The Cosmological Distance-Redshift Relation Revisited

Nick Kaiser, IfA U. Hawaii
ICGC 2015/12/17

NK+John Peacock 2015, MNRAS, 455, 4518
NK+Mike Hudson 2015, MNRAS, 454, 280
NK+Mike Hudson 2015, MNRAS, 450, 883

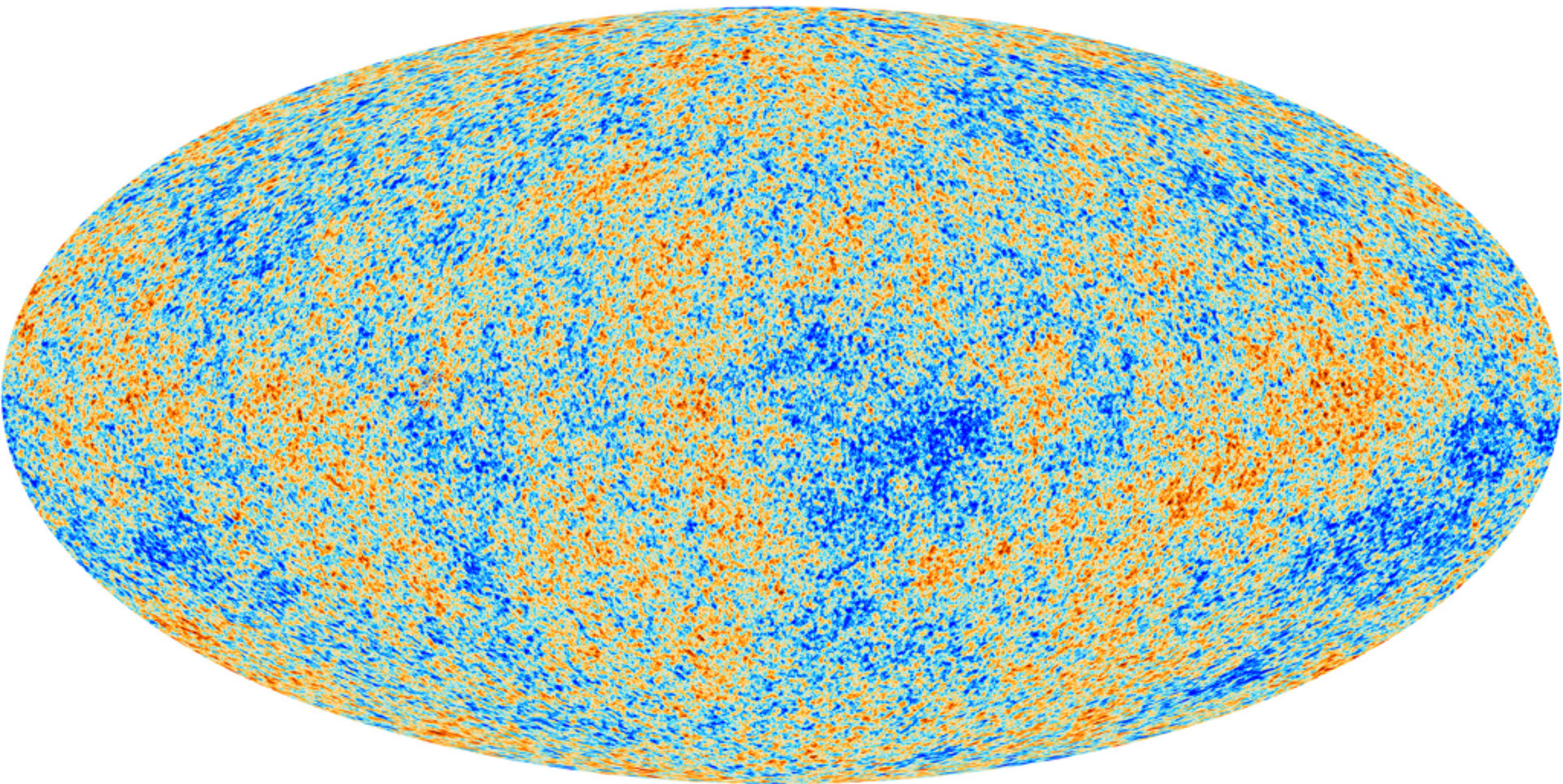
Outline - GR effects on the Cosmological Distance-Redshift relation and implications for cosmological parameter estimation

- Recent works in 2nd order cosmological perturbation theory:
  - claim: non-linear `back-reaction' may have a significant impact
- 2 applications - both relate to biases in D(z)
  - Bias in D(z) at high z from gravitational lensing
    - very long and confusing story
    - subtle relativistic effects (first recognised in '80s)
    - NK+JP: history riddled with misconceptions
      - conventional methods for CMB + SN1a analysis justified
  - Bias in D(z) - and therefore H_0 - at low z from peculiar motions
    - related to "homogeneous Malmquist bias" studied by astronomers
- Also a claimed new probe of cosmology
  - "Doppler lensing" - perturbation to D(z)
  - relation to conventional cosmic-flow studies and SN1a cosmology

Context: cosmological parameters from the CMB

It is usually assumed that we are looking here at a spherical surface at $z \sim 1100$ with $D = D_0(z=1100)$

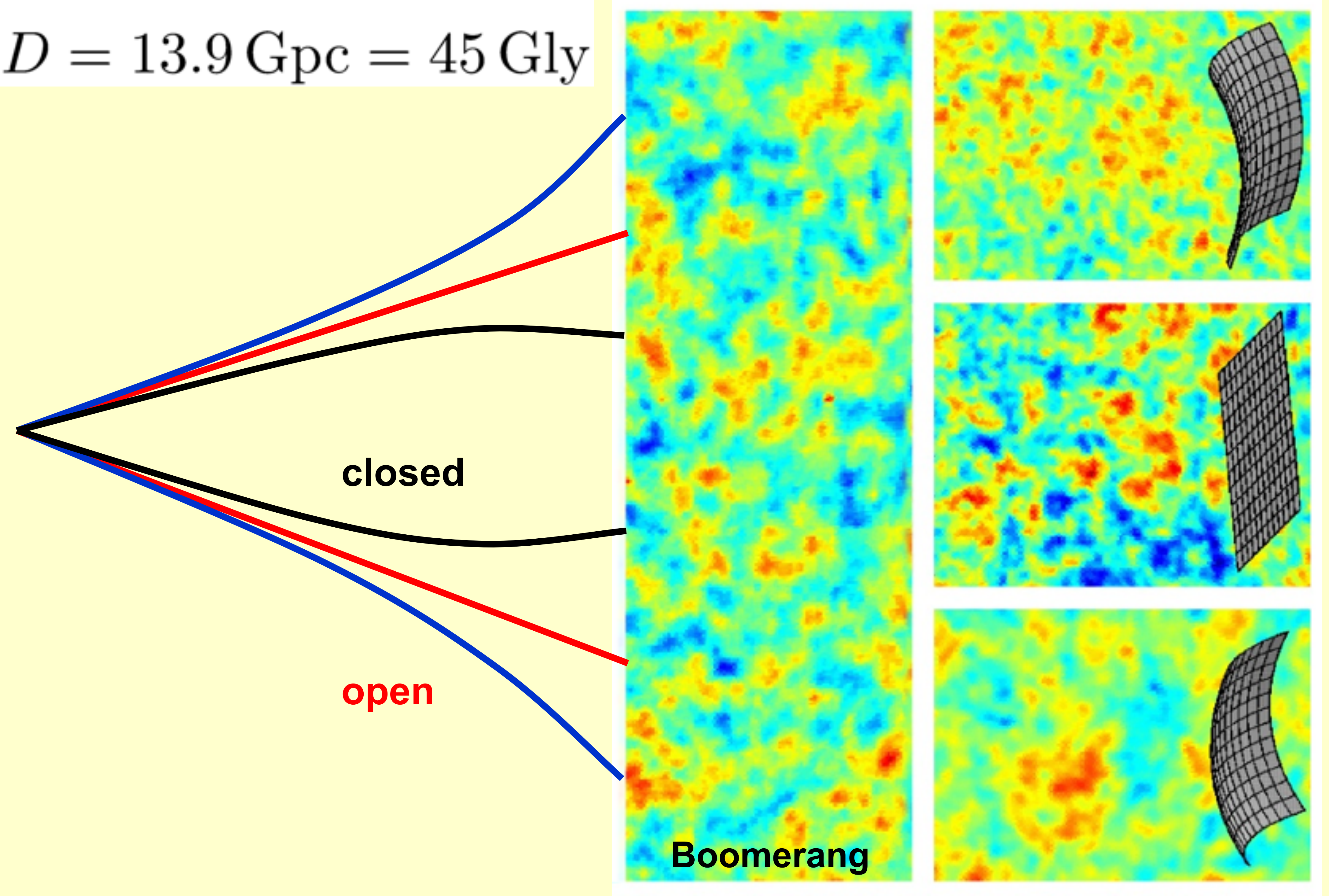
But are we?



How far away is the CMB?

$$D = \int \frac{c}{H(z)}\, dz$$

$$z = 1080 \Rightarrow D = 13.9\,\mathrm{Gpc} = 45\,\mathrm{Gly}$$



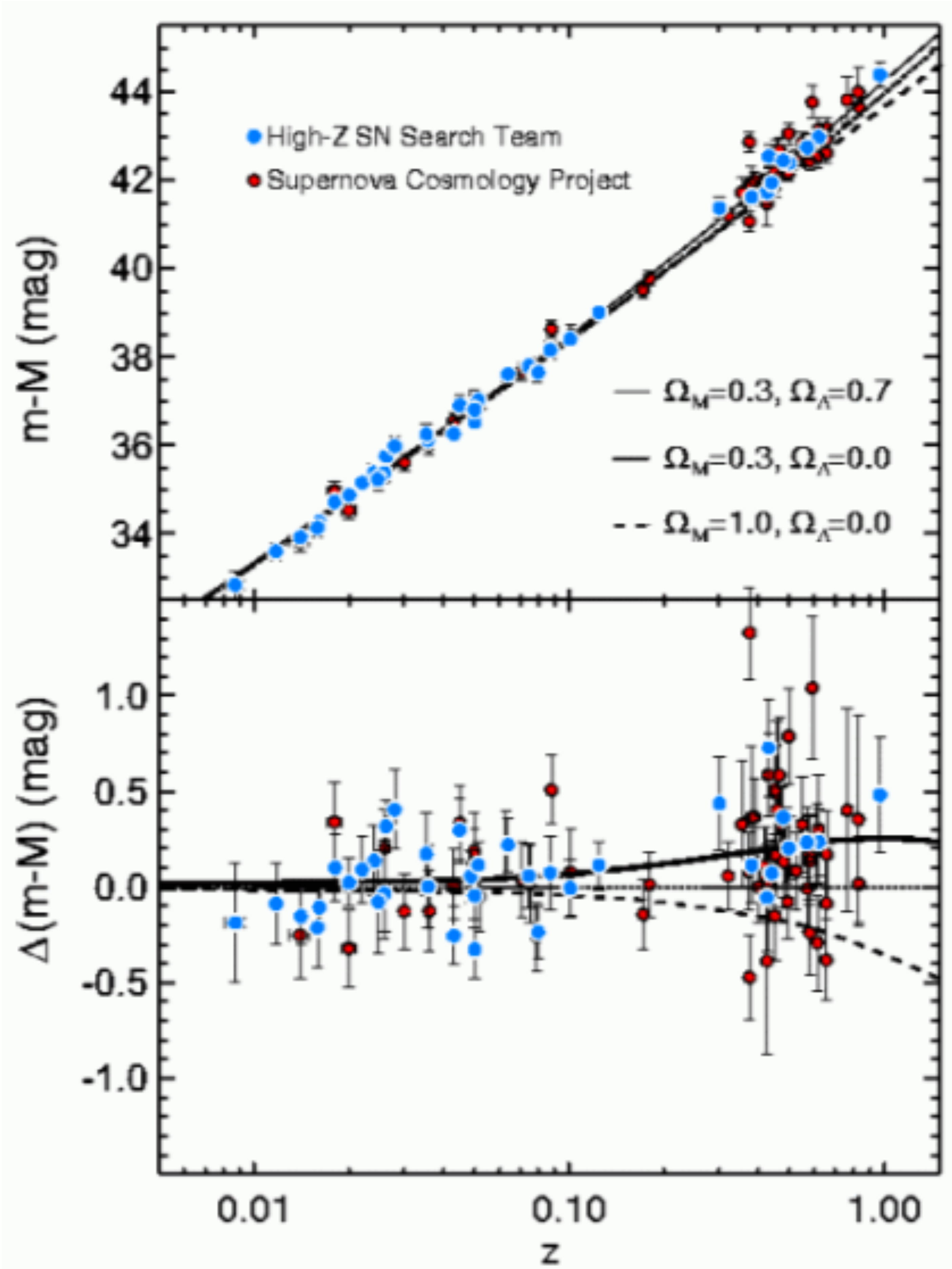
What is the distance to the CMB?
How relativistic corrections remove the tension with local H_0 measurements

Chris Clarkson[1], Obinna Umeh[2], Roy Maartens[2,3] and Ruth Durrer[4]

[1]*Astrophysics, Cosmology & Gravity Centre, and, Department of Mathematics & Applied Mathematics, University of Cape Town, Cape Town 7701, South Africa.*
[2]*Physics Department, University of the Western Cape, Cape Town 7535, South Africa*
[3]*Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom*
[4]*Département de Physique Théorique & Center for Astroparticle Physics, Université de Genève, Quai E. Ansermet 24, CH-1211 Genève 4, Switzerland.*

The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Hubble diagram from SN1a - assumes no flux *bias* from lensing

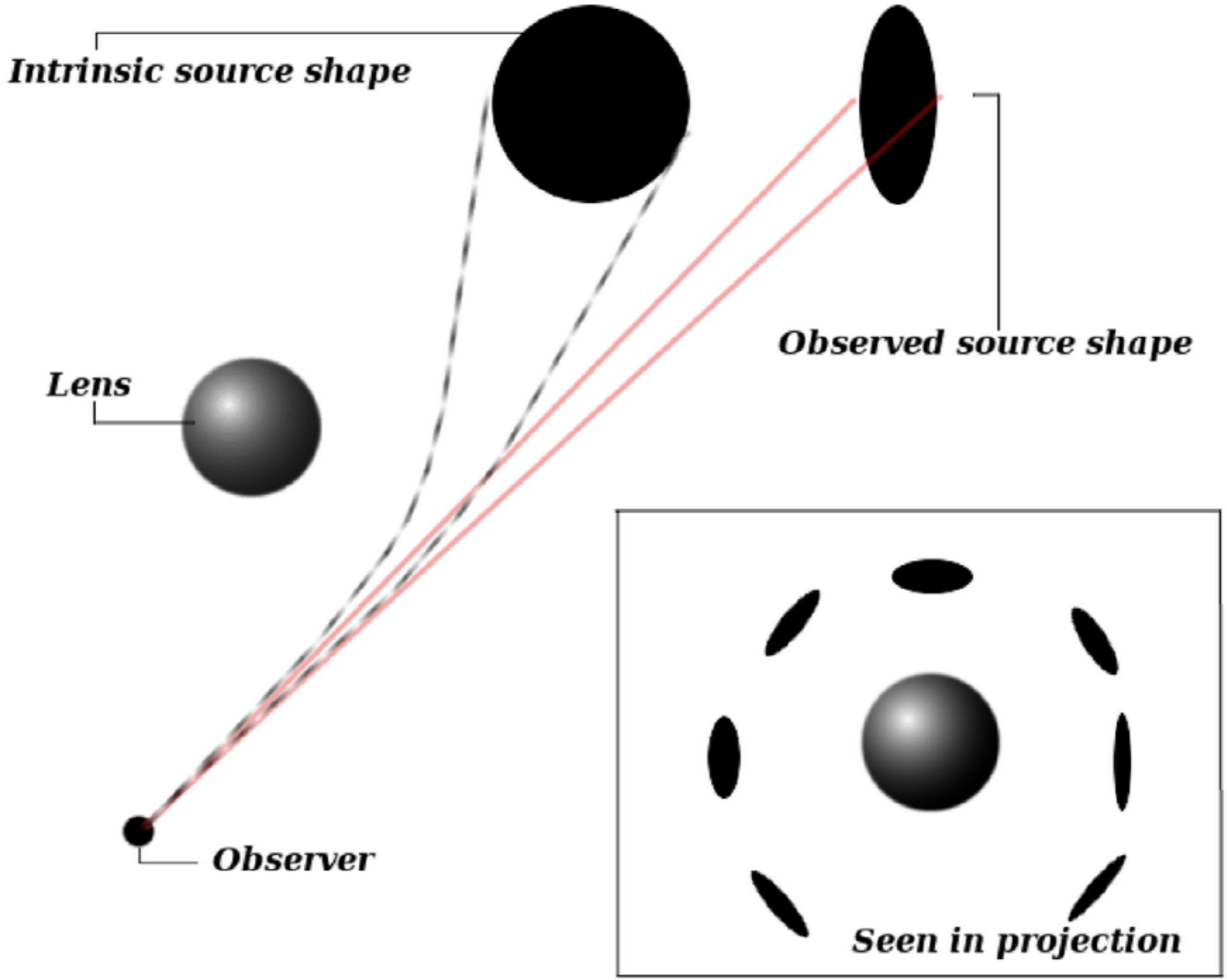


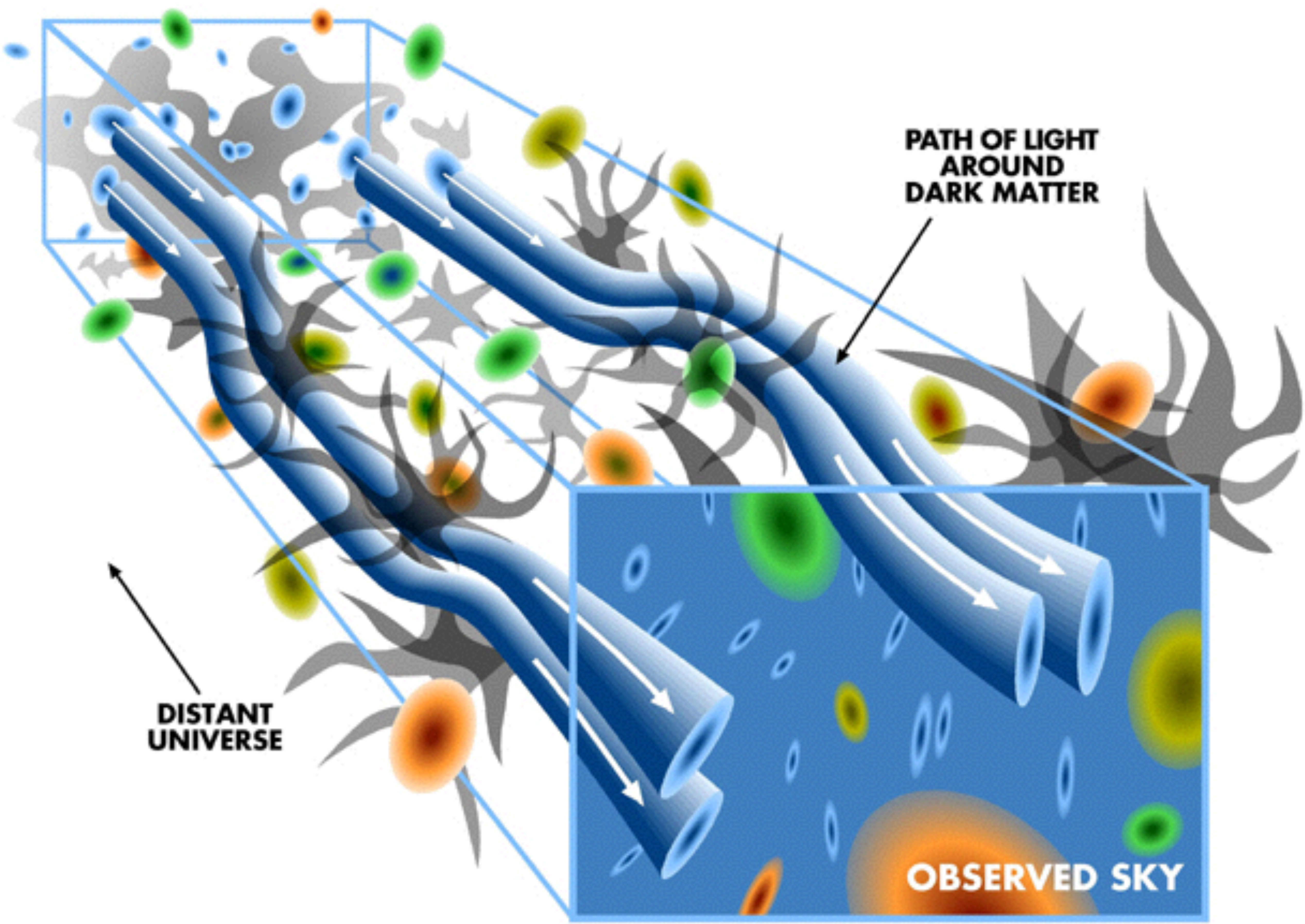
What do we mean by "distance" here?

