Uncovering the physics of loop quantum cosmological and black hole spacetimes (some recent advances)

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What can one learn in this quantum gravity playground?

- Rigorous construction of self consistent model quantum spacetimes.
- Develop and rigorously test different tools and techniques to extract reliable physics.
- How to rule out different quantizations using internal consistency and physical predictions.
- Understand potential quantum gravity implications for early universe and black hole physics.

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Caveat: Quantum mechanics of spacetime, not QFT. Viewpoint that qualitative aspects are captured. In approach to spacelike singularities, local evolution approximated by Bianchi models (BKL conjecture)

Outline:

- Loop quantum cosmology: introduction to key results
- An example of consistency check in loop quantization.
  Black hole interior quantization.
  Problems with approaches so far, and a resolution.
- An example of new tools and techniques.
  Robustness tests on quantum bounce.
  Challenges and recent numerical simulations.
  Tests of effective dynamics.
  Anisotropic models (Cactus implementation)
- Summary

Loop quantum cosmology

A non-perturbative quantization of homogeneous spacetimes using techniques of loop quantum gravity.

Basic variables: Ashtekar-Barbero connection (analogous to vector potential) and the conjugate triad (analogous to electric field). Classical Hamiltonian expressed in terms of the holonomies of the connection and the fluxes of the triad, and quantized.

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Quantum Hamiltonian constraint is a difference equation with discreteness fixed by the underlying quantum geometry **(Bojowald (2001); Ashtekar, Bojowald, Lewandowski (2003))**. Problems with early quantizations. Cured in improved quantization **(Ashtekar, Pawlowski, PS (06))**

Quantum Hamiltonian constraint: $\partial_\phi^2 \Psi = -\Theta\Psi$

$\Theta\Psi := C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi) = \partial_\phi^2\Psi$

Leads to Wheeler-DeWitt equation at classical curvature.
GR recovered in infra-red regime.

How is physics extracted?

- Find physical Hilbert space: self-adjoint Hamiltonian constraint, eigenfunctions and the inner product.
- Identify (Dirac) observables to study relational dynamics. Example: matter field can serve as a clock.
- Construct physical initial states, such as Gaussian states on a classical trajectory in a large macroscopic universe.
- Evolve initial state numerically using quantum difference equation towards the classical singularity.
- Compute expectation values of observables (and their fluctuations). Compare with the classical trajectory.

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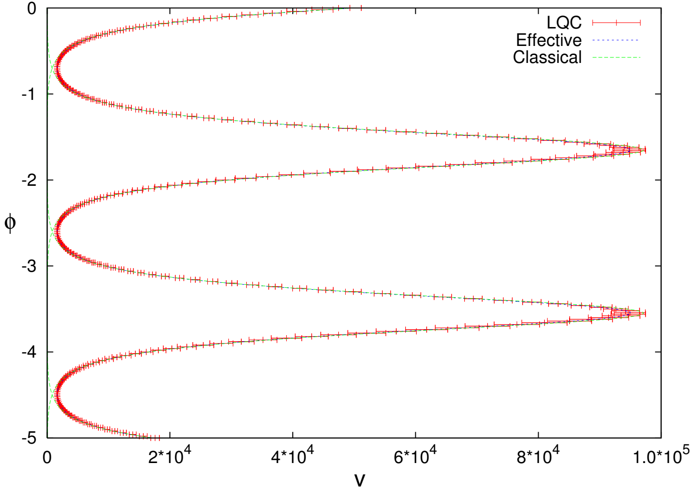
Loop quantum universes do not encounter big bang in the backward evolution. Physical state bounces in Planck regime. Sharply peaked initial states bounce at $\rho = \rho_{\rm b} \approx 0.41 \rho_{\rm Planck}$.

No fine tuning or exotic matter required.

Example: Closed FRW model with a massless scalar

(Ashtekar, Pawlowski, PS, Vandersloot (07))



Perfect agreement with GR at late times for all cycles.

Tight constraints on the growth of fluctuations of state during evolution (Corichi, PS (08); Kaminski, Pawlowski (10); Corichi, Montoya (11))

In the last decade, rigorous quantization performed for a variety of models. Including, spatially closed and open models; with $\pm\Lambda$; in presence of potentials (inflationary as well as cyclic); Bianchi-I, II and IX spacetimes; black hole spacetimes, and Gowdy models (inhomogenities Fock quantized).

