The Long Walk Towards Gravitational Wave Detection

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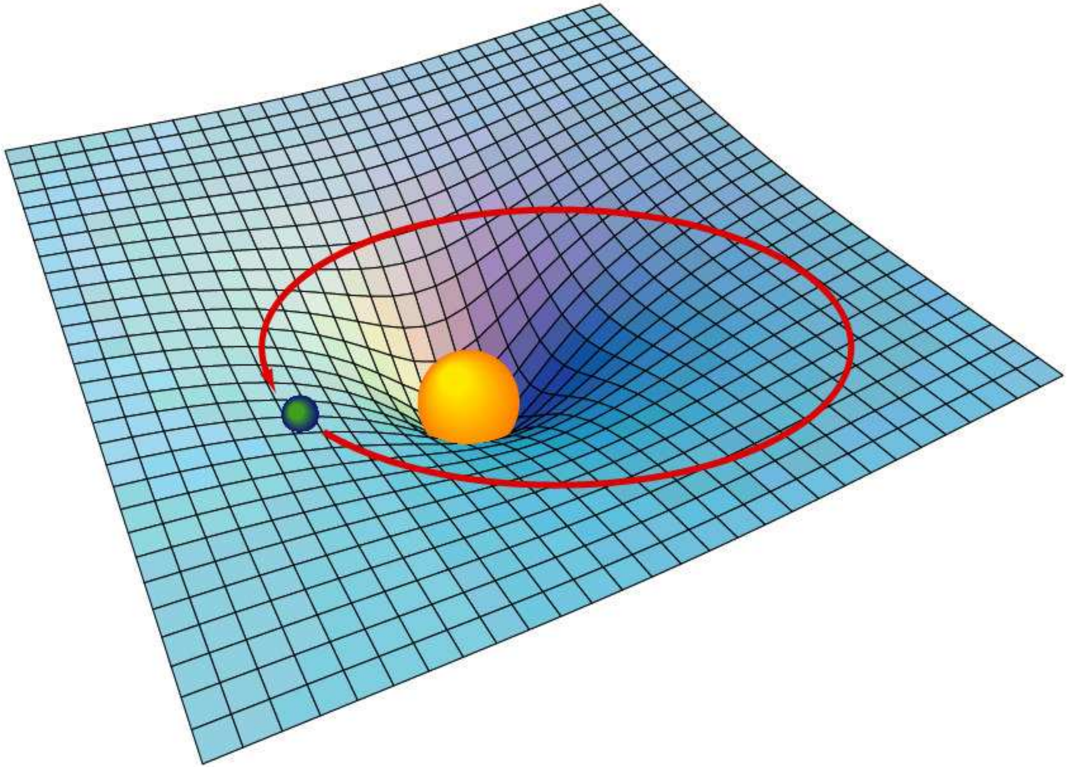
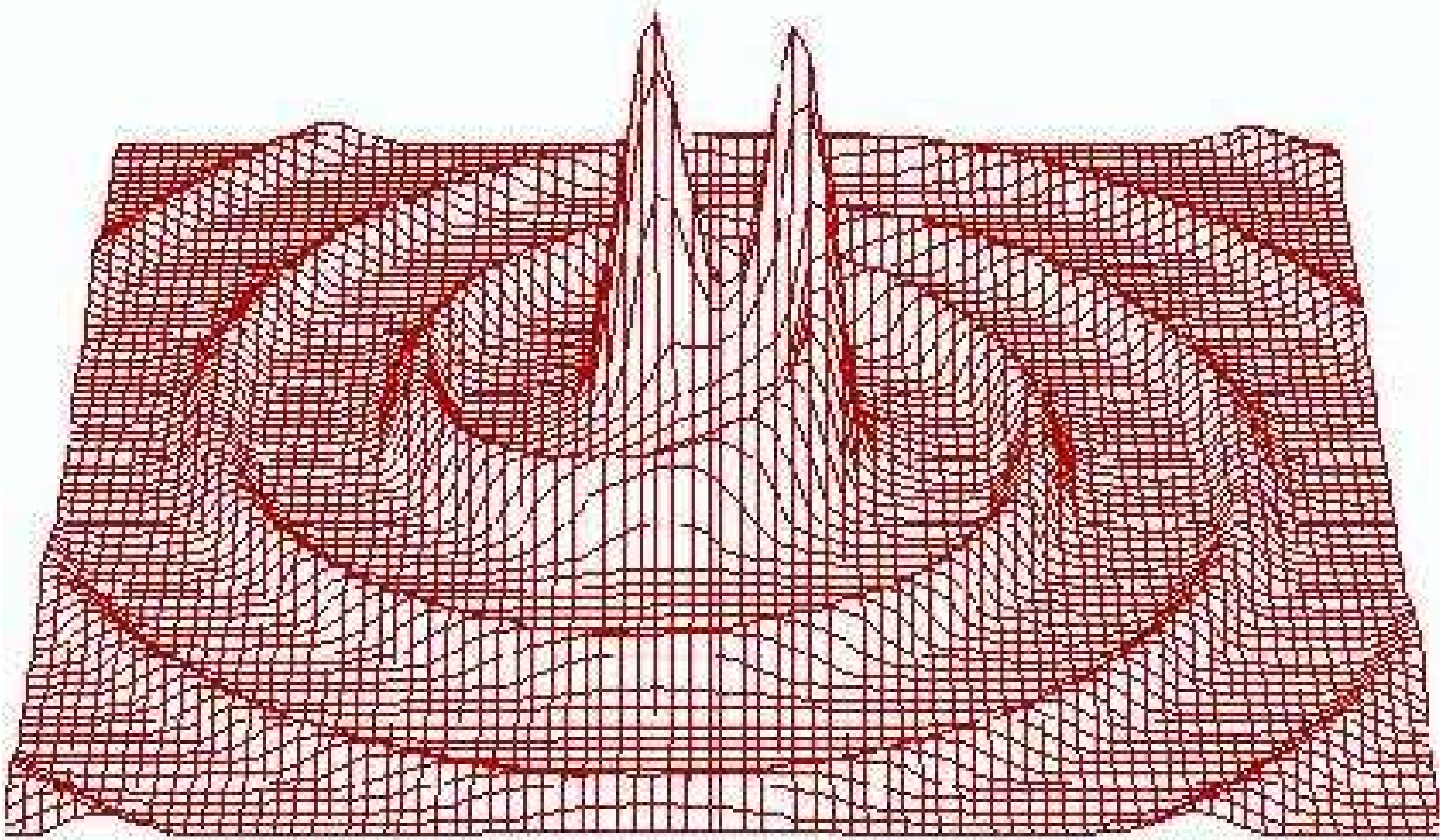
- Effect of Gravity cannot be transmitted Faster that Light
- If Grav Field of an object changes, the changes propagate thro’ space and take a Finite time to reach other objects. These Ripples are called Grav Radn or Grav Waves

Einstein's view: Gravitation ~ Spacetime Geometry ~ Curvature of Spacetime



GW - Ripples in Curvature of Spacetime



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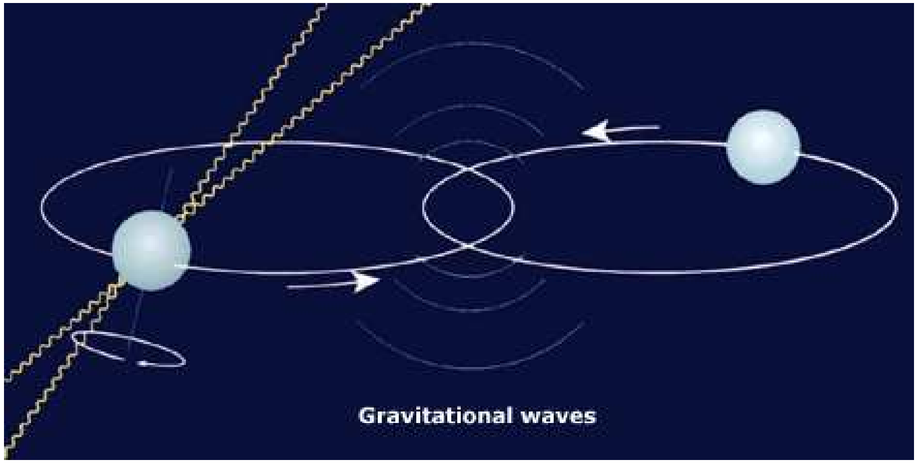

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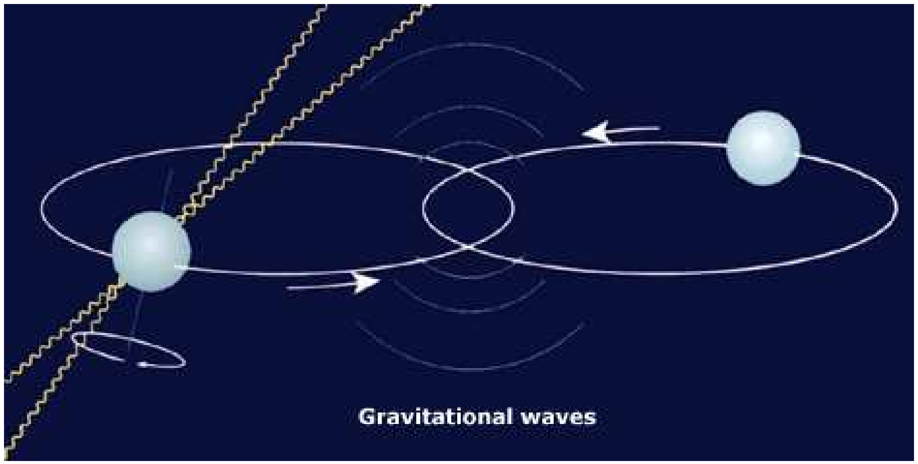
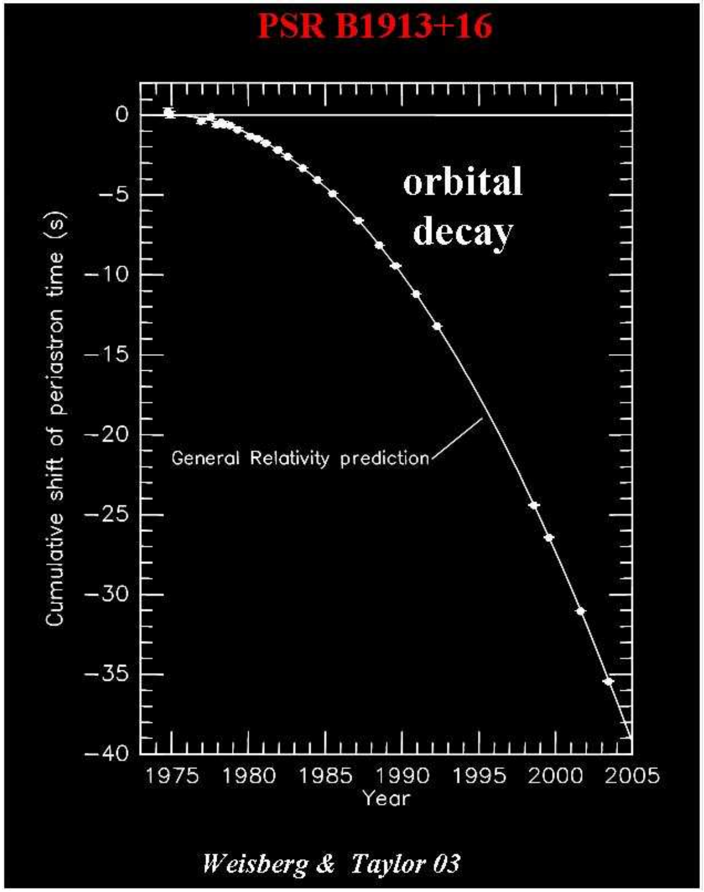


