ATOMS OF SPACE AND THE COSMOS

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SINGULARITIES: BLACK HOLES, UNIVERSE

COSMOLOGICAL CONSTANT

THE THERMODYNAMIC CONNECTION

These challenges involve $\hbar$!

$$A_{Planck} = \frac{G\hbar}{c^3}; \quad \Lambda\left(\frac{G\hbar}{c^3}\right) \approx 10^{-123}; \quad k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

HOW DO WE PUT TOGETHER THE PRINCIPLES OF QUANTUM THEORY AND GRAVITY?

Everybody Wants To Quantize Gravity!

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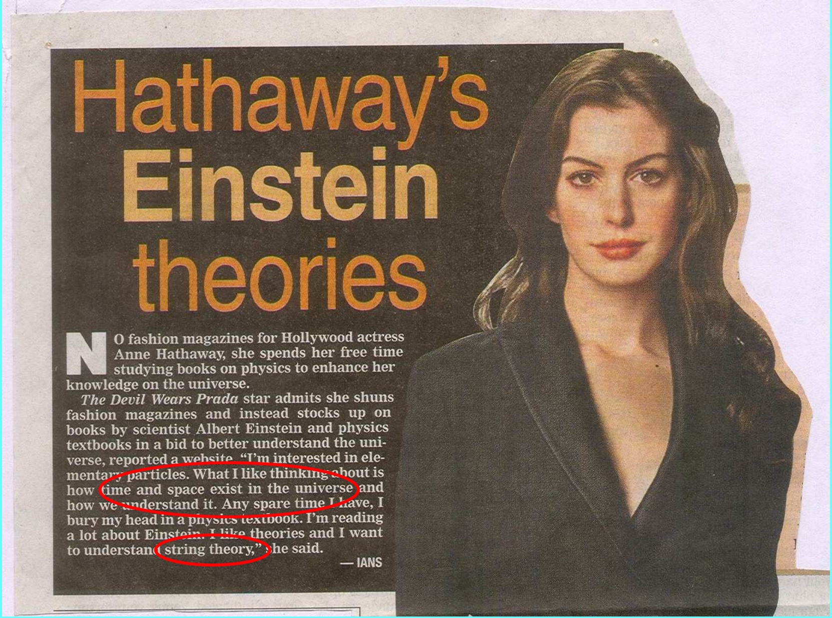

Hathaway's Einstein theories

NO fashion magazines for Hollywood actress Anne Hathaway, she spends her free time studying books on physics to enhance her knowledge on the universe.

The Devil Wears Prada star admits she shuns fashion magazines and instead stocks up on books by scientist Albert Einstein and physics textbooks in a bid to better understand the universe, reported a website. "I'm interested in elementary particles. What I like thinking about is how time and space exist in the universe and how we understand it. Any spare time I have, I bury my head in a physics textbook. I'm reading a lot about Einstein. I like theories and I want to understand string theory," she said.

— IANS

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- ***The perturbative approach does not work***
- ***Virtually every interesting question about gravity is non-perturbative by nature***
- ***Non perturbative approches are too simplistic and not general enough (e.g., minisuperspace QC, CDT...)***
- ***No guiding principle; spacetime metric is assumed to be a quantum variable***

GR: The Next 100 Years

Needs another paradigm shift!

Classical Gravity has the same conceptual status as elasticity/hydrodynamics

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Study spacetime dynamics the way physicists studied fluids before knowing the atomic structure of matter

You could have figured out that water is made of discrete atoms without ever probing it at Angstrom scales!

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it must have micro-structure!

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You can even count the number of atoms

$$N = \frac{E}{(1/2)k_B T}$$

Microphysics leaves a signature at macro-scales

More *IS* Different

The key new variable which distinguishes thermodynamics from point mechanics

$$Heat\ Density = \mathcal{H} = \frac{Q}{V} = \frac{TS}{V} = \frac{1}{V}(E - F)$$

$$\frac{TS}{V} = Ts = p + \rho$$

Normal matter has a heat density

Spacetime also has a heat density!

One can associate a T and s with every event in spacetime, just as you could with a glass of water!

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Note: This fact transcends black hole physics and Einstein gravity

Macroscopic Nature Of Gravity

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Works for a wide class gravitational theories; entropy decides the theory.

