Singularity Resolution in Loop Quantum Cosmology: Recent Developments

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Organization of the Talk

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- Cosmology, Quantum Cosmology and Loop Quantum Cosmology

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- Other Developments

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Wheeler-De Witt Quantum Cosmology, Singularity persists.
Does Lop Quantum Cosmology provide a non-singular model?

Basics

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$$ds^2 \quad := \quad -dt^2 + a^2(t)\left\{dr^2 + r^2 d\Omega^2\right\} \quad .$$

$$S_{EH} \quad = \quad V_0 \int dt \left\{ \frac{3}{8\pi G}(-a\dot{a}^2) + \frac{1}{2}a^3\dot{\phi}^2 - V(\phi)a^3 \right\}$$

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$$p_a \;=\; -\frac{3V_0}{4\pi G}a\dot{a} \;,\; p_\phi \;=\; V_0 a^3 \dot{\phi} \;,\; V_0 \;:=\; \int_{\mathrm{cell}} d^3x \;;$$

$$\begin{aligned} H &= H_{\mathrm{grav}} + H_{\mathrm{matter}} \\ &= \left[-\frac{2\pi G}{3}\frac{p_a^2}{V_0 a}\right] + \left[\frac{1}{2}\frac{p_\phi^2}{a^3 V_0} + a^3 V_0 V(\phi)\right] \\ &= \left(\frac{3V_0 a^3}{8\pi G}\right)\left[-\frac{\dot{a}^2}{a^2} + \left(\frac{8\pi G}{3}\right)\left(\frac{H_{\mathrm{matter}}}{V_0 a^3}\right)\right] \end{aligned}$$

Connection Variables

Basic variables:

$(\tilde{p}, \tilde{c}) : \quad |\tilde{p}| := \frac{a^2}{4}, \; \tilde{c} := \gamma \dot{a}/2, \; \{\tilde{c}, \tilde{p}\} = (8\pi G\gamma)/(3V_0).$

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$$c = \pm \gamma \sqrt{\frac{\kappa}{6} \frac{|p_\phi|}{|p|}} \quad , \quad \dot{p} = \pm \sqrt{\frac{2\kappa}{3}} |p_\phi| |p|^{-1/2} .$$

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$$\frac{dp}{d\phi} = \pm\sqrt{\frac{2\kappa}{3}}|p| \quad \Rightarrow \quad p(\phi) = p_* e^{\pm\sqrt{\frac{2\kappa}{3}}(\phi-\phi_*)}$$

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$$\hat{p}\Psi(c,\phi) = -i\frac{\gamma\ell_{\rm P}^2}{3}\partial_c\Psi(c,\phi)\ ,\ \widehat{p_\phi}\Psi(c,\phi) = -i\hbar\partial_\phi\Psi(c,\phi)\ .$$

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\langle c|\mu\rangle &:= e^{i\mu c/2}\ ,\ \langle \mu'|\mu\rangle \ =\ \delta_{\mu',\mu}\ \ \forall \mu \in \mathbb{R} \\
\hat{p}|\mu\rangle &= \frac{\gamma\ell_{\mathrm{P}}^2}{6}\mu|\mu\rangle\ ,\ \widehat{e^{i\mu' c/2}}|\mu\rangle \ =\ |\mu+\mu'\rangle, \ell_{\mathrm{P}}^2 := \kappa\hbar\ .
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There is no $\hat{c}$ operator $\Rightarrow$ Must use holonomies –

$h_j(c) := e^{\mu_0 c\Lambda\cdot\tau}$ and inverse powers of $\hat{p}$ do not exist so use:

$$
|p|^{-1}\ =\ \left[\frac{3}{8\pi G\gamma l}\{c,|p|^l\}\right]^{1/(1-l)}\ ,\ \ l\in(0,1)\ .
$$