- The system has now been monitored for $\sim$ 30 years

Indeed ..Gravitational Waves exist ..

If General relativity is right (and Newtonian Gravity is incorrect) the system must emit GW. Loss of energy must cause Orbit to shrink by 3 mm/orbit



Nobel Prize (1993).

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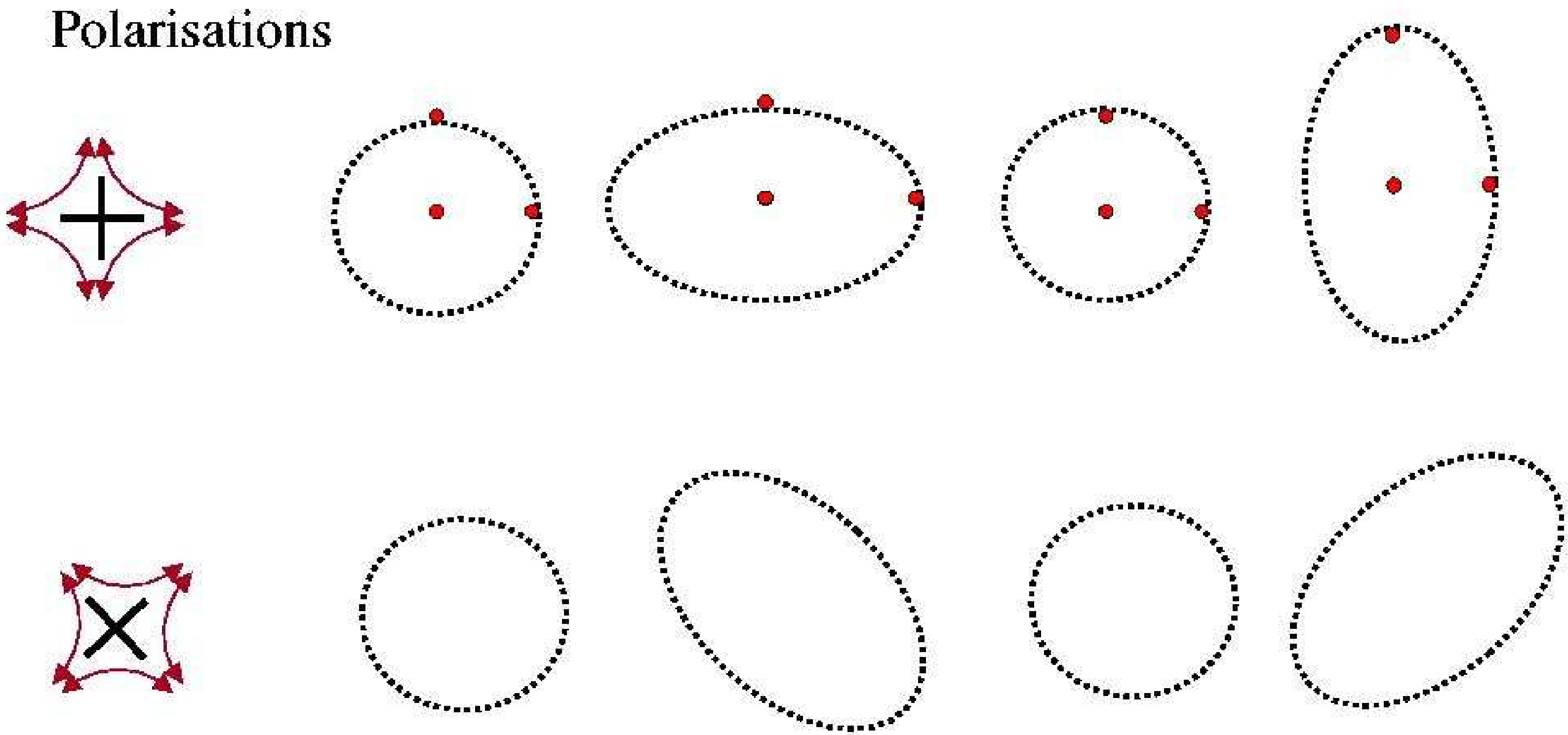
- Consequently, the Physical effect of GWs is observable by monitoring relative motion of two adjacent particles during the passage of the wave - Pirani

Polarisation of GW

Effect on a Ring of Particles



Strain Induced by a GW

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$$h = (\Delta L)/L$$

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- For a typical NS binary at a distance of 200 Mpc (6×10^{21} km);

$$h \sim \frac{4G}{c^4 D} K_{\rm nonsph} \sim 2\,\frac{GM}{Rc^2}\,\frac{GM}{Dc^2} \sim 10^{-21}.$$

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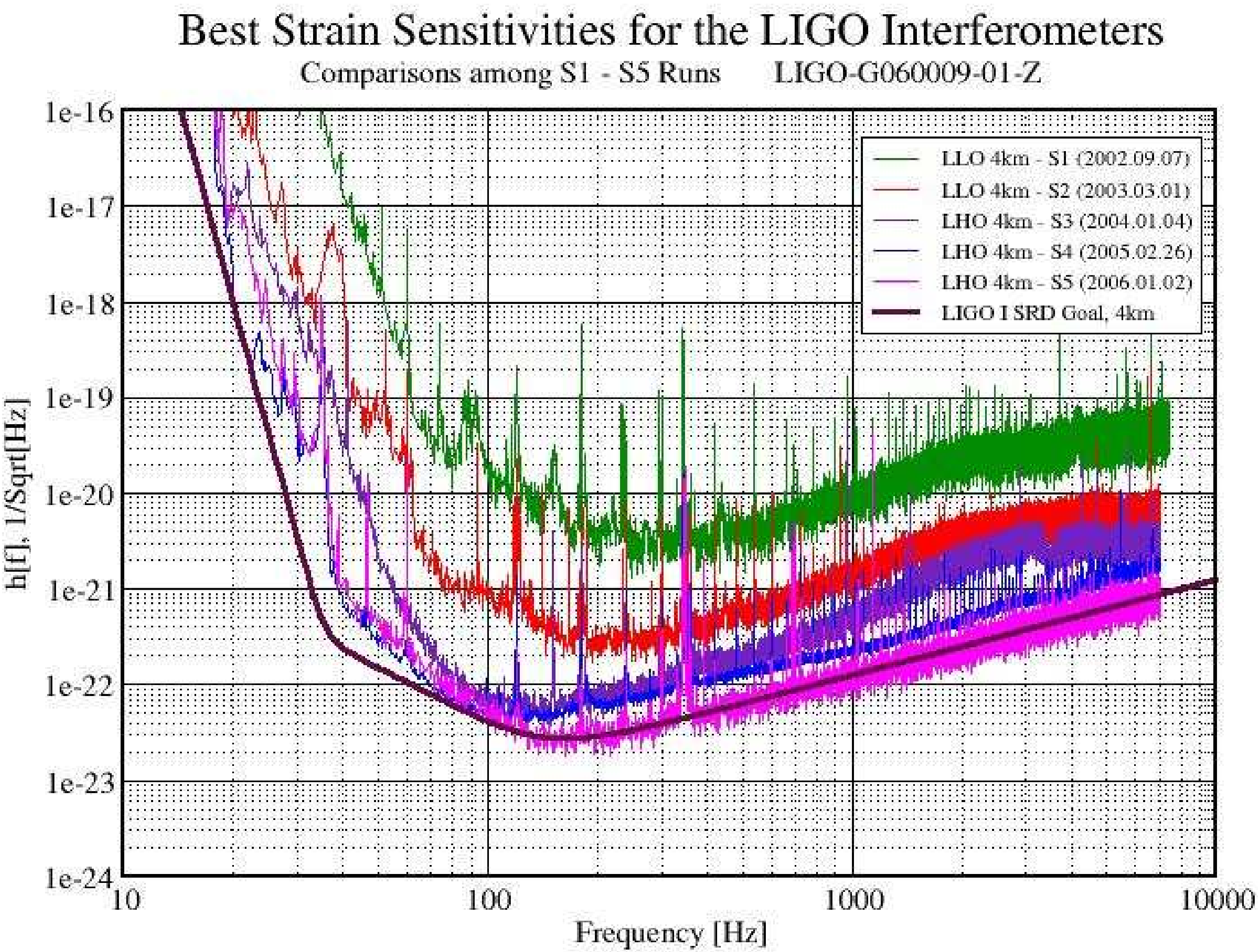
- The miniscule strain and associated tiny displacement must be measured to detect the GW

LIGO TODAY


Best Strain Sensitivities for the LIGO Interferometers
Comparisons among S1 - S5 Runs LIGO-G060009-01-Z
LLO 4km - S1 (2002.09.07)
LLO 4km - S2 (2003.03.01)
LHO 4km - S3 (2004.01.04)
LHO 4km - S4 (2005.02.26)
LHO 4km - S5 (2006.01.02)
LIGO I SRD Goal, 4km
h[f], 1/Sqrt[Hz]
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-22
1e-23
1e-24
10
100
1000
10000
Frequency [Hz]

Can Chirping Binaries be detected?

