BLACK HOLE THERMODYNAMICS : A CURRENT PERSPECTIVE

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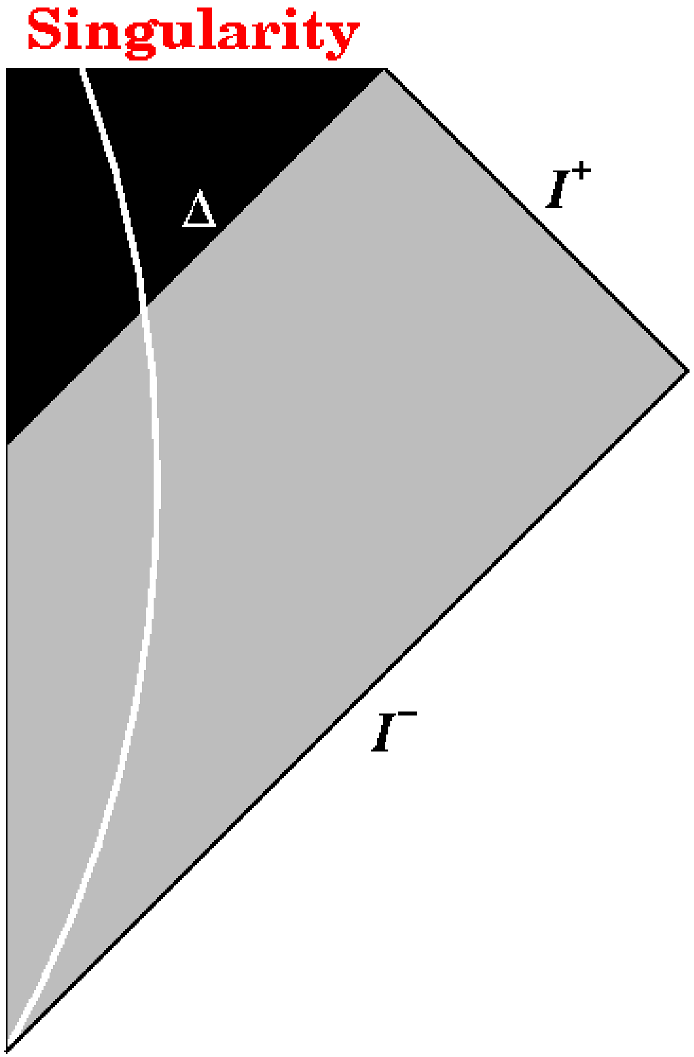
Outline :

- Laws of black hole mechanics
  - Black hole horizons
  - Isolated horizons
  - Dynamical horizons
- Quantization of horizons
  - Loop quantum gravity
  - State counting and entropy
- Summary and the future directions

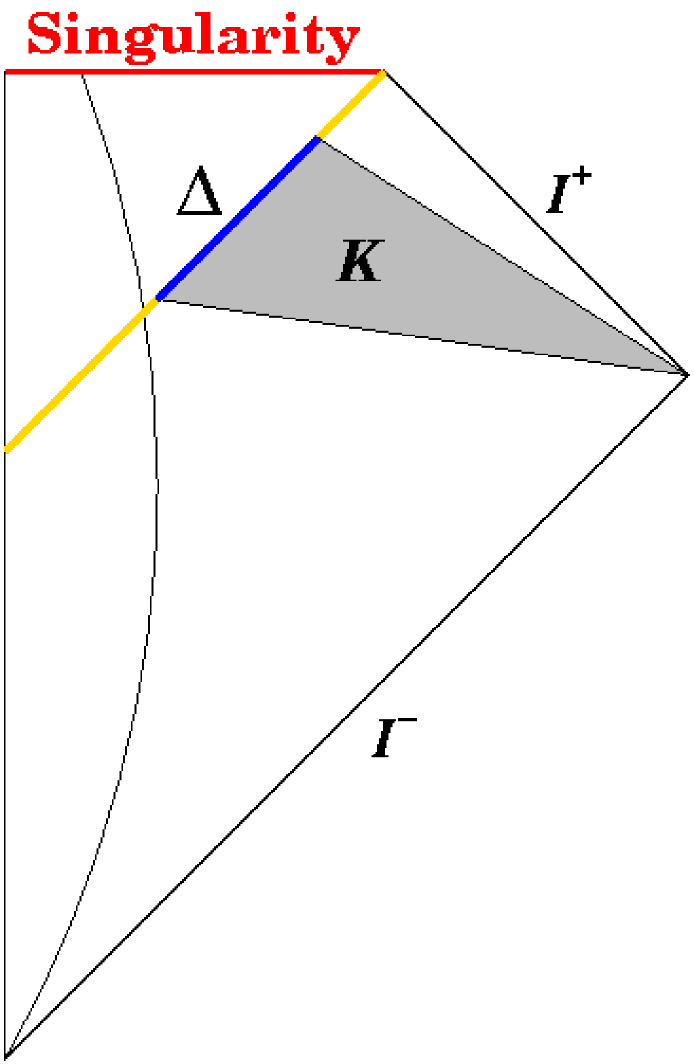
Event horizons :