Inverse Triad Operator

$$\widehat{|p|^{-1}_{(jl)}}|\mu\rangle = \left(\frac{2j\mu_0}{6}\gamma\ell_{\mathrm{P}}^2\right)^{-1}(F_l(q))^{\frac{1}{1-l}}|\mu\rangle \quad , \quad q := \frac{\mu}{2\mu_0 j} \,,$$

$$F_l(q \gg 1) \approx \left[q^{-1}\right]^{1-l} \,, \; F_l(q \approx 0) \approx \left[\frac{3q}{l+1}\right] \,.$$

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Scales and Regimes:

$$\begin{aligned} \text{DeepQuantum} &\;\leftrightarrow\; |p| \ll p_0 := \tfrac{\gamma\ell_{\mathrm{P}}^2}{6}\mu_0, \\ \text{Semiclassical} &\;\leftrightarrow\; p_0 \lesssim |p| \lesssim 2jp_0, \\ \text{Classical} &\;\leftrightarrow\; |p| \gg 2jp_0. \end{aligned}$$

Hamiltonian Constraint

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$$H_{\mathrm{grav}}^{\mathrm{class}} = -\frac{4}{\kappa\gamma^3\mu_0^3}\sum_{ijk}\epsilon^{ijk}\,\mathrm{Tr}\left(h_i h_j h_i^{-1} h_j^{-1} h_k \{h_k^{-1}, V\}\right)$$

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$$\hat{H}_{\mathrm{grav}}^{\mathrm{non-sym}} \sim \sin^2\mu_0 c\left(\sin\frac{\mu_0 c}{2}\hat{V}\cos\frac{\mu_0 c}{2} - \cos\frac{\mu_0 c}{2}\hat{V}\sin\frac{\mu_0 c}{2}\right)$$

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Introduce:

$$\hat{V}|\mu\rangle \;=\; (\frac{1}{6}\gamma\ell_{\rm P}^2|\mu|)^{3/2}|\mu\rangle \;:=\; V_\mu|\mu\rangle$$

$$|\Psi\rangle \;:=\; \sum_\mu \psi(\mu,\phi)|\mu\rangle$$

Difference Equation

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$$A(\mu+4\mu_0)\psi(\mu+4\mu_0,\phi)-2A(\mu)\psi(\mu,\phi)+A(\mu-4\mu_0)\psi(\mu-4\mu_0,\phi)$$

$$= \; -\frac{2\kappa}{3}\mu_0^3\gamma^3\ell_{\mathrm{P}}^2 H_{matter}(\mu)\psi(\mu,\phi)\ , \;\; A(\mu) := V_{\mu+\mu_0} - V_{\mu-\mu_0}\ .$$

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$$f_+(\mu)\psi(\mu+4\mu_0,\phi) + f_0(\mu)\psi(\mu,\phi) + f_-(\mu)\psi(\mu-4\mu_0,\phi)$$

$$= -\frac{2\kappa}{3}\mu_0^3\gamma^3\ell_{\mathrm{P}}^2 H_{matter}(\mu)\psi(\mu,\phi) \quad \text{where,}$$

$$f_+(\mu) := \left|V_{\mu+3\mu_0} - V_{\mu+\mu_0}\right|\ ,$$

$$f_-(\mu) := f_+(\mu-4\mu_0)\ ,\ f_0 := -f_+(\mu) - f_-(\mu)\ .$$

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Specification in a classical regime determines the solutions –
non-singularity!

Effective Hamiltonian

(– Date-Hossain, Banerjee-Date)

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\begin{aligned}
H_{\text{eff}}^{\text{non-sym}} &= -\frac{1}{2\kappa}\left(\frac{6}{\mu_0^3\gamma^3\ell_{\mathrm{P}}^2}\right)\Big[B_+(p)\sin^2(\mu_0 c) + \\
&\qquad \left(A(p) - \frac{1}{2}B_+(p)\right)\Big] + H_{\text{matter}} \ ; \\
B_+(p) &:= A(p+4p_0) + A(p-4p_0) \ , \\
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H^{sym} is obtained by

$$B_+(p)\rightarrow f_+(p)+f_-(p)\ \ ,\ \ 2A(p)\rightarrow f_+(p)+f_-(p).$$

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$$B_+(p) \to f_+(p) + f_-(p) \ , \ 2A(p) \to f_+(p) + f_-(p).$$

These two differ only in the semiclassical regime.

