Einstein’s Dream

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Past

General Relativity

4-diminsional Spacetime

General Covariance

Equivalence Principle

Metric Ansatz

- Gravity ⇒ Matter

$$\eta_{\mu\nu} \quad \Rightarrow \quad g_{\mu\nu}$$

$$\partial_\mu \quad \Rightarrow \quad \nabla_\mu = \partial_\mu + \Gamma_\mu$$

- Matter ⇒ Gravity

Einstein equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu}$

Dawn Era　１９０５－１９４０

1933 **Dark Matter in the Coma Cluster**

1934 Tolman Sol.(P)

1935 Robertson-Walker Metric

Relativistic Cosmological Models

1905 Special Relativity

1915 **General Relativity**

1921 Kaluza-Klein Theory

1926 Quantum Mechanics

1929 Quantum Field Theory

1929 – 1955

Einstein's Unified Theory

Singularity Problem

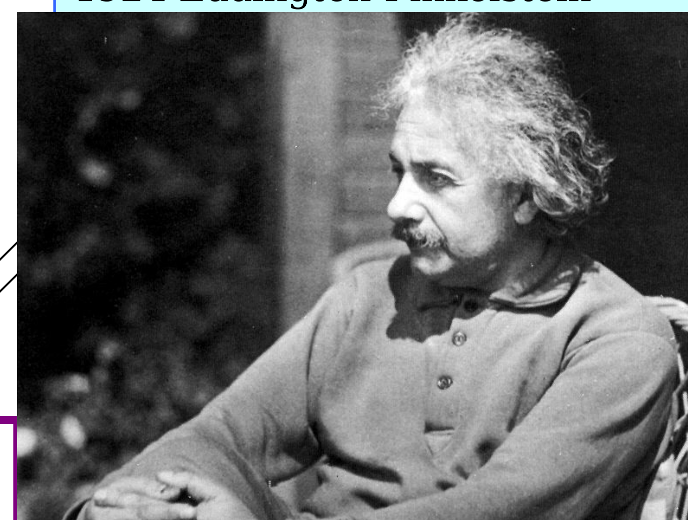
Spherically symmetric vacuum exact solutions

1916 Schwarzschild Sol.

1917 Weyl Formulation

1923 Birkhoff's Theorem

1924 Eddington-Finkelstein



Concepts

Developments in Cosmology

1965 Discovery of CMB

1992 COBE Obs. CMB BB spectrum T anisotropy

2003 WMAP (1st yr)

2005 Boomerang03

Big-Bang model

1946 Gamov theory
Linear perturbation theory
1948 Stationary universe model
1970's Thermal history of

1965-70 **Singularity Theorems**

Quantum Cosmology

1967 WD equation
1968-70 Bianchi cosmology

Precise Cosmology

1980's N-body simulation for LSS
1980 Gauge-invariant perturbation theory

Particle Cosmology

1978 Baryogenesis
1981 **Inflationary Universe Model**
1982 New Inflation Model
Cosmic structure

IC of the Universe

1982 Creation of Universe from Nothing
1983 No boundary proposal for the wave function of the universe
1988 Baby universe model

1940's Developments in **Nuclear Physics**
1960 ADM formalism
1964 Dirac Qz of Gravity
1967 Weinberg-Salam Theory
1973 QCD
1974 **GUT**