- Boundary of causal past of future null-infinity in physical space-time
- Too global – need the knowledge of entire future – not practical
- Laws of black hole mechanics proved



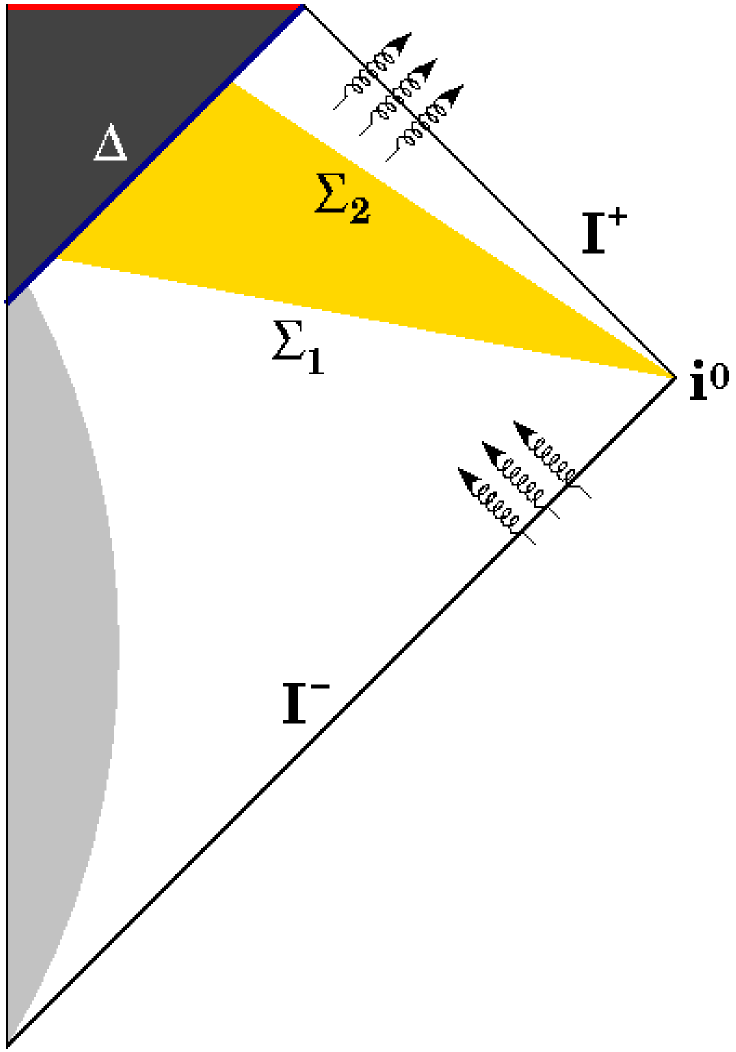
Killing horizons :

- A null-hypersurface and a neighborhood containing a Killing vector field K
- Local, detectable, admits laws of BHM
- Every EH is a KH
- Killing vector can be uniquely fixed



Isolated horizons :

- Equivalence class of null-normals : $[\,\xi \ell^a\,]$
- $\theta_{(\ell)} \equiv q^{ab}\nabla_a \ell_b = 0$
- $\theta_{(n)} < 0,\ n^a$ is shear and twist-free
- Local energy condition :
  - $T_{ab}\ell^b$ is causal
  - $T_{ab}\ell^b n^b = \text{const on } S^2$
- Einstein's eqs hold



Isolated horizons :

- Expansion-free ⇒ area A_Δ is constant
- Raychaudhuri's eq ⇒ ℓ^a shear and twist-free ⇒ it is locally Killing
- Free part of pull-back of SU(2)-connection A is a U(1)-connection $W \sim A^i \tau_i(\theta, \phi)$
- $F^i_{ab}(A) = -\frac{2\pi}{A_\Delta}\epsilon_{abc}E^{ci}, \quad F \sim dW$
- 0th law holds : $\ell^a \nabla_a \ell^b = \kappa \ell^b, \ \kappa =$ const
- 1st law holds : depends on action and symp. st.

