New Perspectives on Gravity
From Gauge/String Duality

RAJESH GOPAKUMAR

Harish-Chandra Research Institute

Allahabad, India

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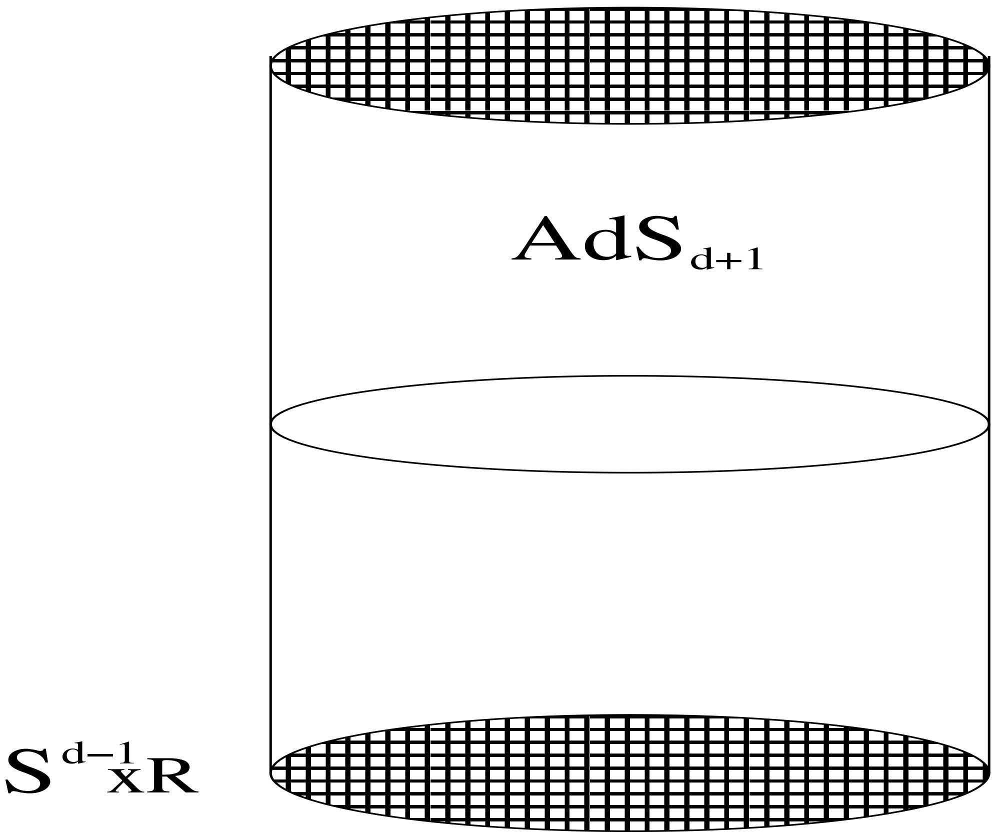
Outline of the talk

- Gauge-String Duality
  - Why it has played a central role in recent developments.
- Going Beyond the Bekenstein Entropy
  - How the microscopic counting matches with higher derivative corrections.
- The Fuzzball Proposal
  - What do the Black Hole microstates actually look like?
- The Hydrodynamics of Black Holes
  - Can we go beyond Thermodynamics?

1 Gauge-String Duality – A Lightning Survey

Gauge-String Duality

✦ Gauge-String duality plays a central role in String Theory's attempt to address non-perturbative issues of Quantum Gravity.



Gives a *definition* of quantum gravity on asymptotically Anti-deSitter (AdS) space-times.

Gauge-String Duality *continued*

SALIENT FEATURES

- Definition is in terms of a d dimensional Quantum Field Theory living on the $S^{d-1} \times R$ boundary of AdS_{d+1} – a realisation of Holography.

- Gauge theory degrees of freedom *are* the microscopic degrees of freedom of Quantum Gravity.

- Correlation functions of gauge invariant operators $\leftrightarrow$ Observables of Quantum Gravity.

- Strongly interacting gauge theory $\leftrightarrow$ Weakly curved gravity.

- Finite temperature field theory $\leftrightarrow$ "Thermal Quantum Gravity" (e.g. physics of black holes).

Gauge/String Duality *continued*

- Particularly well-studied examples are the $d = 2$ and $d = 4$ cases, corresponding to AdS_3 and AdS_5 respectively. (Also the AdS_2 case but there are many subtleties here.)
  - $d = 2$: Two dimensional Conformal Field theories dual to AdS_3 (or locally AdS_3, such as BTZ) geometries.

    Underlies most of the microscopic counts of entropy in String theory such as the Strominger-Vafa black hole. Extremal (or near-extremal) geometries having an AdS_3 factor. (Context of parts 2 and 3 of this talk.)
  - $d = 4$: Four dimensional Yang-Mills theories (in the large N limit) dual to asymptotically AdS_5 geometries.

