### **GR**: Indian Contributions

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Following 1919 brilliant observational support to GR and Eddington's pronouncement

Einstein is right

M N Saha wrote an article on Bending of Light in Kolkata daily, Statesman, explaining this revolutionary phenomenon to public at large

An excellent example of scientist-society interaction and indication of enthusiastic reception of GR

In 1911, there were three brilliant students in Kolkata Univ: S N Bose (boson), M N Saha (ionization equation) & N R Sen (First GR School) N R Sen (NRS) did Ph D with Von Laue at Berlin and established first GR School in Kolkata This is how it all began

### Second School

- In early 1930s, Vishnu Vasudeo Narlikar (VVN father of JVN)
- was a brilliant student at Cambridge
- mentored by Eddington, Fowler, ...
- In 1915, M M Malviya (MMM) established BHU and he had heard of laurels of VVN

- He visited Cambridge to invite VVN to BHU
- In 1932 he had a fellowship from Mt Wilson Caltech, before
- going to the other end of Atlantic,
- he came home for a visit and went to see BHU

MMM showed him around and invited him to lead Maths dept as Professor and Head at the young age of 23 The offer he couldn't refuse He established the second school at Banaras NRS and VVN were the pioneers in GR research and its spread in the country

# Kolkata School Contributions

- B Datt was the first to study homogeneous gravitational collapse in 1938, an year earlier than Oppenheimer and Snyder with general initial velocity prescription (Golden Oldies)
- Nothing heard from him after this remarkable paper
- Who was B Datt a mystery till recently
- He was NRS student but died immediately after the collapse paper

- like his illustrious predecessors, Schwarzschild and Friedmann
- Truly homogeneous collapse should be known as
- Datt-Oppenhaimer-Snyder collapse

NRS obtained exact solutions of Einstein equation for various matter sources including charged fluid

Constructed a static cosmological model as an Einstein cluster having co-counter rotating dust particles with zero total angular momentum. He showed that it would be stable provided its mass is larger than the corresponding mass of Einstein Universe (Golden Oldies)

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Bose initiated work in unified field theory

#### Banaras School

Prahalad Chunnilal Vaidya (PCV) landed in Banaras to work with VVN without any fellowship and left after 10 months when his savings ran out

VVN asked him to find solution for a radiating star, a null radial flow off a spherical massive body

There were three equations, one was solved by VVN and left the other two for PCV to solve

After a week when they met, PCV had the full solution – the

famous Vaidya solution of radiating star (Golden Oldies)

VVN declined to be coauthor – an excellent benchmark example of academic integrity

### 14 Invariants

Mathematician T Y Thomas had shown that 4-dimensional Riemannian space had 14 independent curvature invariants But they were not explicitly constructed In 1949, VVN and K R Karmarkar constructed them explicitly which were rediscovered by Geheniau and Debever in 1956 In 1972, A R Prasanna in a conference in Trieste brought this fact to the notice of Geheniau who readily agreed that these should be called Narlikar-Karmarkar invariants

#### Postscript

In 1930 VVN while at Cambridge obtained homogeneous isotropic expanding model and Eddington offered to communicate it to MNRAS

When the paper was written up, Eddington came across a paper of Lemaitre who in 1927 had done the same work

It was really unfortunate

Lemaitre was also anticipated by Friedmann in 1922

In 1940s VVN enquired with Chandrasekhar for GR related questions in astrophysics

No, there is nothing that asked for GR, replied Chandra

# Conclusion of pre-independence era

This concludes the story until Independence in 1947 Current Science is bringing out special issue on GR Centenary which should be out this weekend For that Jayant Vishnu Narlikar (JVN) has reviewed the pre-independence era Now we go over to post-independence era which has been reviewed by me

At the outset I offer apology to all those colleagues who consider their work important but I have been unable to include that either my inability to appreciate or by oversight

# Cosmology and Raychaudhuri Equation

This is when GR maturing to be a legitimate physical theory beyond its elegance and beauty

The most outstanding contribution was undoubtedly Raychaudhuri equation

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FLRW expanding model – Big-Bang singularity in the past Question: Is it due to symmetry properties – homogeneity and isotropy, or generic to GR equation? Then came Godel universe which had no singularity but it was static with closed timelike lines and large cosmological constant Amal Kumar Raychaudhuri (AKR) thought that one can try to modify Godel model to make it expanding and also remove closed timelike lines

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But all attempts came to a naught

# The Equation

- Then AKR came across a paper by Einstein and Pauli (?) in which  $R_{tt}$  is written as divergence
- This was a crucial clue following that he could write rate of expansion of timelike congruence in terms of shear, rotation and gravitational energy density  $\rho + 3p$  without recourse to any symmetry
- This is how the famous Raychaudhuri equation was born He had considered geodetic congruence thereby missing the crucial acceleration divergence term, this as well as inclusion of null congruence was subsequently done by other authors

This equation is to cosmology what Saha equation is to astrophysics

Amazingly it was discovered in 1953 but got published only in 1955 (GoldenOldies)

It was done under very trying circumstance of threat of being thrown out of job from Indian Association for Cultivation of Science for not working on atomic physics

He was indeed thrown out, came first to Asutosh and finally to Presidency College (now university)

