LOOP QUANTUM GRAVITY

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PLAN :

1. Introductory Remarks
2. Canonical Quantzn of Gravity
3. L Q G framework
4. Quantum dynamics & Interpretation of states
5. Summary and Open Directns

- QG is the effort to find a quantum mechanical description of the gravitational interaction

- QUANTUM GRAVITY

  $\hbar$ (QUANTUM) ; G, c (GRAVITY)

  $l_p = \sqrt{\frac{G}{c^3}\hbar} = 10^{-33}$ cm

  $E_p = \sqrt{\frac{c^5}{G}\hbar} = 10^{19}$ Gev

  Accelerators $\sim 10^3$ Gev

- Why Look for theory of QG?

  Theoretical Consistency
  - $G_{ab} \overset{?}{=} \hat{T}_{ab}$ (classical ; $\sim$ QM)
  - GR is incomplete (singularities)
  - QM is unsatisfactry

  Exptal signatures?
  - Early univ.
  - Local L.I. violation?

→ Just as GR, QM; expect QG gives new conceptual paradigm to view world

Approaches to QG

HEP

↓

Non renormble pert QFT

↓

⇒ gravity = effective field theory

↓

⇒ New degrees of freedom @ high enrgy

↓

STRING THEORY

Relativists

↓

Wrong to set $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

↓

- CAUSAL SETS (Sumati's talk)
- CANONICAL QG

. . . .

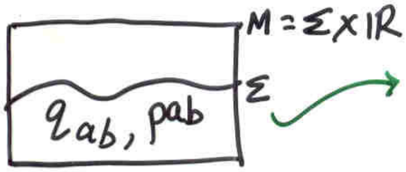
CAUTION : Is SUGRA perturbatively finite ?

2. CANONICAL QG :

(prejudice : QM of gravity w.o. matter can yield robust predctns coming from microstr. of sptime. Add matter later)

- $\{ , \} \longrightarrow \frac{1}{i\hbar}[,]$

- Hamiltonian description of gravity



$\longrightarrow [\hat{q}, \hat{p}] \sim i\hbar\,\delta$

Constraints : $C_a(q,p) = C(q,p) = 0$

(C_a: sp'tl covarnce; C: time reparmtrzn)

⇒ In Quantum theory, physical gge inv states satisfy

$$C_a(\hat{q},\hat{p})|\psi\rangle = C(\hat{q},\hat{p})|\psi\rangle = 0$$

*Wheeler-deWitt eqn.

seems not feasible with $\hat{q}, \hat{p}$

- Interpretation of states thru gge inv observables.

3. LQG : Basic framework:

- QM of formulation of GR in which basic ph. space varbles = (triad, connectn)

- Feautures :- Conservative, Non-perturbative
  - No backgrd sptime on which QM unfolds "QM of sptime"

- Variables: (metric, extr curv)

triad $\tilde{E}^a_i$ ← metric; extr curv → A_a^i

$\sum_{i=1}^{3} \tilde{E}^a_i \tilde{E}^b_i = \tilde{\tilde{q}}^{ab}$

↓ SU(2) YM electricfield

A_a^i ↓ spatial connectn

- Constraints: $C_a(E,A) = C(E,A) = 0$

  New constraint: $\mathcal{D}_a \tilde{E}^a_i = 0 = G_i(E,A)$

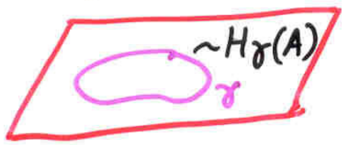
So: Find. repn of $\hat{E}, \hat{A}$

- Quantum constraints $C_a(\hat{E},\hat{A})$, $C(\hat{E},\hat{A})$, $G(\hat{E},\hat{A})$
- physical states:
- Interpretation thru observables which are ggc inv.

Representation of $\hat{E}, \hat{A}$:

- $\hat{E}^a_i$ rotates by SU(2) gge transf.

Choose fn of A which also 'rotates' under SU(2) transf ⇒ Use "WILSON LOOP" or "holonmy fn"



Hγ(A) depends only on values of A along 'γ'

Hγ(A) for all 'γ' allows reconstruction of gge inv info in A.

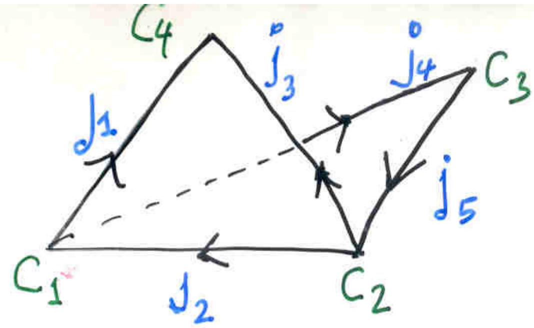
→ Represent $\hat{H}_\gamma$, $\hat{E}$ in quantum theory

- Use "Connection" repn : $\Psi = \Psi(A)$

$$H_\gamma \longrightarrow \hat{H}_\gamma \Psi = H_\gamma(A)\Psi(A)$$
$$E \longrightarrow \hat{E}(x)\Psi = \frac{\hbar}{i}\frac{\delta}{\delta A(x)}\Psi(A)$$

- Not unreasonable to expect that states also labelled by loops

Turns out that o'normal basis is labelled by CLOSED (ORIENTED) GRAPHS.