- Can we detect the GW from 1913+16 today?
  No! Since $P_{orb} \sim 8$ hrs, $f_{GW} \sim 10^{-4}$ Hz.
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- Thus, in 300 million years $f_{GW} \sim 10$ Hz. 15 minutes later $f_{GW} \sim 1000$ Hz. 16,000 cycles in the last three minutes before coalescence. All this brings the system in the sensitivity bandwidths of Earth bound detectors. Eccentricity would reduce from $e = .617$ to $e \to 0$.

Can Chirping Binaries be detected?

- In 2003 a new binary (Double) pulsar J0737-3039 with orbital period of 2.5 hrs ($e = .0877$) which will coalesce in 86 Myrs was discovered. Infall due to Grav radn Damping 7 mm/day! Even more unique Laboratory for relativistic gravitational physics in the strong field regime

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- We have guaranteed sources for the GW detectors if there are *enough* of them.

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- Prototype sources for LISA include supermassive BH binaries ($10^5 - 10^6 M_\odot$)
  or Extreme mass ratio inspirals (EMRI) - NS or stellar mass BH inspiralling into supermassive BH
  or Intermediate mass ration inspirals (IMRI).

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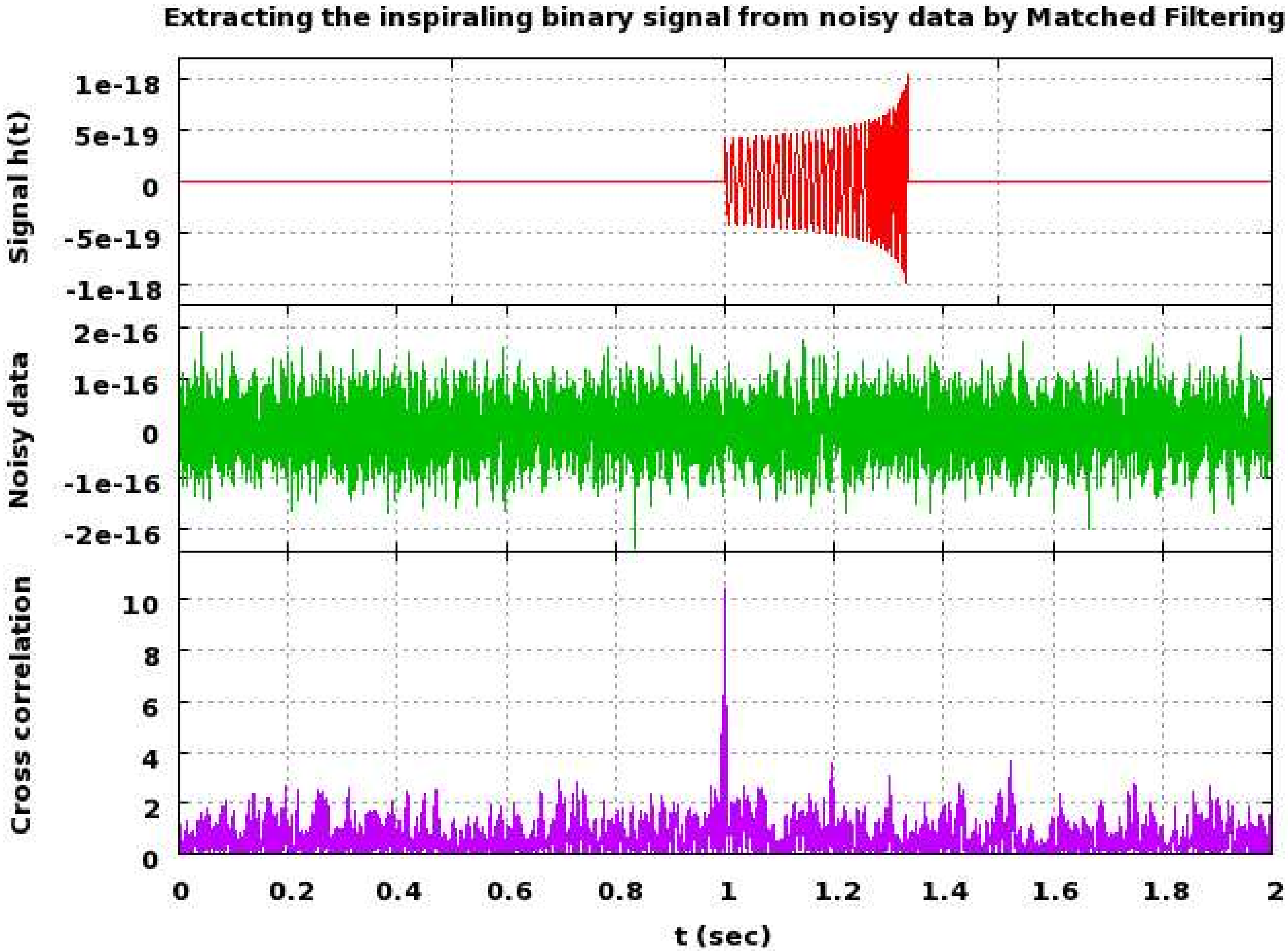
- GW are WEAK SIGNALS buried in NOISE of detector
- Require Matched Filtering (MF) Both for their Detection or Extraction and Parameter Estimation

Chirp Signal, Matched Filtering



From Anand Sengupta (IUCAA)

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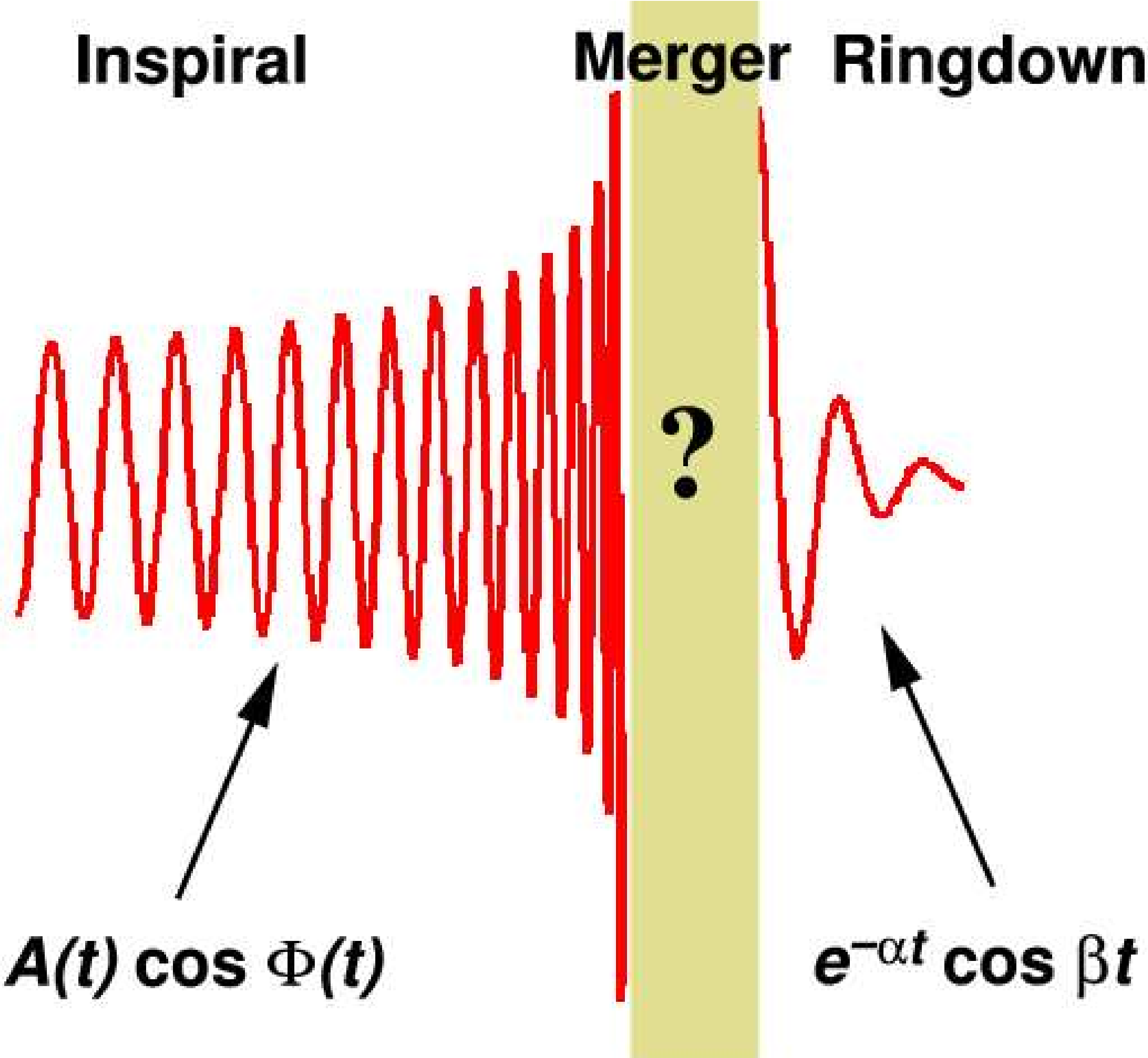
- Favours sources like ICB (NS-NS, BH-BH, NS-BH)
- Chirps (ICB), Bursts (SN, GRB), Periodic (Pulsars), Stochastic (Early Universe)

Stages of Binary Coalescence



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- QNM Ringdown: BH Pertbn Theory

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- Inspiral characterised by the associated PN GW Luminosity

Adiabatic Approximation

$$x = \left(\frac{Gm\omega_{\rm orb}}{c^3}\right)^{2/3}$$

- Use E, the CM energy at Newtonian order and $\mathcal{L}$, GW luminosity or energy flux

$$E = -\frac{1}{2}\mu c^2 x$$

$$\mathcal{L} = \frac{32}{5}\ \frac{c^5}{G}\ \nu^2 x^5$$

$$\frac{c^5}{G} \approx 3.63 \times 10^{52}\ \mathrm{W},$$

- and heuristic Energy Balance equation

$$\frac{dE}{dt} = -\mathcal{L}.$$

Newtonian Phasing ...Contd..

$$x(t) = \frac{1}{4}\tau^{-1/4}$$

$$\tau = \frac{c^3\nu}{5Gm}(t_c - t)$$

$$\phi(t) = \int \omega dt = -\frac{5}{\nu}\int x^{2/3} d\tau$$

GW polarisations

$$h_{+,\times}(t) = \frac{2G\mu}{c^2R}\ x(t)\ \times\{H^{(0)}_{+,\times}(t) + x^{1/2}(t)H^{(1/2)}_{+,\times}(t) + \cdots + x^{5/2}(t)H^{(5/2)}_{+,\times}(t$$

$$H^{(0)}_{+} = -(1 + c_i^2)\cos 2\phi(t),$$

$$H^{(0)}_{\times} = -2c_i \sin 2\phi(t), \cdots$$

PN order required..

