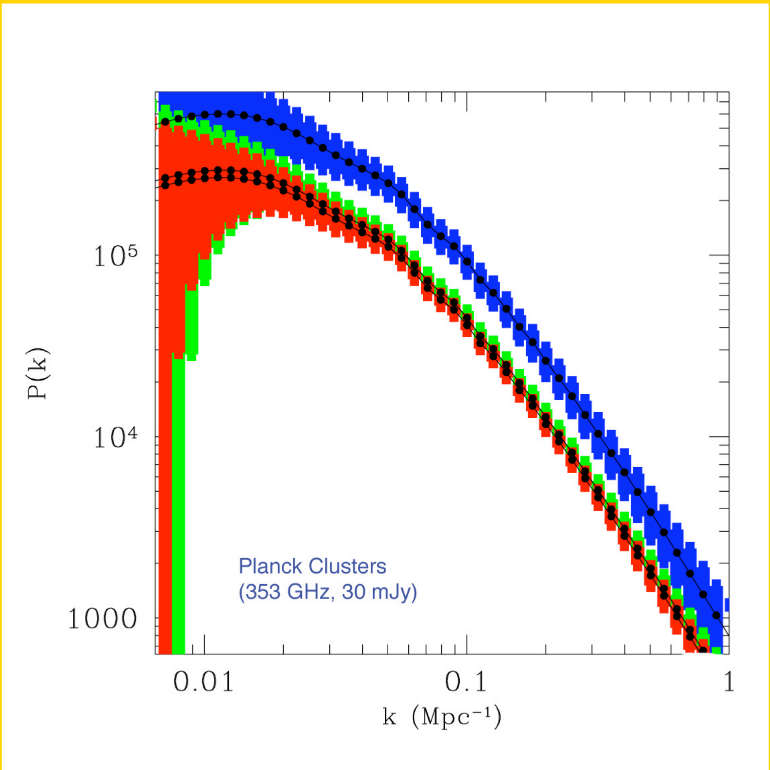


One can estimate 2D, 3D correlation fns and fourier transform them to get the halo power spectrum (more in Dinesh's talk)

Power spectrum of the cluster sample ..

Power spectrum of dark matter density fluctuations P(k)

- **Clusters are biased: 20,000 clusters comparable to ~5x10^5 galaxies**
- **Turnover on large scales- "standard rod" calibrated by primary CMB fluctuations**



From redshift surveys, we will get P(k) *for free* !

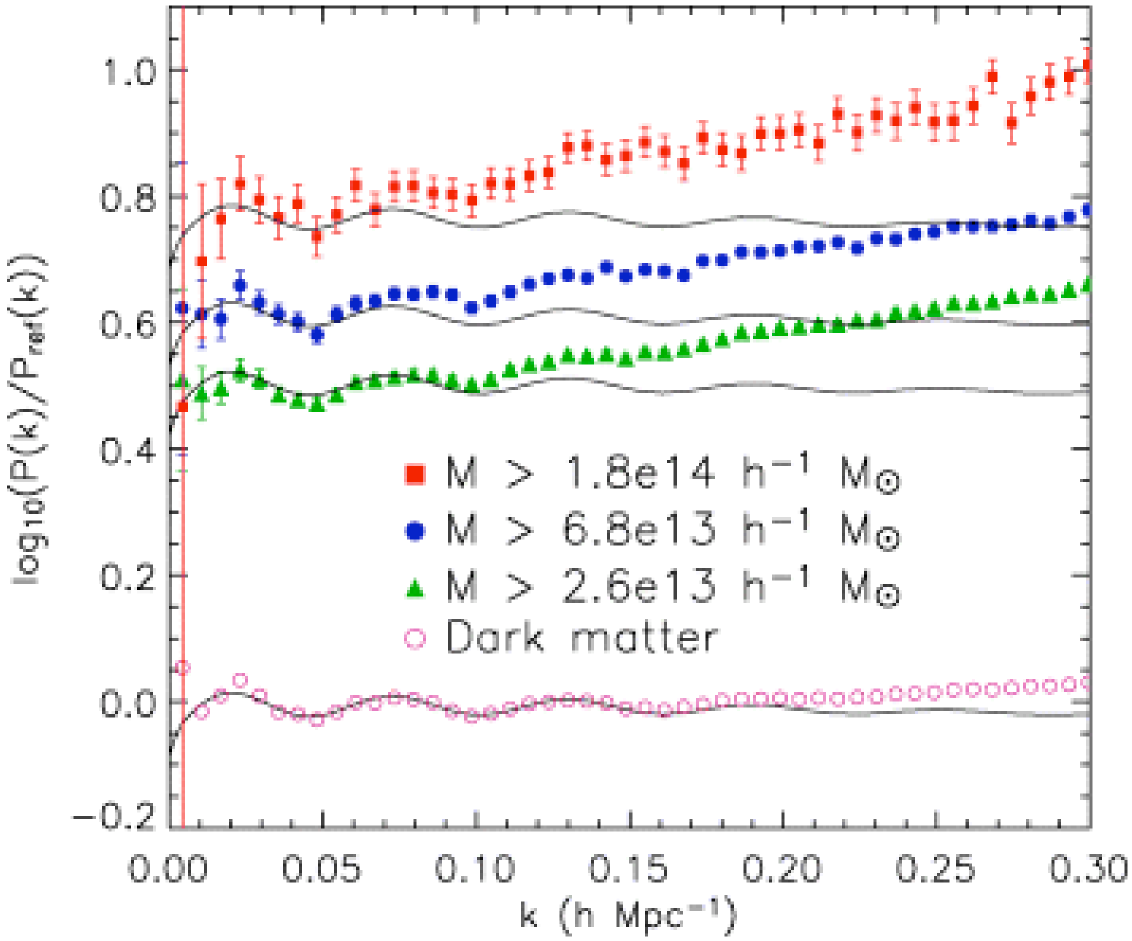
Tight constraints with dn/dz + P(k) together.
Acts like 'internal Mass-calibration' !!