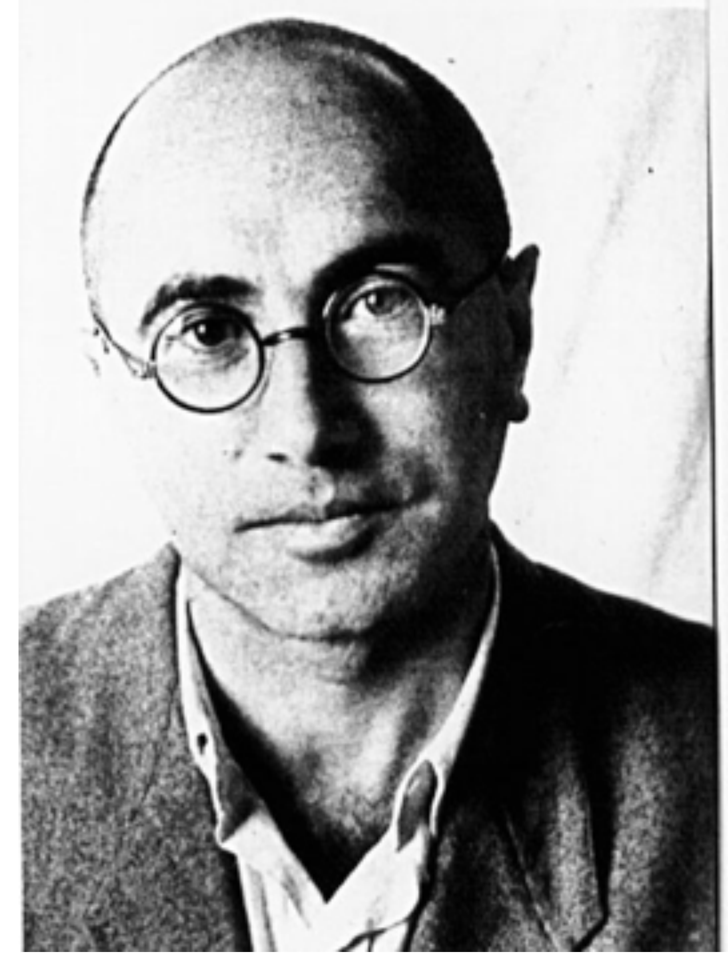
- Distances in cosmology:
  - redshift (monotonic in distance - absent peculiar velocities)
  - `conformal' or `comoving' distance χ
    - as in $ds^2 = -d\tau^2 + a^2(\tau)(d\chi + S_k^2(\chi)\, d\sigma^2)$
    - not observable, but useful to relate other observable "distances"
  - angular diameter distance: $dl = a(\tau(\chi))\, S_k(\chi)\, d\theta = D_A\, d\theta$
  - luminosity distance: $F = L / (4 \pi D_L^2)$
- Here we are interested in D_A and D_L as a function of *redshift*
  - at given z these differ only by a constant
    - since surface brightness depends only on z
- Lensing by structure changes D for any individual object
  - D becomes a random function of direction
- key question here:
  - **does structure *bias* angular sizes or flux densities?**

Preliminaries 2: basics of grav. lensing: deflection & shear

- Basic quantities in gravitational lensing
  - Deflection angle (1 `blob') $\theta_1 \sim \int d\lambda \nabla\Phi/c^2 \sim GM/bc^2 \sim (H\lambda/c)^2\Delta$
  - cumulative deflection $\theta \sim N^{1/2}\,\theta_1 \sim (H\lambda/c)^{3/2}\Delta$
  - where $\Delta = \Delta\varrho/\varrho \sim \xi^{1/2} \sim 1/\lambda$
  - so θ is
    - dominated by *large scales* (~30 Mpc)
    - ~ few arc-minutes ~ 10^{-3} radians at high z
    - but usually thought to be unobservable
  - What is observable is the *gradient* of the deflection angle
    - i.e. the change of the deflection across a source
    - 2x2 image distortion tensor
      - trace: kappa = $\varkappa$ → magnification
      - other 2 d.o.f.: γ → image shear/distortion
    - $\varkappa^2$, $\gamma^2 \sim 10^{-3}$ at ~ degree scales for sources at z >> 1 (e.g. CMB)
    - but scales as $\sim \lambda^{-1}$
    - large, possibly very large, effects small-scale structure


Intrinsic source shape
Observed source shape
Lens
Observer
Seen in projection


PATH OF LIGHT
AROUND
DARK MATTER
DISTANT
UNIVERSE
OBSERVED SKY



OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

A local nonuniformity of density due to the concentration of matter of the universe into separate galaxies produces a significant change in the angular dimensions and luminosity of distant objects as compared to the formulas for the Friedman model.

The propagation of light in a homogeneous and isotropic model of the expanding universe (first studied by A. A. Friedman) has been investigated in a number of papers [1, 2, 3].

In these papers expressions were obtained for the observed angular diameter Θ and the observed brightness of an object with a known absolute diameter and absolute brightness as a function of the distance or, strictly speaking, the red shift of the object $\Delta = (\omega_0 - \omega)/\omega_0$.

In particular, there is a remarkable feature in the function $\Theta(\Delta)$, namely, the presence of a minimum when Δ is approximately equal to 1/2. Formula (10) and Fig. 6 in the appendix show the variation of the function $f(\Delta) = rH/c\Theta$ which is inversely proportional to Θ for a given density of matter. Here r is the radius of the object, H is Hubble's

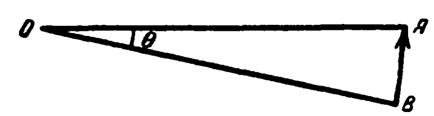


Fig. 1.

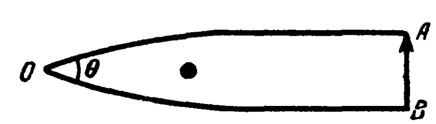
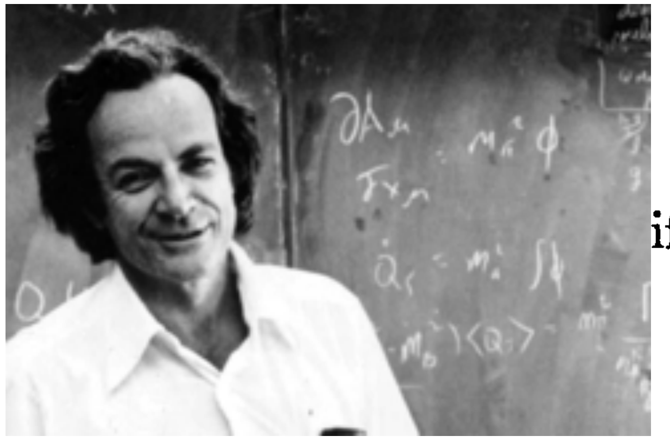


Fig. 2.

A mass situated between these rays bends the latter in such a way that Θ is increased (Fig. 2). What we have in mind is the bending of light rays by the gravitational field predicted by Einstein; this bending amounts to 1.75" for a light ray passing near the limb of the solar disc and has been confirmed by observation.

ON THE PROPAGATION OF LIGHT IN INHOMOGENEOUS COSMOLOGIES. I. MEAN EFFECTS



JAMES E. GUNN
ifornia Institute of Technology and Jet Propulsion Laboratory

Received February 23, 1967; revised May 23, 1967

ABSTRACT

The statistical effects of local inhomogeneities on the propagation of light are investigated, and deviations (including rms fluctuations) from the idealized behavior in homogeneous universes are investigated by a perturbation-theoretic approach. The effect discussed by Feynman and recently by Bertotti of the density of the intergalactic medium being systematically lower than the mean mass density is examined, and expressions for the effect valid at all redshifts are derived.

I. INTRODUCTION

In an unpublished colloquium given at the California Institute of Technology in 1964, Feynman discussed the effect on observed angular diameters of distant objects if the intergalactic medium has lower density than the mean mass density, as would be the case if a significant fraction of the total mass were contained in galaxies. It is an obvious extension of the existence of this effect that luminosities will also be affected, though this was apparently not realized at the time. This realization prompted the conviction that the effect of known kinds of deviations of the real Universe from the homogeneous isotropic models (upon which predictions had been based in the past) upon observable quantities like luminosity and angular diameter should be investigated. The author (1967) has recently made such a study for angular diameters; the present work deals primarily with mean statistical effects upon luminosity. A third paper will deal with possible extreme effects one may expect to encounter more rarely. Some of the results discussed here have been discussed independently by Bertotti (1966) and Zel'dovich (1965).

Kantowski '69

CORRECTIONS IN THE LUMINOSITY-REDSHIFT RELATIONS OF THE HOMOGENEOUS FRIEDMANN MODELS

R. KANTOWSKI*
Southwest Center for Advanced Studies, Dallas, Texas

Received January 22, 1968; revised March 22, 1968

ABSTRACT

In this paper the bolometric luminosity-redshift relations of the Friedmann dust universes ($\Lambda = 0$) are corrected for the presence of inhomogeneities. The "locally" inhomogeneous Swiss-cheese models are used, and it is first shown that the introduction of clumps of matter into Friedmann models does not significantly affect the $R(z)$ or $R(v)$ relations (Friedmann radius versus the redshift or affine parameter) along a null ray. Then, by the use of the optical scalar equations, a linear third-order differential equation is arrived at for the mean cross-sectional area of a light beam as a function of the affine parameter. This differential equation is confirmed by rederiving its small redshift solution from an interesting geometrical point of view. The geometrical argument is then extended to show that "mild" inhomogeneities of a transparent type have no effect on the mean area of a light beam.

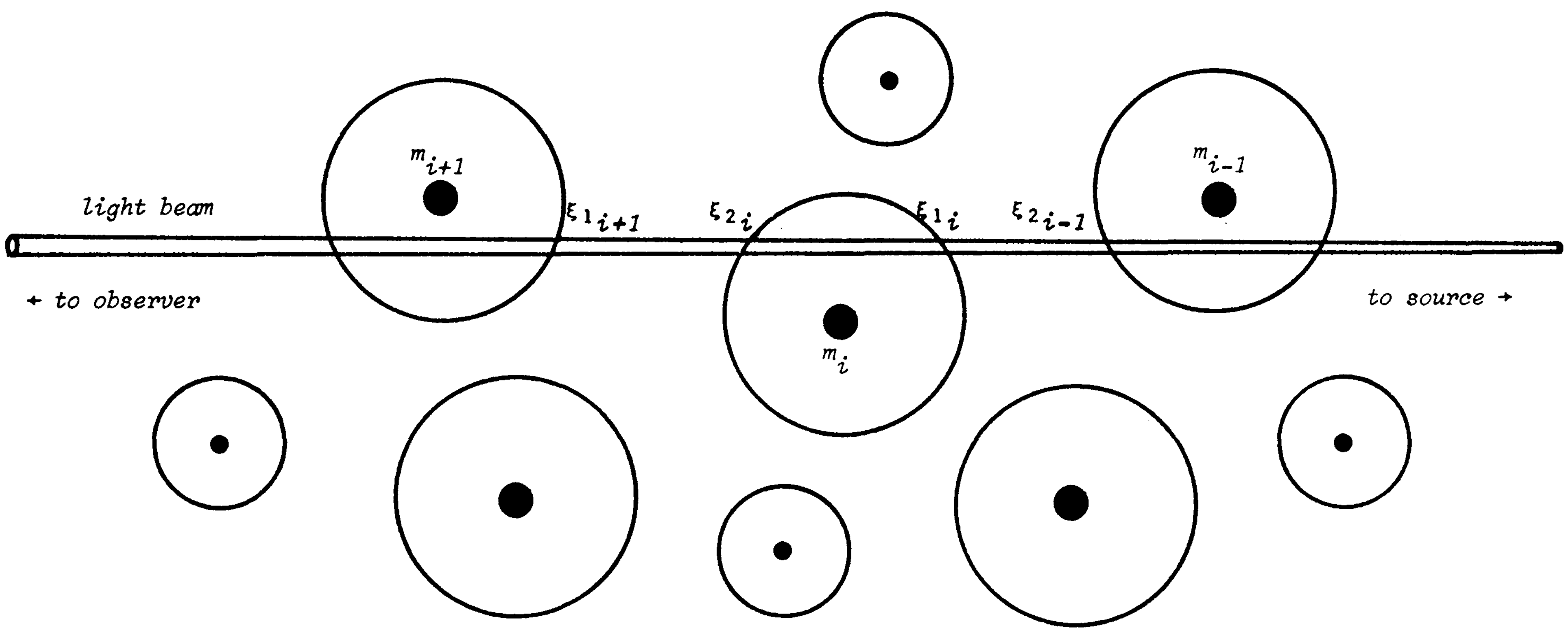


FIG. 1.—Spacelike section of a typical Swiss-cheese universe

Dyer & Roeder '72

THE DISTANCE-REDSHIFT RELATION FOR UNIVERSES WITH NO INTERGALACTIC MEDIUM

C. C. DYER* AND R. C. ROEDER†
Kitt Peak National Observatory,‡ Tucson, Arizona
Received 1972 April 19

ABSTRACT

The distance-redshift relation is derived for model universes in which there is negligible intergalactic matter and in which the line of sight to a distant object does not pass close to intervening galaxies. When fitted to observations, this relation yields a higher value of q_0 than does a homogeneous model.

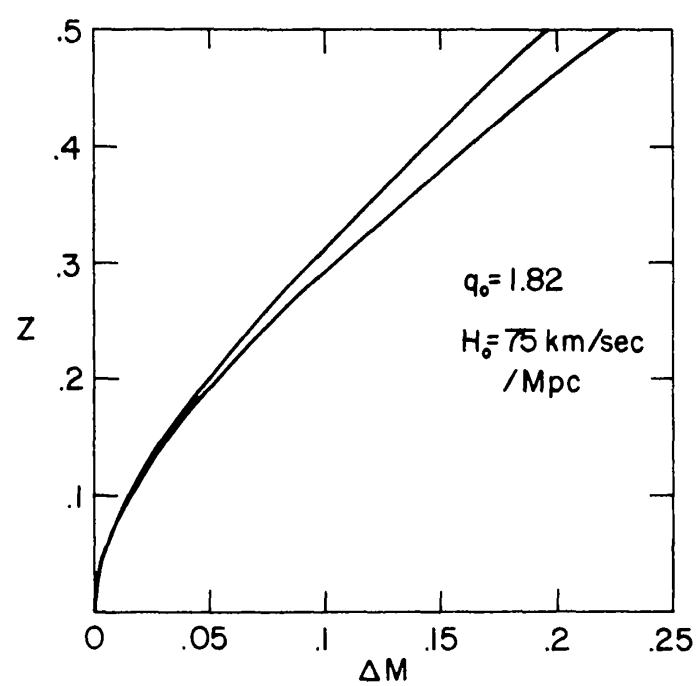
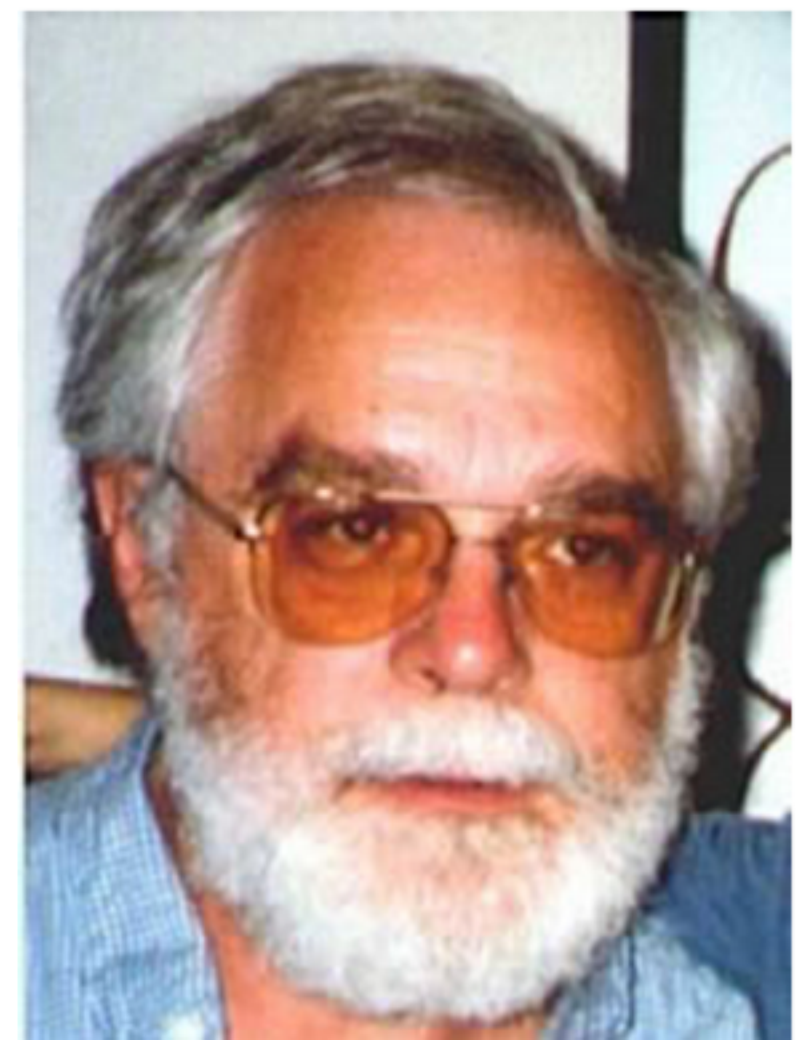


FIG. 1.—The dimming, relative to the homogeneous model, assuming that the beam passes far from any intervening galaxies (*lower curve*) and assuming that the beam passes no closer than 2 kpc to the center of galaxies similar to our own (*upper curve*).