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$$Time\ evolution \propto (N_{\rm sur} - N_{\rm bulk})$$

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All static geometries have

$$N_{\rm sur} = N_{\rm bulk}$$

Gravity responds to heat density
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Cosmological constant arises as an integration constant

Its value is determined by a new conserved quantity for the universe!

WHY ?

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How do we understand this emergent nature of gravitational dynamics at a deeper level?

The Reason

If gravity is immune to zero level of energy it must have a thermodynamic interpretation!

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Connects two features usually thought to be completely separate!

[TP, arXiv:1508.06286]

Since spacetime can be hot, it must have microstructure

We can count the atoms of spacetime without doing Planck scale experiments

The Atoms Of Space

The distribution function for 'atoms of space' provides the microscopic origin for the variational principle

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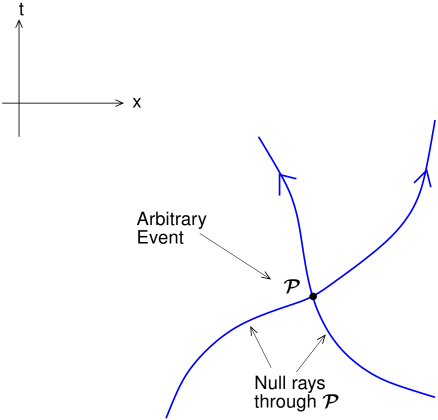
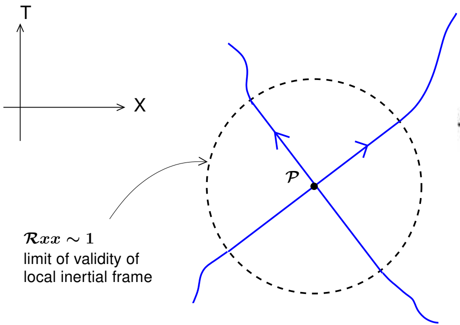
Points in a renormalized spacetime has zero volume but finite area!

Spacetime in Arbitrary Coordinates



Local Inertial Observers



Validity of laws of SR $\Rightarrow$ How gravity affects matter

Matter equations of motion $\Leftrightarrow \nabla_a T^a_b = 0$

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Take non-inertial frames seriously: not "just coordinate relabeling".

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the interface of quantum theory and gravity

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***The** most beautiful result in the interface of quantum theory and gravity*

OBSERVERS WHO PERCEIVE A HORIZON ATTRIBUTE A TEMPERATURE TO SPACETIME

$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

[Davies (1975), Unruh (1976)]

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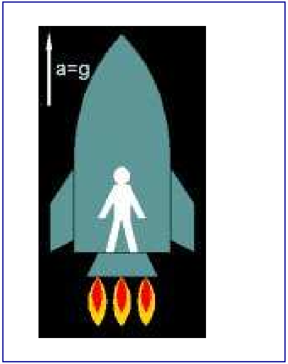
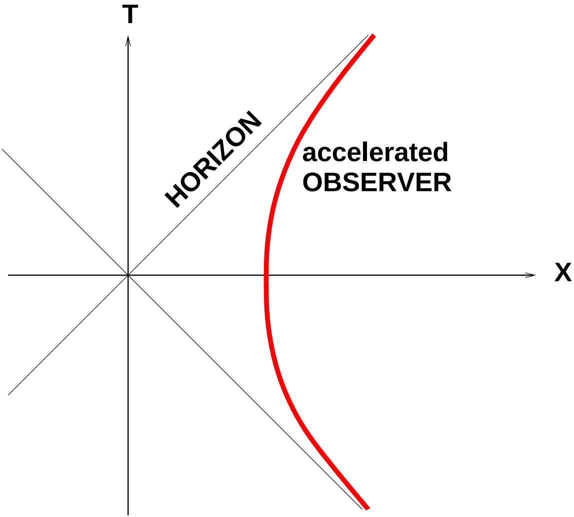
[Davies (1975), Unruh (1976)]