Do Not miss Dinesh Raut's Talk for "real results"

SM & J.Mohr 2003, 2004, Hu 2004, Hu & Haiman 2003

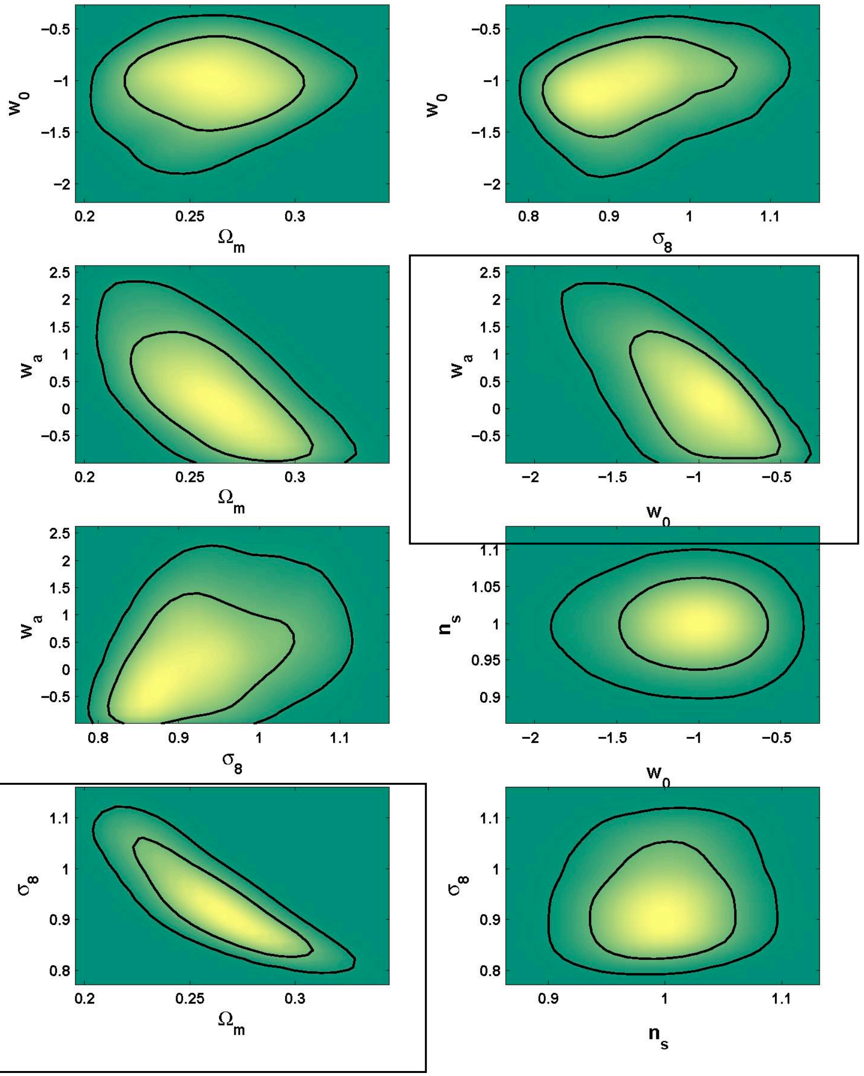
Why did P(k) help in internal mass calibration ?

Ans: Bias is scale (mass dependent) --> rarer peaks in a Gaussian Random field are more biased.



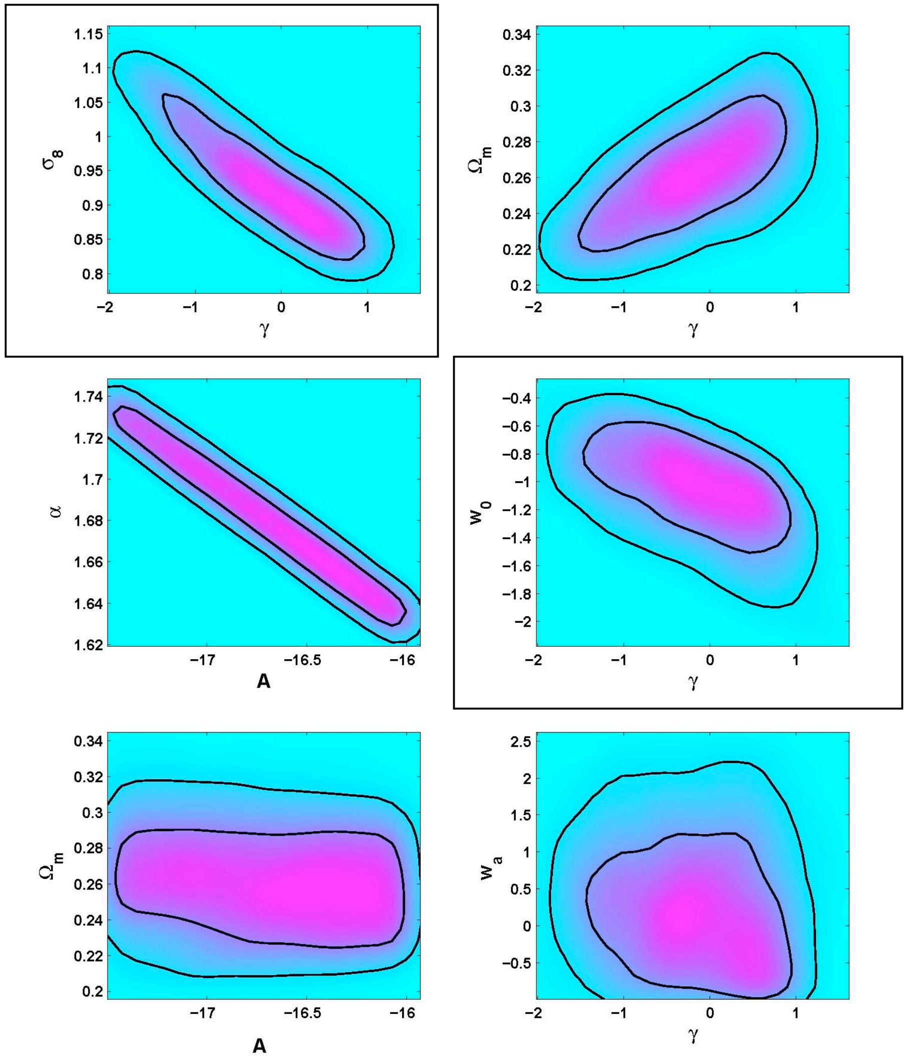
Anguilo etal 2005 simulations

Systematics in cluster probes of DE...