Isolated horizons :

- The appropriate action in presence of Δ

$$S(A,E) = S_{\mathrm{Holst}}(A,E) + \frac{A_\Delta}{4\pi\gamma} S^{CS}_{\Delta}(A) + S_\infty$$

- The boundary action at Δ reduces to U(1) CS-action by IH boundary conditions
- The action gives rise to the symplectic structure

$$\Omega(\delta_1,\delta_2) = \Omega_{\mathrm{vol}}(\delta_1,\delta_2) + \frac{A_\Delta}{8\pi^2 G\gamma} \oint_{S^2} \delta_1 W \wedge \delta_2 W$$

- Phase space : $\Gamma_{\mathrm{vol}} \times \Gamma_\Delta$

Dynamical horizons :

- Unlike IH a DH is a spacelike 3-surface : $S^2 \times R$ on which local dominant energy condition holds
- Sphere is marginally trapped : $\theta_{(\ell)} = 0,\ \theta_{(n)} < 0$ (ensures that area increases), so 2nd-law is built in, $\delta A \geqslant 0$
- Energy-flux exists for $\xi^a = N(r)\ell^a,\ N(r)$ is the lapse of radial normal to the trapped sphere

$$\Delta M_\xi = \Delta E^\xi_{\text{matter}} + \Delta E^\xi_{\text{grav rad}}$$

$$\frac{1}{8\pi}\kappa(r)\delta A = \delta M_\xi$$

IH in Loop quantum gravity :

- $H = H_{\rm vol} \otimes H_\Delta$ because of generalized conn.
- $F^i_{ab}\tau_i(\theta,\phi)$ is a volume-operator, but dW is a boundary-operator; therefore implementation of IH boundary conditions is a nontrivial test of the consistency of loop quantum gravity
- Curvature has a spectrum (in specific units)

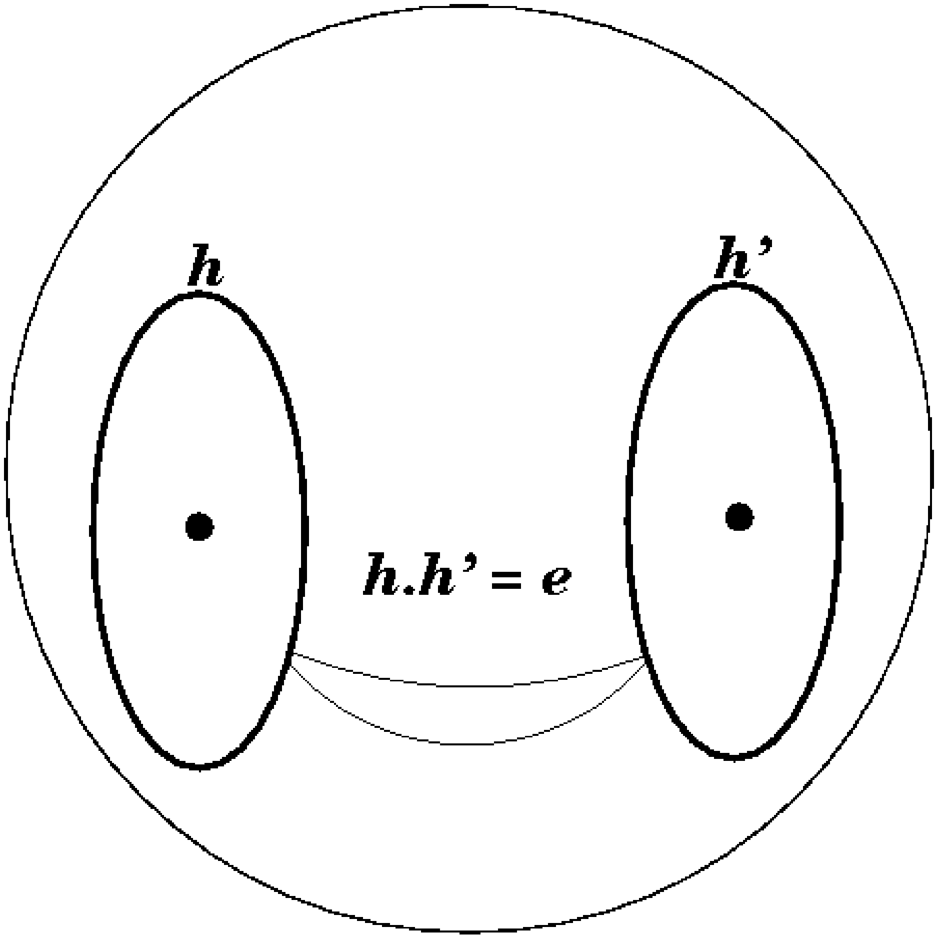
$$[F_{ab}]_{\mathcal{P}} \sim -\frac{4\pi}{A_\Delta}\sum_p m_p \delta^2(p,x)\epsilon_{ab},\ m_p \in \mathbb{Z}/2$$