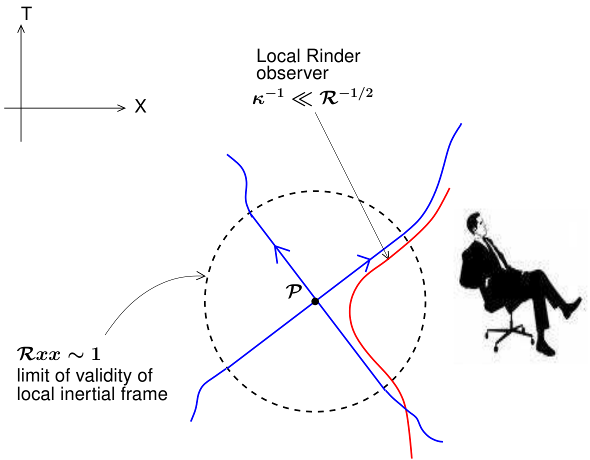
This allows you to associate a heat density $\mathcal{H} = Ts$ with every event of spacetime!

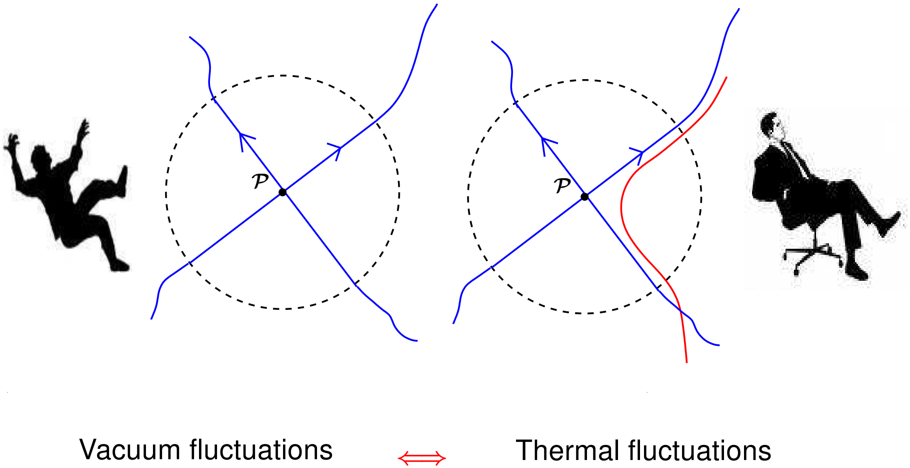


FLAT SPACETIME



Local Rindler Observers





$\mathcal{P}$
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Vacuum fluctuations
$\Longleftrightarrow$
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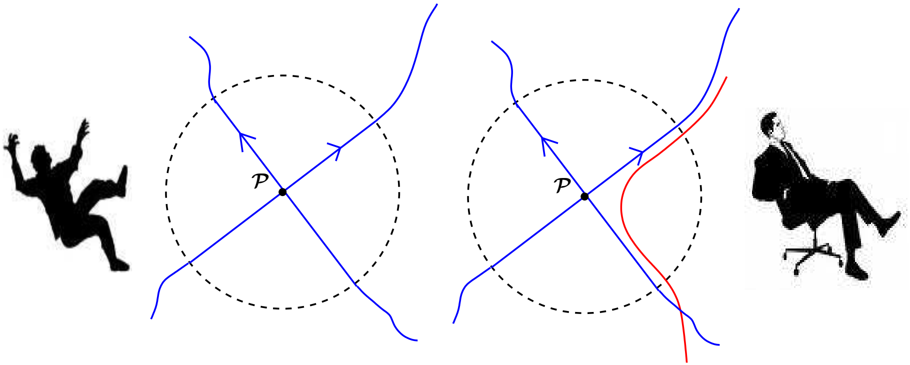

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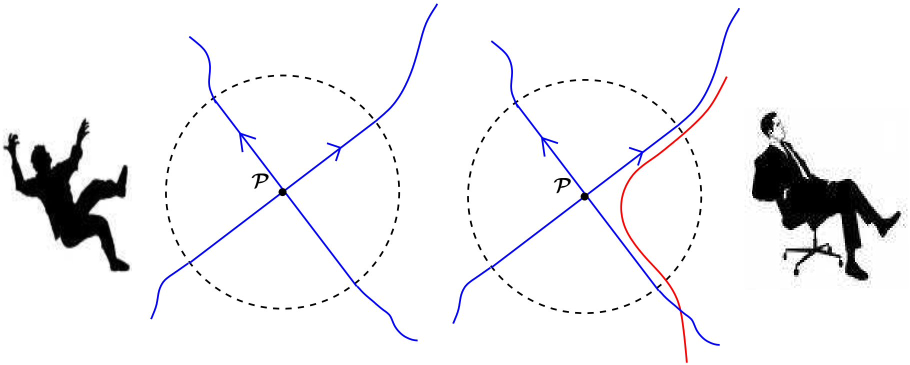
A VERY NON-TRIVIAL EQUIVALENCE!