- Number of GW cycles $\mathcal{N}$ left until coalescence starting at some frequency ω

$$\mathcal{N} = \frac{\phi_c - \phi}{\pi} = \frac{1}{32\pi\nu} x^{-5/2}$$

- $\propto (v/c)^{-5}$ (inverse of $(v/c)^5$ the RR order)
- ~ 16000 cycles for NS-NS binaries
- Matched filtering requires accuracy to about fraction of a cycle.
- Formally (and detailed DA), indicate that one needs to go to relative order 2.5PN or 3PN in $\mathcal{L}$ to achieve the required accuracy.
  (Damour, BRI, Sathyaprakash - 1998, 2000, 2001, 2002)
  (Damour, BRI, Jaranowski, Sathyaprakash - 2003)
  (Buonanno, Chen, Vallisneri - 2003)

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  Analytical and Numerical Approaches - Complementary

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- Generation: Given the motion of the binary system on a fixed orbit, iterate EE to compute multipoles of the Grav field and hence the FZ flux of energy and AM carried by GW. Compute $\mathcal{L}$
- Radiation Reaction: Given the Conserved energy and Radiated Flux of Energy and AM, *ASSUME* the Balance Eqns to Compute the effect of Radiation on the Orbit.
  Compute $x(t)$ and $\phi(t)$
  (GW) Phasing of Binary $\sim$ (EMW) Timing of Pulsars

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- Flux formula at 2.5PN$\sim$ Correction to Newtn accn of rel order 2.5PN. EOM to this order would require one to also control the 1PN and 2PN terms which is more difficult but Done during the Bin Psr Analysis
- Exact solution impossible
  Resort to Approximation Methods

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- Far zone expansion: $\frac{1}{R}$ expansion
- Successful wave-generation formalisms concoct *cocktail* from above options

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- To very high approximation, internal structures of the compact bodies are *effaced* when seen in the external scheme.
- Internal structures affected EOM only starting at the 5PN level ($1/c^{10}$) (influence on orbital motion of Newtonian quadrupole moments induced by tidal interaction between the two compact objects)
- *Effacement result* is the rationale for describing, up to 5PN order, two (non spinning) compact bodies in terms of two point masses.

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- Technical Cost:
  Handle Delta Functions in a Non Linear Theory

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- These regularisation schemes work consistently up to 2.5PN but are *incomplete* at 3PN..
  Results contain numerical constants `ambiguity parameters' that cannot be determined with this self-field regularisation..
  Technical obstacle for many years

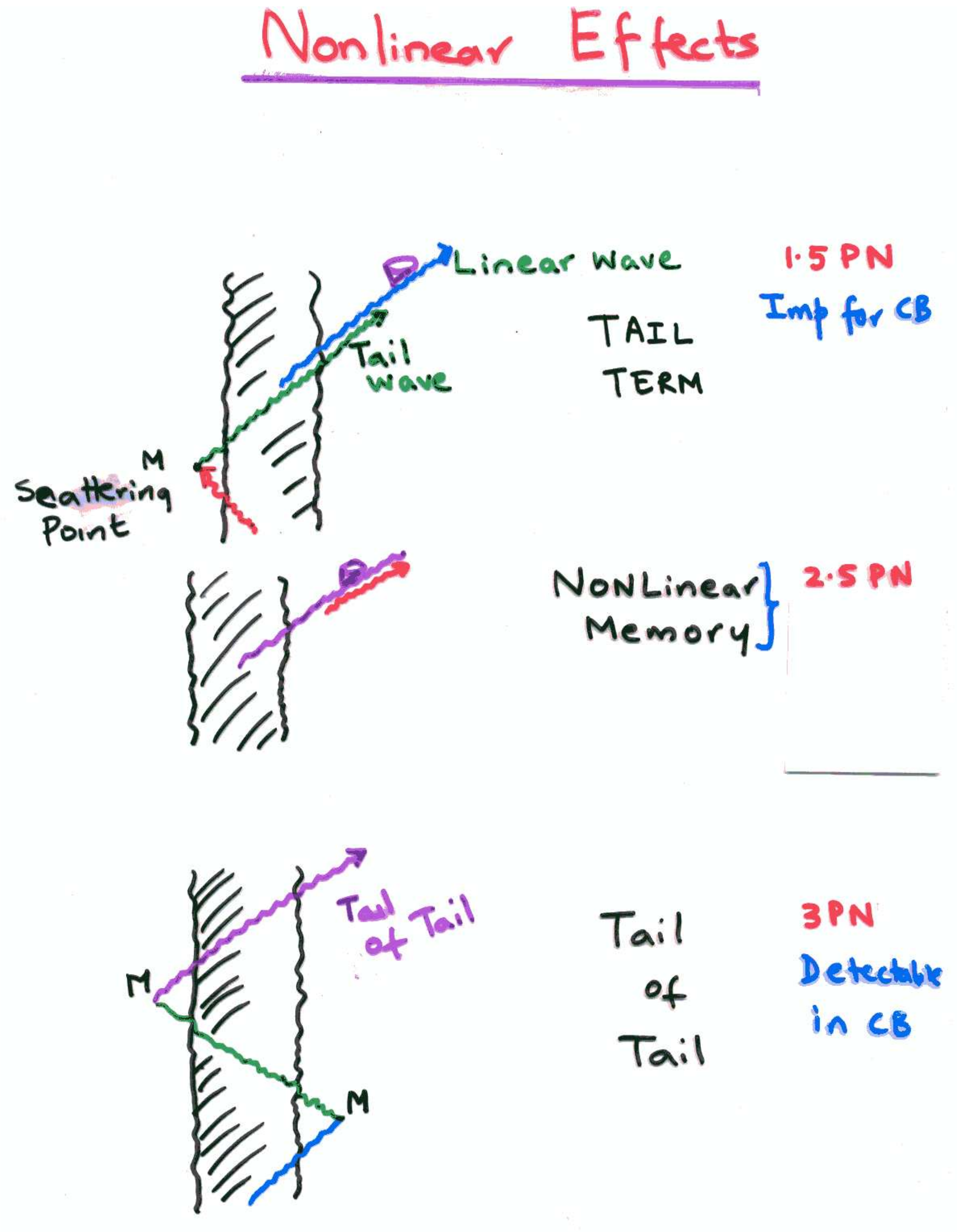
The 3PN Energy Flux

Hadamard Regularisation

$$
\begin{aligned}
\mathcal{L} &= \frac{32c^5}{5G}x^5\nu^2\left\{1+\left(-\frac{1247}{336}-\frac{35}{12}\nu\right)x+4\pi x^{3/2}\right.\\
&+\left(-\frac{44711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right)x^2+\left(-\frac{8191}{672}-\frac{535}{24}\nu\right)\pi x^{5/2}\\
&+\left(\frac{6643739519}{69854400}+\frac{16\pi^2}{3}-\frac{1712}{105}C-\frac{856}{105}\ln(16x)\right.\\
&+\left.\left[-\frac{11497453}{272160}+\frac{41\pi^2}{48}+\frac{176}{9}\lambda-\frac{88}{3}\theta\right]\nu-\frac{94403}{3024}\nu^2-\frac{775}{324}\nu^3\right)x^3\\
&+\left.\left(-\frac{16285}{504}+\frac{176419}{1512}\nu+\frac{19897}{378}\nu^2\right)\pi x^{7/2}+\mathcal{O}(x^4)\right\}\\
\theta &= \xi+2\kappa+\zeta
\end{aligned}
$$

Blanchet, BRI and Joguet

3PN GW Flux includes..



Dimensional regularisation determines the four undetermined parameters

- Imposing the *physical equivalence* between the Dimensional Regularisation result and Hadamard Regularisation one modulo an unique shift in *bare* particle posns $\vec{y}_{1,2}^{\,\mathrm{bare}}$ appearing in DR result from those in HR one uniquely determines the 'ambiguity parameters'

$$\lambda = -1987/3080 \simeq -0.64513$$

(Damour, Jaranowski, Schäfer;
Blanchet, Damour, Esposito-Farese)

$$\xi = -\frac{9871}{9240}\,, \quad \kappa = 0\,, \quad \zeta = -\frac{7}{33}\,,$$

$$\theta = \xi + 2\kappa + \zeta = -11831/9240 \simeq -1.28041$$

$$\hat{\theta} \equiv \theta - 7/3\lambda = 1039/4620 \simeq .22489$$

(Blanchet, Damour, Esposito-Farese, BRI)

Hadamard vs Dimensional

- Hadamard regularization of the self-field of point particles violates the gauge symmetry of perturbative general relativity (diffeomorphism invariance), and thereby breaks the crucial link between Bianchi identities and EOM. This is probably why the Hadamard based works are unable to fix the parameter ω_s, λ, ξ, κ, ζ

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- *Dimensional Regularization* respects the gauge symmetry of perturbative GR; In this sense, a *better* regularization scheme.
- No undetermined parameters!

Are we there???