Weinberg 1976 - no effect (flux conservation)

APPARENT LUMINOSITIES IN A LOCALLY INHOMOGENEOUS UNIVERSE

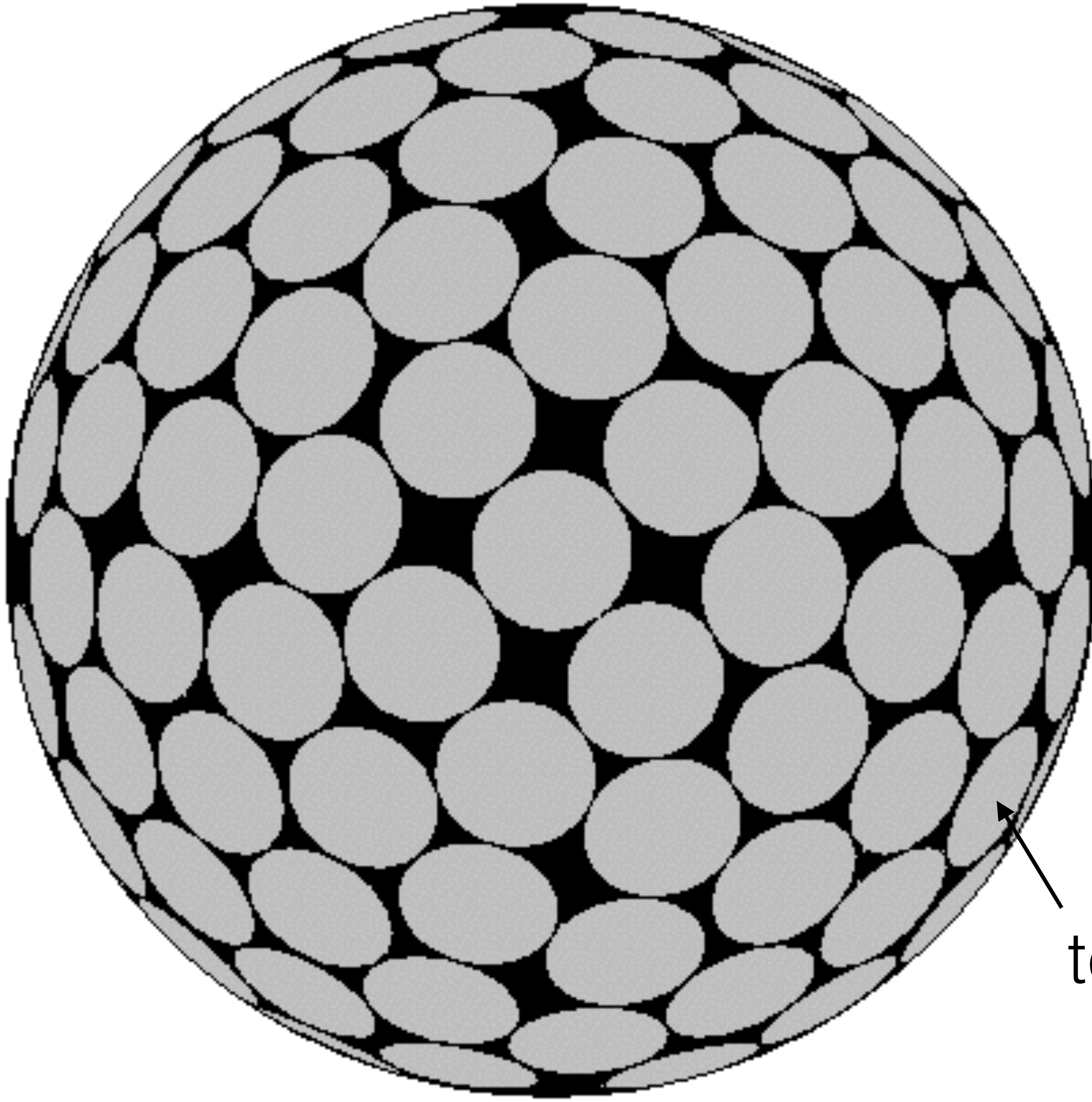
STEVEN WEINBERG
Center for Astrophysics, Harvard College Observatory and Smithsonian Astrophysical Observatory; and Department of Physics, Harvard University
Received 1976 April 6; revised 1976 May 20

ABSTRACT

Apparent luminosities are considered in a locally inhomogeneous universe, with gravitational deflection by individual clumps of matter taken into account. It is shown that as long as the clump radii are sufficiently small, gravitational deflection by the clumps will produce the same average effect as would be produced if the mass were spread out homogeneously. The conventional formulae for luminosity distance as a function of redshift consequently remain valid, despite the presence of any local inhomogeneities of less than galactic dimensions. For clumps of galactic size, the validity of the conventional formulae depends on the selection procedure used and the redshift of the object studied.

Subject headings: cosmology — galaxies: redshifts — gravitation

Weinberg's argument (that <magnification> = 1)



But this *assumes* that the total area is unchanged

Lensing and caustic effects on cosmological distances.

EBD '98

G. F. R. Ellis[1], B. A. Bassett[1,2], and P. K. S. Dunsby[1]

1 *Department of Applied Mathematics, University of Cape Town, Rondebosch 7700, Cape Town, South Africa.*

2 *International School for Advanced Studies, SISSA - ISAS Via Beirut 2-4, 34014, Trieste, Italy.*

December 4, 2013

Abstract

We consider the changes which occur in cosmological distances due to the combined effects of some null geodesics passing through low-density regions while others pass through lensing-induced caustics. This combination of effects increases observed areas corresponding to a given solid angle even when averaged over large angular scales, through the additive effect of increases on all scales, but particularly on micro-angular scales; however angular sizes will not be significantly effected on large angular scales (when caustics occur, area distances and angular-diameter distances no longer coincide). We compare our results with other works on lensing, which claim there is no such effect, and explain why the effect will indeed occur in the (realistic) situation where caustics due to lensing are significant. Whether or not the effect is significant for number counts depends on the associated angular scales and on the distribution of inhomogeneities in the universe. It could also possibly affect the spectrum of CBR anisotropies on small angular scales, indeed caustics can induce a non-Gaussian signature into the CMB at small scales and lead to stronger mixing of anisotropies than occurs in weak lensing.

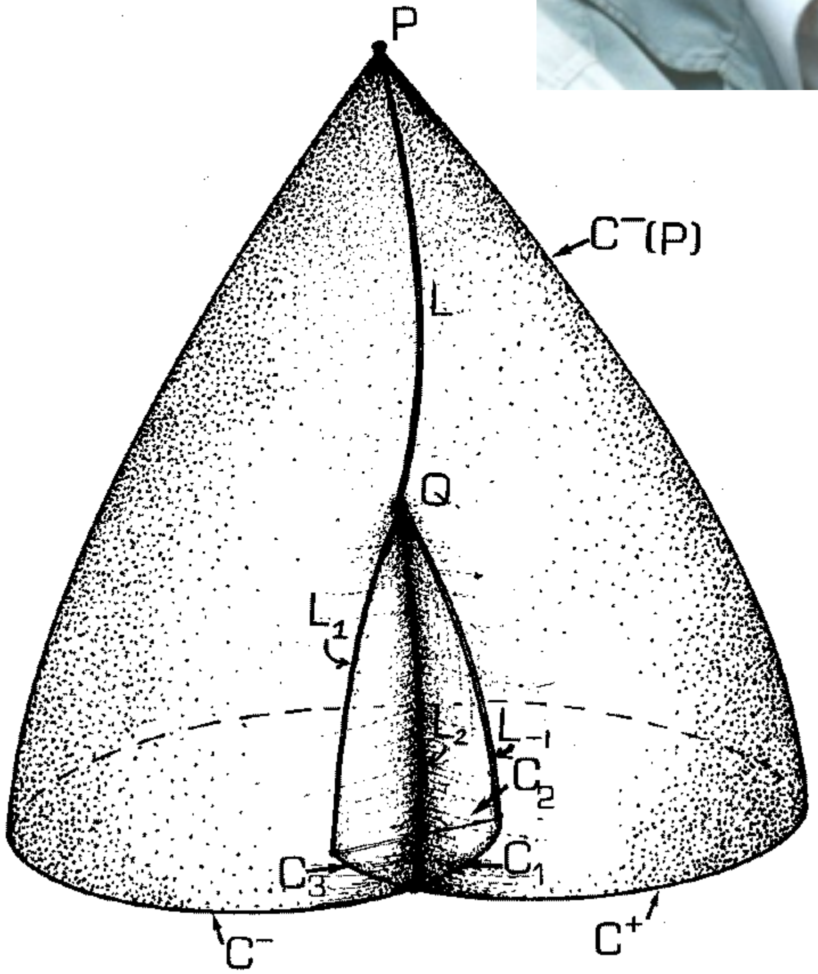
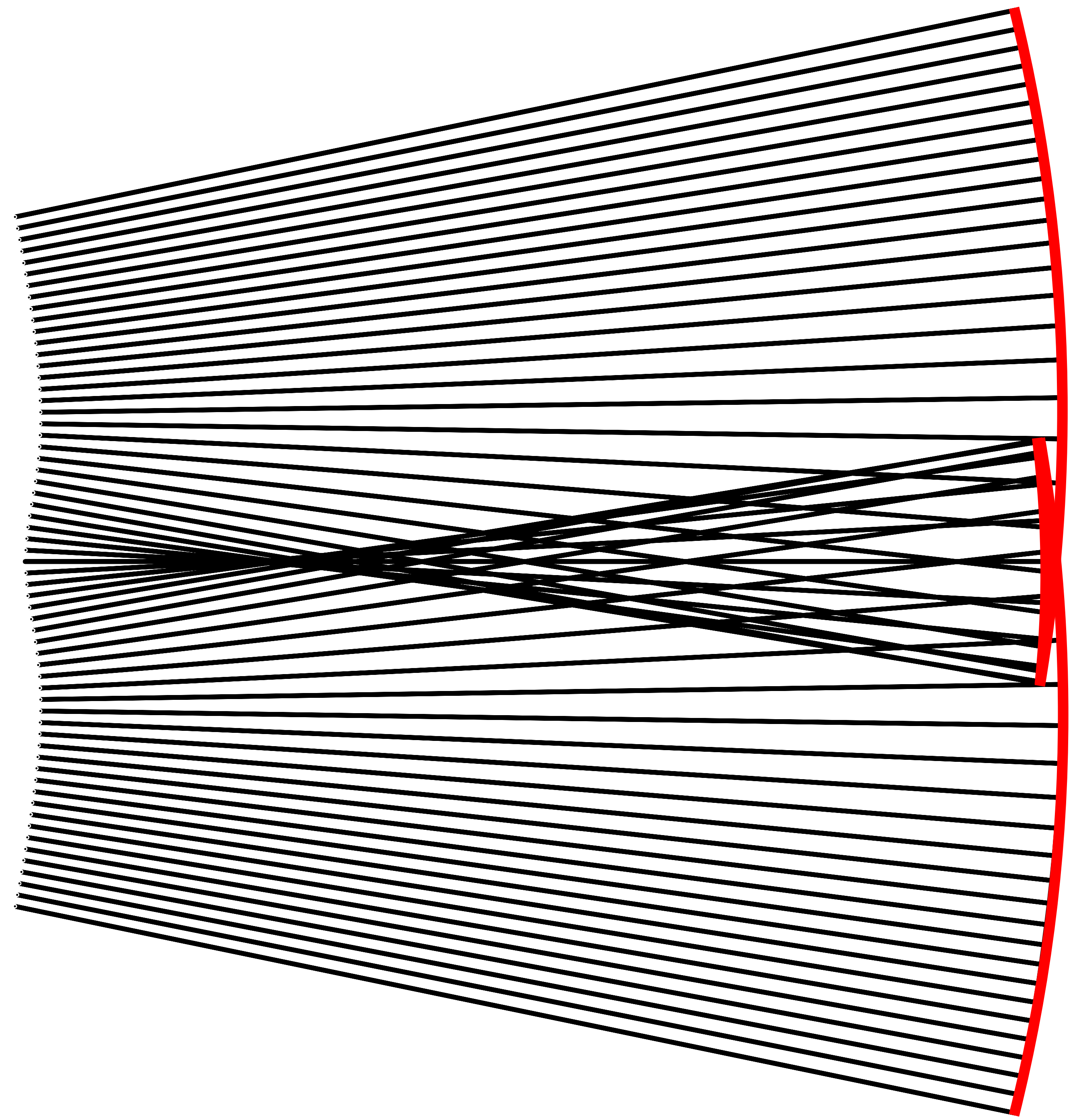


Figure 1: A lens L and resulting caustics on the past light cone $C^{-}(P)$ (2-dimensional section of the full light cone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point Q. The intersection of the past light cone with a surface of constant time defines exterior segments C^{-}, C^{+} of the light cone together with interior segments C_1, C_2, C_3.

Ellis, Bassett & Dunsby '98 critique of Weinberg '76

- EDB98 make two points:
- Weinberg *assumes* that which is to be proven
  - we agree: W76 assumes that the surface of constant z around a source (or observer) is a sphere
- Small scale strong lensing causes the surface to be folded over on itself so total area greatly enhanced
  - quite possibly true
- Thus Weinberg's claim is disproved
  - we disagree: W76 still applies if multiple images are unresolved



Enter Schneider, Ehlers, Seitz etc... ('80s, '90s)

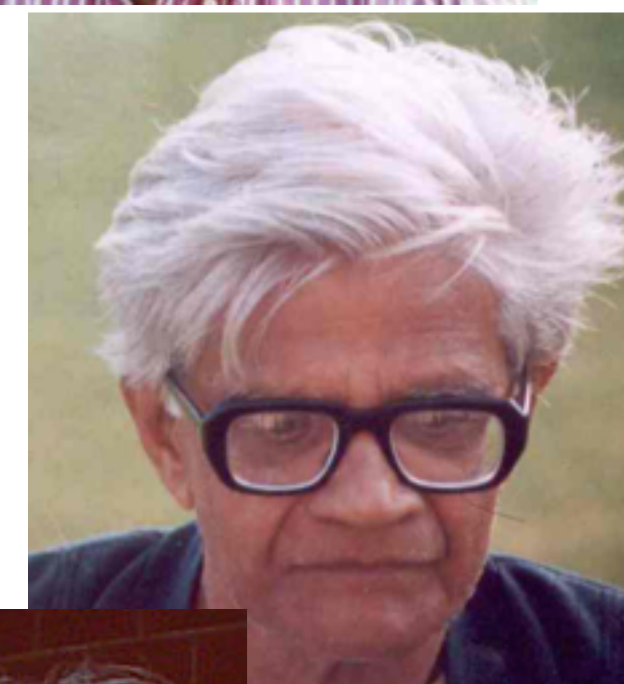
- Two consistent threads:
  - Lens equation:
    - at least one image is made **brighter**
  - Optical scalar equations (Sachs 1961):
    - -> *focusing theorem* (Seitz et al. 1994)
    - Things viewed through 'clumpiness' are further than they appear...

Seitz, Schneider & Ehlers (1994)

Finally, we have derived an equation for the size of a light beam in a clumpy universe, relative to the size of a beam which is unaffected by the matter inhomogeneities. If we require that this second-order differential equation contains only the contribution by matter clumps as source term, the independent variable is uniquely defined and agrees with the χ-function previously introduced [see SEF, eq. (4.68)] for other reasons. This relative focusing equation immediately yields the result that a light beam cannot be less focused than a reference beam which is unaffected by matter inhomogeneities, prior to the propagation through its first conjugate point. In other words, no source can appear fainter to the observer than in the case that there are no matter inhomogeneities close to the line-of-sight to this source, a result previously demonstrated for the case of one (Schneider 1984) and several (Paper I, Seitz & Schneider 1994) lens planes.

The focusing theorem: $\ddot{D}/D = -(R + \Sigma^2)$

- Derived from Sachs '61 "optical scalars"
- from A.K. Raychaudhuri's (Landau) equation
  - transport of expansion, vorticity and shear
- $R = R_{ab}k^a k^b$ where R_{ab} is the *Ricci curvature*
  - local focusing by matter in the beam
- Σ^2 is the cumulative effect of *Weyl curvature*
  - i.e. the tidal effect of matter *outside* the beam
  - Σ being the *rate* of image shearing
- Like cosmological acceleration equation:
  - $d^2a/dt^2 = - 4\pi G(\varrho+3P/c^2)a$
  - so Σ^2 here plays the role of pressure???
- Also like Hawking-Ellis *singularity theorem*
  - both terms are positive => focusing
- e.g. Narlikar (Introduction to Relativity):
  - "*Thus the normal tendency of matter*
  - *is to focus light rays*"



Narlikar on the focusing theorem

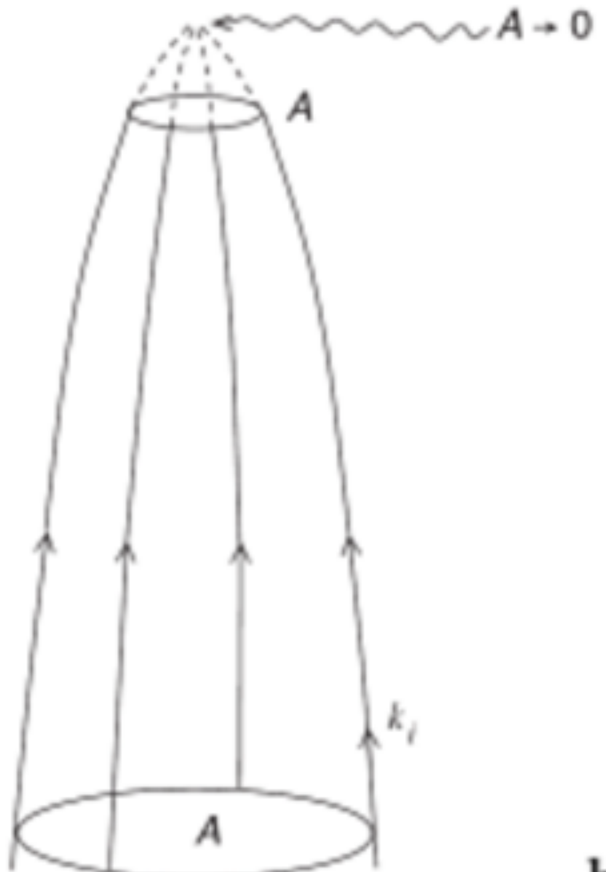


Fig. 18.3. The bundle of geodesics focusses in the future with its cross section A decreasing to zero. This effect was discussed in the context o spacetime singularity by A. K. Raychaudhuri.

The Raychaudhuri equation can be stated in a slightly different form as a *focussing theorem*. In this form it describes the effect of gravity on a bundle of null geodesics spanning a finite cross section. Denoting the cross section by A, we write the equation of the surface spanning the geodesics as f = constant. Define the normal to the cross-sectional surface by $k_i = \partial f/\partial x^i$. Figure 18.3 shows the geometry of the bundle.