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QFT in FFF introduces $\hbar$; we now have $(\hbar/c)$ in the temperature

- **Heat transfered due to matter crossing a null surface:** [T. Jacobson, gr-qc/9504004]

$$Q_m = \int d\mathcal{V}\,(T_{ab}\ell^a\ell^b); \quad \mathcal{H}_m \equiv T_{ab}\ell^a\ell^b$$

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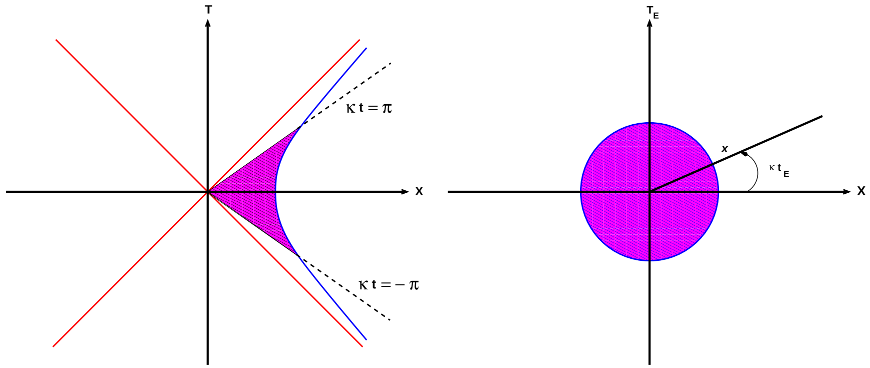
- **Note: Null horizon $\Leftrightarrow$ Euclidean origin**

$$X^2 - T^2 = 0 \Leftrightarrow X^2 + T_E^2 = 0$$



$$T = x \sinh \kappa t,\ X = x \cosh \kappa t \qquad T_E = x \sin\ \kappa t_E,\ X = x \cos\ \kappa t_E$$

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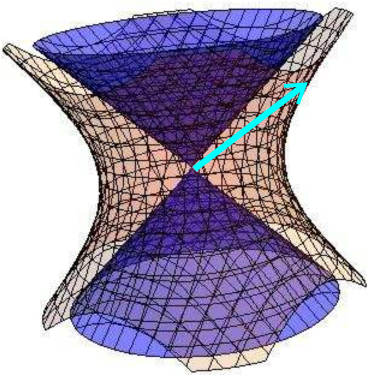

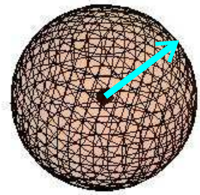

$$X^2 - T^2 = \sigma^2 \Leftrightarrow X^2 + T_E^2 = \sigma^2$$

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Guiding Principle For Dynamics

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Matter equations of motion remain invariant when a constant is added to the Lagrangian

Gravity must respect this symmetry

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The variational principle cannot have metric as the dynamical variable!

Variational principle must have the form

$$Q = \int dV \ \mathcal{H}[T^a_b, g_{ab}, q_A]$$

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How can $\mathcal{H}$ depend on T^a_b but yet be invariant under $T^a_b \rightarrow T^a_b$ + (constant) δ^a_b ?

The Variational Principle

- **Minimal possibility: We must have**

$$Q = \int dV \{\mathcal{H}_g[g_{ab}, n_a] + T^a_b n_a n^b\}$$

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Can one find such a $\mathcal{H}_g[g_{ab}, n_a]$?

