ENTROPY OF SPACETIME AND GRAVITY

T. Padmanabhan

(IUCAA, Pune, India)

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PLAN OF THE TALK

- GRAVITY AND THERMODYNAMICS: HINTS OF A DEEPER CONNECTION
- THE SAKHAROV PARADIGM
- GRAVITY AS AN EMERGENT PHENOMENON
- ENTROPY OF NULL SURFACES AND SPACETIME DYNAMICS
- HORIZON ENTROPY
- PERSPECTIVE AND CONCLUSIONS

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Question:

- Is there a deeper connection between these results and spacetime dynamics [*"Matter tells spacetime how to curve"*]? What is this telling us?

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- Source:

$$T_t^t = T_r^r = \frac{\epsilon(r)}{8\pi}; \quad T_\theta^\theta = T_\phi^\phi = \frac{\mu(r)}{8\pi}$$

- Einstein's equations:

$$\frac{1}{r^2}(1 - f) - \frac{f'}{r} = \epsilon; \quad \nabla^2 f = -2\mu$$

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- Temperature of horizon: $k_B T = \hbar c B/4\pi$.

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- Works for time-dependent horizons, Kerr, Lanczos-Lovelock theory ......; close to Membrane Paradigm in spirit

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Mechanics; Elasticity ($\rho, \mathbf{v}$ ...)	Einstein's Theory (g_{ab} ...)
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- Note: There is no 'quantum thermodynamics'; for example, $TdS = dE + PdV$ is valid with quantum corrections as well.

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- *The form of entropy functional is phenomenological.* Only a microscopic theory can predict it.
- Our state of knowledge of quantum gravity is worse that that of 18th century physicists about microstructure of solids! So we should not hesitate to be phenomenological.

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- We expect P^{abcd} to have a (RG-like) derivative expansion in powers of number of derivatives of the metric:

$$P^{abcd}(g_{ij}, R_{ijkl}) = c_1 \overset{(1)}{P}{}^{abcd}(g_{ij}) + c_2 \overset{(2)}{P}{}^{abcd}(g_{ij}, R_{ijkl}) + \cdots \, ,$$

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- The m-th order term is unique:

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- Explicit form:

$$S_1[\xi] = \int_{\mathcal{V}} \frac{d^Dx}{8\pi}\left((\nabla_c\xi^c)^2 - \nabla_a\xi^b\nabla_b\xi^a\right)$$

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- Closely related to the Lanczos-Lovelock Lagrangian:

$$P_a{}^{ijk} = \frac{\partial\mathcal{L}_m}{\partial R^a{}_{ijk}}\,.$$

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$$\sqrt{-g} L = 2\partial_c \left[\sqrt{-g} Q_a^{\ bcd} \Gamma^a_{bd} \right] + 2\sqrt{-g} Q_a^{\ bcd} \Gamma^a_{dk} \Gamma^k_{bc} \equiv L_{\rm sur} + L_{\rm bulk}$$

  and is 'holographic':

$$[(D/2) - m] L_{sur} = -\partial_i \left[g_{ab} \frac{\delta L_{bulk}}{\delta(\partial_i g_{ab})} + \partial_j g_{ab} \frac{\partial L_{bulk}}{\partial(\partial_i \partial_j g_{ab})} \right]$$

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$$[(D/2) - m] L_{sur} = -\partial_i \left[g_{ab} \frac{\delta L_{bulk}}{\delta(\partial_i g_{ab})} + \partial_j g_{ab} \frac{\partial L_{bulk}}{\partial(\partial_i \partial_j g_{ab})} \right]$$

- The surface term is closely related to horizon entropy in Lanczos-Lovelock theory.

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- In a derivative coupling expansion, Lanczos-Lovelock terms are corrections.

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- This will also make gravity immune to bulk vacuum energy density.

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- Connects with the radial displacements of horizons and $TdS = dE + PdV$ as the key to obtaining a thermodynamic interpretation of gravitational theories.
- The deep connection between gravity and thermodynamics *goes well beyond Einstein's theory*. Closely related to the holographic structure of Lanczos-Lovelock theories.

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- Analogy: horizons in euclidean spacetime $\leftrightarrow$ defects in solids. Clue for a new path integral prescription ?

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