Developments in Black Hole Physics

Exact Solutions

1962 Newman-Penrose formalism

1963 Kerr solution

1965 Kerr-Newman solution

1968 Ernst formalism

1965-70 **Singularity Theorems**

Perturbation Analysis

1957 Regge-Wheeler equation

1962 Landau-Lifshitz formula

1970 Zerilli equation

1972 Teukolsky equation

Cosmic Censorship

1969 SCCH

1979 WCCH

Black Hole Uniqueness

1967 Static BH uniqueness

1972 Stationary BH uniqueness

1982 BH Uniqueness (Charged rotating)

PET

1979 Schoen & Yau

1983 Witten, Nester

1973-78 CygX1 as a BH candidate

GW Astrophysics

BH Astrophysics

Numerical GR

Cosmic Jets

GW detection experiment

BH Thermodynamics

1973 BH area increase theorem

BH entropy

1974 BH evaporation

Microicroscopic origin of BH entropy

Present

Recent Developments

1992 Detection of CMB anisotropy by COBE

1994 GR test by the binary pulsar PSR 1913+16

1998 Accelerating expansion of the present universe (IIa SNe)

1999 BOOMERANG

2003 WMAP (1st yr)

2005 Boomerang03

2006 WMAP (3 yrs)

Confirmation of Inflation

Dark Energy

Higher-Dimensional BH

1993 Topological Censorship Theorem

Gregory-Laflamme Instability

1996 BH Entropy from Quantum Gravity （SST/D, LG)

D

ility

of HD Static BH

2006 5D Black Sat

Higher-Dimensional Cosmology

1999 RS Braneworld Model

2003 KKLT Construction

KKLMMT Model

1995 Duality, Unification of SSTs, M-theory

1996 D-brane, F-theory

Horava-Witten Theory

1997 AdS/CFT

1998 TeV Gravity

2000 Landscape Problem

2002 Moduli Stabilisation by Flux Compactification

Problems To Be Resolved

- Direct experimental tests
  - Tests in the microscopic (< 1mm) and intermediate ranges (～1km, ～1kpc)
- Astrophysics
  - Active galactic/extragalactic objects (BH physics): ***Jet formation***
  - Gravitational collapse (Numerical relativity)
  - Gravitational waves: To be detected soon.
  - Gravitational lenses
- Cosmology
  - Evolutionary model of universe: Specification of the inflationary model
  - Observational cosmology: ***Dark matter and dark energy***
  - The initial condition of the universe and the early evolution
  - Cosmology of exotic relics
- Mathematical physics
  - Exact solutions
  - Classification of solutions (including black holes)
  - Large scale structure of spacetimes
  - Cosmic censorship and spacetime singularity
- Fundamental problems
  - **Modification/ extension of general relativity, QG**

Future

Dark Energy Problem

Why is the expansion of the present universe accelerated at the timescale of order 1/H ?

If we assume that general relativity holds;

- Why is the effective cosmological constant at low energy, *including the quantum contribution*, close to zero with the accuracy $G(10^{-2}\ \mathrm{eV})^4$?
- Why is the effective cosmological constant at present positive and of order $G(10^{-2}\ \mathrm{eV})^4$?

How to Solve the DE Problem

- Modify general relativity in the IR region preserving the four-dimensionality of spactime
  - Modified Gravity (MOND,...)

  Cf. Asymptotic safety (S. Weinberg)

- Modify general relativity in the UV region to make quantum corrections fully under control.
  - Higher-Dimensional Unified Theory (HUNT)

    — Realisation of Einstein’s Dream —

Toward HUNT: GUT

Standard model $\Rightarrow$ (SO(10)) GUT: gauge-sector unification

- hypercharge structure
- gauge-coupling unification
- neutrino mass
- baryon asymmetry
- strong CP(Peccei-Quinn symmetry)

Hypercharge Structure

Standard model $U_Y(1) \times SU(2) \times SU(3)$

$$\begin{array}{cccccc} & \begin{pmatrix} \nu \\ e \end{pmatrix}_L & e_R & \begin{pmatrix} u \\ d \end{pmatrix}_L & u_R & d_R \\ Y = & -1 & -2 & 1/3 & 4/3 & -2/3 \end{array}$$