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Effective density: ($p := a^2/4$)

$$\frac{3}{\kappa}\left(\frac{\dot{a}^2}{a^2}\right) \;:=\; \rho_{\mathrm{eff}} \;=\; \left(\frac{H_{\mathrm{matter}}}{p^{3/2}}\right)\left\{1 - \frac{\kappa\mu_0^2\gamma^2}{3}p\left(\frac{H_{\mathrm{matter}}}{p^{3/2}}\right)\right\}$$

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(ii) semiclassical extremum is a recollapse and classical one is a bounce. p_* varies directly with p_ϕ while ρ_* varies inversely.

back

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(– Bojowald Living Review)

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Basic variables are different; Quantization is very different; Full theory framework is well defined.

Density, curvatures are bounded, thus indicating absence of singularity; Effective dynamics is also non-singular via a bounce; Accommodates inflationary phase naturally.

Most results pre-2005, followed from the inverse volume modifications in the matter sector.

Post-2004: Steps in Physical Level Analysis

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These are the physical predictions.

Constraint equation

$$-\frac{6}{\gamma^2}c^2\sqrt{|p|}+\kappa\ p_\phi^2\ |p|^{-3/2}\ =\ 0\ =\ C_{\rm grav}+C_{\rm matter}\ ;$$

$$\kappa\hat{p}_\phi^2\Psi(p,\phi)\ =\ [\tilde{B}(p)]^{-1}\hat{C}_{\rm grav}\Psi(p,\phi)\ ,\ [\tilde{B}(p)]=\mathrm{Eigen}(\widehat{|p|^{-3/2}})$$

$$\frac{\partial^2\Psi(\mu,\phi)}{\partial\phi^2}\ =\ [B(\mu)]^{-1}\left[\kappa\left(\frac{\gamma}{6}\right)^{3/2}\ell_{\rm P}^{-1}\hat{C}_{\rm grav}\right]\Psi(\mu,\phi)$$

$$\hat{\Theta}_{\rm Sch}(\mu)\Psi(\mu,\phi)\ =\ -\frac{2\kappa}{3}|\mu|^{3/2}\partial_\mu\sqrt{\mu}\ \partial_\mu\Psi(\mu,\phi)$$

$$\hat{\Theta}_{\rm LQC}(\mu)\Psi(\mu,\phi)\ =\ -[B(\mu)]^{-1}\left\{C^+(\mu)\Psi(\mu+4\mu_0,\phi)+\right.$$
$$\left.C^0(\mu)\Psi(\mu,\phi)+C^-(\mu)\Psi(\mu-4\mu_0,\phi)\right\}.$$

General Solution

For each fixed ϕ, on the space of functions $\psi(\mu, \phi)$, $\hat{\Theta}$ is a positive, self-adjoint operator with the kinematical measure scaled by $B(\mu)$. $\Rightarrow$

$$\hat{\Theta} e_k(\mu) = \omega^2(k) e_k(\mu), \ \ k \in \mathbb{R}, \ \ \langle e_k | e_{k'} \rangle = \delta(k, k'). \ \Rightarrow$$

$$\begin{aligned} \Psi(\mu, \phi) &= \int dk \ \tilde{\Psi}_+(k) e_k(\mu) e^{i\omega(k)\phi} + \tilde{\Psi}_-(k) \bar{e}_k(\mu) e^{-i\omega(k)\phi}; \\ &:= \Psi_+(\mu, \phi) + \Psi_-(\mu, \phi) \ . \end{aligned}$$

The spectrum (label k) and $\omega(k)$ differ for different quantizations.