- $H = \oplus_{\mathcal{P},J_p} H_{\mathcal{P},J_p},\ A_\Delta \sim 2\sum_p \sqrt{J_p(J_p+1)}$

IH in LQG :

- For each puncture p, holonomy around p : $h_p \sim \exp(2i\pi n_p/k),\ n_p \in \mathbb{Z}_k,\ k =$ level of CS
- In our units : $k = A_\Delta$
- Thus two spectra are consistent if $n_p = 2m_p$
- Ashtekar-Baez-Krasnov showed in detail how U(1) CS-theory admits such eigenstates of the holonomies in $H_\Delta^{\mathcal{P},n_p}$
- The eigenstates are labeled by $J_p \text{ or } J_{\mathcal{P}}$, these are Jacobi's theta-functions

IH in LQG :



- Since $\prod_p h_p = e$ (join loops by narrow paths) $\sum_p 2m_p = 0$
- States of H_Δ can be described by reduced density op : $\rho_\Delta = \mathrm{Tr}'\rho$
- IH as a microcanonical ensemble has diagonal ρ_Δ, so entropy of BH

$$S_\Delta = \mathrm{Tr}\rho_\Delta \ln \rho_\Delta = \ln N_\Delta$$

IH in LQG :

- Gauss, diffeo and scalar constraints are to be implemented on both Hilbert spaces
- $\boldsymbol{H} = \oplus_{\mathcal{P},J_p} \boldsymbol{H}_{\mathbf{vol}}^{\mathcal{P},J_p} \otimes \boldsymbol{H}_{\boldsymbol{\Delta}}^{\mathcal{P},2m_p}$ is U(1)-invariant
- A sequence $\mathcal{P}$ gauge fixes diffeo completely. Different sequences (# of punctures different) are gauge-inequivalent (MB-statistics)
- Scalar constraint has to be smeared by a lapse that vanishes on the horizon (related to IH b.c.)
- Physical states : spin-states satisfying IH b.c.

State counting and entropy :

- Choose units : $4\pi\gamma\ell_P^2 = 1$
- Area-spectrum (in the spin-network basis) :

$$A_{\{J_p\}} = 2\sum_p \sqrt{J_p(J_p+1)}$$

- Counting of spin states under two constraints :

$$1) \qquad A_\Delta - \epsilon \leqslant A_{\{J_p\}} \leqslant A_\Delta + \epsilon$$

$$2) \qquad \sum_p 2m_p = 0 \mod A_\Delta$$

- # states = # states of the effective CS-theory obeying 1), 2)

First attempts :

- Maximum entropy $\Leftarrow$ Largest # punctures $\Leftarrow$ Each puncture carrying spin-1/2 (semiclassical)
- Ashtekar-Baez-Krasnov : $S = \frac{\ln 2}{\sqrt{3}} \, A_\Delta$
- Kaul-Majumdar : SU(2) Chern-Simons theory of level k (= area) on punctured sphere
- # states = # conformal blocks (Witten, Verlinde)

$$S = \frac{\ln 2}{\sqrt{3}} \, A_\Delta - \frac{3}{2} \ln A_\Delta$$

- Careful analysis show that dominant configuration is not spin-1/2 alone

Recursive method :

- $\nu(A_\Delta, N)$ = # states obeying $\sum_p 2m_p = N$
- Puncture #1 carries spin-1/2 : # states
  $$\nu(A_\Delta - \sqrt{3}, N-1) + \nu(A_\Delta - \sqrt{3}, N+1)$$
- Puncture #1 carries spin-1 : # states
  $$\sum_{-2}^{+2} \nu(A_\Delta - \sqrt{8}, N-n)$$
  $$\vdots$$
- Puncture #1 carries spin-J : # states
  $$\sum_{-2J}^{+2J} \nu(A_\Delta - 2\sqrt{J(J+1)}, N-n)$$