Contributions to the accumulated number $\mathcal{N} = \frac{1}{\pi}(\phi_{\rm ISCO} - \phi_{\rm seismic})$ of gravitational-wave cycles. Frequency entering the bandwidth is $f_{\rm seismic} = 10$ Hz; terminal frequency is assumed to be at the Schwarzschild innermost stable circular orbit $f_{\rm ISCO} = \frac{c^3}{6^{3/2}\pi G m}$.

A$\equiv 2 \times 1.4 M_\odot$ B$\equiv 10 M_\odot + 1.4 M_\odot$ C$\equiv 2 \times 10 M_\odot$

	A	B	C
Newtonian	16031	3576	602
1PN	441	213	59
1.5PN	−211	−181	−51
2PN	9.9	9.8	4.1
2.5PN	−12.2	−20.4	−7.5
3PN	2.6	2.3	2.2
3.5PN	−1.0	−1.9	−0.9

Blanchet, Faye, BRI and Joguet

Blanchet, Damour, Esposito-Farese and BRI

The Test Particle Case

The corresponding Number of Cycles in the Test Mass Case

	A	B	C
Newtonian	16034	3577	602
1PN	357	216	55
1.5PN	-228	-208	-61
2PN	-12	-15	-5
2.5PN	13	21	7.5
3PN	-12	-23	-9
3.5PN	1.5	2.7	1.1
4PN	-0.21	-0.29	-0.16
4.5PN	-0.43	-1.0	-0.4
5PN	-0.02	0.23	0.14
5.5PN	0.33	-0.02	-0.07

Mino, Sasaki, Shibata, Tagoshi and Tanaka
(Ajit Parameshwaran - Recheck ..Signs)

Current Status

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  Tanaka, Tagoshi, Sasaki (1996)

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- Restricted Wave Approximation: Phase at best PN accuracy; Amplitude Newtonian; Dominant Harmonic at Twice Orbital Frequency

$$h(t) = \frac{4A\eta m}{D} v^2(t) \cos[\varphi(t) + \varphi_C],$$

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$$h(t) = \frac{4A\eta m}{D} v^2(t) \cos[\varphi(t) + \varphi_C],$$

- Full Waveform - Fourier Domain -SPA
  2.5 PN Amplitude, 3.5 PN Phase
  Van Den Broeck and Sengupta 05, 06

Detection with 3.5PN phasing

- Detection Problem Standard adiatatic approximants will provide an effectualness > 0.965 at order 3PN. (Ajith, BRI, Robinson, Sathyaprakash)

Statistical Vs Systematic errors

- Matched filtering $\Rightarrow$ detector output is `filtered' using pre-calculated waveforms with different signal parameters.
- The `measured' values of the signal parameters correspond to that of the template which has maximum SNR.
- They need not be the `actual' value of the parameters:
  - Systematic errors due to approximate signal model we use
  - Statistical errors due to the noise present

Parameter Estimation with 3.5PN Phasing

- Parameter estimation aims at calculating the probability distribution for the measured values of a signal and to compute the interval in which the true parameters of the signal lie (at a specified confidence level).
- Error estimates are obtained using covariance matrix and hence provides a lower bound on the errors.
- Parameter estimation for nonspinning binaries:
- **Ground based detectors**: Estimation of $\mathcal{M}$ and η improves up to 10% and 50% (depending on the system) when 3.5PN phasing is used instead of 2PN one.
  (Arun, BRI, Sathyaprakash,Sundararajan, 2005)
- **LISA**: Trends are similar to that of the ground based ones.
  Arun, 2006

The Rocket effect: GW Recoil

- GW take away energy and angular momentum leading to the inspiral. It occurs at leading order 2.5PN

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- In an unequal mass binary, the lighter mass moves with a higher velocity and more efficiently forward beams its radiation. This leads to momentum flux in the direction of its velocity and thus a recoil opposing this.
- 2PN Linear Momentum Flux from ICB and the associated recoil velocity (See Moh'd Qusailah's talk for more details)

Testing GR with Binary Pulsar

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- Binary pulsar measurements are performed by fitting the pulse arrival times to a 'relativistic' timing model which is a function of the Keplerian parameters (orbital period, eccentricity, projected semi-major axis of the pulsar orbit) and post-Keplerian parameters (periastron advance, time dilation, secular change of orbital period)

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- Different theories of gravity have different predictions for the values of the PK parameters as functions of the individual masses m_1 and m_2 of the binary

Testing GR with Binary Pulsar

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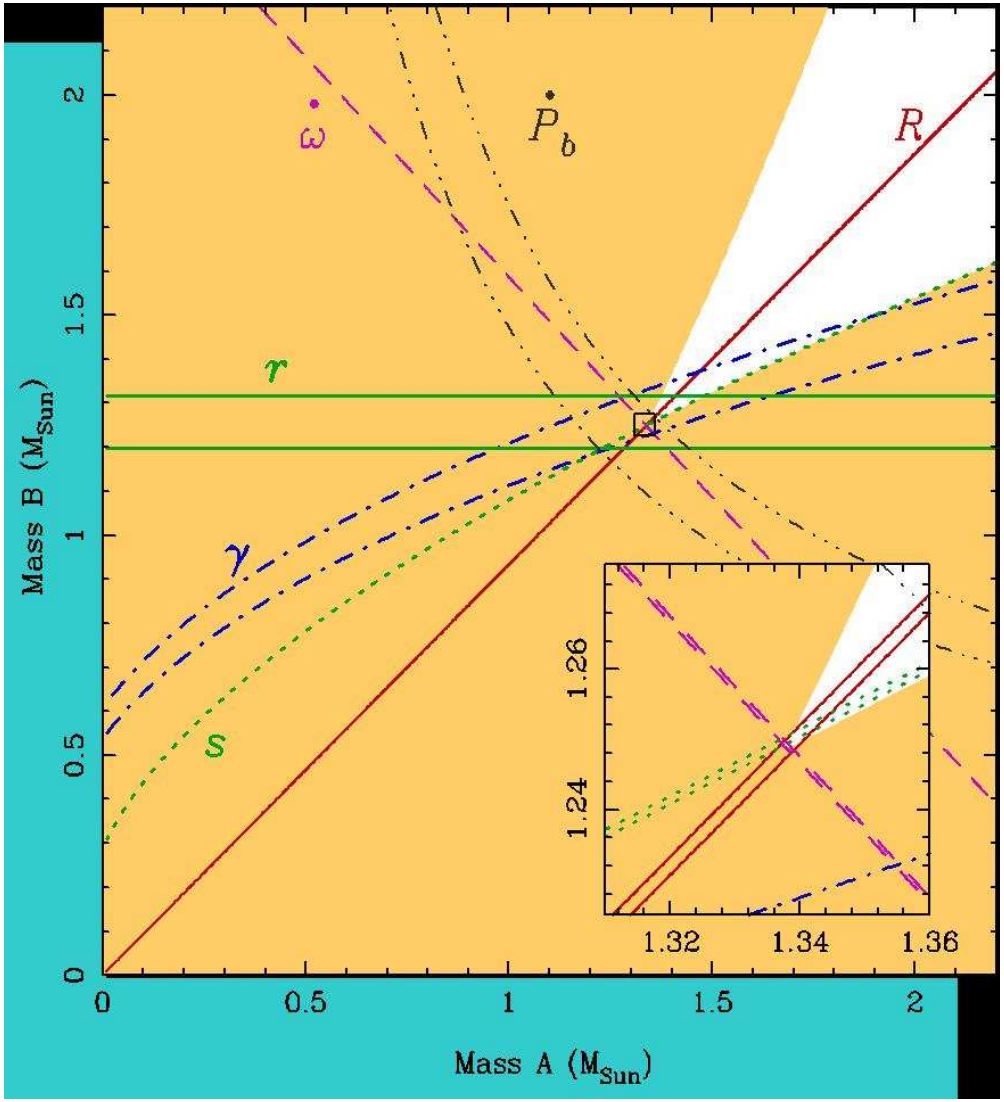
- A measurement of three or more PK parameters leads to a test by requiring consistency, within observational errors, in the estimation of the masses of the two bodies as determined by the various parameters
- For J0737-3039 measurement of five PK parameters together with additional measurement of the mass ratio determine and check consistency of pulsar masses in the $m_1 - m_2$ plane

Testing GR with Binary Pulsar



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Testing PN Gavity by GW observations

- GW observations of the coalescence of binary black holes will provide an unique opportunity to test PN theory to very high orders. Velocities of the system close to merger is as high as $v/v \sim 0.2 - 0.4$
- Suppose the GW signal is detected with a high SNR satisfying all detection criteria
- Assuming the 3.5PN restricted waveform is a good representation of the actual signal, fit the signal by a template bank where *ALL* PN coefficients are treated as *independent* parameters