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\begin{aligned}
\hat{p}_\phi \Psi(\mu, \phi) &:= -i\hbar\partial_\phi \Psi(\mu, \phi) \ , \\
\widehat{|\mu|_{\phi_0}} \Psi(\mu, \phi) &:= e^{i\sqrt{\hat{\Theta}}(\phi-\phi_0)} |\mu| \Psi_+(\mu, \phi_0) + \\
&\quad e^{-i\sqrt{\hat{\Theta}}(\phi-\phi_0)} |\mu| \Psi_-(\mu, \phi_0)
\end{aligned}
$$

On an initial datum, $\Psi(\mu, \phi_0)$:

$$
\widehat{|\mu|_{\phi_0}} \Psi(\mu, \phi_0) = |\mu| \Psi(\mu, \phi_0) \ , \ \hat{p}_\phi \Psi(\mu, \phi_0) = \hbar\sqrt{\hat{\Theta}} \Psi(\mu, \phi_0) \ .
$$

Physical Inner Product

These operators are self-adjoint on the space of solutions provided,

$$\langle\Psi|\Psi'\rangle_{\rm phys} \;:=\; \text{“}\int_{\phi=\phi_0} d\mu B(\mu)\text{”} \;\; \bar{\Psi}(\mu,\phi)\Psi'(\mu,\phi)\ .$$

For Schrodinger quantization, the integral is really an integral while for LQC it is actually a sum over μ taking values in a lattice.

Semiclassical states

Semiclassical states are physical states in which a chosen set of observables have specified expectation values with specified tolerances.

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For example, in Schrodinger quantization, a state peaked at $p_\phi = p_\phi^*$ and $|\mu|_{\phi_0} = \mu^*$ is given by,

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\begin{aligned}
\Psi_{\rm semi}(\mu, \phi_0) \; &:= \; \int dk e^{-\frac{(k-k^*)^2}{2\sigma^2}} e_k(\mu) e^{i\omega(\phi_0 - \phi^*)} \\
k^* \; &:= \; -\sqrt{3/2\kappa}\hbar^{-1} p_\phi^* \; , \\
\phi^* \; &:= \; \phi_0 + -\sqrt{3/2\kappa}\ell n|\mu^*| \; .
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Evolve with $e^{i\sqrt{\hat{\Theta}}(\phi-\phi_0)}$.

Evolution and Results

For a semiclassical solution $\Psi_{\rm semi}(p_\phi^*, \mu^* : \phi)$, one obtains a curve in the (μ, ϕ) plane computed as:

$$|\mu|_{p_\phi^*, \mu^*}(\phi) := \langle \widehat{|\mu|_{\phi_0}} \rangle (\phi).$$

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Schrodinger quantization: curves passes through zero volume;

Loop quantization: curves bounce away from zero volume; are well approximated by the effective dynamics and Bounce persists even if $B_{\rm LQC}(\mu) \to B_{\rm Sch}(\mu)$. Density at bounce varies inversely with p_ϕ^* – undesirable. – Ashtekar-Pawlowski-Singh.

Hints for improved quantization

From the expression for effective density, one can see that the bounce occurs for $|p_*| = \sqrt{\frac{\kappa\mu_0^2\gamma^2}{6}}|p_\phi|$ and

$$\rho_* := \rho_{cl}(p_*) = (\tfrac{\kappa\mu_0^2\gamma^2}{3}p_*)^{-1} = \sqrt{2}(\frac{\kappa\mu_0^2\gamma^2}{3})^{-3/2}|p_\phi|^{-1}.$$

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If $\mu_0 \to \bar{\mu}(p) := \sqrt{\Delta/|p|}$, Δ a constant, then

$$\rho_{\mathrm{eff}} = 0 \Rightarrow \rho_{cl} = \rho_{\mathrm{crit}} := \left(\frac{\kappa\Delta\gamma^2}{3}\right)^{-1} \ , \ |p_*| = \left(\frac{p_\phi^2}{2\rho_{\mathrm{crit}}}\right)^{1/3}.$$

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Improved quantization proposes this replacement in all holonomies. This is actually viable and the description simplifies if one uses eigenbasis of $\hat{V} = |p|^{3/2}$.