Using a calculation similar to that which led to the geodetic deviation equation in Chapter 5, we get the focussing equation as

$$\frac{1}{\sqrt{A}}\frac{\mathrm{d}^2\sqrt{A}}{\mathrm{d}\lambda^2} = \frac{1}{2}R_{im}k^i k^m - |\sigma|^2, \tag{18.10}$$

Equation (18.10) is similar to the Raychaudhuri equation with $|\sigma|^2$ being the square of the magnitude of shear. With Einstein's equations, we can rewrite (18.10) as

$$\frac{1}{\sqrt{A}}\frac{\mathrm{d}^2\sqrt{A}}{\mathrm{d}\lambda^2} = -4\pi G\left(T_{im} - \frac{1}{2}g_{im}T\right)k^i k^m - |\sigma|^2. \tag{18.12}$$

For dust we have $T_{im} = \rho u_i u_m$ and this condition is satisfied with the left-hand side equalling $\rho(u_i k^i)^2$. (Remember that k_i is a null vector, so $g_{im}k^i k^m = 0$.) Thus the normal tendency of matter is to focus light rays by gravity.

more on the focusing theorem: $\ddot{D}/D = -(R + \Sigma^2)$

- In a cosmological context we are interested in how D deviates from the background value $D = D_0 + D_1 + \ldots$
- Focussing vanishes in the background
- If we take the average, and linearise,
  - and assuming $\langle \delta R \rangle = 0$ we have the *averaged focusing theorem*

$$\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$$

- There is an inevitable tendency for structure to cause beams to focus
  - related to the (rate of) shearing of the beam
- predicts decrease of distance *qualitatively* the same as Clarkson et al.
  - i.e. a large - and possibly ultra-violet divergent - effect!
- So Weinberg was wrong?

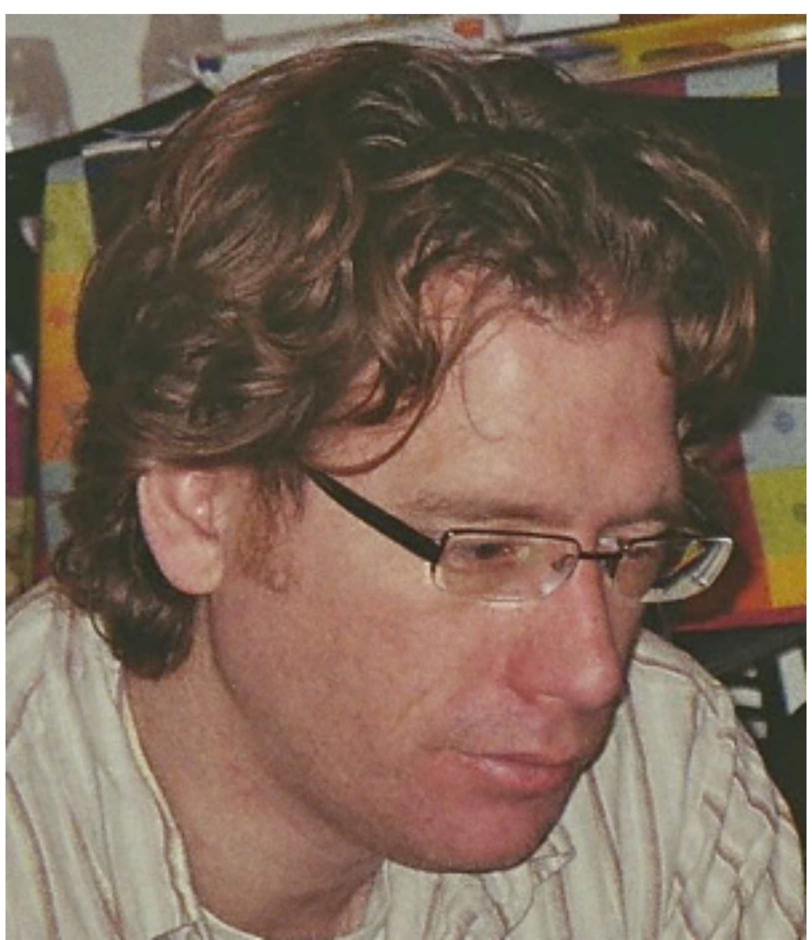
GRAVITATIONAL MAGNIFICATION OF THE COSMIC MICROWAVE BACKGROUND

R. BENTON METCALF AND JOSEPH SILK
Departments of Physics and Astronomy and Center for Particle Astrophysics, University of California, Berkeley, Berkeley, CA 84720
Received 1996 November 6; accepted 1997 June 12

ABSTRACT

Some aspects of gravitational lensing by large-scale structure are investigated. We show that lensing causes the damping tail of the cosmic microwave background (CMB) power spectrum to fall less rapidly with decreasing angular scale than previously expected. This is because of a transfer of power from larger to smaller angular scales, which produces a fractional change in power spectrum that increases rapidly beyond $\ell \sim 2000$. We also find that lensing produces a nonzero mean magnification of structures on surfaces of constant redshift if weighted by area on the sky. This is a result of the fact that light rays that are evenly distributed on the sky oversample overdense regions. However, this mean magnification has a negligible affect on the CMB power spectrum. A new expression for the lensed power spectrum is derived, and it is found that future precision observations of the high-ℓ tail of the power spectrum will need to take lensing into account when determining cosmological parameters.

Subject headings: cosmic microwave background — gravitational lensing



Kibble & Lieu (2005)

AVERAGE MAGNIFICATION EFFECT OF CLUMPING OF MATTER

T. W. B. Kibble
Blackett Laboratory, Imperial College, London SW7 2AZ, UK; kibble@imperial.ac.uk

AND

Richard Lieu
Department of Physics, University of Alabama at Huntsville, Huntsville, AL 35899; lieur@cspar.uah.edu
Received 2004 December 9; accepted 2005 June 20

ABSTRACT

The aim of this paper is to reexamine the question of the average magnification in a universe with some inhomogeneously distributed matter. We present an analytic proof, valid under rather general conditions, including clumps of any shape and size and strong lensing, that as long as the clumps are uncorrelated, the average "reciprocal" magnification (in one of several possible senses) is precisely the same as in a homogeneous universe with an equal mean density. From this result, we also show that a similar statement can be made about one definition of the average "direct" magnification. We discuss, in the context of observations of discrete and extended sources, the physical significance of the various different measures of magnification and the circumstances in which they are appropriate.

Subject headings: cosmology: miscellaneous — distance scale — galaxies: distances and redshifts — gravitational lensing

Kibble & Lieu 2005

There is another important distinction to be made. We may choose at random one of the sources at redshift *z*, or we may choose a random direction in the sky and look for sources there. These are not the same; the choices are differently weighted. If one part of the sky is more magnified, or at a closer angular-size distance, the corresponding area of the constant-*z* surface will be smaller, so fewer sources are likely to be found there. In other words, choosing a source at random will give on average a smaller magnification or larger angular-size distance.

- Weinberg: $<\mu> = 1$ when averaged over *sources* (or area)
- Kibble & Lieu: $<1/\mu> = 1$ when averaged over *directions on the sky*
  - latter is more relevant for CMB observations

Recent developments...

- Relativists raised the concern that physical cosmologists have erred in failing to take into account the inherent non-linearity of Einstein's equations
  - the *backreaction* "mantra": averaging and non-linearity (of Einstein's equations) do not commute
  - so maybe cosmic acceleration is a mirage
- requires calculations in 2nd order perturbation theory...
- now mostly accepted that effects are too small to get rid of Λ
- but maybe there are still appreciable impacts:
  - Clarkson, Ellis++ '12 - large ($O(\kappa^2)$) source magnification
  - Clarkson++ '14 - similarly large *area* increase
    - "backreaction" strikes back?
- and the size of the effect is qualitatively consistent with expectation of
  - *singularity theorem* (Sachs... Raychaudhury... Narlikar ....)
  - and *focusing theorem* (Schneider, Ehlers & Sietz)




What is the distance to the CMB? How relativistic corrections remove the tension with local H_0 measurements

Chris Clarkson[1], Obinna Umeh[2], Roy Maartens[2,3] and Ruth Durrer[4]

[1] *Astrophysics, Cosmology & Gravity Centre, and, Department of Mathematics & Applied Mathematics, University of Cape Town, Cape Town 7701, South Africa.*
[2] *Physics Department, University of the Western Cape, Cape Town 7535, South Africa*
[3] *Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom*
[4] *Département de Physique Théorique & Center for Astroparticle Physics, Université de Genève, Quai E. Ansermet 24, CH-1211 Genève 4, Switzerland.*

The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Clarkson et al. 2014

$$\langle \Delta \rangle \simeq \frac{3}{2} \left\langle \left(\frac{\delta d_A}{\chi_s} \right)^2 \right\rangle = \frac{3}{2} \left\langle \kappa^2 \right\rangle \,, \tag{1.5}$$

where κ is the usual linear lensing convergence. This is actually the leading contribution to the expected change to large distances. We prove this remarkably simple and important result in a variety of ways in several appendices. It implies that the total area of a sphere of constant redshift will be larger than in the background. Physically this is because a sphere about us in redshift space is not a sphere in real space — lensing implies that this 'sphere' becomes significantly crumpled in real space, and hence has a larger area. When interpreted

4 Conclusions

We have demonstrated an important overall shift in the distance redshift relation when the aggregate of all lensing events is considered, calculated by averaging over an ensemble of universes. This result is a consequence of flux conservation at second-order in perturbation theory. This is a purely relativistic effect with no Newtonian counterpart — and it is the first quantitative prediction for a significant change to the background cosmology when averaging over structure [21]. The extraordinary amplification of aggregated lensing comes mainly from the integrated lensing of structure on scales in the range 1–100 Mpc, although structure down to 10kpc scales contributes significantly. We have estimated the size of the effect using

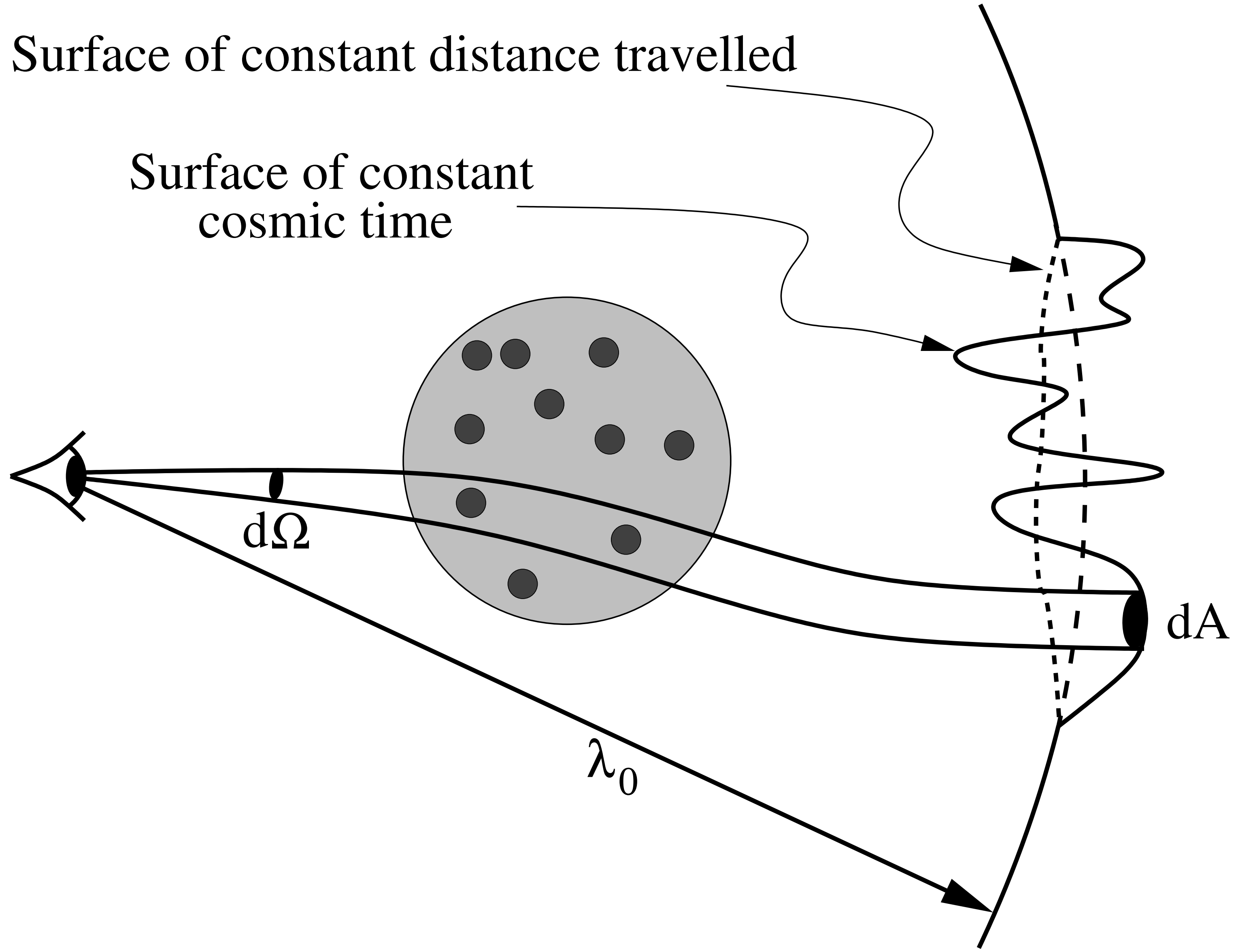
NK + Peacock 2015

- Weinberg *assumes* that the area of a surface of constant redshift is unperturbed by lensing by intervening structures
  - seems reasonable since *static* lenses do not affect redshift
  - and leads to conservation of e.g. source-averaged flux density
    - but not strictly true and breaks down at some level
- But if we assume this assumption is valid the claims for large effects can all be readily understood as *purely statistical effects*:
  - The mean flux magnification μ of a source is unity
    - so $\langle \Delta\mu \rangle_{\rm source} = 0$
  - but μ is a fluctuating quantity
  - so any non-linear function of μ (e.g. $D/D_0 = 1 / \sqrt{\mu}$) will *not* average to unity

KP15: Statistical biases...

- Example: Source averaged distance bias:
  - $D/D_0 = \mu^{-1/2} = (1 + \Delta\mu)^{-1/2} = 1 - \Delta\mu / 2 + 3(\Delta\mu)^2/8 + ...$
  - so $\langle D/D_0 \rangle_{source} = 1 + 3\langle(\Delta\mu)^2\rangle/8 + ... = 1 + 3\langle\kappa^2\rangle/2 + ...$
- Similarly for source averaged mean inverse magnification
  - $\langle D^2/D_0^2 \rangle_{source} = 1 + 4 \langle\kappa^2\rangle + ...$
- *These are precisely the results for the mean perturbation to the distance and distance squared found by Clarkson et al. 2014*
- But e.g. the latter is not the perturbation to the constant z surface area
  - that would be the average over *directions* rather than over sources
- Similarly, Clarkson et al. 2012 claim mean source averaged flux magnification is $\langle\mu\rangle = 1 + \langle 3\kappa^2 + \gamma^2\rangle + ... = 1 + \langle 4\kappa^2\rangle + ...$
  - but this is the *direction* averaged magnification
- *These come from non-commutativity of averaging and non-linearity*
  - $\langle f(x)\rangle$!= $f(\langle x\rangle)$ if x is a fluctuating quantity
  - but have *nothing* to do with the non-linearity of Einstein's equations

KP2015: closing the loophole in Weinberg's argument



2 effects:

1) wiggly lines don't get as far as straight lines
2) wrinkly surface has more area than a smooth one

but both effects are ~(bending angle)^2 ~ 10^{-6}

Key features of KP15 calculation of area of photosphere

- Calculations are rather technical, some key features are:
  - Weak field assumption:
    - we model the metric as weak field limit of GR
      - but we allow for non-rel motion of sources
        - these have negligible effects
      - similarly for gravitational waves
        - "photons can't surf a gravitational wave"
      - going beyond 1st order can be estimated and is tiny effect
    - result is isomorphic to light propagation in "lumpy glass"
  - Boundary conditions:
    - Perturbation theory calculations assume photosphere is constant z
    - Not true. It is more realistically a surface of constant cosmic time
    - Pert. theo. results may be qualitatively OK, but fail quantitatively
  - Final result for perturbation to the area of the photosphere is

$$\langle \Delta A \rangle / A_0 = \frac{1}{\lambda_0^2} \int_0^{\lambda_0} d\lambda \, (2\lambda(\lambda_0 - \lambda) + \lambda^2) J(\lambda).$$

where

$$J \equiv -8 \int_{-\infty}^{0} dy \, \xi'_\phi(y)/y = 2\pi \int k \, \Delta_\phi^2(k) \, d\ln k,$$

but $J = d\langle\theta^2\rangle/d\lambda$ and $J\lambda$ is on the order of 10^{-6}

What about the "*focusing theorem*"? $\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$

- 2 lessons from foregoing:
  - 1) The theorem applies to a bundle of rays fired along a given direction
    - i.e. a *direction* - not *source*-averaged quantity
    - and paths to sources avoid over-densities
    - so care is needed in interpreting this
  - 2) D is a non-linear function of A
    - so, because A is a fluctuation quantity, we automatically expect a statistical bias in D
    - and the size of the effect is ~ $\langle \varkappa^2 \rangle$
- So is the "normal tendency of matter to focus light rays"?
  - as inferred from the averaged focusing theorem
- or simply this statistical effect?

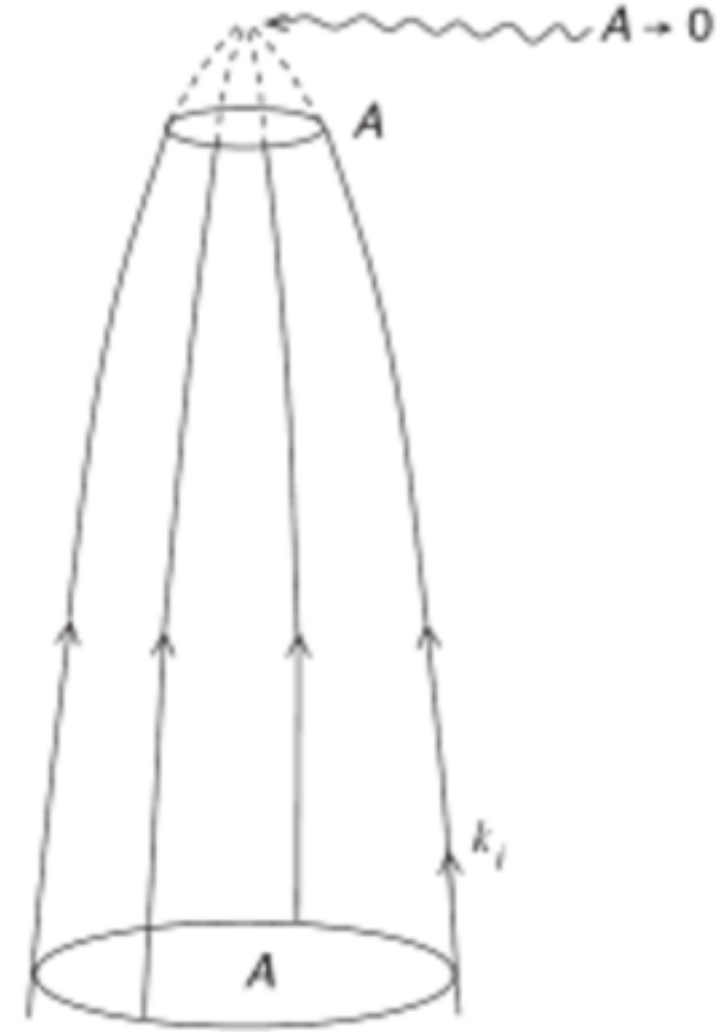


Fig. 18.3. The bundle of geodesics focusses in the future with its cross section *A* decreasing to zero. This effect was discussed in the context of spacetime singularity by A. K. Raychaudhuri.