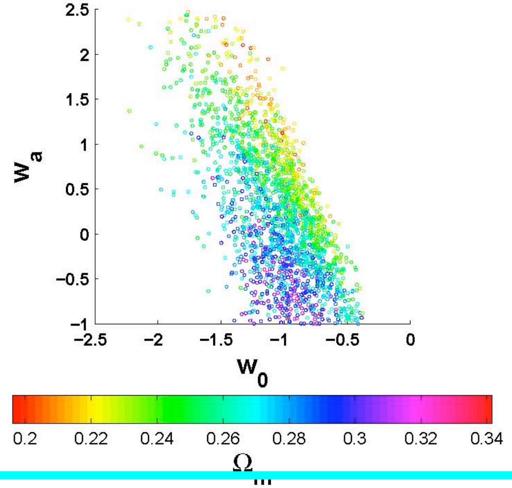
cosmological

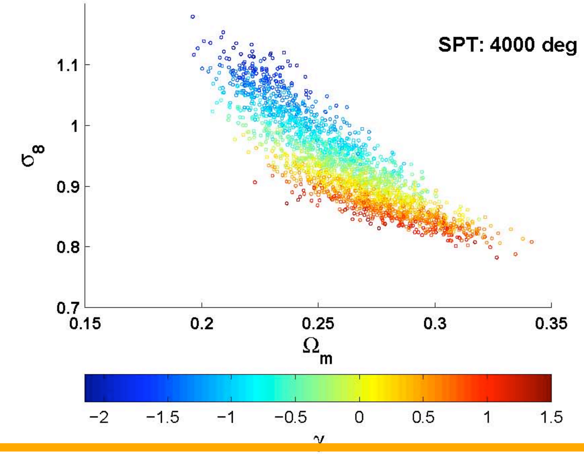
γ → uncertain evolution M-Flux

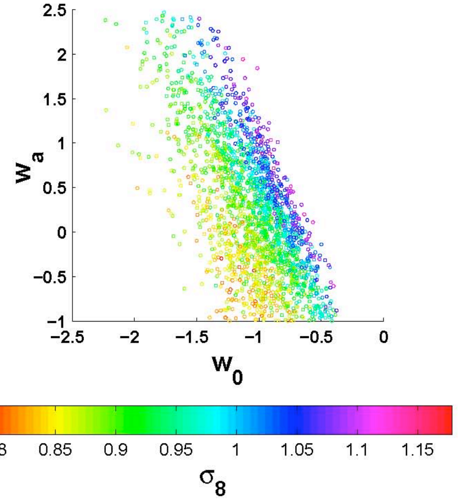


astrophysica

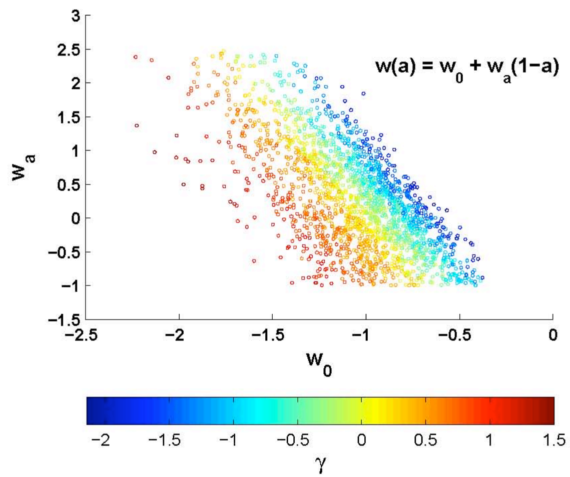
Need for extra information..