Improved quantization Expressions

Ambiguity parameters $j = 1/2, l = 3/4$ are chosen.

$$
\begin{aligned}
v &:= K\mathrm{sgn}(\mu)|\mu|^{3/2} \quad , \quad K := \frac{2\sqrt{2}}{3\sqrt{3\sqrt{3}}} \ ; \\
\hat{V}|v\rangle &= \left(\frac{\gamma}{6}\right)^{3/2} \frac{\ell_{\mathrm{P}}^3}{K}|v||v\rangle \ , \\
\widehat{e^{ik\frac{\bar{\mu}}{2}c}}\Psi(v) &:= \Psi(v+k) \ , \\
\widehat{|p|^{-\frac{1}{2}}}\Psi(v) &= \frac{3}{2}\left(\frac{\gamma\ell_{\mathrm{P}}^2}{6}\right)^{-1/2} K^{1/3}|v|^{1/3} \\
&\quad \left||v+1|^{1/3} - |v-1|^{1/3}\right| \Psi(v) \\
B(v) &= \left(\frac{3}{2}\right)^{3/2} K|v| \left||v+1|^{1/3} - |v-1|^{1/3}\right|^3
\end{aligned}
$$

Improved Quantization: Cont...

$$
\begin{aligned}
\hat{\Theta}_{\mathrm{imp}}\Psi(v,\phi) &= -[B(v)]^{-1}\left\{C^{+}(v)\Psi(v+4,\phi)+\right. \\
&\quad \left. C^{0}(v)\Psi(v,\phi)+C^{-}(v)\Psi(v-4,\phi)\right\} \ , \\
C^{+}(v) &:= \frac{3\pi KG}{8}|v+2|\,\big|\,|v+1|-|v+3|\,\big| \ , \\
C^{-}(v) &:= C^{+}(v-4)\ ,\ C^{0}(v) \ := \ -C^{+}(v)-C^{-}(v).
\end{aligned}
$$

Bounce Results

Detailed analysis establishes singularity resolution via a bounce – Ashtekar-Pawlowski-Singh.

The close model with free, massless scalar leads to a cyclic universe – Ashtekar-Pawlowski-Singh.

A general, exactly solvable effective model has been given which can serve as a perturbative basis for analysing bounce scenarios. Its bouncing solutions "explain" the bounces seen numerically and the model also allows several quantum effects which can be incorporated perturbatively – Bojowald.

Other Developments: Effective Models

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Despite energy condition violations, stability of matter as well as causal propagation of perturbations holds – Hossain;

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Computation of density perturbations and their power spectra has been explored – Hossain, Calcagni-Cortes;

A systematic approach to constructing effective theories has been initiated – Bojowald-Skirzewski.

Other Developments: Anisotropic models

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Anisotropic models also show non-singularity at the difference equation level. At the effective level, for vacuum, Bianchi I, there are no inverse triad corrections. But the $\sin^2$ modifications suffice to give non-singular Kasner – Date;

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Bianchi I with free massless scalar has been analyzed in the improved quantization – Chiou.

Bianchi IX models are also being analysed with "improved quantization".

Other Developments: Inhomogeneities

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Inhomogeneities is a fact of nature although these are small in the early universe. These can thus be treated perturbatively. Work on "cosmological perturbation theory" in connection variables has already begun.

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One can also try to understand how starting from an inhomogeneous model one can obtain a homogeneous one as a good approximation. In particular qualitative implications of the parent model for the homogeneous approximation has been explored in a simplified lattice model and an alternative argument for the $\bar{\mu}(p)$ improvement has been advanced along with removing the V_0 dependence of the homogeneous models – Bojowald.

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$\boxed{\text{Thank You.}}$