(Ashtekar, Bentivegna, Chiou, Corichi, Diener, Gambini, Gupt, Karami, Kaminski, Lewandowski, Martin-Benito, Megevand, Mena-Marugan, Olmedo, Pawlowski, PS, Pullin, Szulc, Vandersloot, Wilson-Ewing (2006-14))

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Universal maximum on energy density predicted by an exactly solvable model **(Ashtekar, Corichi, PS (08))** (verified in all of the numerical simulations carried out so far).

Fundamental issues on consistent quantum probabilities addressed. The probability for the bounce computed to be unity **(Craig, PS (13))**

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- Till recently all the numerical simulations performed for isotropic models with sharply peaked Gaussian states only, bouncing far away from Planck volume.
- So far, only one numerical study for quantum anisotropic models.

Effective spacetime description

Under appropriate conditions, quantum evolution can be approximated by a continuum effective spacetime description. For isotropic model, Friedmann equation modified by a ρ^2 term (Taveras (08)). Numerical simulations so far showed that the effective dynamics is an excellent approximation for sharply peaked states.

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Effective dynamics primary tool to extract phenomenology.

- Inflationary spacetimes: Ashtekar, Bojowald, Corichi, Karami, Lidsey, Mulryne, Nunes, Sloan, PS, Tavakol, Vandersloot, Vereshchagin
- Cyclic models: Bojowald, Cailleteau, Maartens, PS, Vandersloot
- Stringy scenarios: De Risi, Garriga, Gupt, Maartens, PS, Vilenkin, Zhang
- Anisotropic spacetimes: Chiou, Corichi, Dadhich, Gupt, Joe, Montoya, PS, Vandersloot
- Cosmological perturbations: Barrau, Bojowald, Cailleteau, Calcagni, Dapor, Grain, Hossain, Kagan, Mielczarek, Shankarnarayanan, Wilson-Ewing
- Exotic singularities: Cailleteau, Cardoso, Gumjudpai, Sami, PS, Tsujikawa, Vandersloot, Vidotto, Wands, Ward
- Genericness of singularity resolution in isotropic and Bianchi-I spacetimes: PS

Restriction on quantizations using consistency arguments

A detour on viability of models

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A large class of these quantum ambiguities are eliminated demanding the following three conditions **(Corichi, PS (08))**

- Physical predictions of quantities which are classically invariant under change in fiducial structure must be free of dependence on any fiducial scale after quantization.
- Well defined infra-red (classical) limit at small spacetime curvature.
- An unambiguous scale at which quantum gravitational effects become important.

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Leads to unique quantization in isotropic and Bianchi models (Corichi, PS (08-09)), and Kantowski-Sachs spacetimes with matter (Joe, PS (14))

Quantization of Schwarzschild interior

Schwarzschild interior corresponds to Kantowski-Sachs vacuum. Can be quantized using loop quantum cosmological techniques. Spatial manifold: $\mathbb{R} \times \mathbb{S}^2$. Non-compactness implies need of a fiducial scale (L_o) to define symplectic structure. Quantum Hamiltonian constraint non-singular.

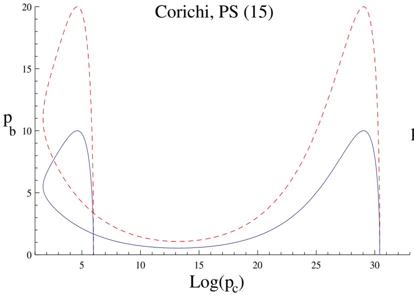
However, problems with quantizations so far:

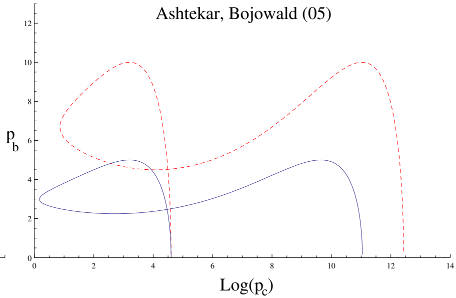
- Ashtekar-Bojowald (05): Dependence on fiducial structure, arbitrariness in physical predictions. White hole forms after singularity resolution. But the white hole mass is arbitrary, and can be changed by changing L_o.