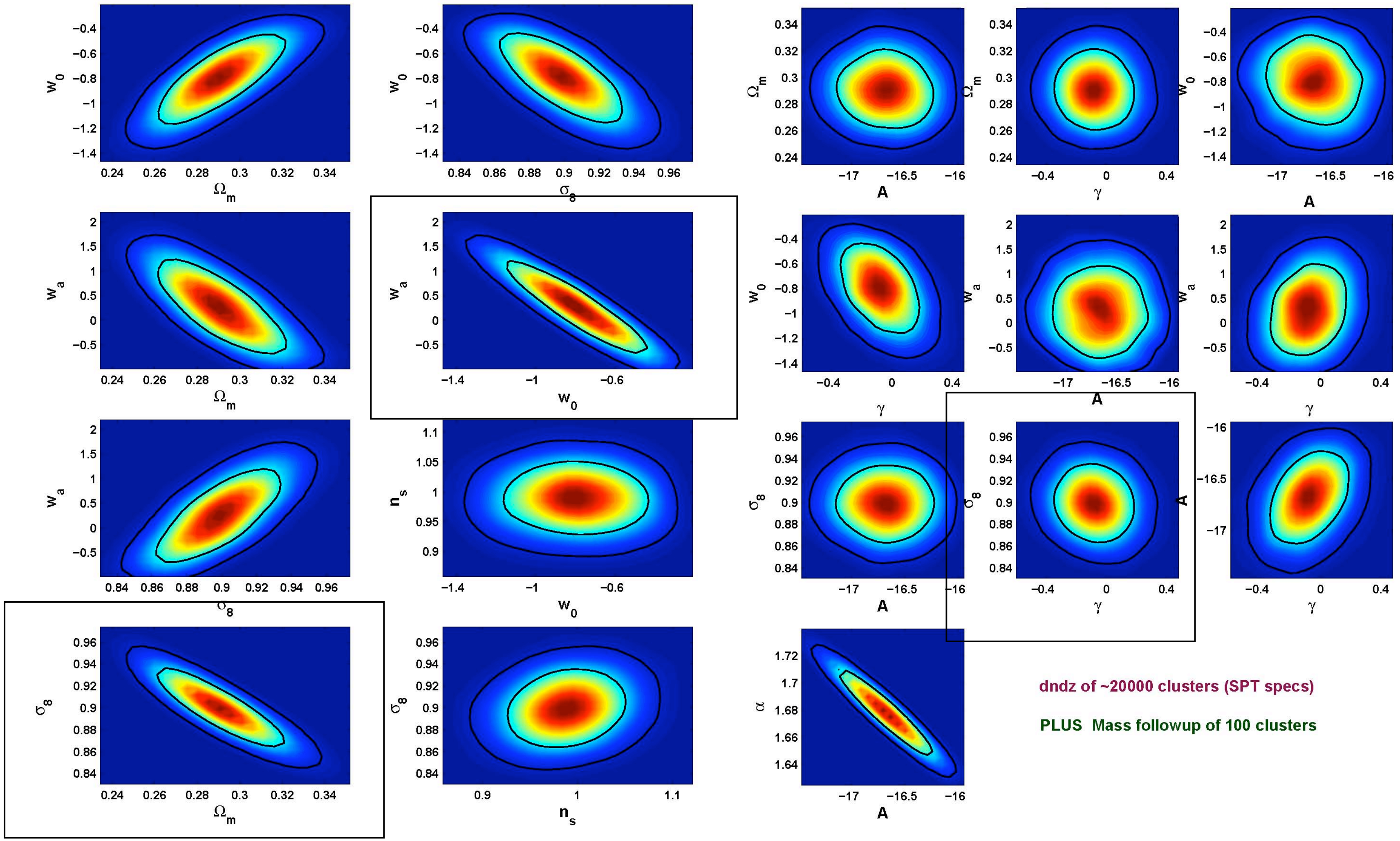


CMB is needed

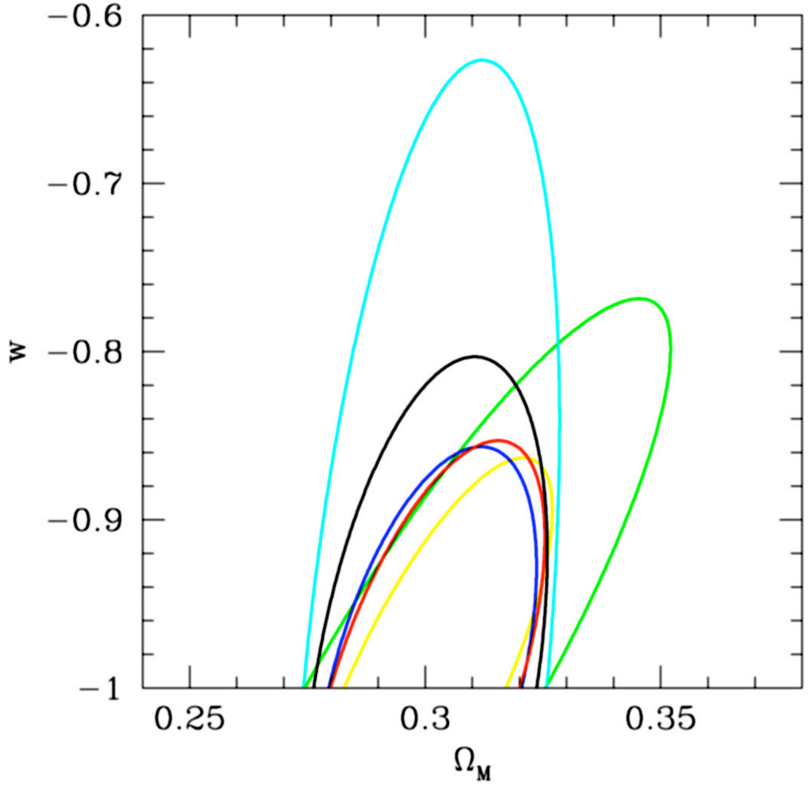


High-z obs needed, training set

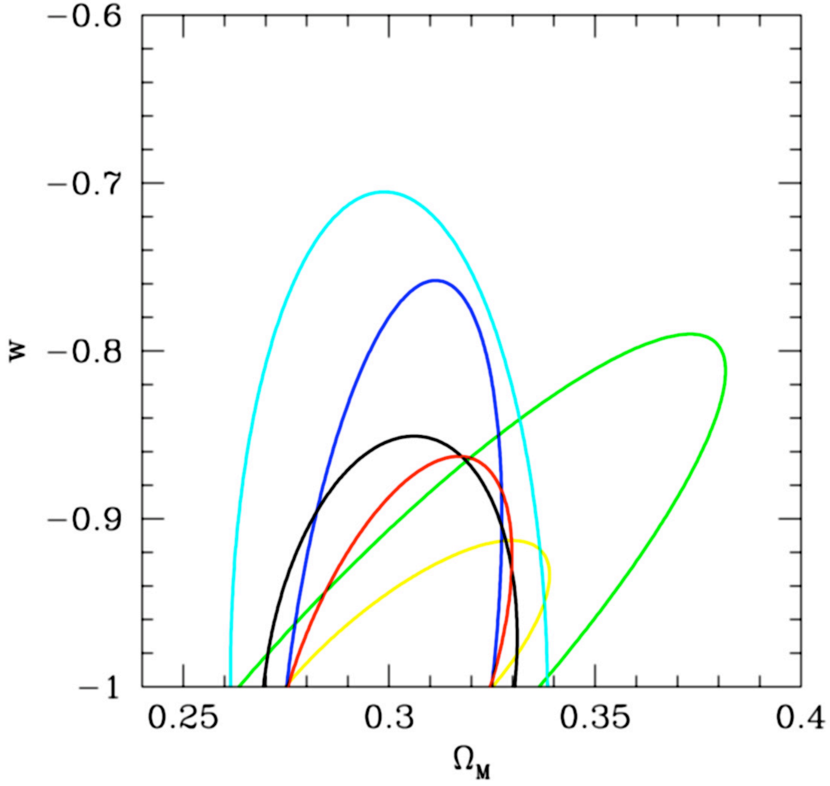
Effect of external mass calibration:



Voila! The effect of calibration…

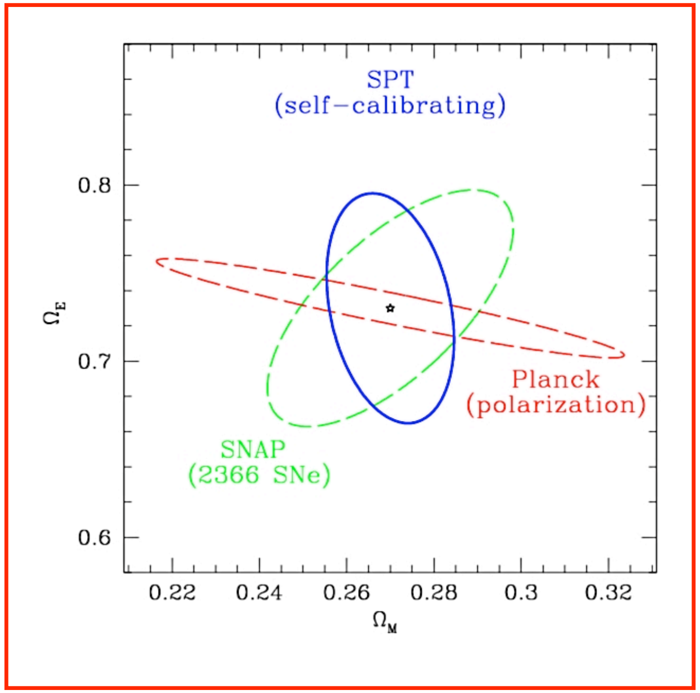


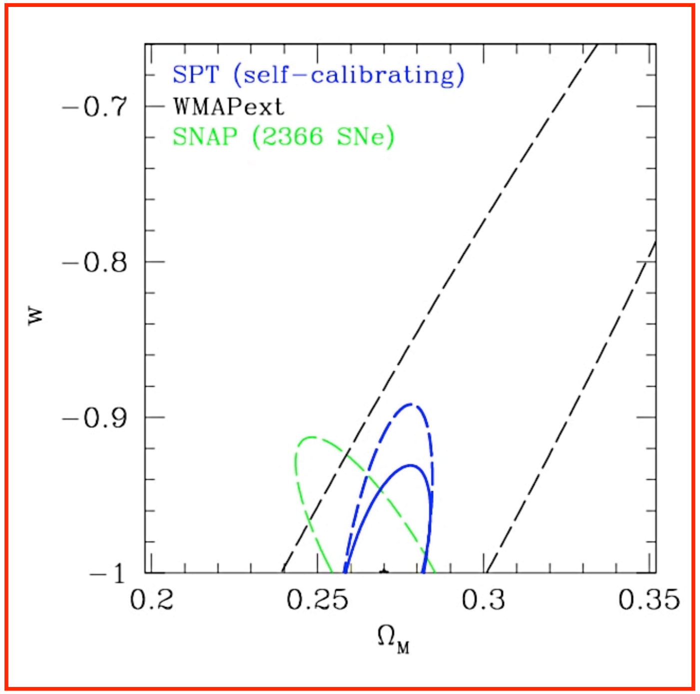
Planck



South Pole Telescope

And the hope is tight cosmological constraints….