Dynamics Of Gravity

T.P., A. Paranjape [gr-qc/0701003]; T.P. [arXiv:0705.2533]

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$$\mathcal{H}_g = -\left(\frac{1}{16\pi L_P^2}\right)\left(4P^{ab}_{cd}\nabla_a n^c \nabla_b n^d\right)$$

$$P^{ab}_{cd} \propto \delta^{aba_2b_2\ldots a_m b_m}_{cdc_2d_2\ldots c_m d_m} R^{c_2d_2}_{a_2b_2}\ldots R^{c_m d_m}_{a_m b_m}$$

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In $d = 4$, it leads uniquely to GR

$$G^a_b = (8\pi L_P^2) T^a_b + \Lambda \delta^a_b$$

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Microscopically $\mathcal{H}_g(x^i, n_a)$ is the 'distribution function for atoms of space with momentum' n_a

The Thermodynamic Connection

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- Macroscopically, identify $\boldsymbol{n_a} \leftrightarrow \boldsymbol{\ell_a}$ and

$$\boldsymbol{Q_{\mathrm{tot}} \equiv \int \sqrt{\gamma}\, d^2 x d\lambda \left(\mathcal{H}_g[\ell] + \mathcal{H}_m[\ell]\right)}$$

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Extremizing the heat densities of all null surfaces leads to gravitational dynamics!

Boltzmann: If you can heat it,
it must have micro-structure!

To store energy $\boldsymbol{\Delta E}$ at temperature $\boldsymbol{T}$, you need

$$\boldsymbol{\Delta n = \frac{\Delta E}{(1/2) k_B T}}$$

degrees of freedom. Connects microphysics with thermodynamics!

Boltzmann: If you can heat it,
it must have micro-structure!

You can heat up spacetime!

Do we have an equipartition law for the microscopic spacetime degrees of freedom? Can you count the atoms of space?

The Quantum of Area

TP, [gr-qc/0308070]; [0912.3165]; [1003.5665]

Equipartition with a surface-bulk correspondence

$$E_{\rm bulk} = \int_{\partial\mathcal{V}} \frac{dA}{L_P^2}\left(\frac{1}{2}k_B T_{loc}\right) \equiv \frac{1}{2}k_B \int_{\partial\mathcal{V}} dn\, T_{\rm loc}$$

Associates $dn = dA/L_P^2$ atoms (microscopic degrees of freedom) with an area dA

We must be able to express — and interpret — the field equation in a purely thermodynamic language !

Geometry $\Leftrightarrow$ Thermodynamics

K. Parattu, B.R. Majhi, T.P. [arXiv:1303.1535]

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$$p^a_{bc} \equiv -\Gamma^a_{bc} + \frac{1}{2}(\Gamma^d_{bd}\delta^a_c + \Gamma^d_{cd}\delta^a_b)$$

These variables have a thermodynamic interpretation

$$(q\delta p, p\delta q) \Leftrightarrow (s\delta T, T\delta s)$$

What Makes Spacetime Evolve ?

T.P., Gen.Rel.Grav (2014) [arXiv:1312.3253]

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$$\int \frac{d\Sigma_a}{8\pi L_P^2} \left[q^{\ell m} \partial \, p^a_{\ell m} \right] = -\frac{1}{2} k_B T_{\rm av} \, \left(N_{\rm sur} - N_{\rm bulk} \right)$$

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This replaces the field equation for gravity

Three constants: $\hbar, c, L_P^2$

Newton's Law of Gravitation

T.P. [hep-th/0205278]

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$$F = \left(\frac{c^3 L_P^2}{\hbar}\right)\left(\frac{m_1 m_2}{r^2}\right)$$

Gravity, like matter, is intrinsically quantum and cannot exist in the limit of $\hbar \to 0$!

What Next?

T.P. [arXiv:1508.06286]

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Understand this from a deeper, microscopic, level:

- Origin of the auxiliary vector field n_a
- Why are null vectors selected out?
- Determine $\mathcal{H}_g$; use alternative, dimensionless, form:

$$\mathcal{H}_g \equiv -\frac{1}{8\pi}(L_P^2 R_{ab} n^a n^b)$$

The Challenge

How can we get $\mathcal{H}_g$ from a microscopic theory without knowing the full QG?

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We need to recognize discreteness and yet use continuum mathematics!