Optical scalars (in weak-field GR or lumpy glass)

$\ddot{\mathbf{r}} = \boldsymbol{\nabla}_\perp \tilde{n}$ Geodesic equation

$$n = [(1 - 2\phi(\mathbf{r})/c^2)/(1 + 2\phi(\mathbf{r})/c^2)]^{1/2}$$

Optical *tensor* transport equation:

$$\dot{\mathbf{K}} = (\boldsymbol{\nabla}_\mathbf{x}\boldsymbol{\nabla}_\mathbf{x} - \mathbf{K}\partial_z)\tilde{n} - \boldsymbol{\nabla}_\mathbf{x}\tilde{n}\boldsymbol{\nabla}_\mathbf{x}\tilde{n} - \mathbf{K}\cdot\mathbf{K}$$

Optical scalar transport equations:

$$\dot{\theta} = \left(\frac{{\nabla_\perp}^2}{2} - \theta\partial_\lambda\right)\tilde{n} - |\boldsymbol{\nabla}_\perp\tilde{n}|^2/2 - \theta^2 - \Sigma^2$$

$$\dot{\boldsymbol{\Sigma}} = (\{\boldsymbol{\nabla}_\perp\boldsymbol{\nabla}_\perp\} - \boldsymbol{\Sigma}\partial_\lambda)\tilde{n} - \{\boldsymbol{\nabla}_\perp\tilde{n}\boldsymbol{\nabla}_\perp\tilde{n}\} - 2\theta\boldsymbol{\Sigma}$$

Solve for θ

The solution of $\dot{A}/2A = \theta(\lambda) = \lambda^{-1} + \Delta\theta(\lambda)$ is

$$A = \Omega\lambda^2 \exp\left(2\int_0^\lambda d\lambda'\,\Delta\theta(\lambda')\right)$$

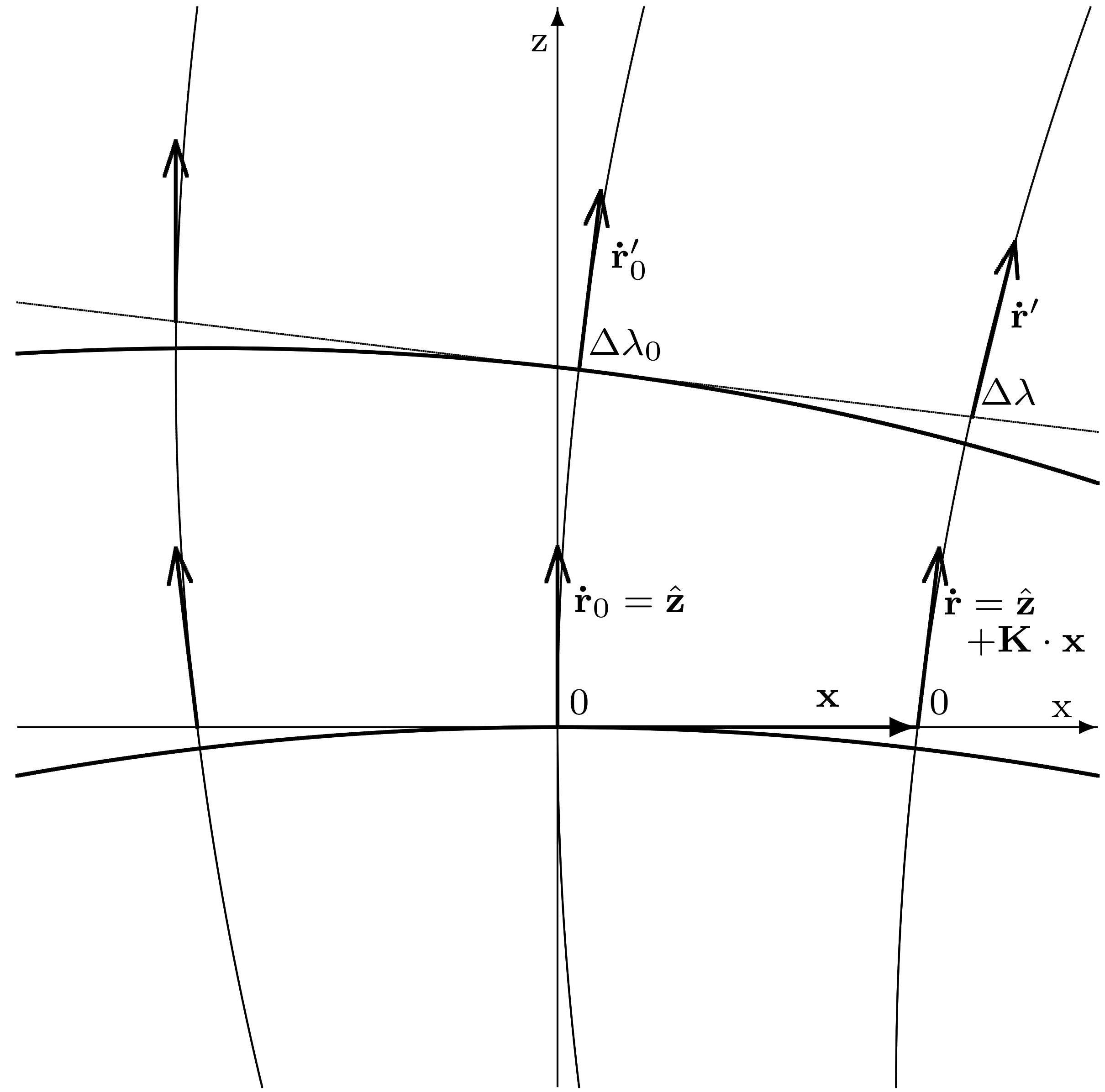


Figure D1. Illustration of a bundle of rays (thin curves) and associated wave-fronts (thick curves) and ray direction vectors $\dot{\mathbf{r}} = d\mathbf{r}/d\lambda$ (arrows). The base of each arrow is labelled by distance (physical for lumpy glass, background conformal for perturbed FRW) along the path. Close to the guiding ray the ray vectors will vary linearly with transverse displacement. The optical tensor $\mathbf{K}$ is the derivative of the ray direction with respect to coordinates $\mathbf{x}$ on the plane that is tangent to the wavefront at the location of the guiding ray. The optical tensor transport equation tells us how $\mathbf{K}$ evolves as the bundle propagates through any metric or refractive index fluctuations. Since rays are perpendicular to the

Part II: relativistic effects on D(z) at low z

- Two recent results from relativity:
  - "Doppler lensing" as a new probe of structure
    - related to SN1a error analysis
  - Bias in H_0 at low-z
    - another 2nd order effect
- But things are again not quite as they might appear…..

Cosmology with Doppler lensing

David J. Bacon[1], Sambatra Andrianomena[2], Chris Clarkson[2], Krzysztof Bolejko[3], Roy Maartens[1,4]
[1] *Institute of Cosmology & Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX*
[2] *Astrophysics, Cosmology Gravity Centre, and Department of Mathematics and Applied Mathematics, University of Cape Town, Cape Town, 7701, South Africa.*
[3] *Sydney Institute for Astronomy, The University of Sydney, NSW 2006, Australia*
[4] *Physics Department, University of the Western Cape, Cape Town 7535, South Africa*

Accepted —-. Received —-; in original form —-.

ABSTRACT
Doppler lensing is the apparent change in object size and magnitude due to peculiar velocities. Objects falling into an overdensity appear larger on its near side, and smaller on its far side, than typical objects at the same redshifts. This effect dominates over the usual gravitational lensing magnification at low redshift. Doppler lensing is a promising new probe of cosmology, and we explore in detail how to utilize the effect with forthcoming surveys. We present cosmological simulations of the Doppler and gravitational lensing effects based on the Millennium simulation. We show that Doppler lensing can be detected around stacked voids or unvirialised over-densities. New power spectra and correlation functions are proposed which are designed to be sensitive to Doppler lensing. We consider the impact of gravitational lensing and intrinsic size correlations on these quantities. We compute the correlation functions and forecast the errors for realistic forthcoming surveys, providing predictions for constraints on cosmological parameters. Finally, we demonstrate how we can make 3-D potential maps of large volumes of the Universe using Doppler lensing.

Key words: Cosmology: theory; cosmology: observations; gravitational lensing: weak

arXiv:1401.3694

Cosmology with Doppler lensing

David J. Bacon[1], Sambatra Andrianomena[2], Chris Clarkson[2], Krzysztof Bolejko[3], Roy Maartens[1,4]

1 INTRODUCTION

Light rays from distant sources are focused by overdensities (or defocused by underdensities) along the line of sight, leading to apparent magnification (or demagnification) of images. But besides this *gravitational lensing*, there is a further effect which appears to magnify or demagnify the images of objects in the Universe. This *Doppler lensing* effect arises from the peculiar velocity of the source, and was first highlighted and investigated in general by Bonvin (2008) (see also Bonvin et al. (2006)). Bolejko et al. (2013) then showed that the effect can dominate over gravitational lensing, and even reverse its effect, leading to an 'anti-lensing' phenomenon. Doppler lensing gives a new window into the peculiar velocity field in addition to the usual redshift space distortion measurements.

The effect is a consequence of the distortion introduced by mapping from redshift-space to real space, as illustrated in Figure 1. Imagine we have three spherical galaxies with the same physical size, and (as an extreme case) the same

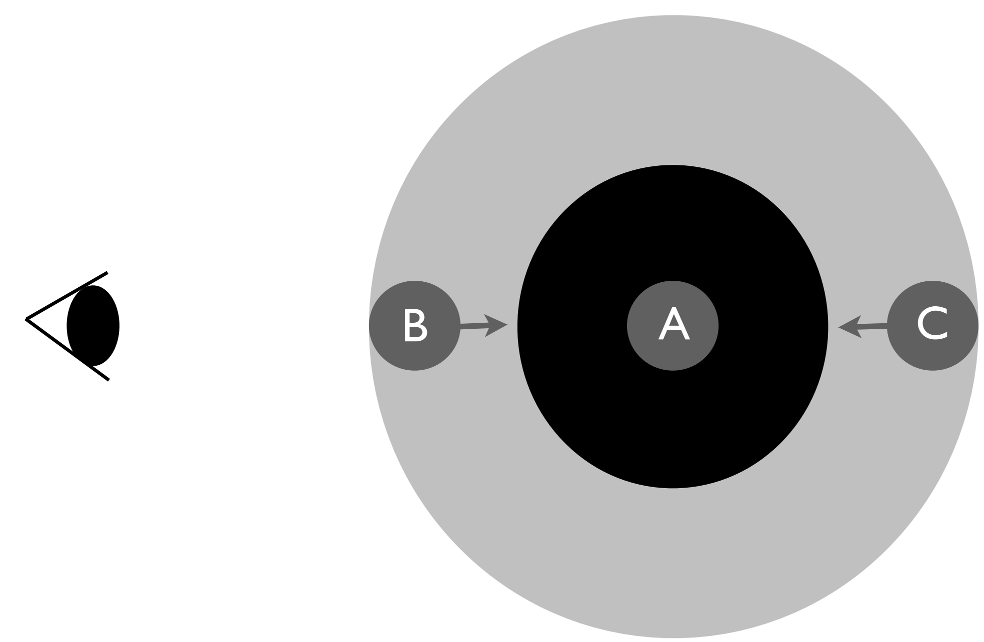


Figure 1. Three spherical galaxies of the same physical size and same observed redshift. A is at the centre of a spherical overdensity while B and C are falling towards the centre.

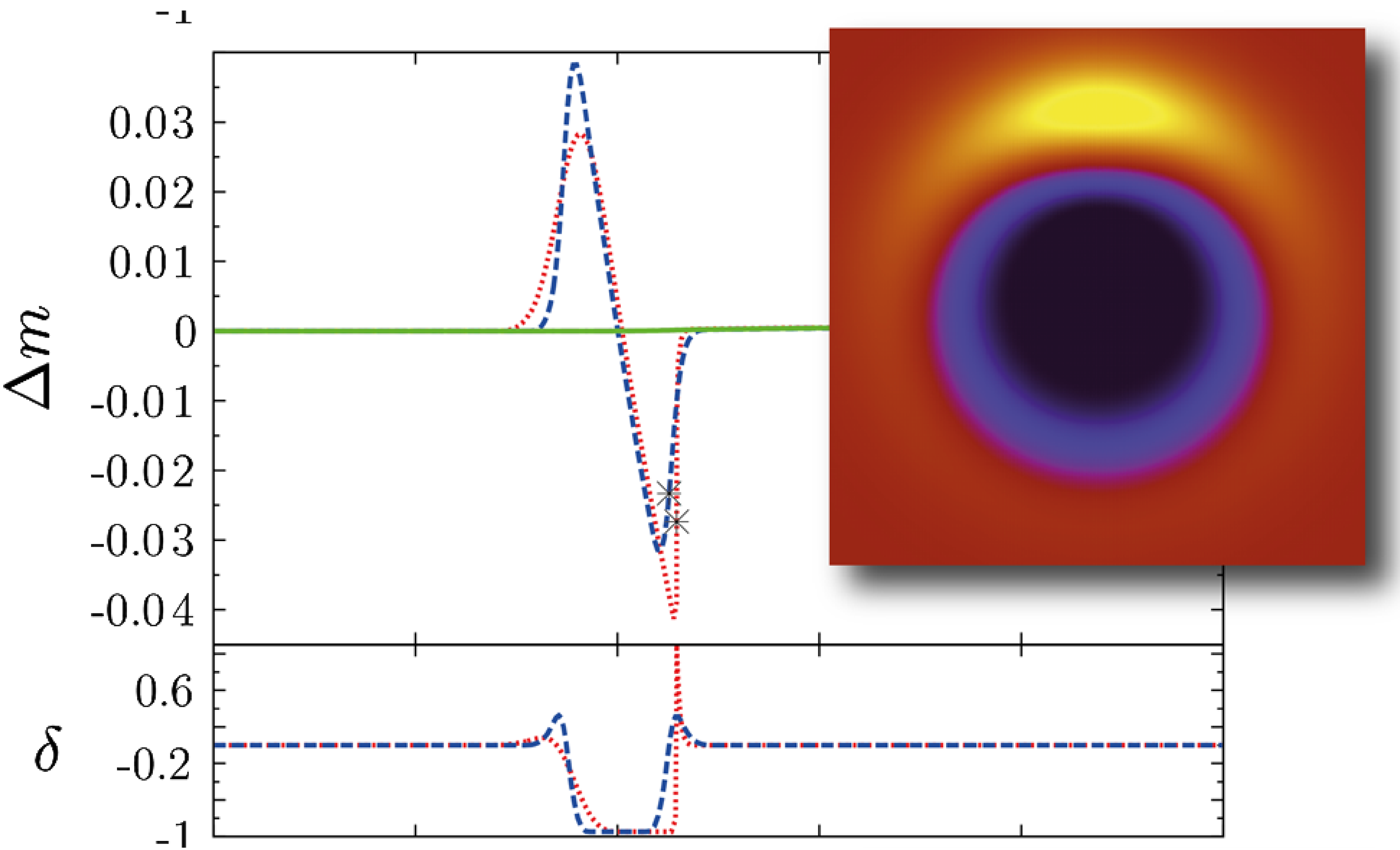
Antilensing: The Bright Side of Voids

Krzysztof Bolejko,[1] Chris Clarkson,[2] Roy Maartens,[3,4] David Bacon,[4] Nikolai Meures,[4] and Emma Beynon[4]
[1]*Sydney Institute for Astronomy, The University of Sydney, Sydney, New South Wales 2006, Australia*
[2]*Centre for Astrophysics, Cosmology and Gravitation and, Department of Mathematics and Applied Mathematics, University of Cape Town, Cape Town 7701, South Africa*
[3]*Physics Department, University of the Western Cape, Cape Town 7535, South Africa*
[4]*Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom*
(Received 1 October 2012; published 10 January 2013)

More than half of the volume of our Universe is occupied by cosmic voids. The lensing magnification effect from those underdense regions is generally thought to give a small dimming contribution: objects on the far side of a void are supposed to be observed as slightly smaller than if the void were not there, which together with conservation of surface brightness implies net reduction in photons received. This is predicted by the usual weak lensing integral of the density contrast along the line of sight. We show that this standard effect is swamped at low redshifts by a relativistic Doppler term that is typically neglected. Contrary to the usual expectation, objects on the far side of a void are *brighter* than they would be otherwise. Thus the local dynamics of matter in and near the void is crucial and is only captured by the full relativistic lensing convergence. There are also significant nonlinear corrections to the relativistic linear theory, which we show actually underpredicts the effect. We use exact solutions to estimate that these can be more than 20% for deep voids. This remains an important source of systematic errors for weak lensing density reconstruction in galaxy surveys and for supernovae observations, and may be the cause of the reported extra scatter of field supernovae located on the edge of voids compared to those in clusters.

DOI: 10.1103/PhysRevLett.110.021302 PACS numbers: 98.62.Sb

Boleko et al. 2013



PHYSICAL REVIEW D **78,** 123530 (2008)

Effect of peculiar motion in weak lensing

Camille Bonvin*

Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH–1211 Genève 4, Switzerland

(Received 1 October 2008; published 29 December 2008)

We study the effect of peculiar motion in weak gravitational lensing. We derive a fully relativistic formula for the cosmic shear and the convergence in a perturbed Friedmann universe. We find a new contribution related to galaxies' peculiar velocities. This contribution does not affect cosmic shear in a measurable way, since it is of second order in the velocity. However, its effect on the convergence (and consequently on the magnification, which is a measurable quantity) is important, especially for redshifts $z \leq 1$. As a consequence, peculiar motion modifies also the relation between the shear and the convergence.