GUT

$$SU(5) \supset U(1) \times SU(2) \times SU(3)$$

$$5^* = (1, 3^*)_{2/3} + (2^*, 1)_{-1} : \quad (d^c, e^-, -\nu)_L$$

$$10 = (1, 3^*)_{-4/3} + (2, 3)_{1/3} + (1, 1)_2 : \quad \begin{pmatrix} [u^c] & -u & -d \\ u & 0 & -e^+ \\ d & e^+ & 0 \end{pmatrix}_L$$

$$SO(10) \supset SU(5)$$

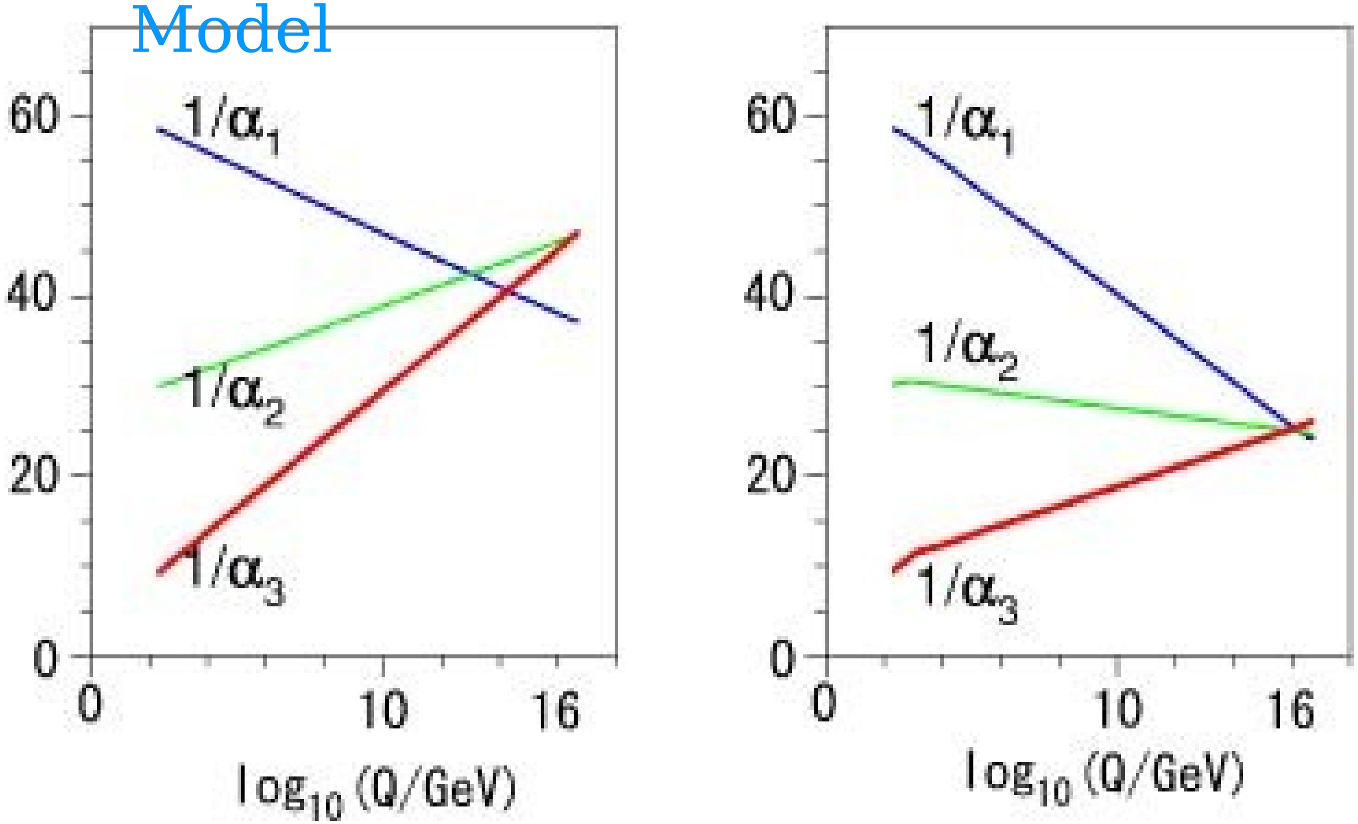
$$16 = 5^* + 10 + 1 \Rightarrow \quad \text{neutrino mass}$$