Atoms Of A Fluid

- Continuum fluid mechanics: $\boldsymbol{\rho(x^i), \mathrm{U}(x^i), ...}$ ignores discreteness and *velocity dispersion.*

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  - Recognizes discreteness, yet uses continuum maths!
  - Atom at x^i has an extra attribute p_i.
  - Many atoms with different p_i can exist at same x^i

The $\mathcal{H}_g$ is proportional to the $f(x^i, n_j)$ for the number of atoms of space "at" x^i with "momentum" n_i. In dimensionless form:

$$\frac{d(Q/E_P)}{d(V/L_P^3)} \equiv \mathcal{H}_g(x^i, n_j) = f(x^i, n_j)$$

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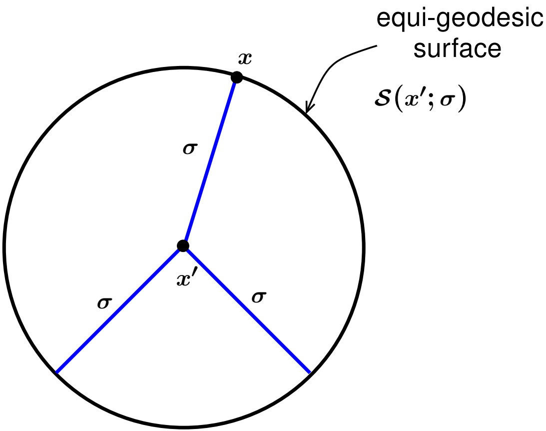
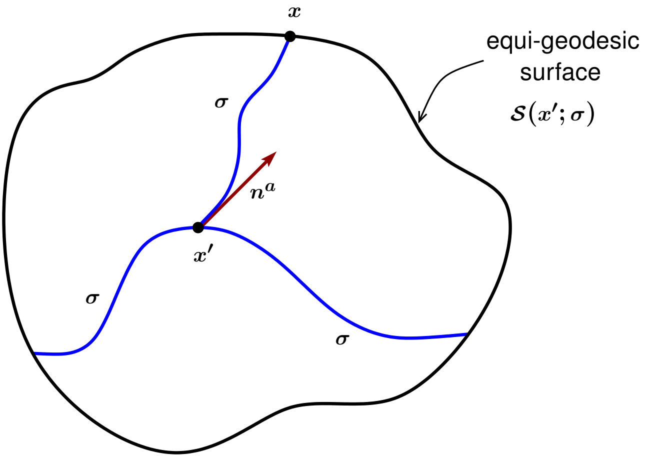
Use equi-geodesic surfaces to make this idea precise



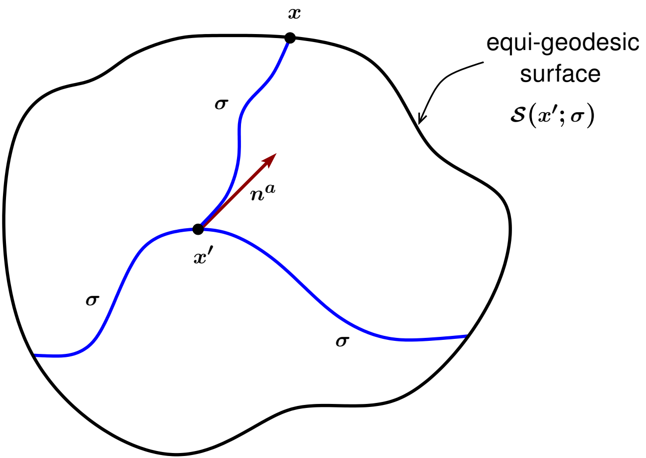
$$ds^2 = d\sigma^2 + \sigma^2 d\Omega^2_{(S3)}$$

$$\sqrt{g} \propto \sigma^3 \qquad \sqrt{h} \propto \sigma^3$$



$$ds^2 = d\sigma^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

The $\sqrt{g} = \sqrt{h}$ will pick up curvature corrections



$$ds^2 = d\sigma^2 + h_{\alpha\beta}dx^\alpha dx^\beta$$

$$\sqrt{h}(x,x') = \sqrt{g}(x,x') = \sigma^3\left(1 - \frac{\sigma^2}{6}\mathcal{E}\right)\sqrt{h_\Omega}; \quad \mathcal{E} \equiv R_{ab}n^a n^b$$