DOI: 10.1103/PhysRevD.78.123530 PACS numbers: 98.80.Es, 95.30.Sf, 95.35.+d, 98.62.Sb

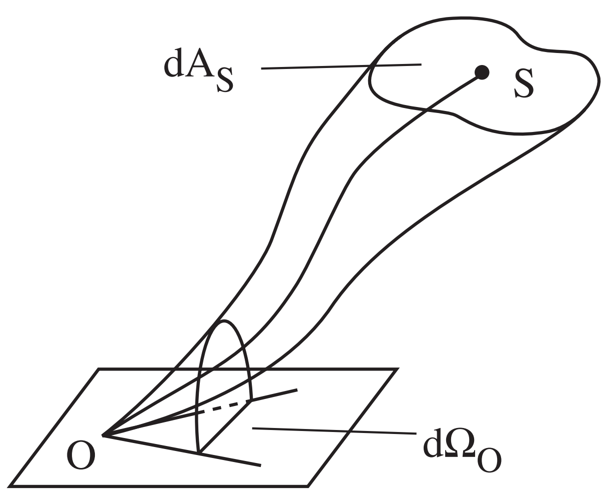


FIG. 1. A light beam emitted by a galaxy at spacetime position S and received by an observer at O. At the observer position, the plane normal to the observer four-velocity is indicated.

$$\kappa = \int_{\eta_S}^{\eta_O} d\eta \frac{(\eta - \eta_S)(\eta_O - \eta)}{\eta_O - \eta_S} \nabla_\perp^2 \Psi + 3\Psi_S - 2\int_{\eta_S}^{\eta_O} d\eta \frac{\eta - \eta_S}{\eta_O - \eta_S} \dot{\Psi} - \frac{4}{(\eta_O - \eta_S)} \times \int_{\eta_S}^{\eta_O} d\eta \Psi + \left(1 - \frac{1}{\mathcal{H}_S(\eta_O - \eta_S)}\right) \times \left[(\mathbf{v}_S - \mathbf{v}_O)\cdot \mathbf{n} + \Psi_S - \Psi_O + 2\int_{\eta_S}^{\eta_O} d\eta \dot{\Psi}\right], \quad (31)$$

PHYSICAL REVIEW D **73,** 023523 (2006)

Fluctuations of the luminosity distance

Camille Bonvin,* Ruth Durrer,† and M. Alice Gasparini‡

Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH-1211 Genève 4, Switzerland

(Received 7 November 2005; published 27 January 2006)

We derive an expression for the luminosity distance in a perturbed Friedmann universe. We define the correlation function and the power spectrum of the luminosity distance fluctuations and express them in terms of the initial spectrum of the Bardeen potential. We present semianalytical results for the case of a pure CDM (cold dark matter) universe. We argue that the luminosity distance power spectrum represents a new observational tool which can be used to determine cosmological parameters. In addition, our results shed some light into the debate whether second order small scale fluctuations can mimic an accelerating universe.

$$\begin{aligned}\tilde{d}_L(z_S,\mathbf{n}) = (1+z_S)(\eta_O-\eta_S)\Big\{ 1 &- \frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\mathbf{v}_O\cdot\mathbf{n} - \Big(1-\frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\Big)\mathbf{v}_S\cdot\mathbf{n} \\ &- \Big(2-\frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\Big)\Psi_S + \Big(1-\frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\Big)\Psi_O \\ &+ \frac{2}{(\eta_O-\eta_S)}\int_{\eta_S}^{\eta_O} d\eta\Psi + \frac{2}{(\eta_O-\eta_S)\mathcal{H}_S}\int_{\eta_S}^{\eta_O} d\eta\dot{\Psi} - 2\int_{\eta_S}^{\eta_O} d\eta\frac{(\eta-\eta_S)}{(\eta_O-\eta_S)}\dot{\Psi} + \int_{\eta_S}^{\eta_O} d\eta\frac{(\eta-\eta_S)(\eta_O-\eta)}{(\eta_O-\eta_S)}\ddot{\Psi} \\ &- \int_{\eta_S}^{\eta_O} d\eta\frac{(\eta-\eta_S)(\eta_O-\eta)}{(\eta_O-\eta_S)}\nabla^2\Psi\Big\}. \end{aligned} \qquad (59)$$

$$\mathcal{H} \equiv \dot{a}/a = a^{-1}\frac{da}{d\eta} \equiv Ha$$

We now consider a Friedmann universe with scalar perturbations. In longitudinal (or Newtonian) gauge the metric is given by

$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = a^2[-(1+2\Psi)d\eta^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j]. \qquad (35)$$

So what’s new in Doppler (or anti) lensing?

THE ASTRONOMICAL JOURNAL VOLUME 81, NUMBER 9 SEPTEMBER 1976

Motion of the Galaxy and the Local Group determined from the velocity anisotropy of distant Sc I galaxies. II. The analysis for the motion

Vera C. Rubin,*‡ Norbert Thonnard,‡ and W. Kent Ford Jr.†‡
Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, DC 20015

Morton S. Roberts
National Radio Astronomy Observatory, Green Bank, West Virginia
(Received 18 May 1976; revised 28 June 1976)

For an all-sky sample of 96 Sc I–Sc II galaxies, $3500 < V_c < 6500$ km sec^{-1}, for which radial velocities and magnitudes have been obtained, the quantity $\mathrm{HM} = \log V_c - 0.2 m_c = \log H - 0.2 M_0 - 5$ varies across the sky. The form of the variation is consistent with a motion of the Sun of $V_\odot = 600 \pm 125$ km sec^{-1} toward $\alpha = 32° \pm 20°$, $\delta = +53° \pm 11°$, ($l = 135°$, $b = -8°$), corresponding to a motion of the Galaxy and the Local Group of galaxies of $V_{GM} = 454 \pm 125$ km sec^{-1} toward $l = 163°$, $b = -11°$. The mean error arises from the scatter of the data and does not take possible systematic errors into account. The Galaxy is moving almost edge-on; the leading edge is in the anticenter direction. Alternative explanations which might account for the observed anisotropy are examined: (1) that apex galaxies are intrinsically fainter than antapex galaxies; (2) that apex (anticenter) galaxies are more obscured; (3) that the Hubble constant varies by 20% across the sky. Each of these explanations is shown to be less likely than a motion of the observer. It is also demonstrated that a Malmquist bias does not produce the observed anisotropy. Additionally, undetected systematic errors in the magnitude system are probably no larger than $0^m.1$, so can account for no more than one-fourth of the observed effect. Moreover, 22 nearer galaxies, $1600 < V_c < 3500$ km sec^{-1} exhibit a more pronounced anisotropy in HM than the sample $3500 < V_c < 6500$ km sec^{-1}. Of the explanations considered above, only a motion of our Galaxy is consistent with the variation in HM observed at both distances. Support for this explanation comes also from a sample of E and S0 galaxies, $3500 < V_c < 6500$ km sec^{-1} (Sandage 1975). After correction for the motion of the observer, the random motions of these Sc galaxies are small, $\sigma(\Delta V)_{\mathrm{radial}} < 200$ km sec^{-1}, and the Hubble flow is uniform, $\sigma(\Delta H/H) < 0.04$.

What’s new in Doppler (or anti) lensing?

- Long history of observations
  - Rubin-Ford effect (1976)
  - …… Tully-Fisher … Faber-Jackson … Dn-sigma …
  - Cosmic flows II; 6df survey…
- What’s new in theory?

The magnitude–redshift relation in a perturbed Friedmann universe

Misao Sasaki *Research Institute for Theoretical Physics, Hiroshima University, Takehara, Hiroshima 725, Japan*

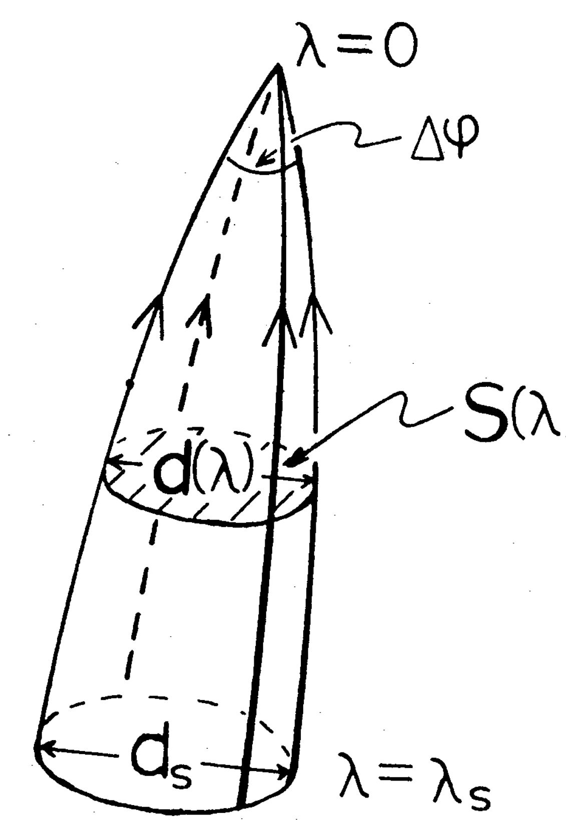
Accepted 1987 April 30. Received 1987 April 29; in original form 1987 March 2

Summary. A general formula for the magnitude–redshift relation in a linearly perturbed Friedmann universe is derived. The formula does not assume any specific gauge condition, but the gauge-invariance of it is explicitly shown. Then the application of the formula to the spatially flat background model is considered and the implications are discussed.

$$d\hat{s}^2 = a(\eta)^2\, ds^2 = a(\eta)^2 g_{\mu\nu}\, dx^\mu\, dx^\nu \qquad g_{\mu\nu} = g^{(b)}_{\mu\nu} + \delta g_{\mu\nu}$$

$$\frac{\Delta d_L(z,\gamma^i)}{d_L(z)} = \frac{1}{\lambda_s}\int_0^{\lambda_s} d\lambda \left[(\lambda-\lambda_s)\lambda \overset{(3)}{\Delta}\Psi + 2(\Psi-\Psi_0)\right] + \frac{1}{2}\left(\frac{\eta_0}{\lambda_s}-3\right)(\Psi_s-\Psi_0)$$

$$+\frac{1}{6}\left(\frac{\eta_0}{\lambda_s}-3\right)[\eta_s(\psi_{|i}\gamma^i)_s - \eta_0(\psi_{|i}\gamma^i)_0] - \frac{1}{3}\eta_0(\psi_{|i}\gamma^i)_0 + \Psi_0 + \frac{3}{\eta_0}\delta_*\eta_0,$$



What's new in Doppler (or anti) lensing?

- Long history of observations
  - Rubin-Ford effect (1976)
  - …… Tully-Fisher … Faber-Jackson … Dn-sigma …
  - Cosmic flows II; 6df survey…
- What's new in theory?
  - long history back to Zel'dovich '64
  - classic paper by Sasaki et al '87
- Wasn't this all thrashed out in relation to SN1a cosmology?

PHYSICAL REVIEW D **73,** 123526 (2006)

Correlated fluctuations in luminosity distance and the importance of peculiar motion in supernova surveys

Lam Hui[1,2,*] and Patrick B. Greene[1,3,†]

[1]*Institute for Strings, Cosmology and Astro-particle Physics (ISCAP)*
[2]*Department of Physics, Columbia University, New York, New York 10027, USA*
[3]*Department of Physics and Astronomy, University of Texas at San Antonio, Texas 78249, USA*
(Received 7 December 2005; published 23 June 2006)

Large scale structure introduces two different kinds of errors in the luminosity distance estimates from standardizable candles such as supernovae Ia (SNe)—a Poissonian scatter for each SN and a coherent component due to correlated fluctuations between different SNe. Increasing the number of SNe helps reduce the first type of error but not the second. The coherent component has been largely ignored in forecasts of dark energy parameter estimation from upcoming SN surveys. For instance it is commonly thought, based on Poissonian considerations, that peculiar motion is unimportant, even for a low redshift SN survey such as the Nearby Supernova Factory (SNfactory; $z = 0.03$–0.08), which provides a useful anchor for future high redshift surveys by determining the SN zero point. We show that ignoring coherent peculiar motion leads to an underestimate of the zero-point error by about a factor of 2, despite the fact that SNfactory covers almost half of the sky. More generally, there are four types of fluctuations: peculiar motion, gravitational lensing, gravitational redshift and what is akin to the integrated Sachs-Wolfe effect. Peculiar motion and lensing dominates at low and high redshifts, respectively. Taking into account all significant luminosity distance fluctuations due to large scale structure leads to a degradation of up to 60% in the determination of the dark energy equation of state from upcoming high redshift SN surveys, when used in conjunction with a low redshift anchor such as the SNfactory. The most relevant fluctuations are the coherent ones due to peculiar motion and the Poissonian ones due to lensing, with peculiar motion playing the dominant role. We also discuss to what extent the noise here can be viewed as a useful signal, and whether corrections can be made to reduce the degradation.

DOI: 10.1103/PhysRevD.73.123526 PACS numbers: 98.80.−k, 95.30.Sf, 98.80.Es, 98.80.Jk

Hui & Greene 2006

In summary, the total peculiar motion and lensing contributions to δ_{d_L} are

$$\delta_{d_L}(z, \mathbf{n}) = \mathbf{v_e} \cdot \mathbf{n} - \frac{1}{\chi_e}\left[\frac{a}{a'}\right]_e (\mathbf{v_e} \cdot \mathbf{n} - \mathbf{v_0} \cdot \mathbf{n}) - \int_0^{\chi_e} d\chi \frac{(\chi_e - \chi)\chi}{\chi_e} \nabla^2 \phi(\chi). \tag{18}$$

To reiterate: $\mathbf{v_e}$ and $\mathbf{v_0}$ are the peculiar velocities of the emitter and observer, and $\mathbf{n}$ is the line-of-sight unit vector pointing away from the observer ($\mathbf{n}$ here plays the role of $\boldsymbol{\theta}$ in Eq. (7)); the comoving distance to emitter χ_e, the scale factor at emission a_e and its derivative with respect to conformal time a'_e are evaluated at redshift z. One can see from above that for small χ_e or at a low redshift, the peculiar motion term proportional to $1/\chi_e$ becomes important, while at a large redshift, the lensing term (second line) is more important. A more rigorous derivation of δ_{d_L},

$$(\sigma_i^{\text{Poiss.,vel.}})^2 \equiv \left[\frac{5}{\ln 10}\right]^2 \left[1 - \frac{a_i}{a'_i \chi_i}\right]^2 (D'_i)^2 \times \int \frac{d^3k}{(2\pi)^3} \frac{k_z^2}{k^4} P(k, a = 1),$$

PHYSICAL REVIEW D **73,** 103002 (2006)

Large-scale bulk motions complicate the Hubble diagram

Asantha Cooray[1]
[1]*Center for Cosmology, Department of Physics and Astronomy, University of California, Irvine, California 92697, USA*

Robert R. Caldwell[2]
[2]*Department of Physics and Astronomy, Dartmouth College, 6127 Wilder Laboratory, Hanover, New Hampshire 03755, USA*
(Received 17 January 2006; published 15 May 2006)

We investigate the extent to which correlated distortions of the luminosity distance-redshift relation due to large-scale bulk flows limit the precision with which cosmological parameters can be measured. In particular, peculiar velocities of type 1a supernovae at low redshifts, $z < 0.2$, may prevent a sufficient calibration of the Hubble diagram necessary to measure the dark energy equation of state to better than 10%, and diminish the resolution of the equation of state time-derivative projected for planned surveys. We consider similar distortions of the angular-diameter distance, as well as the Hubble constant. We show that the measurement of correlations in the large-scale bulk flow at low redshifts using these distance indicators may be possible with a cumulative signal-to-noise ratio of order 7 in a survey of 300 type 1a supernovae spread over 20 000 square degrees.

combination, we obtain (see, Ref. [28] for details including their equation 3.15; also [29,31]):

$$\frac{\delta d_L}{d_L} = \hat{\mathbf{n}} \cdot \left[\mathbf{v}_{\rm SNe} - \frac{a}{a'\chi}(\mathbf{v}_{\rm SNe} - \mathbf{v}_{\rm obs}) \right], \qquad (1)$$

where $\hat{\mathbf{n}}$ is the unit vector along the line-of-sight, $\mathbf{v}_{\rm SNe}$ is the SN velocity, $\mathbf{v}_{\rm obs}$ is the velocity of the observer, χ is the comoving radial distance to the SN, and the prime denotes the derivative with respect to the conformal time. Unless otherwise stated, here and throughout, we take a unit system in which $c = 1$. The covariance matrix of errors in luminosity distance is

$$\mathrm{Cov}_{ij} \approx \sigma^2_{\rm int}(z_i)\delta_{ij} + C^{vv}(z_i, z_j, \theta_{ij}), \qquad (2)$$

where $\sigma^2_{\rm int}(z_i)$ is the variance term that affects each distance individually (e.g. due to random velocities, or the intrinsic uncertainty in the calibration of SN light curves).

$$C^{HH}(z_i, z_j, \theta_{ij}) = \sum_{\text{even}\,\ell} \frac{2l+1}{4\pi} \cos\theta_{ij} \frac{2}{\pi} F_l \times \int dk P_{mm}(k, z_i, z_j) j_l(k[\chi_i - \chi_j \cos\theta_{ij}]) \times j_l(k\chi_j \sin\theta_{ij}) \left\{ D'(z_i)\left(1 - \frac{a}{a'\chi}\right)_i + \chi_i \left[D'(z_i)\left(1 - \frac{a}{a'\chi}\right)_i \right]' \right\} \left\{ D'(z_j) \times \left(1 - \frac{a}{a'\chi}\right)_j + \chi_j \left[D'(z_j)\left(1 - \frac{a}{a'\chi}\right)_j \right]' \right\},$$

The Astrophysical Journal, 741:67 (15pp), 2011 November 1 doi:10.1088/0004-637X/741/1/67

THE EFFECT OF PECULIAR VELOCITIES ON SUPERNOVA COSMOLOGY

Tamara M. Davis[1,2], Lam Hui[3], Joshua A. Frieman[4,5,6], Troels Haugbølle[7], Richard Kessler[4,5], Benjamin Sinclair[1], Jesper Sollerman[7,8], Bruce Bassett[9,10,11], John Marriner[6], Edvard Mörtsell[12], Robert C. Nichol[13], Michael W. Richmond[14], Masao Sako[15], Donald P. Schneider[16], and Mathew Smith[17]

We analyze the effect that peculiar velocities have on the cosmological inferences we make using luminosity distance indicators, such as Type Ia supernovae. In particular we study the corrections required to account for (1) our own motion, (2) correlations in galaxy motions, and (3) a possible local under- or overdensity. For all of these effects we present a case study showing the impact on the cosmology derived by the Sloan Digital Sky Survey-II Supernova Survey (SDSS-II SN Survey). Correcting supernova (SN) redshifts for the cosmic microwave background (CMB) dipole slightly overcorrects nearby SNe that share some of our local motion. We show that while neglecting the CMB dipole would cause a shift in the derived equation of state of $\Delta w \sim 0.04$ (at fixed Ω_m), the additional local-motion correction is currently negligible ($\Delta w \lesssim 0.01$). We then demonstrate a covariance-matrix approach to statistically account for correlated peculiar velocities. This down-weights nearby SNe and effectively acts as a graduated version of the usual sharp low-redshift cut. Neglecting coherent velocities in the current sample causes a systematic shift of $\Delta w \sim 0.02$. This will therefore have to be considered carefully when future surveys aim for percent-level accuracy and we recommend our statistical approach to down-weighting peculiar velocities as a more robust option than a sharp low-redshift cut.