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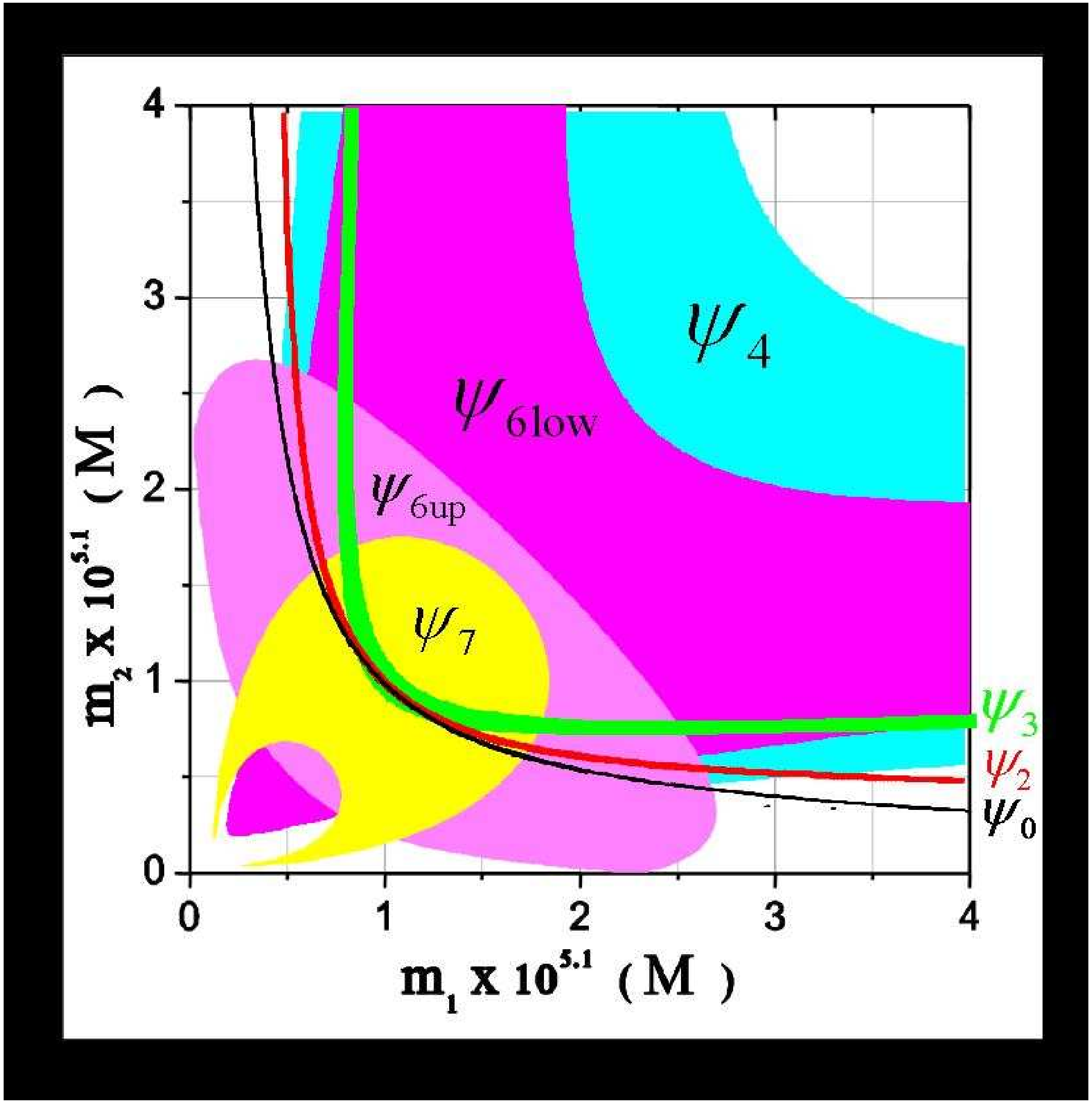
Testing GR with Binary Pulsar

- If the PN theory that approximates GR correctly describes the dynamics of the binary the (mass) parameters must be consistent with each other within their respective error bars
- All PN parameters can be estimated with $1-\sigma$ error bar for LISA sources at 3Gpc. LISA provides an unique opportunity to probe the non-linear structure of PN approximation to GR and possible its alternatives.
  Adv. LIGO can test up to 1.5PN
  (Arun, BRI, Qusailah, Sathyaprakash, 2006).

Testing PN Theory with GW Observations



Model for the orbit: Circular vs Elliptic

- For long lived compact binaries, the quasi-circular approximation is quite appropriate. GRR decreases the orbital eccentricity to negligible values by the epoch the emitted gravitational radiation enters the sensitive bandwidth of the interferometers.
- However, there exist Ap scenarios e.g. involving Kozai oscillations, that lead to compact binaries in *quasi-eccentric* orbits that are excellent sources for LISA.
- 3PN Energy and Angular momentum Fluxes for ICB's in quasi-eccentric orbits
- 3PN (5.5PN) secular evolution of orbital elements (Arun, Blanchet, BRI and Qusailah, 2007)
- Data analysis for the eccentric binaries is more involved. Three timescales: Orbital, periastron precession and radiation reaction. Secular GRR + Fast orbital oscillations. Damour, Gopakumar, BRI, 2005

Spin of the binary: Spinning vs Nonspinning

- Spin effects are more important for black hole binaries. Including the spin effects is an important step towards constructing more realistic and general templates. Theoretically, computation of waveforms with spin effects is more complex. Till date spin effects are computed in the phase up to 2.5PN order (Apostolatos et al 94; Kidder, Will, Wiseman 93; Kidder 95; Blanchet, Buonanno, Faye 06) and in the amplitude up to 2PN order (Kidder 95; Ohashi, Tagoshi, Owen 98).
- Apostolatos et al gave a prescription to incorporate orbital precession effects and the consequent modulations in the model of the gravitational waveform.

Restricted WF vs Full WF

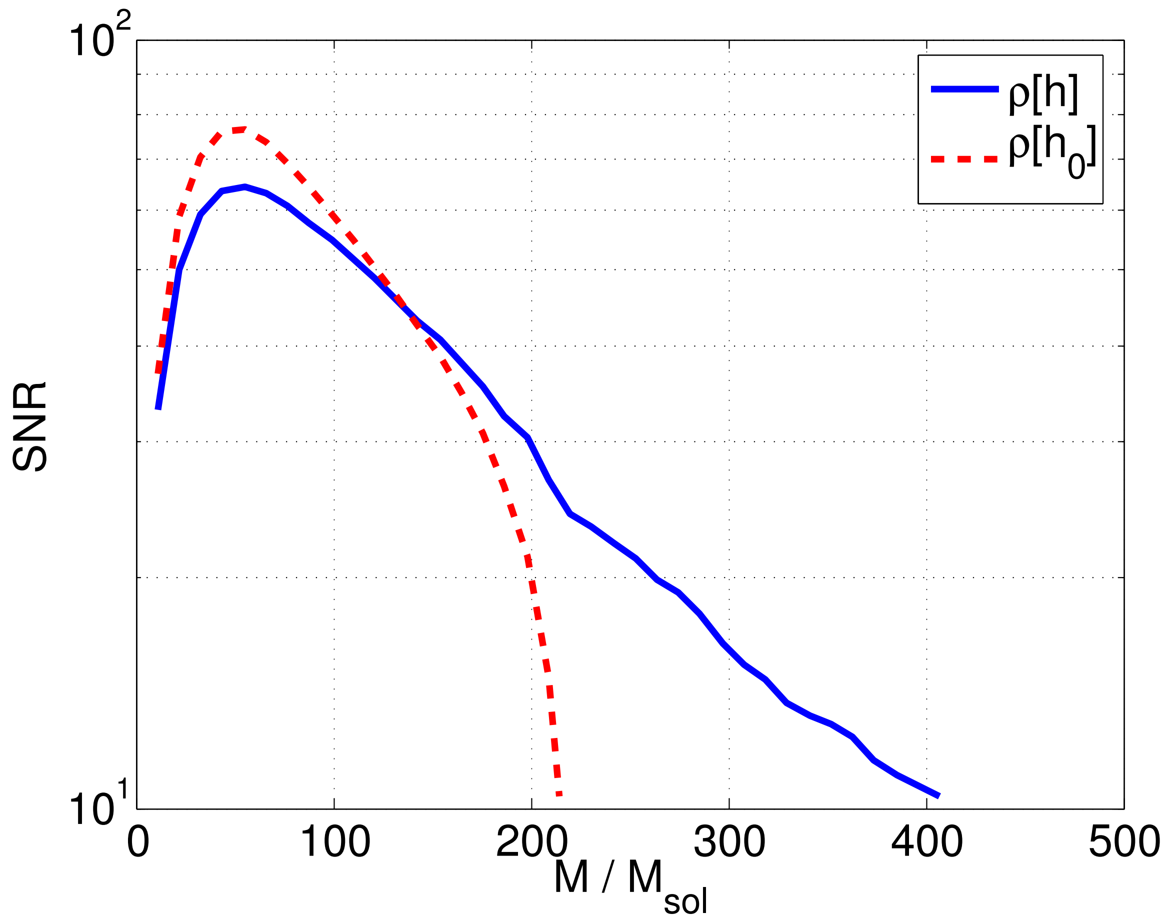
- GWDA uses 'restricted waveform approximation'. Very high PN accurate *phasing* of the binary, keeping the *amplitude* of the wave to be at leading Newtonian order. This is justified by the argument that matched filtering is more sensitive to the phase of the GW than its amplitude.
- Recently Van Den Broeck 06; Van Den Broeck, Sengupta 06; looked into the validity of the restricted waveform approximation for *detection* as well as *parameter estimation*.

Restricted WF vs Full WF

- Restricted waveforms overestimate the SNR relative to the Full waveform since they ignore the higher harmonics.
- This is because of 'destructive' interference between harmonics.
- One may need to account for this while constructing the templates for data analysis
- Advanced detectors will be sensitive at lower frequencies and higher harmonics may enter BW even if the dominant one does not. Detector's mass reach may increase by factors of two (adv LIGO) or three (EGO). Allow for detection of inspirals with higher total mass.