Zero-Point Length

T.P. Ann.Phy. (1985), 165, 38; PRL (1997), 78, 1854

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Quantum spacetime has a zero-point length:

$$\sigma^2(x,x') \;\to\; S(\sigma^2) = \sigma^2(x,x') + L_0^2$$
$$g_{ab}(x) \;\to\; q_{ab}(x,x';L_0^2)$$

D. Kothawala, T.P. [arXiv:1405.4967]; [arXiv:1408.3963]

Origin of Null Vectors

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The number of atoms of space at x^i with attribute ("momentum") n_i scales as volume or area measure of the equigeodesic surface in the quantum Euclidean space when $x' \to x$

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The $\sigma^2 \to 0$ limit picks null vectors! Euclidean origin maps to local Rindler horizons.

Area Of A Point

T.P. [arXiv:1508.06286]

$$\sqrt{q} = \sigma \left(\sigma^2 + L_0^2\right) \left[1 - \frac{1}{6}\mathcal{E} \left(\sigma^2 + L_0^2\right)\right] \sqrt{h_\Omega}$$

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Aside: Spacetime becomes two-dimensional at Planck scales

Distribution Function For Atoms Of Space

The area measure gives exactly what we need, along with a zero-point-contribution

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Zero-point contribution is important; degrees of freedom of Planck 2-sphere: $4\pi L_P^2/L_P^2 = 4\pi$

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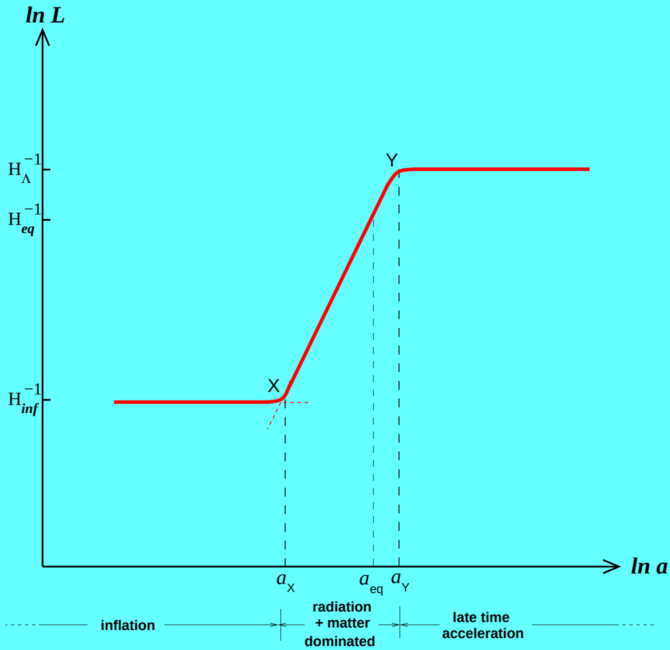

ln L
H_Λ^{-1}
H_{eq}^{-1}
H_{inf}^{-1}
Y
X
ln a
a_X
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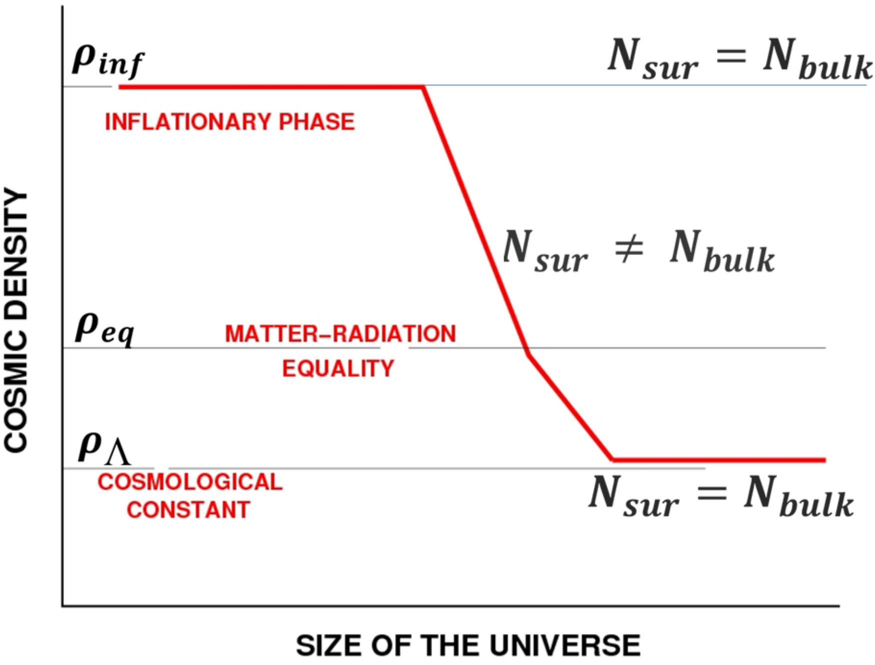
But we have no clue why