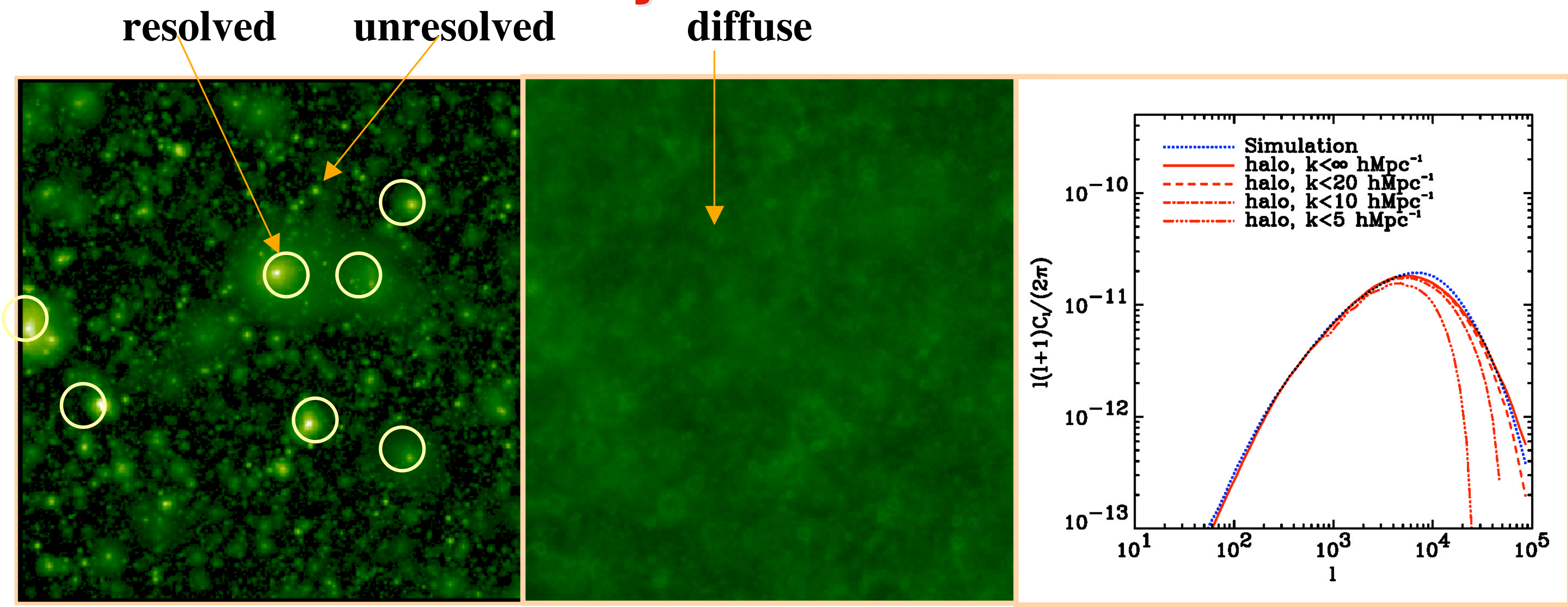




Example: The amount and nature of DE

PART THREE:
WHEN THE CLUSTERS ARE UNRESOLVED …

Contribution to temperature anisotropy from SZ only simulation…

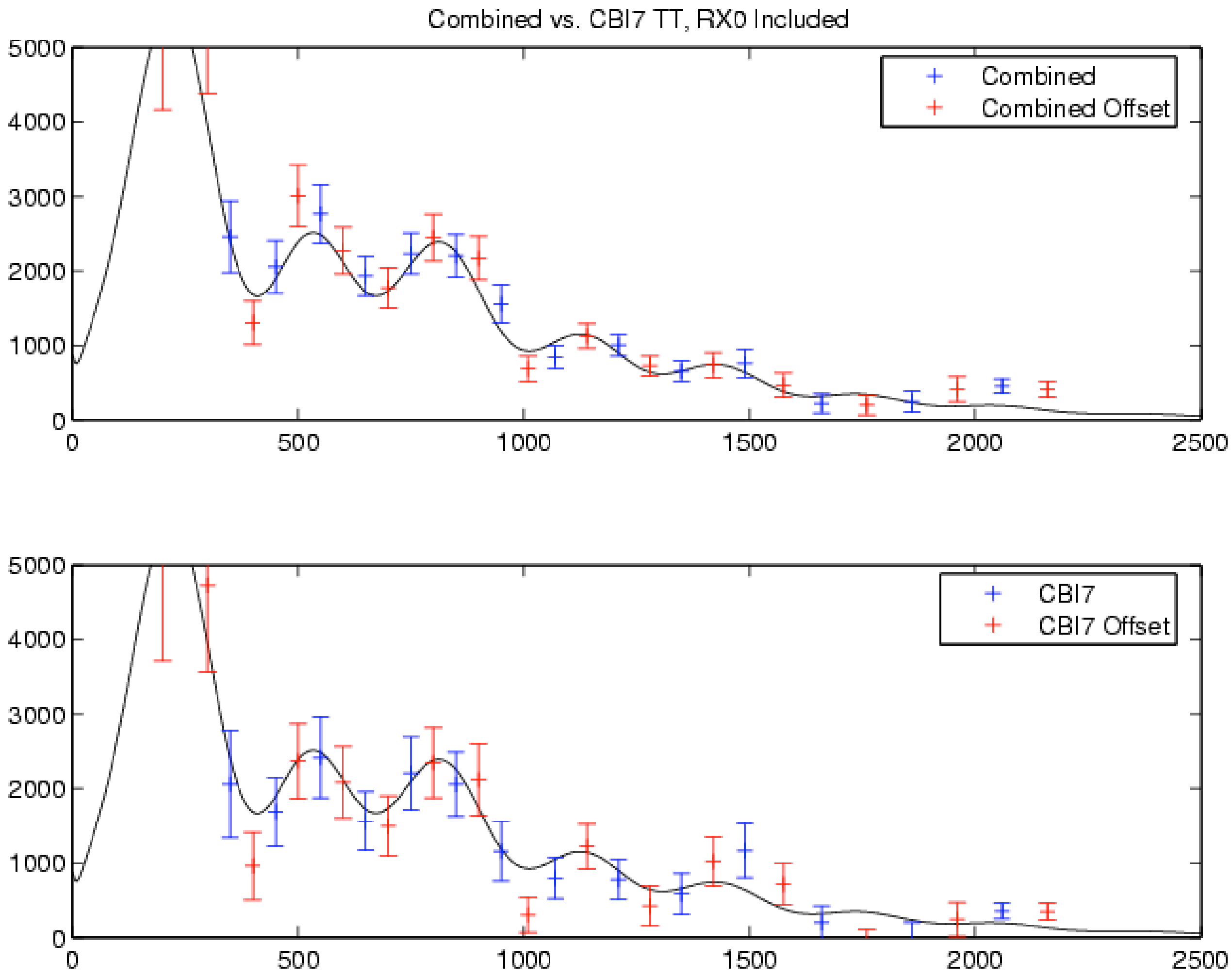


Contribution to the final SZE anisotropy will be from both diffuse gas and unresolved haloes

Simulations of SZE captures contribution from both. Analytical works depend on the so called `halo model'

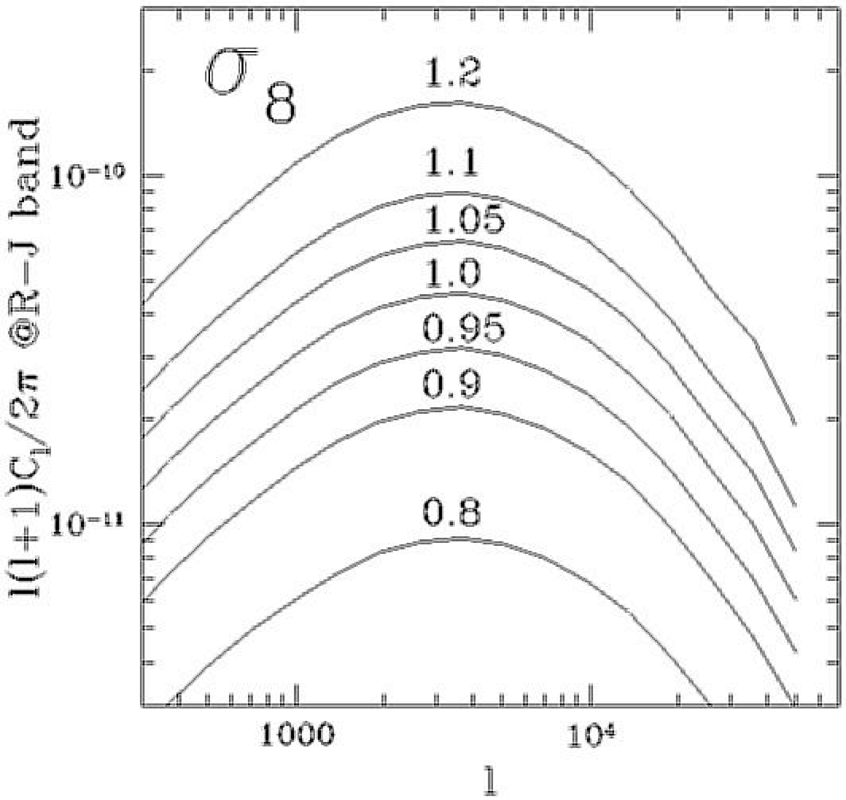
da Silva etal 2001, Refregier & Tessier 2002

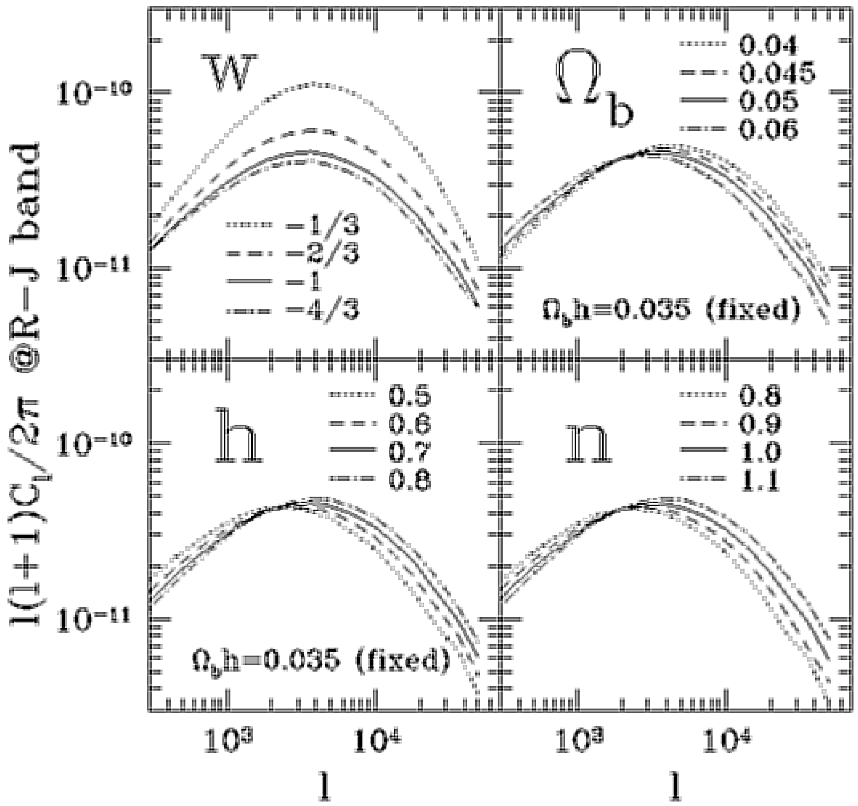
Beyond WMAP range - the rule of the Secondaries...