$\rightarrow \Psi_{\gamma, \{j\}, \{C\}}(A)$

j: ang mom quant# #s ($\in \frac{1}{2}$ integers) $\rightarrow$ EDGES

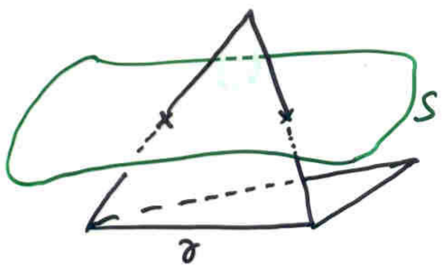
C: Tensors $\rightarrow$ vertices

SPIN NETWRK STATE $\quad \gamma$: Graph

2 states Ognal unless $\gamma, \{j\}$ coincide. Then scalar product $\langle \Psi_{\gamma\{j\}C} | \Psi_{\gamma\{j\}C'} \rangle$ only depends on C, C'.

• Interesting oprtrs:

$\hat{E} \rightarrow \sqrt{\hat{q}} = \sqrt{q(\hat{E})} \rightarrow \widehat{Area}(S) = \int_S \sqrt{\hat{q}}\, d^2x$



: $\widehat{Area}(S)\, \Psi_{\gamma\{j\}\{C\}}$

$\sim \sum_{\substack{\text{intersectns} \\ S \cap \gamma}} \sqrt{j(j+1)}\, l_P^2\, \Psi_{\gamma\{j\}\{C\}}$

Similarly $\widehat{Volume}(R)$ counts vertices

• Intuitively: "Discrete microstr. under-lying smooth sptl geometry". Smooth spatl geometry @ large distances $\gg l_P$ $\equiv$ "<u>weave</u>" CAUTION: Not based on gge inv. objects.

· Quantum Constraints (Diffeo, SU(2))

- SU(2) Gauss Law: Imposes SU(2) gge inv.
  → restricts choice of 'C' in $\Psi_{\gamma,\{j\}\{c\}}$

- Sptl Diffeos:

$\Psi_{\gamma,\{j\},\{c\}}$ $\xrightarrow{\text{unitary!}}$ $\Psi_{\gamma'\{j\}\{c\}}$

Diff inv states ≃ "generalized knot classes"

4. QUANTUM DYNAMICS, Interpretation:

- Hamiltonian Constraint: $C(E,A) = 0$

Problem: $C(E,A)$ = non polynomial in E (polynomial in A)

→ How to write $C(\hat{E},\hat{A})$?

Thiemann's soln :

- First Step: Re-express $C(E,A)$ as polynomial in E, A & $\{A, \text{Volume}\}$

  ↳ $\hat{C}$ = polynomial in $\hat{E}, \hat{A}, [\hat{A}, \widehat{\text{Volume}}]/i\hbar$

- Second Step: Get $\hat{A}$ from $\widehat{H_\gamma}(A)$ via shrinking of loops, edges.

  ⇒ Get sequence of oprtrs "$\hat{C}_\varepsilon$", can show limit exists on diff inv. states!

However: (Infinitely?) many choices for shrinking procedure ⇒ Have to justify, physically, one unique choice, OR

choose some other strategy.

- Gge inv observables: Must commute with constraints. E.g. In E&M, gge inv observable

$O(E,A) \equiv \{O, \nabla\cdot\vec{E}\} = 0$

BUT: In any gen cov. theory, constrts generate evolution as well as gge!!

$\dot{O} = \{O, \int NC + N^a C_a\}$

⇒ OBSERVBLES = constants of motn of Einst. Eqn.

Not available!

What to do?

→ Must develop novel, physically inspired approx methods which take into account what we know about "1-dimnal, loopy" nature of physical states and make some robust physical predictions without going thru "royal route".

(Note that in classical theory, we interpret after gge fixing......

Can discuss this later...)

Summary, Other Results, Open dirns:

I Summary:

- Repn. of $\hat{H}_\gamma$, $\hat{E}^a_i(x)$ on $\Psi(\bar{A}) \in \mathcal{L}^2(\bar{A}, d\mu(\bar{A}))$ — diff inv, SU(2) inv.
- Uniqueness thm (LOST-Fleischack)
- Spin network basis, $\widehat{\text{Area}}$, $\widehat{\text{Volume}}$
- SU(2) gge inv ✓
- Diffeo inv Hilbert space by grp avging
- WdW eqn. partly understood.

II Other Results:

- Surface = BH horizon, detailed e-fns of $\widehat{\text{Area}}$(horizon) beautifully dovetail with C.S. states on horizon to give entropy proportional to Area → Amit's talk
- LQC: mimic LQG constructions for homogeneous grav fields → Quantum resoln of singularities → Shyam's Talk

III Open Directions:

- GRAND PROBLEMS: < quantum dynamics / interpret. of states
  - spectrum of $\widehat{\text{volume}}$
  - Constraint algebra $[\hat{C}(x), \hat{C}(y)]$
  - study model systems → may suggest new ways of thinking about $\hat{C}$.
    (LQC, CGHS —LADDHA, cyl waves, Gowdy, 2+1 gravity)
  - Develop approxmtn methods to describe flat spacetime + gravitons ......
- LQC & early universe physics
- Hawking Radtn.
- LQG for asymp flat / cosmlgl const.

→ SPIN FOAM models ←.

Review CQG 20 R43 (2003)
Alejandro Perez
gr-qc/0301113

stuff in this talk:

- Backgrd Indep QG: A status Report
  CQG 21 R53, (2004)
  A. Ashtekar, J. Lewandowski
  gr-qc/0404018
- LQG: Living Rev Rel 1:1 '98
  C. Rovelli, grqc/9710008