- Boehmer-Vandersloot (08): Physics free of fiducial scale but Planck scale effects at horizon! No classical limit. "Charged" Nariai spacetime emerges after bounce **(Dadhich, Joe, PS (15))**

Schwarzschild interior quantization revisited

New quantization proposed which is free of fiducial structures, resolves central singularity leading to a white hole, and gives GR at small curvature scales **(Corichi, PS (15))**.





(Different curves correspond to different L_o)
Mass of the black hole/white hole determined by p_c when $p_b = 0$.

Bounce highly asymmetric (unlike isotropic models).
Important new caveat for existing black hole phenomenology works inspired by LQG.

Development of new tools and techniques
to extract physics

Some of the open questions on robustness of physics

- Is quantum bounce an artifact of choosing special kinds of initial states? Does bounce occur if the initial state has very large quantum fluctuations?
- Only simulations with sharply peaked Gaussian states considered so far, which bounce at volumes much larger than the Planck volume. How do we probe deeper quantum geometry? Is the effective spacetime description still a good approximation?
- Due to heavy computational costs, many details of quantum bounce and validity of effective dynamics in anisotropic models remain unexplored.

Numerical challenges

Loop quantum Hamiltonian constraint (difference equation) is extremely well approximated by the second order Wheeler-DeWitt differential equation at small spacetime curvatures (large volumes).

$$\frac{\partial^2 \Psi}{\partial \phi^2} = 12\pi G v \left(\frac{\partial}{\partial v} \left(v \frac{\partial \Psi}{\partial v} \right) \right)$$

Characteristic speeds: $\lambda^{\pm} = \pm\sqrt{12\pi G} v$
The stability of evolution constraints the maximum time step $\Delta\phi$:

$$\Delta\phi \leq \frac{\Delta v}{|\lambda^{\pm}|} \propto \frac{1}{v}$$

Quantum geometry fixes volume discreteness. Maximal time step inversely proportional to maximal volume on the grid.

States which are highly quantum, and which probe deep Planckian geometry require a very large grid in volume. Computational cost of such numerical simulations is extremely high.

Computational cost

- Isotropic models:
  - For states wich are sharply peaked in a macroscopic universe, typical simulations consider boundary in volume at $v \sim 10^5$. On a single core 'Intel-i7' workstation, such a simulation can be performed in approximately 15 minutes.
  - For widely spread states, typical simulations require volume boundary at $v \sim 10^{12}$ (and higher). This requires 10^7 more spatial grid points. Stability requirements lead to 10^7 finer time steps. On a single core 10^{10} years needed.

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- Anisotropic models:
  - Non-hyperbolicity encountered for Bianchi-I vacuum model. However, one can evaluate the entire physical wavefuntion by integration

$$\chi(b_1, v_2, v_3) = \int d\omega_2 d\omega_3 \tilde{\chi}(\omega_2, \omega_3) e_{\omega_1}(b_1) e_{\omega_2}(v_2) e_{\omega_3}(v_3)$$

  - For a state sharply peaked at $\omega_2 = \omega_3 = 10^3$, a typical simulation requires 10^{14} floating-point operations.
  - For wider states, and states probing deep quantum geometry, typical simulations require 10^{19} flop. Memory needed $\sim$ 5 Tb.

Chimera scheme and test of effective dynamics

(Diener, Gupt, PS (14)) Use two grids: An inner grid where the LQC difference equation is solved, and a carefully chosen outer grid in logarithmic coordinate at large volumes where the WDW theory is an excellent approximation. With $v_{\rm int} = 12,500$ and $v_{\rm outer} = 2 \times 10^{12}$ evolution takes only 5 min on a single core.

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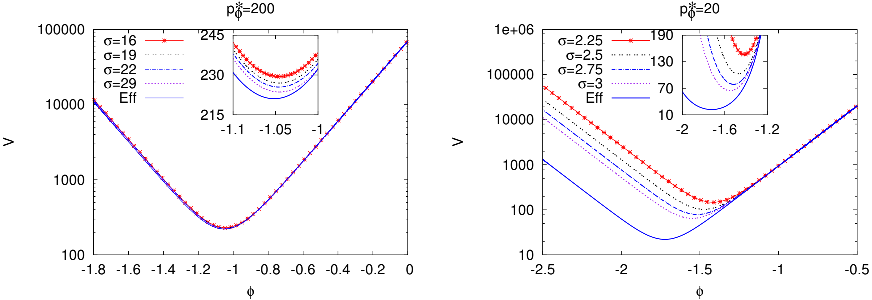
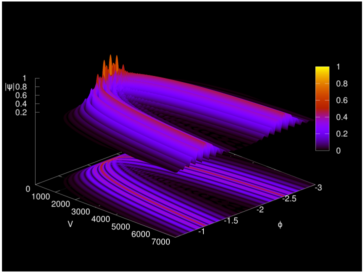