Latest CBI data

The cosmological power of SZ power spectrum:





$C_l \sim \sigma_8^{6-7}$

Komatsu & Seljak 2002

Effect of other parameters are subdued under the effect of σ_8.
But can break degenracies.

Calculating the SZ fluctuations:

Adding cluster form factor to cluster distriution:

$$C_l = g_\nu^2 \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz} \int_{M_{\min}}^{M_{\max}} dM \frac{dn(M,z)}{dM} \left| y_l(M,z) \right|^2$$

Start with some analytic/simulation pressure profile of cluster (normalized to the total mass of the cluster), say **P_{gas}(M, z, x=r/r_0,)**

$$P_{gas}(M,x,z) \propto \rho_{gas}(M,x,z) k_B T_{gas}(M,x,z)$$

Then, the 3D Compton y-parameter is

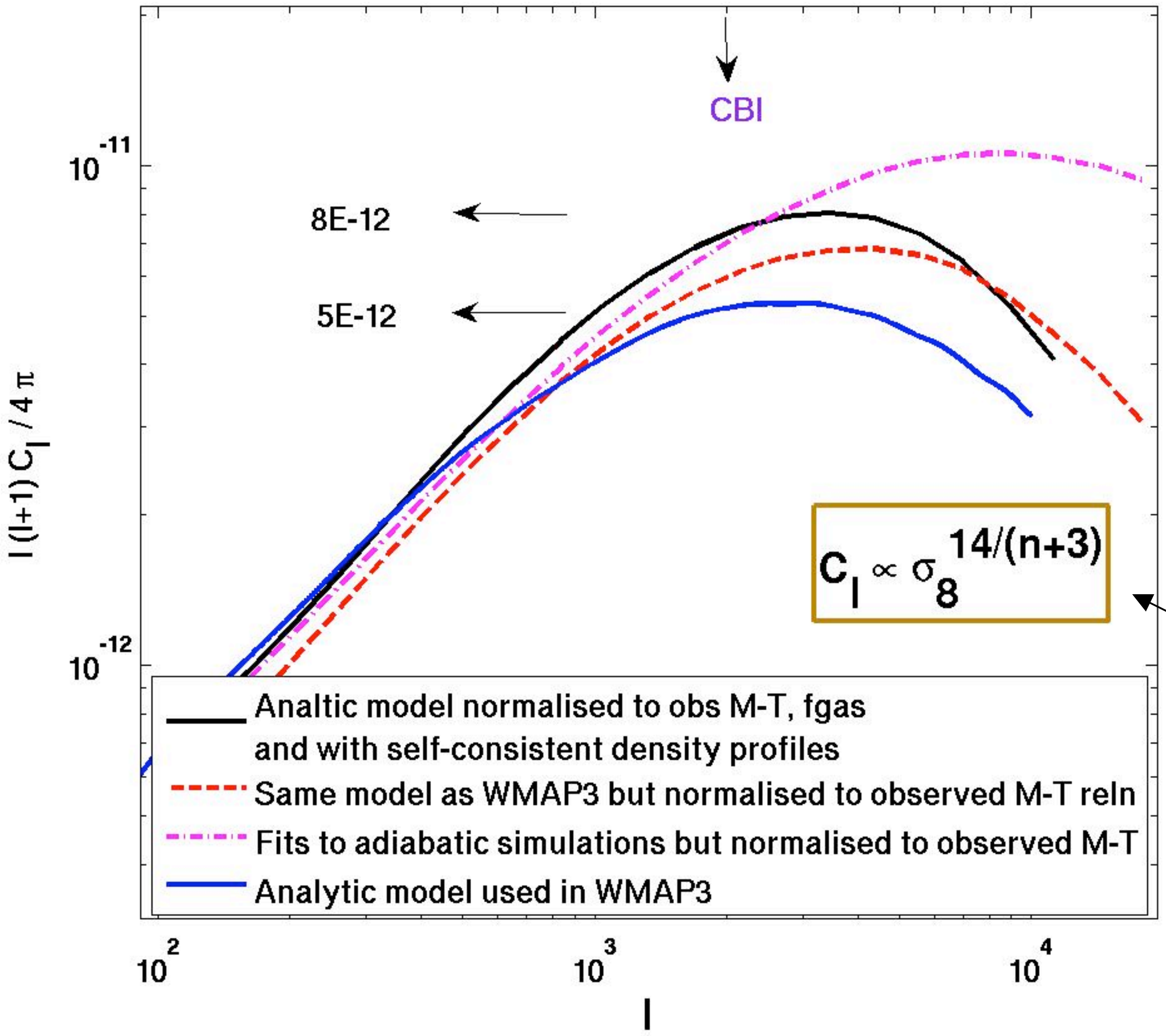
$$y(M,x,z) = \frac{\sigma_T}{m_e c^2} P_{gas}(M,x,z)$$

Its 2D fourier transform is

$$y_l(M,z) = \frac{4\pi r_0}{l_0^2} \int_0^\infty dx x^2 y(M,x,z) \frac{\sin(lx/l_0)}{lx/l_0}; l_0 = \frac{d_A}{r_0}$$