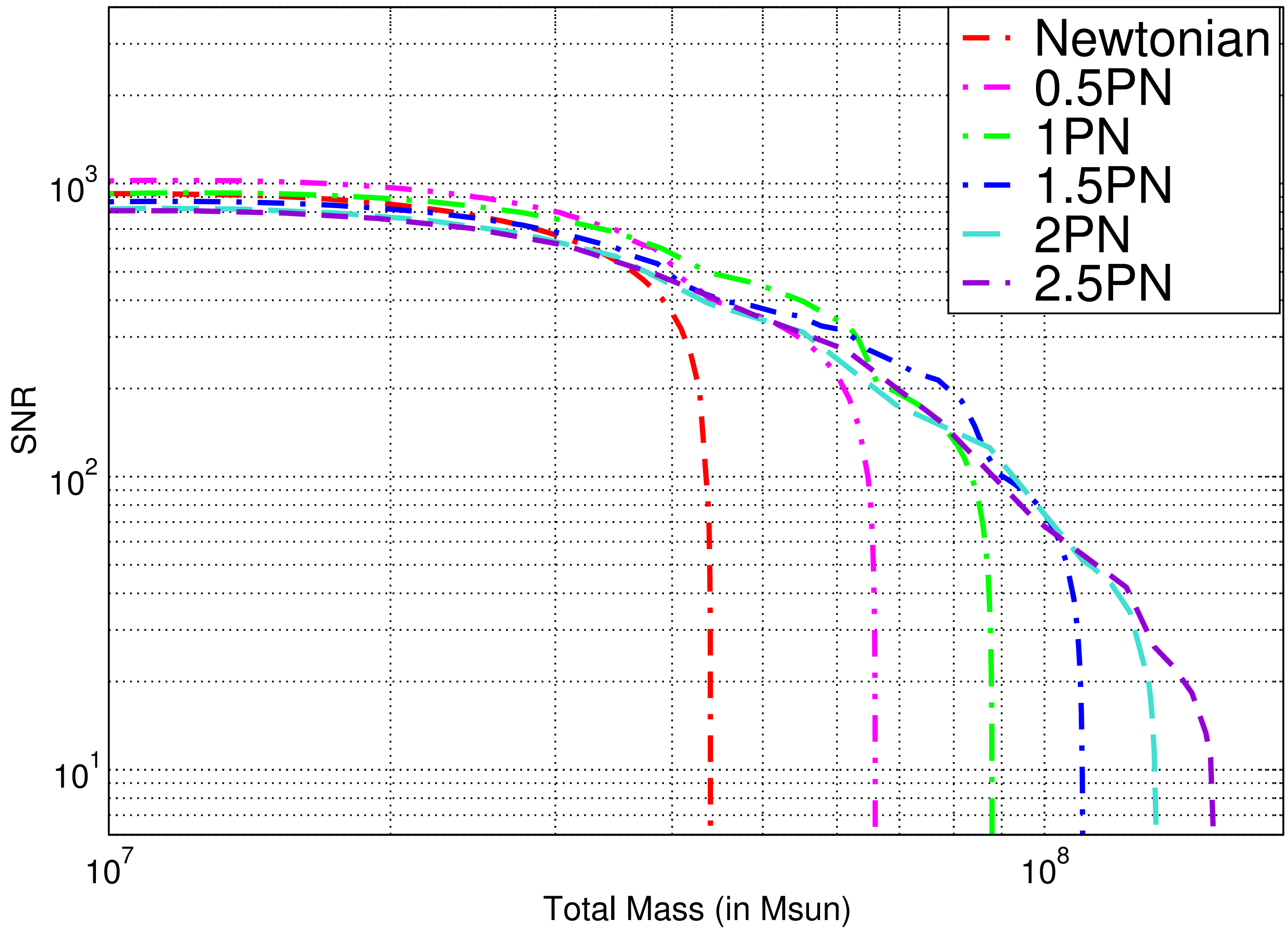
Non-Restricted Full Waveform - Mass reach

Advanced LIGO



Van Den Broeck and Sengupta (2006)

Non-Restricted Full Waveform - SNR - LISA



Arun, BRI, Sathyaprakash, Sinha

Late inspiral: Adiabatic vs Non-adiabatic

- Computation of the waveforms for the inspiralling compact binary systems are implemented using the PN approximation to general relativity. One assumes here that though the orbital frequency of the system changes with time, the change in frequency per orbital period is negligible compared tothe orbital frequency itself, i.e., $\frac{\dot{\omega}}{\omega^2} \ll 1$.
- Strictly speaking, this *adiabatic approximation* is valid only in the early part of the inspiral and not during the very late inspiral and merger phases. Hence the standard PN approximation is expected to break down towards the very late part of the inspiral.

Late inspiral: Adiabatic vs Non-adiabatic

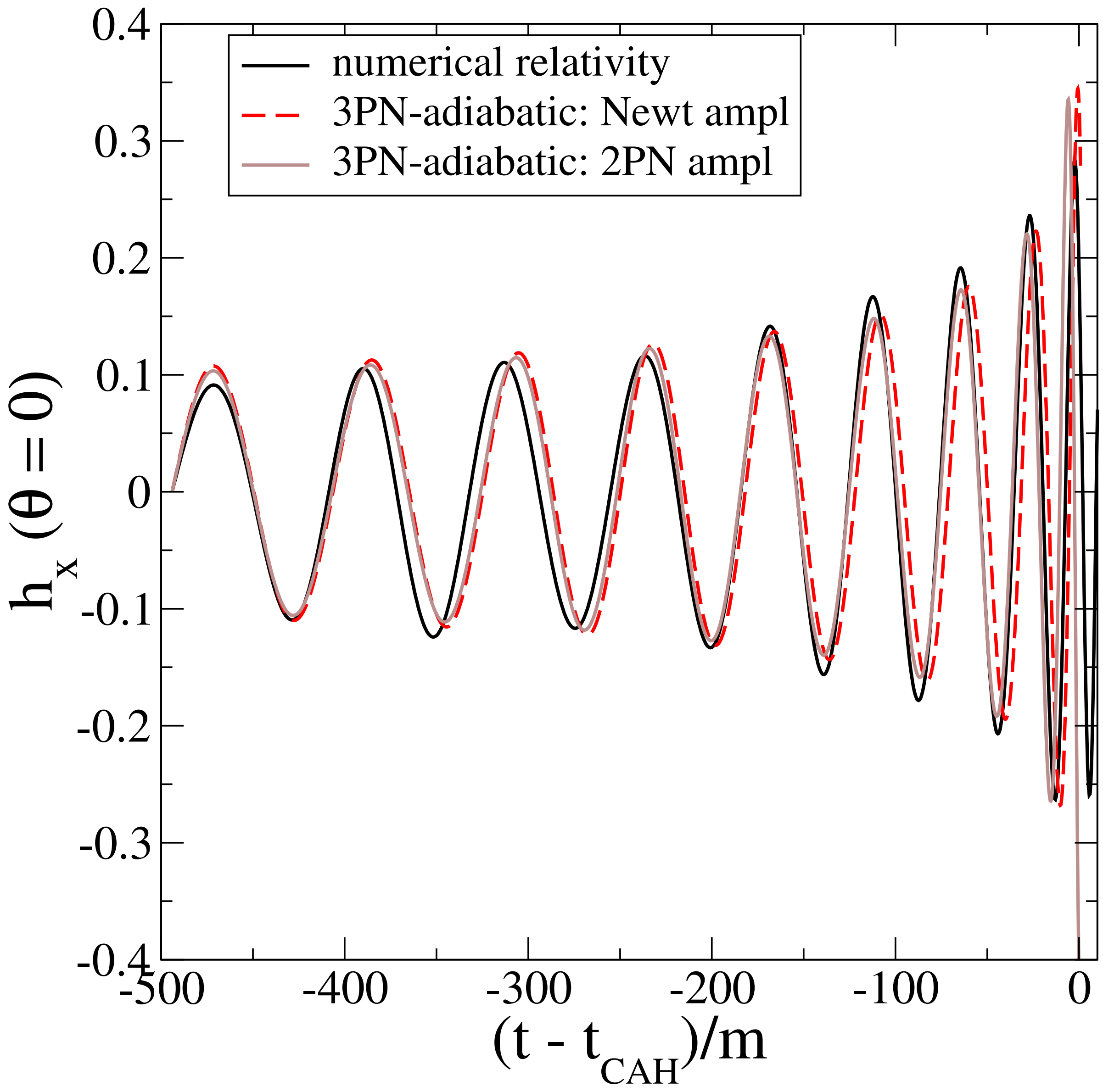
- Alternatives have to be explored to include the effects of non-adiabaticity and to model the plunge and merger phases.
- Effective one body (EOB) approach first proposed by Buonanno and Damour is one of the most important among them. This method, for the first time, does not assume adiabaticity anymore and provides an analytical description of the transition from plunge to merger and subsequent 'ringing'. 3PN: Damour Jaranowski Schäfer; Spin EOB: Damour 01
- Other approaches to go beyond the adiabatic approximation, have been made by Buonanno, Chen, Vallisneri, Pan.. (Phenomenological Templates) and Ajith, BRI, Robinson, Sathyaprakash (Complete Adiabatic Templates)

Numerical Relativity Arrives

- With the amazing progresses in numerical relativity recently ushered by work of Pretorius (2005), one will have better waveforms for the late inspiral and merger parts of the binary evolution which can be used for constructing templates as well as to test the robustness of the analytical adiabatic and non-adiabatic models.
- Other groups have followed (Baker et al 06, Campanelli et al 06, Herman et al 06); Sperhake 06;
- There is exciting progress in matching the PN waveforms to the Numerical Relativity ones (Buonanno, Cook, Pretorius 06; Damour, Nagar 06)