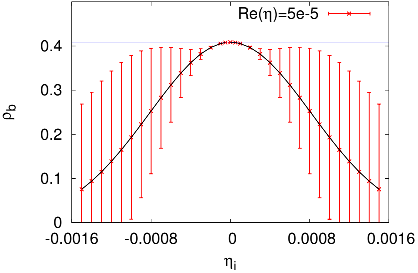
At a coarse level, effective theory captures underlying quantum evolution quite well, especially for sharply peaked states. But departures present for wide states. Subtle features revealed.

Quantum bounce for highly quantum states

Bounce not restricted to any special states. Even occurs for states which are highly non-Gaussian or squeezed.

(Diener, Gupt, Megevand, PS (2014))





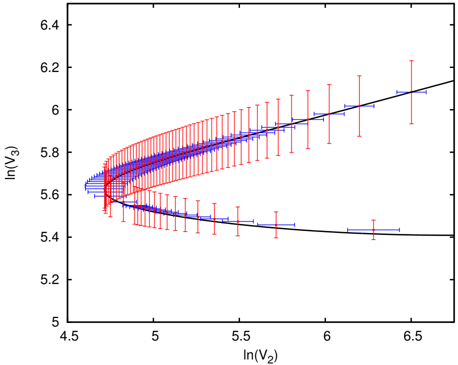
In the isotropic model, quantum fluctuations are found to always lower the curvature scale at which the bounce occurs. Quantum fluctuations in the state enhance the "repulsive nature of gravity" in the quantum regime.

Anisotropic quantum bounce

Till now only limited evidence of bounces in Bianchi-I vacuum model **(Martin-Benito, Mena Marugan, Pawlowski (2008))**.

Using Cactus framework, physics of quantum bounce in Bianchi-I vacuum spacetime now rigorously understood

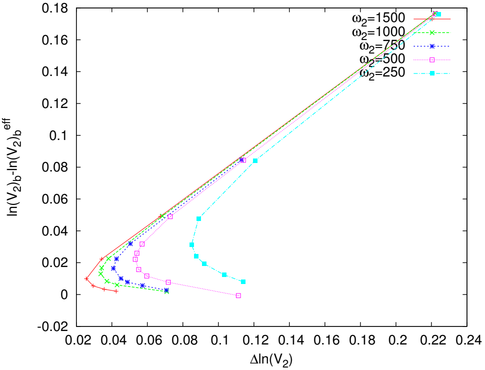
(Diener, Joe, Megevand, PS (To appear))



Anisotropic shear remains bounded throughout the evolution.

Effective description turns out to be a good approximation for sharply peaked states in the Bianchi-I model.

Agreement between the quantum evolution and the effective theory depends non-monotonically on the relative fluctuations (similar features found in isotropic models).



More work at analytical and numerical level needed to understand various details and improve effective theory. Important exercise for robustness and details of phenomenology.

Summary

- LQC provides a glimpse of how non-perturbative quantum gravity corrections may resolve singularities. Rigorous tools being developed to extract reliable physics.

  Many questions remain to be answered:
- Singularity resolution an artifact of simplifications? Singularities in homogenous and isotropic spacetimes were believed to be artifacts of symmetry reduction **(Eddington 1940's)**. But, turned out to be present in more general situations on inclusion of anisotropies **(Raychaudhuri 1950's)**. Shown to be generic in GR **(Hawking, Penrose, Geroch 1960's)**. (Will history repeat itself?)
- Inhomogenieities? Insights from Gowdy model **(Mena-Marugan, Olmedo, Pawlowski, ...)**, Spinfoam cosmology **(Bianchi, Rovelli, Vidotto)**, group field theory **(Gielen, Oriti, Sindoni)** and full LQG **(Alesci, Cianfrani, Lewandowski, ...)**
- Distinct signatures? A lot of ongoing interesting work by various groups on perturbations **(Agullo, Ashtekar, Barrau, Bojowald, Bonga, Cailleteau, Calcagni, Dapor, Grain, Gupt, Hossain, Lewandowski, Mielczarek, Nelson, Wilson-Ewing ...)**