That's where he had the most remarkable innings of legendary teacher and mentor to galaxy of students who had done proud to both him and the College

### Hoyle-Narlikar theory

In 1963, Fred Hoyle and JVN proposed a new theory of gravitation by incorporating Mach's Principle

It was an elegant theory but unfortunately accorded to steady state cosmology which observations refuse to favour

They however proved the first cosmic no-hair theorem by showing stability of de Sitter spacetime against scalar perturbations Homogeneity and isotropy were natural for steady state while for big bang it had to be laboured in

Had they further persisted on, they could have perhaps predicted that constant curvature density perturbation was only consistent with steady state cosmology

# Gravitational Collapse

Inhomogeneous dust collapse need not always result into black hole – may violate CCH – naked singularity, Christodoulou in 1983 Tracing null geodesic backward in time, it is shown that it ends at singularity indicating NS

There has been lot of work led by Pankaj Joshi (PJ), and number of university researchers

PJ and I H Dwivedi considered a general initial prescription for end product being BH or NS

Inhomogeneity gives rise to shear which makes collapse incoherent and it is that which may lead to formation of NS Roy Marrtens, PJ and ND envisioned that very near NS, curvatures would be diverging, that could result into forming a fireball giving rise to intense radiation like GRB, and soon it would be gulped by apparent horizon

GRBs as birth cries of black hole was the title of our paper Piran et al later on use the same phrase in the abstract without citing us!

### Black Hole

Black hole is the most bizarre and remarkable prediction of GR, is it stable?

C V Vishveshwara, a student of Charlie Misner at Maryland did this pioneering study of stability of Schwarzschild black hole under scalar perturbations

This was the beginning of study of quasi-normal modes as the most effective and efficient gravitational probes

People hear black holes ringing and singing

Gravitational wave adds its own charm in application of this tools

#### **Black Hole Energetics**

Penrose Process: Extracting rotational energy from rotating BH because of existence of ergosphere with negative energy orbits A novel – purely geometric process, could it power guasars, AGNs? It turned out it is not efficient enough – relative velocity between fragments > 1/2c, astrophysically unsustainable Beautiful but not real In 1985, S M Wagh, S V Dhurandhar and ND argued that a rotating BH was always surrounded by magnetic field, energy required to put a fragment on negative energy orbit could now come from electromagnetic field without any constraint on relative velocity

Revival of PP for astrophysics as magnetic PP – MPP It is very efficient, efficiency could exceed 100 percent, the prediction which had been borne out by recent studies by Ramesh Narayan et al

There is competing Blandford-Znajek mechanism of 1977 in which rotating magnetic field creates a potential difference between pole and equator and discharge of which drives energy out In high magnetic field limit, the two approximate to be the same It is a different matter that the present authors call MPP as BZ process

They are indeed different as low magnetic field limit is quite distinct for the two

# Gravitational wave and INDIGO-LIGO

There is a strong air of gravity wave all around, I need hardly to add anything more

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### Exact solutions

S Data-Majumdar by generalizing Weyl's solution obtained an interesting solution for axially symmetric charged perfect fluid solution (Golden Oldies)

It was also independently obtained by Papapetrou – known as Papapetrou-Majumdar metric

Vaidya and R S Tikekar considered an interesting metric ansatz: constant time slice being spheroidal rather than spherical. This gives rise to a class of solutions which could describe compact stars. There has been a good bit of activity on Vaidya-Tikekar model

There had been great activity in solving Einstein equation but there are very few physically interesting solutions

Following J M M Senovilla's discovery of singularity free cylindrically symmetric model, some singularity free models have been found, in particular with stiff equation of state  $p = \rho$  by L K Patel and ND

Gravity and Thermodynamics - talks by Paddy and his students

Brane World gravity: Our Universe (3-brane) being a hypersurface in higher dimensional bulk space which is effectively compact by its curvature

Randall-Sundrum gravity model: There had been good bit of work by Sayan Kar and others

Black hole on brane: It was shown by that Reissner - Nordstrom Charged black hole solution could describe black hole on brane by Roy Maartens, Philipos Pappolous, V Rezania and ND. Here Charge is not Maxwellian but Weyl – projection of bulk Weyl on the brane

There is very important study by Romesh Kaul, Ghanshyam Date and Sandipan Sengupta on Nieh-Yan invariant providing a topological interpretation of Barbero Immirzi parameter

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# What made the difference

- 1. Summer Schools
- 2. IAGRG
- 3. ICGC
- 4. IUCAA

5. GR in Institutes: TIFR, RRI, PRL, IIA, IISc, IITs, IISERs, CTP(jamia), ...

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### **Final Words**

It is fair to say that GR research has come of age in the country is keeping pace with the recent developments in almost all areas //[2ex]

Reflection: Had AKR had conducive mathematical backup, could he or someone around him not proved the powerful singularity theorems

The same could be said of quasi-normal modes and MPP, ...?

We are perhaps mentally wired as to feel satisfied by establishing an idea in principle but seriously lack in the nut-bolt work to take it to fruition - logical conclusion

That's why we have been missing the bus often

Hope young researchers would overcome this mental block and won't miss the bus any longer!

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Thank you