The peculiar-motion-induced magnitude covariance is related to the velocity correlation function $\xi_{ij}^{\rm vel}$ by

$$C_{ij}^{\rm vel} = \left[\frac{5}{c\ln 10}\right]^2 \left[1 - \frac{a_i}{a_i'}\frac{c}{\tilde{\chi}_i}\right]\left[1 - \frac{a_j}{a_j'}\frac{c}{\tilde{\chi}_j}\right]\xi_{ij}^{\rm vel}, \quad (26)$$

where c is the speed of light, $\tilde{\chi} \equiv R_0\chi$ is the radial comoving distance, $a = R/R_0$ is the normalized scale factor, and the prime denotes the conformal time derivative. All quantities with

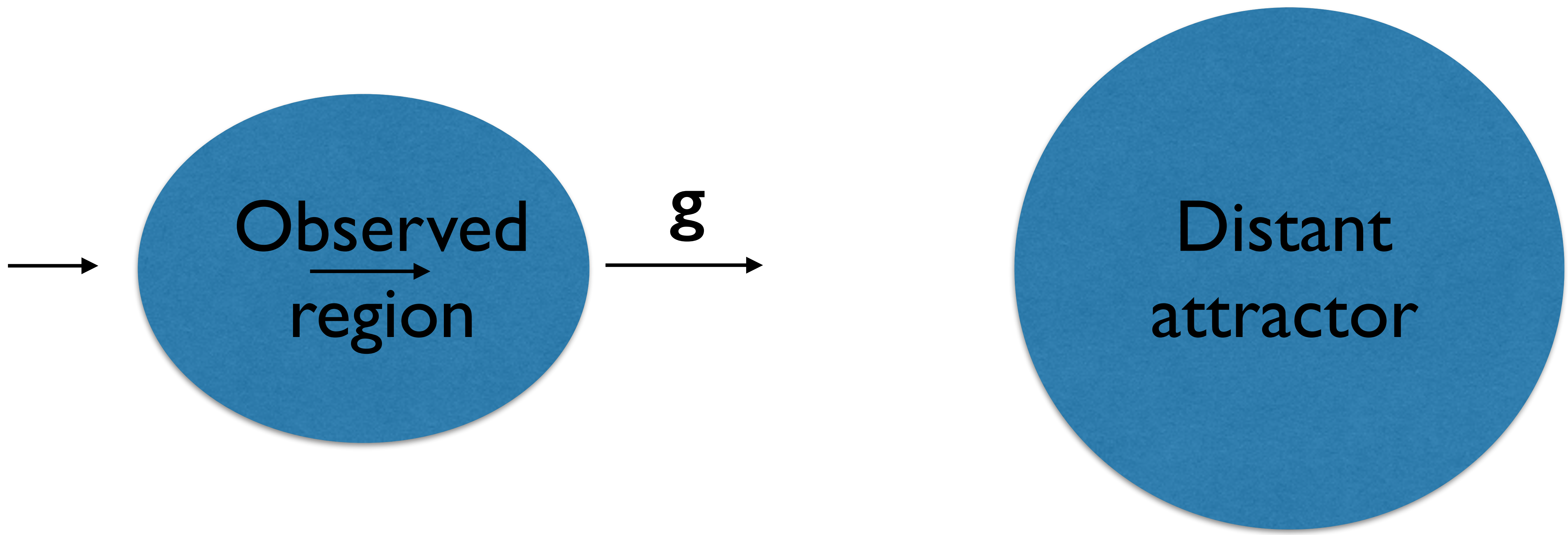
$$C_{ij}^{\rm vel} = \left[\frac{5}{c\ln 10}\right]^2 \left[1 - \frac{a_i}{a_i'}\frac{c}{\tilde{\chi}_i}\right]\left[1 - \frac{a_j}{a_j'}\frac{c}{\tilde{\chi}_j}\right] \times D_i' D_j' \int_0^\infty \frac{dk}{2\pi^2} P(k)_{z=0} \times \sum_{\ell=0}^{\infty}(2\ell+1) j_\ell'(k\tilde{\chi}_i) j_\ell'(k\tilde{\chi}_j)\mathcal{P}_\ell(\hat{\mathbf{x}}_\mathbf{i}\cdot\hat{\mathbf{x}}_\mathbf{j}),$$

What's new in Doppler (or anti) lensing?

- Long history of observations
  - Rubin-Ford effect (1976)
  - Tammann, Sandage & Yahil (1979)
  - …… Tully-Fisher … Faber-Jackson … Dn-sigma …
  - Cosmic flows II; 6df survey…
- What's new in theory?
  - long history back to Zel'dovich '64
  - classic paper by Sasaki et al '87
- Wasn't this all thrashed out in relation to SN1a cosmology?
  - Hui & Greene '06; Cooray & Caldwell '06; Davis et al 2011
- So it's "not even wrong"?
  - not quite… lowest order effect is traditional pec. vel.
  - but next order (finite z) effect depends on absolute motion
  - violates Equivalence Principle!

Kaiser & Hudson, 2014

- Perturbation to the distance (at fixed z) from velocities (alone)
  - $(\delta d/d)_z = -(a/a'\chi)(\mathbf{v}_S.\mathbf{n} - \mathbf{v}_O.\mathbf{n}) + \mathbf{v}_S.\mathbf{n}$
- At low z, $a'\chi/a=z$, so first term dominates
  - it depends only on relative velocity
- But for finite z we need 2nd term
  - this depends on *absolute* peculiar motion of sources
  - what if observer and sources share a common motion?
    - perhaps caused by the attraction of a distant mass excess
  - would we see a dipole in $(\delta d/d)_z = \mathbf{v}_S.\mathbf{n}$?
- Ans: no
  - theorists have kept only velocity driven term
  - have discarded another term of the same order (SW '67)
  - KH14: consistent analysis respects EP
  - needed to allow for effect of large-scale motions in SN1a H0


Observed
region
g
Distant
attractor

2) Bias in H_0 from 2nd order pert^n theory

PHYSICAL REVIEW D **78,** 083531 (2008)

Scale dependence of cosmological backreaction

Nan Li* and Dominik J. Schwarz+

Fakultät für Physik, Universität Bielefeld, Universitätsstraße 25, D-33615 Bielefeld, Germany

(Received 2 November 2007; published 23 October 2008)

Because of the noncommutation of spatial averaging and temporal evolution, inhomogeneities and anisotropies (cosmic structures) influence the evolution of the averaged Universe via the cosmological backreaction mechanism. We study the backreaction effect as a function of averaging scale in a perturbative approach up to higher orders. We calculate the hierarchy of the critical scales, at which 10% effects show up from averaging at different orders. The dominant contribution comes from the averaged spatial curvature, observable up to scales of ∼200 Mpc. The cosmic variance of the local Hubble rate is 10% (5%) for spherical regions of radius 40 (60) Mpc. We compare our result to the one from Newtonian cosmology and Hubble Space Telescope Key Project data.

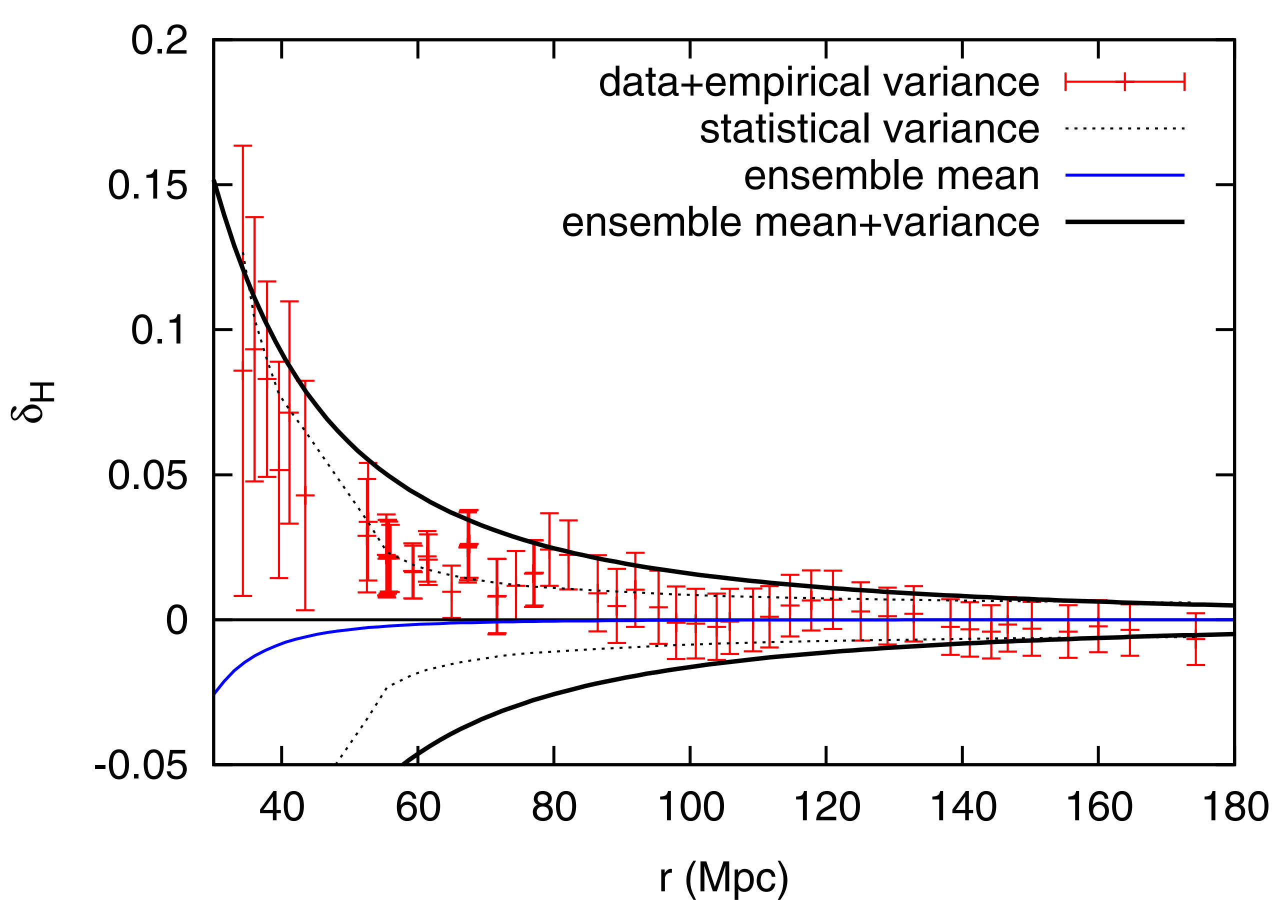


FIG. 2 (color online). Relative fluctuation of the Hubble rate from cosmological backreaction and its cosmic variance band (thick lines) compared to the empirical mean and variance of δ_H obtained from the HST Key Project data [5] as a function of averaging radius. The thin line shows the ensemble mean of δ_H. The band enclosed by the thick lines indicates the effect of the inhomogeneities ($\propto 1/r^2$), and the dashed lines are the effect from sampling with given measurement errors in an otherwise perfectly homogeneous Universe.

PHYSICAL REVIEW D **80,** 083525 (2009)

Influence of structure formation on the cosmic expansion

Chris Clarkson,* Kishore Ananda,† and Julien Larena‡

Cosmology & Gravity Group, Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa

(Received 4 August 2009; published 23 October 2009)

We investigate the effect that the average backreaction of structure formation has on the dynamics of the cosmological expansion, within the concordance model. Our approach in the Poisson gauge is fully consistent up to second order in a perturbative expansion about a flat Friedmann background, including a cosmological constant. We discuss the key length scales which are inherent in any averaging procedure of this kind. We identify an intrinsic homogeneity scale that arises from the averaging procedure, beyond which a residual offset remains in the expansion rate and deceleration parameter. In the case of the deceleration parameter, this can lead to a quite large increase in the value, and may therefore have important ramifications for dark energy measurements, even if the underlying nature of dark energy is a cosmological constant. We give the intrinsic variance that affects the value of the effective Hubble rate and deceleration parameter. These considerations serve to add extra intrinsic errors to our determination of the cosmological parameters, and, in particular, may render attempts to measure the Hubble constant to percent precision overly optimistic.

The Hubble rate in averaged cosmology

Obinna Umeh, Julien Larena and Chris Clarkson

Astrophysics, Cosmology and Gravity Center and
Department of Mathematics and Applied Mathematics, University of Cape Town,
Rondebosch 7701, South Africa

E-mail: umeobinna@gmail.com, julien.larena@gmail.com, chris.clarkson@uct.ac.za

Received November 23, 2010
Revised January 27, 2011
Accepted March 3, 2011
Published March 21, 2011

Abstract. The calculation of the averaged Hubble expansion rate in an averaged perturbed Friedmann-Lemaître-Robertson-Walker cosmology leads to small corrections to the background value of the expansion rate, which could be important for measuring the Hubble constant from local observations. It also predicts an intrinsic variance associated with the finite scale of any measurement of H_0, the Hubble rate today. Both the mean Hubble rate and its variance depend on both the definition of the Hubble rate and the spatial surface on which the average is performed. We quantitatively study different definitions of the averaged Hubble rate encountered in the literature by consistently calculating the backreaction effect at second order in perturbation theory, and compare the results. We employ for the first time a recently developed gauge-invariant definition of an averaged scalar. We also discuss the variance of the Hubble rate for the different definitions.

Keywords: cosmic flows, cosmological perturbation theory, dark energy theory

The second-order luminosity-redshift relation in a generic inhomogeneous cosmology

Ido Ben-Dayan,[a,b] **Giovanni Marozzi,**[c,d] **Fabien Nugier**[e] **and Gabriele Veneziano**[c,f]

Published November 22, 2012

Abstract. After recalling a general non-perturbative expression for the luminosity-redshift relation holding in a recently proposed "geodesic light-cone" gauge, we show how it can be transformed to phenomenologically more convenient gauges in which cosmological perturbation theory is better understood. We present, in particular, the complete result on the luminosity-redshift relation in the Poisson gauge up to second order for a fairly generic perturbed cosmology, assuming that appreciable vector and tensor perturbations are only generated at second order. This relation provides a basic ingredient for the computation of the effects of stochastic inhomogeneities on precision dark-energy cosmology whose results we have anticipated in a recent letter. More generally, it can be used in connection with any physical information carried by light-like signals traveling along our past light-cone.

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Backreaction on the luminosity-redshift relation from gauge invariant light-cone averaging

I. Ben-Dayan,[a,b] M. Gasperini,[c,d] G. Marozzi,[e] F. Nugier[f] and G. Veneziano[e,g]

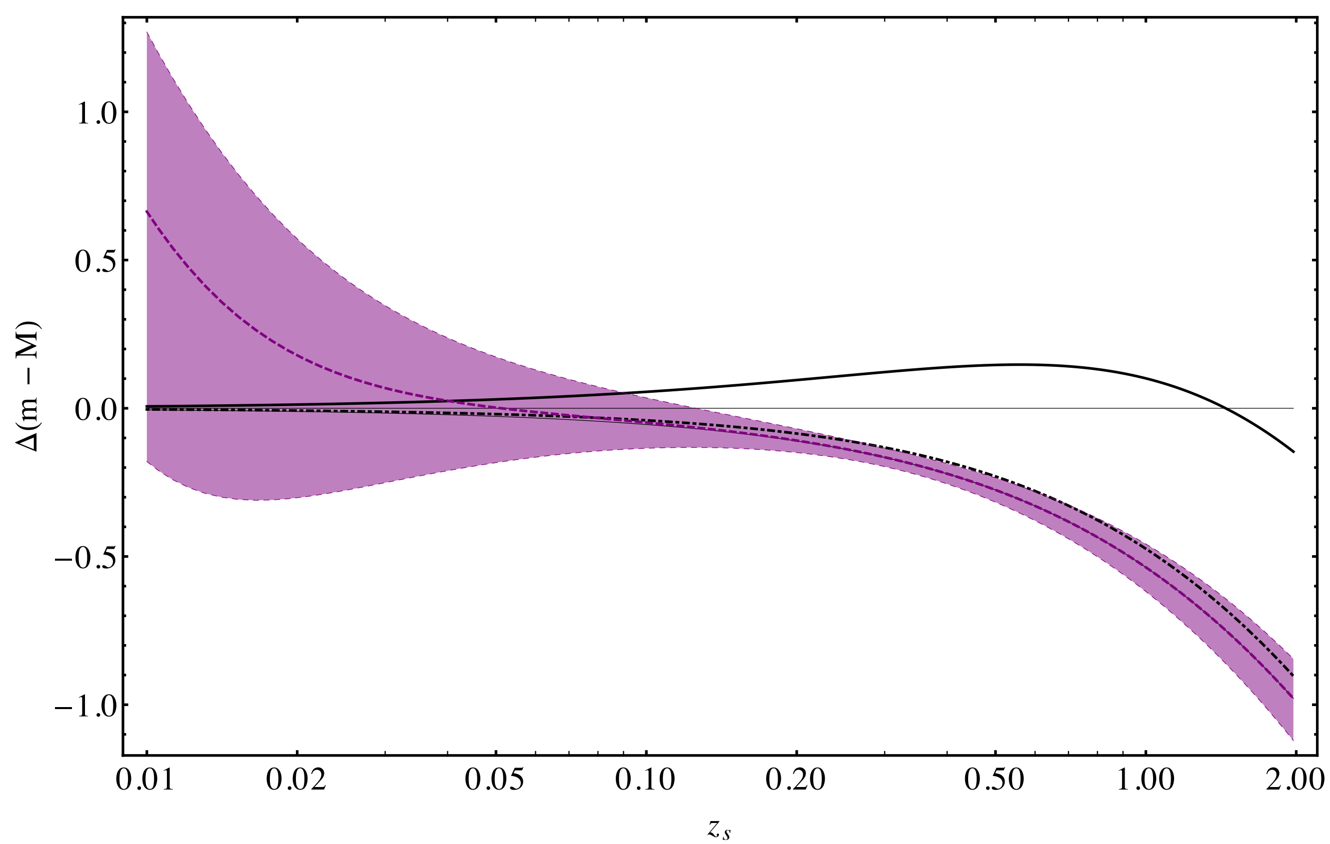


Figure 4. The distance-modulus difference of eq. (6.3) is plotted for a pure CDM model (thin line), for a CDM model including the contribution of IBR_2 (dashed blue line) plus/minus the dispersion (coloured region), and for a ΛCDM model with $\Omega_\Lambda = 0.73$ (thick line) and $\Omega_\Lambda = 0.1$ (dashed-dot thick line). We have used for all backreaction integrals the cut-off $k = 1\,\mathrm{Mpc}^{-1}$.

