Ultraviolet and Infrared Divergences in Superstring Theory

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1. Summary of this talk

2. Review of ultraviolet (UV) and infrared (IR) divergences in quantum field theory (QFT)

3. Absence of UV divergence in superstring theory

4. Recent progress on understanding IR divergences in superstring theory

1. Summary of the talk

QFT's are the standard tools for describing the physics of elementary particles.

– a tool for computing physical quantities, e.g. scattering amplitudes of elementary particles.

But most QFT's suffer from UV and IR divergences

– infinities that appear in the expressions for various physical quantities – unless we are careful.

UV divergences arise from quantum fluctuations of small wavelength modes, and are ‘bad’

– must be eliminated in order to get a sensible theory.

There is a class of QFT’s where UV divergences can be removed by a standard procedure known as renormalization.

– renormalizable QFT.

We use only these kinds of QFT’s for describing theories of elementary particles.

IR divergences arise from quantum fluctuations of long wavelength modes and have physical origin

– indicates that we are asking the wrong question.

e.g. they arise when we do not take into account the effect of change of quantum ground state and/or masses of elementary particles due to interaction.

⇒ tadpole divergences and mass renormalization divergences.

Once we ask the right questions, these divergences automatically disappear.

QFT's come with an in built mechanism that tells us how to ask the right questions and get rid of the IR divergences.

Gravity

Applying standard QFT techniques to general theory of relativity runs into difficulties with UV divergence.

The theory is not renormalizable.

Superstring theory resolves this problem by regarding the elementary constituents of matter as one dimensional objects – strings.

This theory has no UV divergences!

– gives finite results without any need of renormalization.

However superstring theory has IR divergences similar to those which appear in QFT's.

Since IR divergences in QFT's disappear once we ask the right questions, one might expect that the same may be true in superstring theory.

However conventional formulation of superstring theory does not tell us how to ask the right questions so that we get finite answers.

e.g. how to systematically take into account the effect of change of the quantum ground state and/or masses of 'elementary particles' due to interaction.

Often we work with

1. Ground states with large amount of supersymmetry which do not get modified due to interactions

2. Massless particles whose masses are not modified by interactions due to symmetry.

In such cases there are no IR divergences.

But for ground states with less supersymmetry and/or massive particles we do have IR divergences that need to be removed

– a necessary criteria for a consistent theory.

Recent progress

Construction of a quantum field theory whose scattering amplitudes agree with that of superstring theory.

– superstring field theory

This theory is free from UV divergences but has all the IR divergences of superstring theory.

However, since this is a QFT, there is a systematic procedure for taking into account shift of quantum ground state / masses due to interaction

$\Rightarrow$ results free from IR divergences.

Conclusion

We now have a formulation of superstring theory that gives results free from UV and IR divergences.

2. UV and IR divergences in QFT

Most commonly used approach for studying scattering amplitude in QFT's is perturbation theory.

Take all the interaction effects to be small and carry out a Taylor series expansion in the parameters that label the interaction strengths.

The coefficients of the Taylor series expansion are given by sum of Feynman diagrams.

In d space-time dimensions, contribution from a typical Feynman diagram looks like

$$\int \mathbf{d^d}\ell_1 \cdots \mathbf{d^d}\ell_\mathbf{g} \prod_{\mathbf{j=1}}^{\mathbf{r}} (\mathbf{k_j^2} + \mathbf{m_j^2})^{-1} \mathcal{N}$$

each ℓ_i: a d-dimensional vector labelling loop momenta

each $\mathrm{k_j}$: a d-dimensional vector given by appropriate linear combination of the ℓ_i's and $\mathrm{p_1}, \cdots \mathrm{p_n}$

$\mathrm{p_1}, \cdots \mathrm{p_n}$: the momenta carried by the incoming and outgoing particles whose scattering amplitude we are trying to calculate

$\mathrm{m_j}$: the mass of one of the particles in the theory

$\mathcal{N}$: a polynomial in $\{\ell_\mathrm{i}\}$ and $\{\mathrm{p_k}\}$

$$\int \mathbf{d^d}\ell_\mathbf{1} \cdots \mathbf{d^d}\ell_\mathbf{g} \prod_{\mathbf{j=1}}^{\mathbf{r}} (\mathbf{k_j^2} + \mathbf{m_j^2})^{-\mathbf{1}} \mathcal{N}$$

UV divergences: divergences from the region of integration where one or more of the $\ell_\mathbf{i}$'s become large

IR divergences: arise from the vanishing of one or more factors of $(\mathbf{k_j^2} + \mathbf{m_j^2})$

$$\int d^d\ell_1 \cdots d^d\ell_g \prod_{j=1}^{r} (k_j^2 + m_j^2)^{-1} \mathcal{N}$$

1. Use $(k_j^2 + m_j^2)^{-1} = \int_0^\infty ds_j \exp[-s_j(k_j^2 + m_j^2)]$

2. Carry out integration over ℓ_j's explicitly using rules of gaussian integration

Result

$$\int_0^\infty ds_1 \cdots \int_0^\infty ds_r\, F(\{s_i\})$$

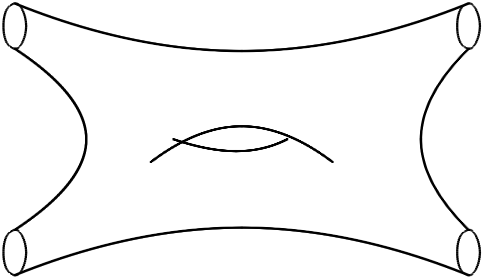
for some function $F(\{s_i\})$.