subha@tifr

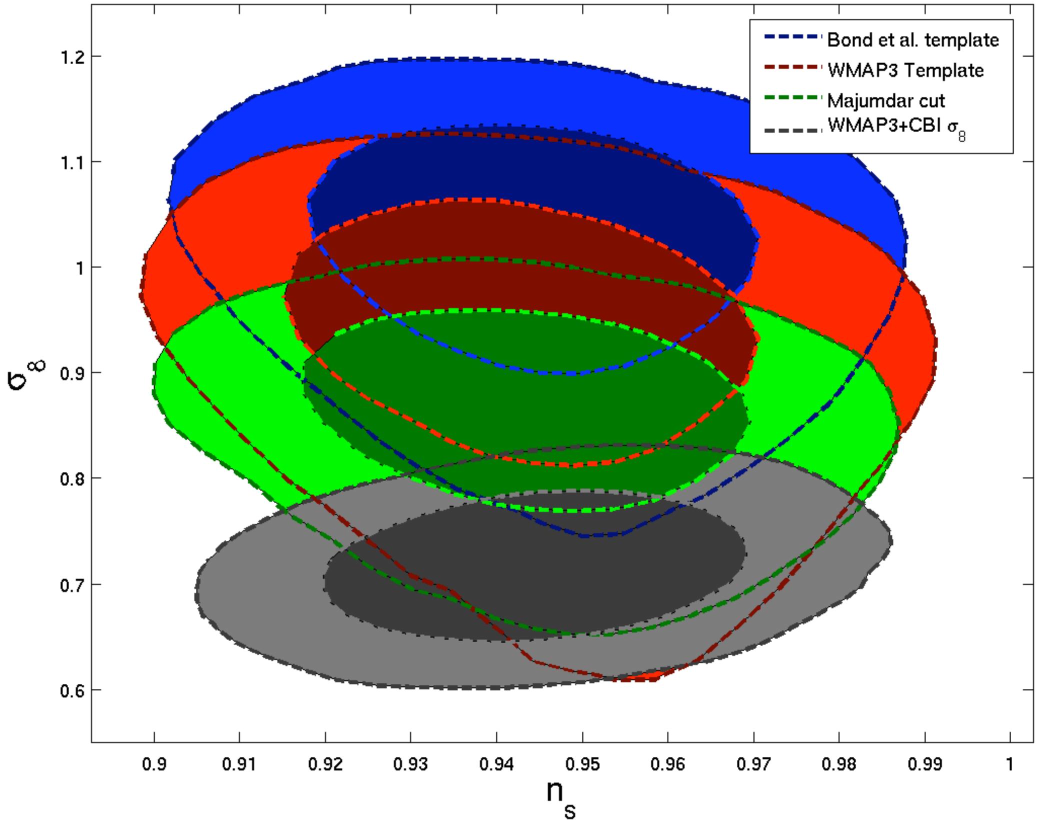
Phenomenological modelling of SZ fluctuations…



SM, Sievers etal

More weightage on lower mass haloes changes the C_l-σ_8 scaling reln.

Improved' Inferred σ_8



Inferred σ_8 **= 0.83 ± 0.07**

consistent with WMAP3 to 1-sigma

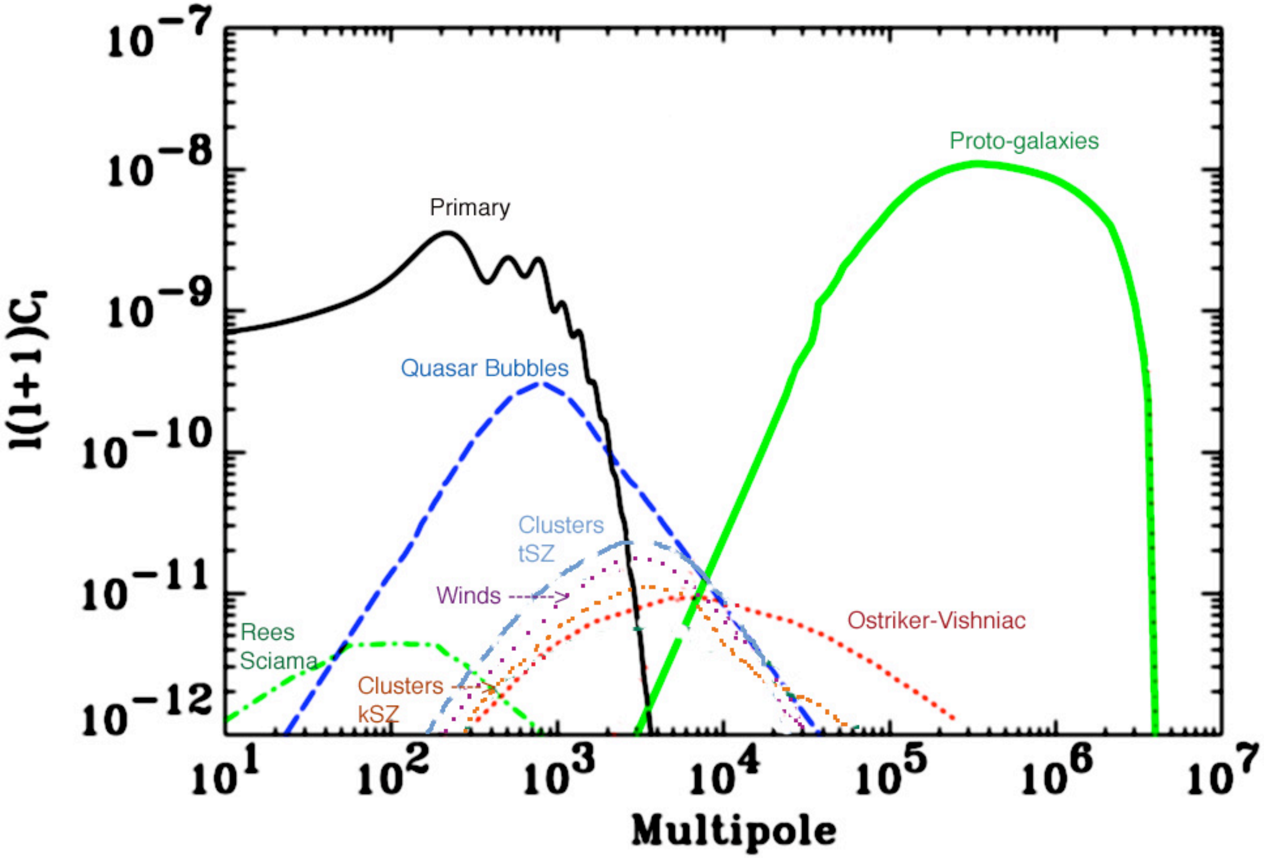
(Our result mellows down a long standing discordance In cosmology Previous,
consistent at 2-sigma with WMAP3.)

SM, Sievers etal

Its still a simplified treatment….

The future …

The secondary CMB anisotropy from clusters is just the tip of the iceberg. The secondary avalanche is coming. Let me leave you with the possibilities for the future!



Aghanim, SM & Silk, 2007

Conclusions -

Compton scattering of CMB photons by hot intracluster gas gives secondary distortions. The clusters appear as holes in the sky at freq < 217 GHz. Many many observational efforts now.

If these holes (clusters) are fully resolved, then one can use them as standard rulers. This gives us clue on the expansion history of our Universe.

If the clusters are not fully resolved, but only detected, then their number density and distribution depends on cosmology. Can be used as strong probes (Example of DM & DE).

If the clusters are not resolved, they still contribute to new CMB temperature anisotropies at arc-min scales. The power spectrum is sensitive to cosmology (Example of sigma8).

Future: Behold the emergence of the secondaries!

Studying 'Holes in the Sky' (their distribution, their imprint on CMB fluctuations etc) will continue to be a major intiative in both theoretical and observational cosmology in the coming decade.