JCAP04(2012

Average and dispersion of the luminosity-redshift relation in the concordance model

I. Ben-Dayan,[a] **M. Gasperini,**[b,c] **G. Marozzi,**[d,e] **F. Nugier**[f] **and G. Veneziano**[d,g,h]

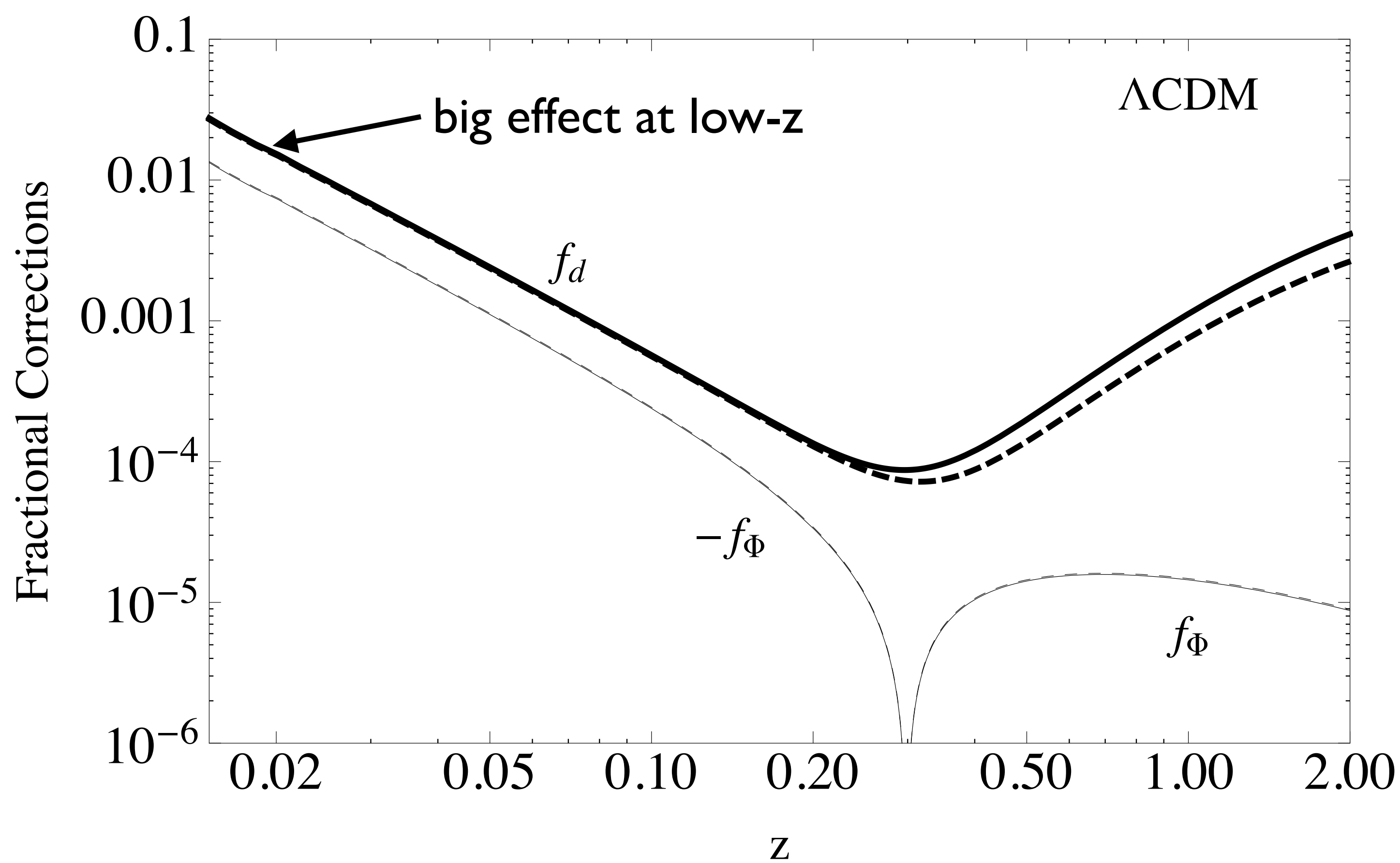


Figure 6. The fractional correction to the flux (f_Φ, thin curves) and to the luminosity distance (f_d, thick curves), for a perturbed ΛCDM model with $\Omega_{\Lambda 0} = 0.73$. Unlike in figure 3, we have taken into account the non-linear contributions to the power spectrum given by the HaloFit model of [17] (including baryons), and we have used the following cutoff values: $k_{UV} = 10h\,\mathrm{Mpc}^{-1}$ (dashed curves) and $k_{UV} = 30h\,\mathrm{Mpc}^{-1}$ (solid curves).

JCAP06(2013

Do Stochastic Inhomogeneities Affect Dark-Energy Precision Measurements?

I. Ben-Dayan,[1,2] M. Gasperini,[3,4] G. Marozzi,[5] F. Nugier,[6] and G. Veneziano[5,7]

The effect of a stochastic background of cosmological perturbations on the luminosity-redshift relation is computed to second order through a recently proposed covariant and gauge-invariant light-cone averaging procedure. The resulting expressions are free from both ultraviolet and infrared divergences, implying that such perturbations cannot mimic a sizable fraction of dark energy. Different averages are estimated and depend on the particular function of the luminosity distance being averaged. The energy flux being minimally affected by perturbations at large z is proposed as the best choice for precision estimates of dark-energy parameters. Nonetheless, its irreducible (stochastic) variance induces statistical errors on $\Omega_\Lambda(z)$ typically lying in the few-percent range.

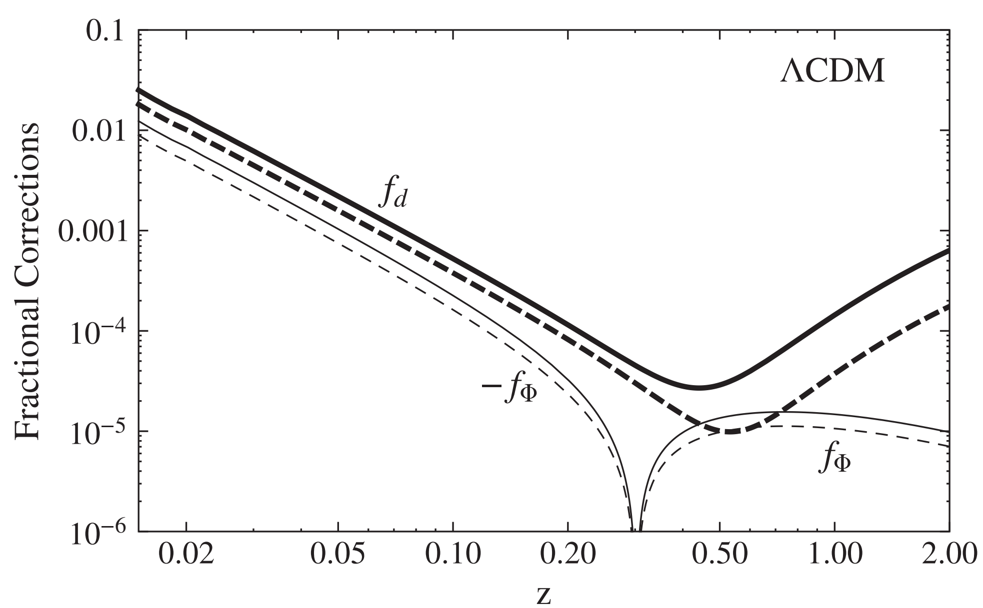


FIG. 2. The fractional correction to the flux f_Φ of Eq. (7) (thin curves) is compared with the fractional correction to the luminosity distance f_d of Eq. (13) (thick curves) for a ΛCDM model with $\Omega_\Lambda = 0.73$. We have used two different cutoff values: $k_{UV} = 0.1\ \text{Mpc}^{-1}$ (dashed curves) and $k_{UV} = 1\ \text{Mpc}^{-1}$ (solid curves). The spectrum is the same as that of Fig. 1 adapted to ΛCDM.

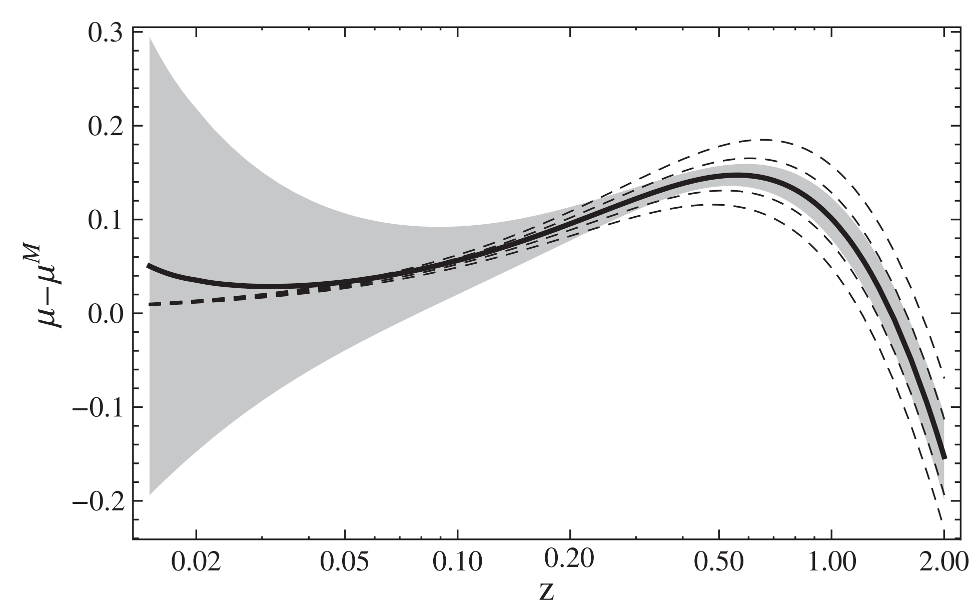


FIG. 3. The averaged distance modulus $\overline{\langle\mu\rangle} - \mu^M$ (thick solid curve) and its dispersion of Eq. (15) (shaded region) are computed for $\Omega_\Lambda = 0.73$ and compared with the homogeneous value for the unperturbed ΛCDM models with $\Omega_\Lambda = 0.69, 0.71, 0.73, 0.75, 0.77$ (dashed curves). We have used $k_{UV} = 1\ \text{Mpc}^{-1}$ and the same spectrum as in Fig. 2.

Value of H_0 in the Inhomogeneous Universe

Ido Ben-Dayan,[1] Ruth Durrer,[2] Giovanni Marozzi,[2] and Dominik J. Schwarz[3]
[1]*Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany*
[2]*Université de Genève, Département de Physique Théorique and CAP, 24 quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland*
[3]*Fakultät für Physik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany*

(Received 5 February 2014; published 6 June 2014)

Local measurements of the Hubble expansion rate are affected by structures like galaxy clusters or voids. Here we present a fully relativistic treatment of this effect, studying how clustering modifies the mean distance- (modulus-)redshift relation and its dispersion in a standard cold dark matter universe with a cosmological constant. The best estimates of the local expansion rate stem from supernova observations at small redshifts ($0.01 < z < 0.1$). It is interesting to compare these local measurements with global fits to data from cosmic microwave background anisotropies. In particular, we argue that cosmic variance (i.e., the effects of the local structure) is of the same order of magnitude as the current observational errors and must be taken into account in local measurements of the Hubble expansion rate.

$$\overline{\langle d_L^{-2}\rangle}(z) = (d_L^{\rm FL})^{-2}[1 + f_\Phi(z)], \tag{4}$$

where for $z \ll 1$,

$$f_\Phi(z) \simeq -\left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \overline{\langle(\vec{v}_s\cdot\vec{n})^2\rangle}. \tag{5}$$

would nearly double the effect in Eq. (5). The dominant peculiar velocity contribution at low redshift gives

$$f_\Phi(z) \simeq -\left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \frac{\tau^2(z)}{3}\int_{H_0}^{k_{\rm UV}}\frac{dk}{k}k^2\mathcal{P}_\psi(k), \tag{6}$$

The brightness of supernovae is typically expressed in terms of the distance modulus μ. Because of the nonlinear function relating μ and Φ, one obtains different second order contributions,

$$\overline{\langle\mu\rangle} - \mu^{\rm FL} = -\frac{2.5}{\ln(10)}\left[f_\Phi - \frac{1}{2}\overline{\langle(\Phi_1/\Phi_0)^2\rangle}\right], \tag{7}$$

where at $z \ll 1$, we also find

$$\overline{\langle(\Phi_1/\Phi_0)^2\rangle} \simeq -4f_\Phi. \tag{8}$$

Bias in H_0 from 2nd order pert^n theory

- Backreaction causes systematic bias in H measurement
  - interesting bias in flux density, distance etc at low-z
- But isn’t this just the residual “homogeneous Malmquist bias” in “inverse + type II” method?

Malmquist bias?

- Objects in a region of estimated distance space will have a distance that is biased
  - because of (large) uncertainty in distance
- But "Schechter's method" largely avoids that
  - don't measure velocity as a function of distance
  - do it the other way round
  - small scatter in distance for objects at same redshift
- but not completely free from bias
  - analysed by Lynden-Bell '92 and Willick & Strauss '97

DLB '92

Eddington–Malmquist Bias, Streaming Motions, and the Distribution of Galaxies

D. Lynden-Bell

ABSTRACT Schechter's method of eliminating Malmquist bias is reviewed and presented in the context of the $D_n - \sigma$ relationship for elliptical galaxies. A Malmquist-like correction occurs which is dependent on the dispersion in the velocity field of galaxies; however, this correction does not increase with distance so it is much less important than the normal Malmquist bias that this method eliminates. The method is applied to a bulk flow model of the ellipticals and gives almost identical results to those found using the other reduction method which employs the Malmquist corrections. Ways of using the method to model the density and velocity fields out to 10,000 km/sec are briefly indicated.

is already small.

Solving for R, we obtain the value R_m at which the maximum occurs

$$R_m = \tfrac{1}{2}\left\{w + \sqrt{w^2 + 4\sigma_v^2\left[3 + \frac{d\ln(n/\sigma_v)}{d\ln r}\right][1+u'(v)]^{-2}}\right\}. \qquad (9.16)$$

Equations (9.16) and (9.15) constitute our solution for R_m. Notice that when $w >> \sigma_v$, then

$$R_m = w\left\{1 + \frac{\sigma_v^2}{w^2}\left[3 + \frac{d\ln(n/\sigma V)}{d\ln r}\right][1+u'(v)]^{-2}\right\} \qquad (9.17)$$

Willick et al 1997 (astro-ph vs ApJ)

2.2.2. Further discussion of the VELMOD *likelihood*

The physical meaning of the VELMOD likelihood expressions is clarified by considering them in a suitable limit. If we take σ_v to be "small," in a sense to be made precise below, the integrals in Eqs. (11) and (12) may be approximated using standard techniques. If in addition we neglect sample selection ($S = 1$) and density variations ($n(r) =$ constant), and assume that the redshift-distance relation is single-valued, we find for the forward relation:

10 = 2*5

$$P(m|\eta, cz) \simeq \frac{1}{\sqrt{2\pi}\sigma_e} \exp\left\{-\frac{1}{2\sigma_e^2}\left(m - \left[M(\eta) + 5\log w + \frac{10}{\ln 10}\Delta_v^2\right]\right)^2\right\}, \quad (15)$$

$$P(m|\eta, cz) \simeq \frac{1}{\sqrt{2\pi\sigma_e}} \exp\left[\!\left[-\frac{1}{2\sigma_e^2}\left\{m - \left[M(\eta) + 5\log w + 3 \times \frac{5}{\ln 10}\Delta_v^2\right]\right\}^2\right]\!\right], \quad (15)$$

We thank Marc Davis, Carlos Frenk, and Amos Yahil for extensive discussions of various aspects of this project, as well as the support of the entire Mark III team: David Burstein, Stéphane Courteau, and Sandra Faber. We also thank the referee, Alan Dressler, for an insightful report that improved the quality of the paper. J. A. W. and M. A. S. are grateful for the

- KH15: The "3" here comes from the standard formula for HMB.
- The right answer is 1.5
  - as found by the relativistic backreaction folks!

Kinematic Bias in Cosmological Distance Measurement

Nick Kaiser & Michael J. Hudson
[1] *Institute for Astronomy, University of Hawaii*
[2] *Department of Physics and Astronomy, University of Waterloo*

ABSTRACT
Recent calculations using non-linear relativistic cosmological perturbation theory show biases in the mean luminosity distance and distance modulus at low redshift. We show that these effects may be understood very simply as a non-relativistic, and purely kinematic, Malmquist-like bias, and we describe how the effect changes if one averages over sources that are limited by apparent magnitude. This effect is essentially identical to the distance bias from small-scale random velocities that has previously been considered by astronomers, though we find that the standard formula overestimates the homogeneous bias by a factor 2.

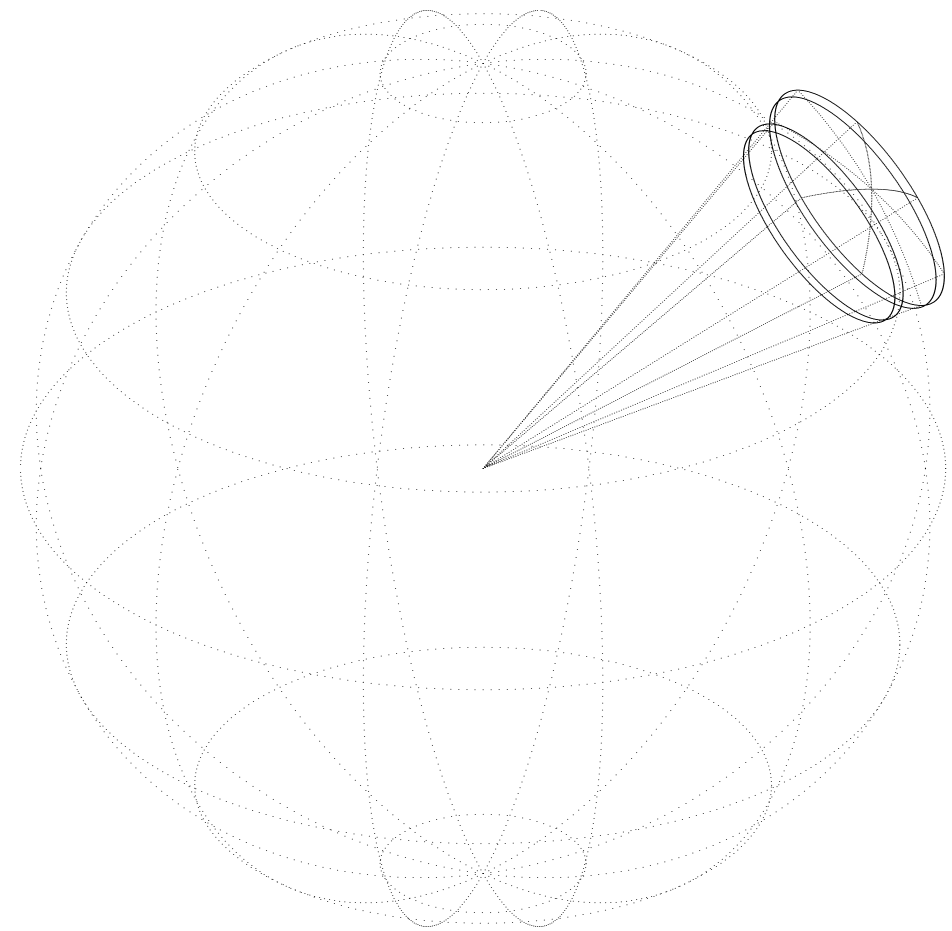


Figure 1. Dotted lines are lines of longitude and latitude on the surface of constant redshift. On this surface, peculiar velocities are equally likely to be positive as negative. The cone illustrates how a section of this sphere maps to real space for the case of a negative peculiar velocity. The section is pushed out radially away from the observer – who resides at the centre of the sphere – and consequently is expanded in area. Similarly, for a positive peculiar velocity the section would be compressed. The result of this is that the average of the distance, when weighted by real-space area, is positive. This is the cause of the bias found in the relativistic perturbation theory analyses. More relevant to real observations is the bias in distance averaged over the sources that lie in a shell of given redshift. We consider this in §2.2. There we find that there are some relatively minor differences that arise from the clustering of sources and from the Jacobian involved in transforming volumes from redshift to real space, but the main difference is that the generalisations of (8) have different numerical pre-factors when the sources are subject to selection based on flux density.

Conclusions - 1. biases in cosmological parameters

- Recent advances in non-linear relativistic perturbation theory
  - claimed to find significant biases in D(z)
    - from lensing at high z
    - from velocities at low z -> biased H_0
  - attributed to `backreaction'
- But effects found are purely statistical in nature
  - regarding lensing:
    - Weinberg was *almost* right - SN1a flux densities are unbiased
    - conventional CMB analysis is legitimate
    - focusing theorem has been misunderstood
      - no large systematic focusing - area decrease - of beams
  - regarding velocities:
    - bias in H_0 is `homogeneous Malmquist bias'
      - but reveals a subtle error in standard formula

Concluding 2. Doppler lensing

- Perturbations to D(z) from peculiar motions
  - not a new probe of large-scale structure
    - rehash of cosmic-flow analysis from 70's, 80's etc.
- relativistic calculations go beyond lowest order ($\sim \delta v/cz$) terms
  - previously used in SN1a error analysis
  - but violate equivalence principle
- Consistent treatment requires including gravitational redshift
  - no observable effect from ultra-large scale structure (beyond tide)
  - consistent method for SN1a covariance analysis