Ref: Wilczek, F: in Physics in the 21st Century, eds. K.Kikkawa et al.(1997, World Scientific)

Coupling Constant Unification

Standard Model

MSSM



De Boer, W & Sander, C: PLB585, 276(2004)[hep-ph/0307049]

Toward HUNT: SGUT

GUT $\Rightarrow$ SGUT: boson-fermion correspondence

- Dark matter
- Λ problem
- hierarchy problem
- gauge-coupling unification

Toward HUNT: HD Sugra GUT

SGUT $\Rightarrow$ Sugra GUT: inclusion of gravity

- Flat inflaton potential

Sugra GUT $\Rightarrow$ HD Sugra GUT: matter sector unification

- Repetition of families
  - Cabibbo/neutrino mixing
  - CP violation
- Incomplete (split) multiplets

Family Repetition

Standard Model

$$\boxed{\begin{matrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & e_R \\ \begin{pmatrix} u \\ d \end{pmatrix}_L & u_R \; d_R \end{matrix}} \qquad \boxed{\begin{matrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & \mu_R \\ \begin{pmatrix} c \\ s \end{pmatrix}_L & c_R \; s_R \end{matrix}} \qquad \boxed{\begin{matrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & \tau_R \\ \begin{pmatrix} t \\ b \end{pmatrix}_L & t_R \; b_R \end{matrix}}$$

Higher-dimensional model

$$M = X_4 \times Y_6$$

$$\Psi = \sum_a \psi_a(x)\chi_a(y); \quad \gamma^p D_p \chi_a = m_a \chi_a$$

$$A_\mu = A_\mu(x) \in H, \quad A_p = A_p(y) \in K; \quad H \times K \subset G$$

$$\Rightarrow \quad \int_Y d\mu(y) i\bar{\Psi}\Gamma^M D_M \Psi = \sum_a \bar{\psi}_a i(\gamma^\mu D_\mu - m_a)\psi_a$$

The number of zero modes for χ ⇒ # of family

This number is often determined by topology.

Triplet-Doublet Splitting

Baryon number violation by Higgs

$$5^* \times 10 = 5 + 45 \Rightarrow \quad \bar{\Phi}_5 \psi_{5^*} \psi_{10}$$

$$\Phi_5 = (\Phi_2, \Phi_3)$$

Orbifold GUT (Kawamura model [2000])

$$M = X_4 \times S^1/\mathbb{Z}_2 \ni (x, y)$$

$$\Phi_5(x, -y) = P\Phi_5(x, y); \quad P = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

$$\Phi_2 = \phi_2(x) + \phi_2^1(x) \cos(\pi y/L) + \cdots$$

$$\Phi_3 = 0 + \phi_3^1(x) \sin(\pi y/L) + \cdots$$

Ref: Nilles, H.P.: hep-th/0410160

Toward HUT: Superstring/M-theory

HD Sugra GUT ⇒ Superstring/M theory

- Consistency as a quantum theory, finite control parameters
- No Λ freedom (M-theory)
- Quantum corrections produce potentials for moduli fields and do not contribute to cosmological constant in four dimensions.

For example,

$$\langle S\rangle_{\mathrm{vac}} = \frac{1}{2\kappa_D^2}\int d^D x(-\tilde{g})^{1/2}\left(\tilde{R} - 2\Lambda_0 - \sum_j e^{-\alpha_j\phi}\langle|F_j|^2\rangle\right)$$

$$\Rightarrow \frac{1}{2\kappa^2}\int d^4x(-g)^{1/2}\left(R - \Lambda_0 e^{-\beta\psi} - \sum_j e^{-\alpha_j\phi-\beta_j\psi}\langle|F_j|^2\rangle\right)$$

Hence, the moduli stabilisation mechanism and the dark energy problem is tightly connected.