Recursive method :

- Total # states : $\nu(A_\Delta, N) =$

$$\sum_{J=1/2}^{J_{\max}} \sum_{-2J}^{+2J} \nu(A_\Delta - 2\sqrt{J(J+1)}, N-n)$$

- To solve Fourier transform $\nu(A_\Delta, N) \mapsto \nu_\omega(A_\Delta)$
- Recursion eq of $\nu_\omega(A_\Delta)$ is solved by the ansatz

$$\nu_\omega(A_\Delta) = \exp(\lambda(\omega) A_\Delta)$$

- Solve (num) $\lambda(\omega) = 0.861 - 0.61\omega^2 - o(\omega^4)$
- # states : $\nu(A_\Delta, 0) = \int_{-\pi}^{\pi} \frac{d\omega}{2\pi} \exp(\lambda(\omega) A_\Delta)$

$$= \frac{o(1)}{\sqrt{A}} \exp(0.861 A_\Delta)$$

Statistical method :

- Configuration : $\{N_J\}$ where N_J-punctures carry spin-J (AG-Mitra)
- Area : $A_{\{N_J\}} = 2\sum_{J=1/2}^{J_{\max}} N_J\sqrt{J(J+1)}$
- Each choice of $\{N_J\}$ has # states $\prod_J (2J+1)^{N_J}$
- Each $\{N_J\}$ can be chosen in $(\sum_J N_J)!/\prod_J N_J!$ many ways
- # states for a fixed configuration

$$d_{\{N_J\}} = \frac{(\sum_J N_J)!}{\prod_J N_J!}\prod_J (2J+1)^{N_J}$$

Statistical method :

- Total # states $d(A_\Delta) = \sum_{\{N_J\}} d_{\{N_J\}}$
- Maximizing it subject to the area constraint

$$N_J = (\sum_J N_J)(2J+1)\exp(-2\lambda\sqrt{J(J+1)}$$
$$\sum_J (2J+1)\exp(-2\lambda\sqrt{J(J+1)}) = 1$$

- Thus $\lambda = 0.861,\ \sum_J N_J = 0.34 A_\Delta$

$$d = \exp(\lambda A_\Delta) + o(1)$$

expand total # states around dominant config and integrate the Gaussian fluctuations

Statistical method :

- It is not difficult to incorporate spin-projection constraint – configuration $\{N_{J,m}\}$
- Proceeding as before the total # states

$$d = \sum_{\{N_{J,m}\}} d_{\{N_{J,m}\}} = \frac{o(1)}{\sqrt{A_\Delta}} \ \exp(0.861 A_\Delta)$$

- Another recursive calculation (Lewandowski-Domagala, Meissner) gives a slightly lower # states, hence of the γ-parameter
- The differences occur because we used more quantum states : $|m\rangle$ vs. $|J, m\rangle$

Statistical method :

- Pure surface states can also be counted using statistical method – relevant configuration $\{N_m\}$
  $$N_m = \sum_J N_{J,m},\ J = |m|, |m|+1, \ldots$$
- Maximize entropy to find the dominant config

$$d_{\{N_m\}} = \frac{(\sum_m N_m)!}{\prod_m N_m!},\ \frac{N_m}{\sum_m N_m} = \exp(-\lambda\sqrt{J_m(J_m+1)})$$

$$J_m = J_{\min} = |m| \text{ for } m \neq 0,\ J_0 = 1$$

- This gives $\lambda = 0.790$ slightly higher than one of Meissner ($|m=0\rangle$ states are counted here)
- Nevertheless, two methods/groups converge

Summary and future directions :

- Horizons play a key role in describing BH microstates – near-horizon or pure-horizon?
- IH does not describe a single solution – so what point of view should one adopt for a BH?
- String theory is too tied to SUSY, hence extremal BHs – can corrections be computed?
- Hawking radiation in LQG :
  - Quantize dynamical horizons?
  - Calculate density matrix of canonical ensemble?

Summary and future directions :

- LQG gives a microcanonical temperature $T_H = \frac{\kappa_{(\ell)}\gamma}{2\lambda} + o(1/A_\Delta)$ which for $\lambda = \pi\gamma$ gives Hawk-temperature; can it be made canonical?
- Which are the correct states - $|m\rangle$ or $|J, m\rangle$?
- What are IH microstates in full LQG?
- Singularity resolution in the singlet-sector – how can it be addressed from full LQG or even from this effective theory?
- What is the final-state of IH (if unstable in QM)?