$$\rho_\Lambda L_P^4 \approx 1.4 \times 10^{-123} \approx 1.1 \times e^{-283}.$$



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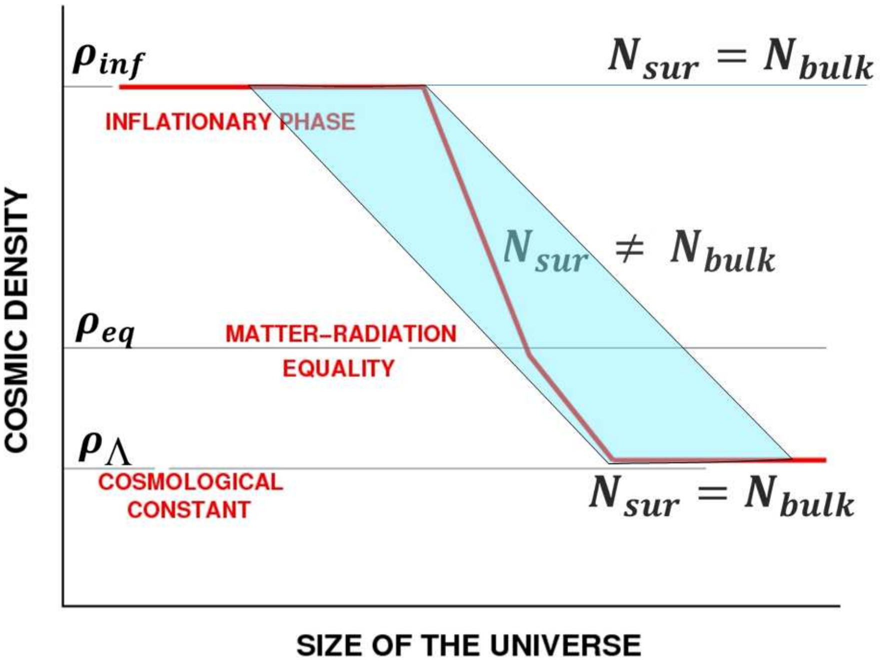

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Value of the Cosmological Constant

Hamsa Padmanabhan, T.P. [arXiv:1302.3226]

$$\rho_\Lambda = \frac{4}{27} \frac{\rho_{inf}^{3/2}}{\rho_{eq}^{1/2}} \exp(-36\pi^2)$$

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MAY BE ONE SHOULD NOT THINK OF COSMOLOGY AS PART OF GENERAL RELATIVITY!

Open Questions

- Matter and Geometry need to emerge together for proper interpretation of $\boldsymbol{T^{ab}n_an_b}$ at the microscopic scale. How do we do this?

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- Generalisation to Lanczos-Lovelock models with $R_{ab} \to \mathcal{R}_{ab}$: What happens at microscopic scales?

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- If we take number of atoms of spacetime at a point to be proportional to the area measure in a spacetime with zero-point length, we get the correct variational principle
- A Planck scale 2-sphere has $\boldsymbol{4\pi}$ degrees of freedom which allows the determination of the cosmological constant

References

T.P, *General Relativity from a Thermodynamic Perspective*, *Gen. Rel. Grav.*, **46**, 1673 (2014) [arXiv:1312.3253].

T.P, *Distribution function of the Atoms of Spacetime and the Nature of Gravity*, *Entropy* **17**, 7420 (2015) [arXiv:1508.06286].

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