Difficulties of HUNT

Choice of theory

- M-theory or 10D superstring theories
- Ambiguity due to duality and branes

Compactification

- What determines the type of compactification?
- Moduli stabilisation
- No-Go theorem against accelerating expansion
- Identification of our four-dimensional universe

SUSY breaking

- Mechanism
- Control

No-Go Theorem

For any (warped) compactification with a compact closed internal space, if the strong energy condition holds in the full theory and all moduli are stabilized, no stationary accelerating expansion of the four-dimensional spacetim

Proof

For the geometry $ds^2 = h^{-1/2}(y) ds^2(X_4) + ds^2(Y_n)$,

from the relation $R_{VV}(X) = R_{VV} - \frac{1}{4} h \triangle_Y h^{-1}$

for any time-like unit vector V on X, we obtain

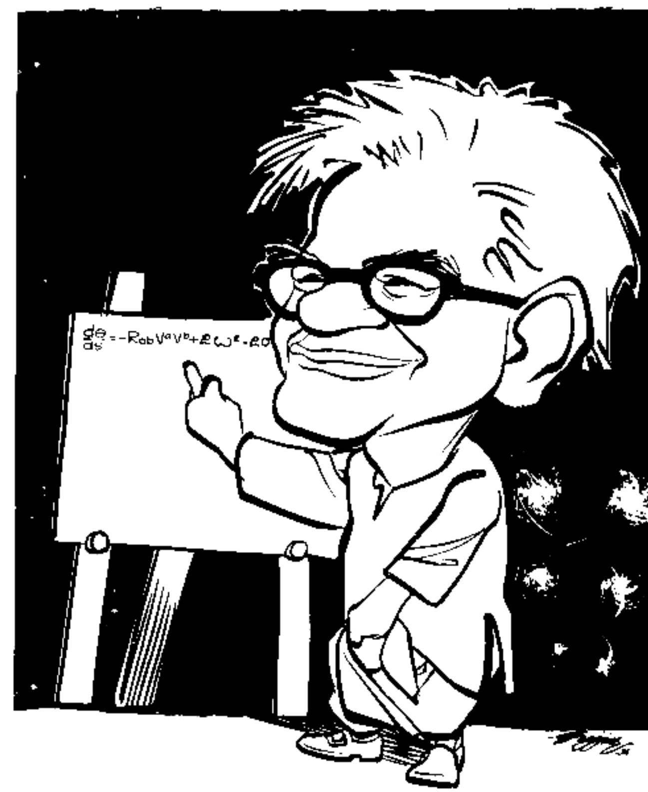
$$R_{VV}(X) \int_Y d\Omega(Y) h^{-1} = \int_Y d\Omega(Y) \left[h^{-1} R_{VV} - \frac{1}{4} \triangle_Y h^{-1} \right]$$

Hence, if Y is a compact manifold without boundary, h^{-1} is a smooth function on Y, and the strong energy condition $R_{VV} \geq 0$ is satisfied in the (n+4)-dimensional theory, then the strong energy condition $R_{VV}(X) \geq 0$ is satisfied on X.

$$\frac{3\ddot{a}}{a} \equiv \dot{\theta} + \frac{1}{3}\theta^2 = -R_{VV}(X) - 2\sigma^2 \leq 0$$

Hence, from the celebrated Raychaudhuri equation

the accelerated expansion of the universe cannot occur.



A. K. Raychaudhuri

The Cosmic Converger

Summary

Summary

Predictions preceded observations in the last century:

General relativity has played a leading role in cosmology and astrophysics in these 90 years and produced new predictions, often by interplays with developments in microscopic physics. Many of them, although regarded as exotic at first, have been successfully confirmed by observations.

Now, general relativity is facing a new situation:

We are experiencing a new situation in which observations precede theoretical predictions. In particular, the Dark Energy/Cosmological Constant problem is clearly beyond the scope of general relativity.

Resolution of this problem will certainly lead to a drastic change in our understanding of Nature.

I believe that HUNT will play a crucial role in the resolution, but only young people here will be able to know the real answer.