UV divergence: one or more $s_i \to 0$

IR divergence: one or more $s_i \to \infty$

3. Absence of UV divergence in superstring theory

Just as a particle trajectory gives a curve in space-time, the trajectory of a string gives a surface in space-time.



⇒ simple expression for scattering amplitudes

$$\textbf{scattering amplitude} = \sum_{g=0}^{\infty} \int_{M_{g,n}} \mathcal{I}_{g,n}$$

$M_{g,n}$: moduli spaces of two dimensional Riemann surfaces of genus g with n marked points

Different points in $M_{g,n}$: Surfaces of different shape, each of genus g and with n marked points

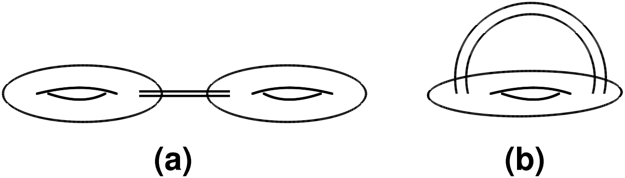
genus g: number of handles of the surface

n: total number of incoming and outgoing particles

Integrand $\mathcal{I}_{g,n}$: depends on the states that are being scattered and also the coordinates of $M_{g,n}$

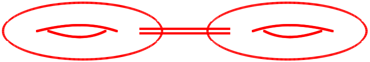
Possible divergences now come from divergences in the integration over $M_{g,n}$

– arise from singular Riemann surfaces

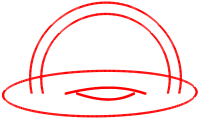


– the Riemann surface either becomes a pair of Riemann surfaces connected by an infinitely narrow tube (a)

or develops an infinitely narrow handle connecting two points on a single Riemann surface (b)



(a)



(b)

In this limit the integration over $M_{g,n}$ resembles integration over the parameters s_i in the QFT's with

$$s_i \sim 1 \,/\, \text{radius of the narrow tube}$$

In the singular limit, radius of the tube → 0

$s_i \to \infty$

– IR divergence

This shows that all divergences in string theory are IR divergence and there are no UV divergences in the theory.

However unlike in a QFT, conventional superstring perturbation theory does not give us a systematic mechanism for removing IR divergences.

4. Recent progress on understanding IR divergences in superstring theory

If we could construct a QFT whose scattering amplitudes give us the amplitudes of superstring theory, then we would have a systematic procedure for removing IR divergences in string theory.

– had been attempted earlier

– successfully formulated for a cousin of superstring theory – the bosonic string theory. **Zwiebach**

For superstrings there is an apparent no go theorem.

Low energy limit of a superstring theory gives type IIB supergravity for which we cannot write down an action.

Resolution

It is possible to construct a QFT that gives the correct scattering amplitudes of string theory, but contains an additional set of particles which are free.

Pius, Rudra, A.S.; A.S., Witten; A.S.

These additional particles are unobservable since they do not scatter.

Scattering amplitude for the interacting part is given by a sum of Feynman diagrams as in conventional QFT's.

Each Feynman diagram gives integration over a part of $M_{g,n}$, and the sum of all contribution gives integral over full $M_{g,n}$.

All IR divergences come from $s \to \infty$ limit for one or more propagators as in conventional QFT's.

On the other hand this theory has no UV divergence since its scattering amplitudes are the same as that of string theory.

With the help of this theory one can successfully remove the IR divergences of the theory following the usual procedure followed in a QFT.

Therefore we have a formulation of string theory free from all divergences.

Structure of the action

Two sets of string fields, ψ and ϕ

Each is an infinite component field, represented as a vector

Action takes the form

$$\mathbf{S} = \int \left[-\frac{\mathbf{1}}{\mathbf{2}}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

Q, X: commuting operators (matrix with differential operators as entries)

(,): Lorentz invariant inner product

f(ψ): a functional of ψ describing interaction term.

$$\mathbf{S} = \int \left[-\frac{1}{2}(\phi, \mathbf{Q}\,\mathbf{X}\,\phi) + (\phi, \mathbf{Q}\,\psi) + \mathbf{f}(\psi) \right]$$

Equations of motion:

$$\mathbf{Q}(\psi - \mathbf{X}\,\phi) = 0$$

$$\mathbf{Q}\phi + \mathbf{f}'(\psi) = 0$$

first + X × second equation gives

$$\mathbf{Q}\psi + \mathbf{X}\,\mathbf{f}'(\psi) = 0$$

ψ**: interacting fields,** $\mathbf{X}\phi - \psi$**: free fields**

Quantization of ψ gives the usual scattering amplitudes of string theory while quantization of $\mathbf{X}\phi - \psi$ produces particles which do not scatter.

Future prospects

Once we have a QFT description of string theory scattering amplitudes, we can use QFT methods to prove various other desirable features of the scattering amplitudes.

1. Unitarity (conservation of probability)

2. Crossing symmetry, analyticity etc.