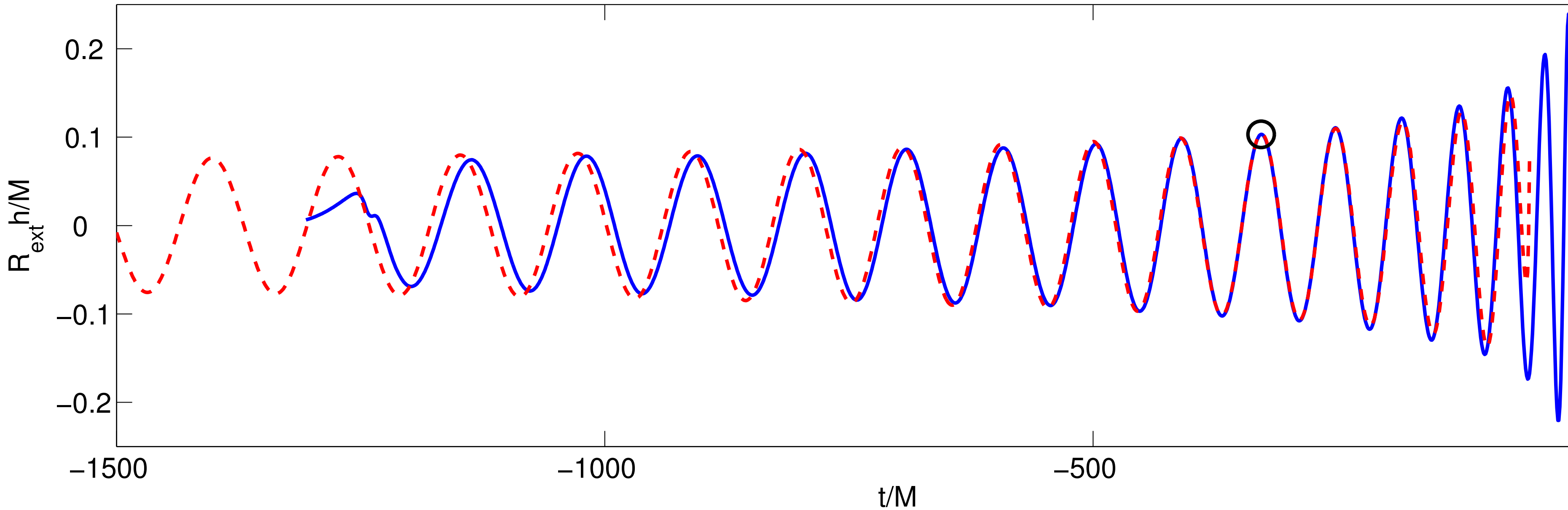
Inspiral, Merger, Ringdown

Buonanno, Cook, Pretorius

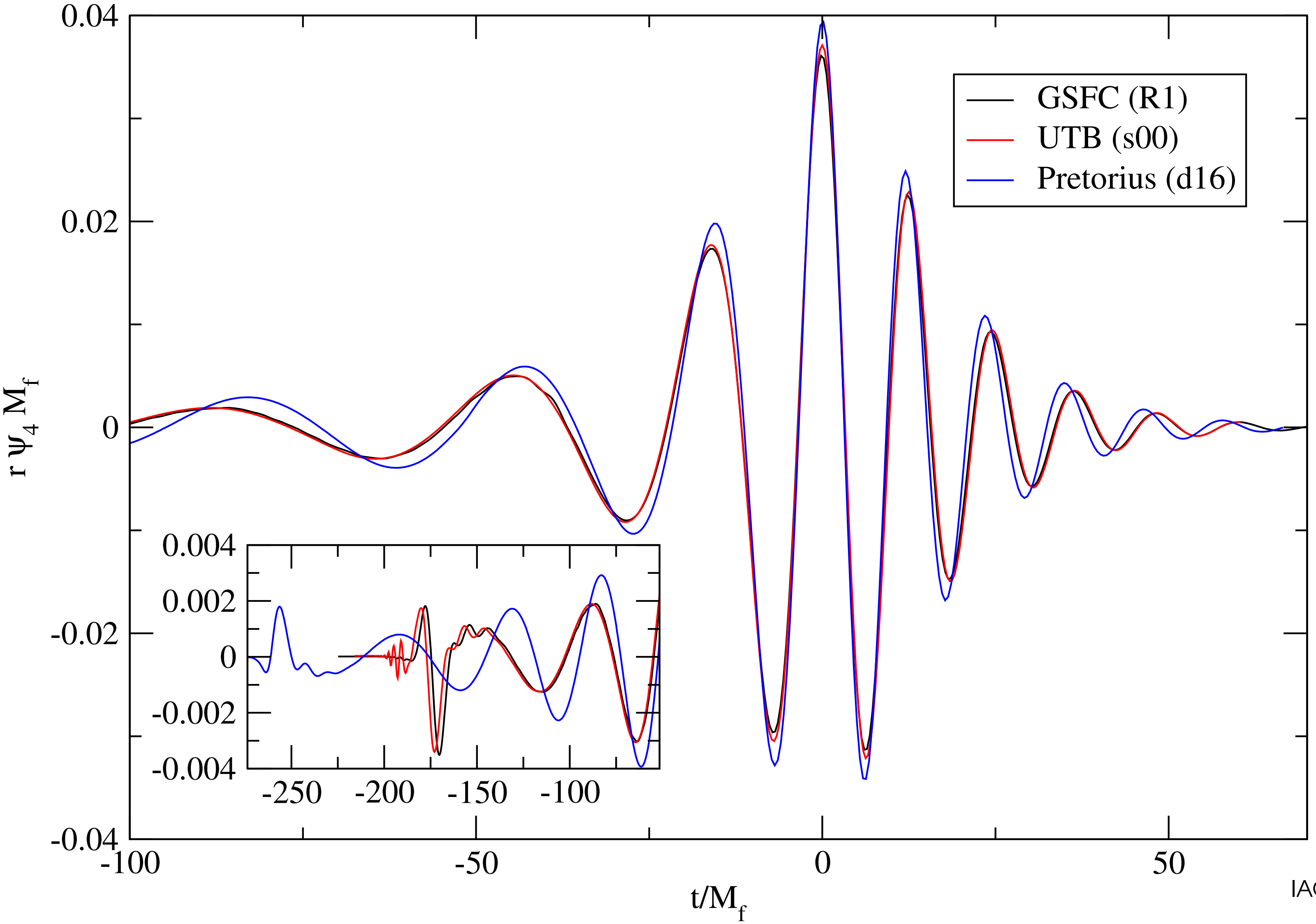


Binary BH Inspiral

Baker et al



Comparison of three codes



Results

- Merger period is relatively brief
- The dominant GW frequency rises quickly generating in Fourier domain a signal spread in frequency
- Inclusion of merger ringdown signal increases SNR for larger binary masses (as for EOB)
- BBH of masses 40 $M_\odot$ can be detected by LIGO at 100 Mpc with SNR 15
- SMBBH of masses $2 \times 10^6 - 10^7$ can be detected by LISA with SNR 10^4 at 1 Gpc

Conclusion and Open Issues

- The Long Walk towards Templates for ICB is thankfully complete as of now. From 1PN to 2PN took only about Three Years; From 2PN to 3.5PN it was Nine long years!
- Can one determine the EOM without Regularisation using Extended Bodies?
- Finally, can one determine the Radiation Field without Regularisation using Extended Bodies
- The problem of radiation reaction for Extreme Mass Ratio Inspirals (EMRI) and the evolution of the Carter consant has made progress over the last few years and interesting results are emerging. They need to be consolidated
- Numerical Relativity runs are hoped to bring insights of the merger phase and subsequent QNM ringing

Approximation Methods

- PN approximation valid under assumption of weak gravitational field inside the source (weakly stressed) and slow internal motion. Domain of validity limited to near zone of the source i.e. exterior region small w.r.t wavelength of the waves. *A priori* unable to incorporate (no incoming) boundary conditions at infinity
- PM approximation for weakly self gravitating sources uniformly valid all over spacetime
- PM approximation is 'upstream' relative to the PN approximation, multipole approximation and far-zone approximation.
  Each of the latter can be implemented at the second stage after the PM approximation is implemented as the first stage.

Model of Source

- Accurate relativistic description of binary neutron stars or binary black holes requires General method for dealing with the gravitational interaction of two (non spinning) *compact* bodies: bodies whose radii are of the same order as their gravitational radii
- Following Damour, in his analysis of the Binary pulsar, one employs a *matching* approach which combines two different approximation methods
- (i) "external perturbation scheme": iterative, weak-field (post-Minkowskian) approximation scheme valid in a domain outside two world-tubes containing the two bodies
- (ii) "internal perturbation scheme" describing the small perturbations of each body by the far-field of its companion.

Problem of Self-Field Regularisation

- Consider the Newtonian Potential at $\vec{x}$ due to two point particles situated at $\vec{y_1}$ and $\vec{y_2}$

$$U(\vec{x}) = \frac{Gm_1}{r_1} + \frac{Gm_2}{r_2}$$

$$r_1 = |\vec{x} - \vec{y}_1|; \; r_2 = |\vec{x} - \vec{y}_2|;$$

- How does one compute the value of this singular function at a singular point? $U(\vec{y_1})$??? Is $U(\vec{y_1}) = \frac{Gm_2}{r_{12}}$??
- What does one mean by $U(\vec{x})\delta(\vec{x} - \vec{y_1})$? An ill-defined object in distribution theory..
- How are partial derivatives of such objects defined?

$$\partial_i \partial_j U = (\partial_i \partial_j U)_{\mathrm{Ord}} + \frac{4\pi}{3}\delta_{ij}\left[Gm_1 \delta(\vec{x} - \vec{y_1}) + 1 \leftrightarrow 2\right]$$

3PN Generation of GW

Dimensional Regularisation

- Work in $d+1$ spacetime dimensions, d considered as a continuous complex number
- Introduced by 't Hooft & Veltman (1972) to respect gauge symmetry of perturbative quantum gauge theories. Here we use it to respect gauge symmetry associated with diffeomorphism or Gen coord invariance of classical general relativistic description of interacting point masses
- $d = (3 - A)$ equivalent to Riesz upto 2.5PN
- Respects basic properties of Calculus Associativity, commutativity, Distributivity, Leibniz $\partial(FG) = \partial F.G + F\partial G$, Schwarz $\partial_\mu \partial_\nu F = \partial_\nu \partial_\mu F$, integration by parts *etc*

$$\mathrm{Eg}: \; U(\vec{x}) = \frac{m_1}{r_1^{d-2}} + \frac{m_2}{r_2^{d-2}} \implies U(\vec{y_1}) = \frac{m_2}{r_{12}^{d-2}}$$

Similarly for $U^n, U^n \partial_i U, \ldots$

Regularisation and Renormalisation

- Regularization: Give a definite meaning to divergent integrals. Needed to deal with point particles.
  In ADM coords there are no poles
  In harmonic coords there are poles and hence divergence $\frac{1}{d-3}$,
- Renormalisation: Change bare variables so that observable results are finite. Needed to deal with both point particles or extended bodies
- Hadamard regularisation $\approx$ " Throw away divergencies"
- Dimensional Regularisation $\approx$
  Much cleaner$\Rightarrow$ Unambiguous final results

Physical Interpretation

- Two ways of thinking of this renormalization
- First: Add to the usual point-particle action a counter-term describing a possible (infinitesimal) shift of the world-lines y_a^μ (*infinite dipole* term $\propto \frac{1}{d-3}$). Counter-term exactly what is needed to absorb the $1/\varepsilon$ poles and to leave a finite answer for both EOM and the bulk metric (metric outside world-lines)
- Second: Use only usual point-particle action but to consider that the "bare" world-lines y_a^μ entering the point particle action can be decomposed into a renormalised part, finite as $\varepsilon \to 0$, and a shift though being formally "small", namely of 3PN order, containing a pole part $\propto 1/\varepsilon$ which absorbs all the poles appearing in harmonic-coordinates calculations.