    Largely been used to study strongly coupled gauge dynamics from gravity side. But also a good laboratory for studying generic non-supersymmetric black holes. (Context of Part 4 of this talk.)

Gauge/String Duality *continued*

What are the new things we can learn about Quantum Gravity by having a candidate definition of the theory in these examples?

Focus on the physics of black holes and try to shed light on many of its mysteries.

THEME: Take the underlying microscopic description of black holes seriously, as in any many body statistical system. Explore the consequences of going beyond the macroscopic description. Check whether it is consistent with all that we know.

Beginning to give some interesting new perspectives, perhaps will lead to qualitatively new insights and phenomena. Analogy with Brownian motion in conventional statistical mechanics.

CAVEAT: Many of these ideas are still being developed and still not completely well understood.

2 Going Beyond the Bekenstein Entropy

Going Beyond the Bekenstein entropy

- For a large class of supersymmetric (and thus extremal) blackholes, in five (or four) dimensions, the geometry has a (locally) AdS_3 (or AdS_2) factor.
- There is then a microscopic description of these black holes in terms of states in a two dimensional interacting conformal field theory.
- Thus a microcanonical description exists. The asymptotic behaviour (for large charge N) of the density of states $\rho_{CFT}(N)$ agrees with the Bekenstein entropy

$$S_{micro} = \ln \rho_{CFT}(N) \approx \frac{A_H}{4G_N} = S_{Bek}.$$

- Can now consider systematic corrections from the asymptotic answer in the exact microcanonical expression for $\rho_{CFT}(N)$.

Going Beyond the Bekenstein entropy *continued*

An illustration of the kind of corrections (toy example):

- Consider the number of ways $P(n)$ to partition an integer n (arises as the $\rho(n)$in the microcanonical ensemble of free bosons or fermions in one spatial direction)

$$Z(\beta) = \sum_{n=0}^{\infty} P(n) e^{-n\beta} = \prod_{k=1}^{\infty} (1 - e^{-k\beta})^{-1}$$

.

- Hardy-Ramanujan formula gives saddle point value for asymptotic behaviour of $P(n)$.

$$P(n) \approx \frac{1}{4\sqrt{3}n} e^{2\pi\sqrt{\frac{n}{6}}}$$

- Therefore

$$S_{micro} = \ln P(n) \approx 2\pi\sqrt{\frac{n}{6}} - \ln(4\sqrt{3}n) + \mathcal{O}(\frac{1}{\sqrt{n}}).$$

Going Beyond the Bekenstein entropy *continued*

- Hardy-Ramanujan formula is only the saddle point value. A systematic asymptotic expansion of $P(n)$ given by the Rademacher series:

$$P(n) = 8\pi\left(\frac{6}{n-\frac{1}{24}}\right)^{\frac{3}{4}} \sum_{k=1}^{\infty} \frac{1}{k} G_{n,k} \mathcal{I}_{\frac{3}{2}}\left(\frac{2\pi}{k}\sqrt{\frac{1}{6}\left(n-\frac{1}{24}\right)}\right).$$

  ($\mathcal{I}_{\frac{3}{2}}(z)$ is a Bessel function, while $G_{n,k}$ is a phase.)

- Similarly, in the dual 2d CFT's $\rho_{CFT}(N)$ has a saddle point answer which agrees with S_{Bek}.

- But there is a generalised Rademacher series expansion giving an asymptotic expansion with a complicated dependence on the (multiple) charges.

- Can we understand the origin of these corrections from the gravity point of view?

- Higher derivative corrections to the Einstein action would correct S_{Bek}.

Going Beyond the Bekenstein entropy *continued*

- Wald's formula for the entropy of a general covariant lagrangian:

$$S_{Wald} = -2\pi \int_{Hor} \frac{\delta \mathcal{L}}{\delta R_{abcd}} \epsilon_{ab}\epsilon_{cd}.$$

- In string theory there are specific corrections to the Einstein (supergravity) action that change the leading order solution for these extremal black holes.

$$\Delta \mathcal{L} \propto R_{abcd}R^{abcd} + \text{other terms}.$$

- Computing the contribution to Wald's entropy from these corrections gives striking match to subleading terms in the microscopic counting [a]

$$S_{Wald} = S_{micro}$$

[a] Cardoso, de Wit, Mohaupt; Ooguri, Strominger, Vafa; Dabholkar .....

Going Beyond the Bekenstein entropy *continued*

A very fruitful approach in these computations has been the entropy function defined by Sen for charged extremal black holes (with AdS_2 factor in the geometry, for example)

- Parametrise the horizon geometry by the values u_A of various scalar fields (e.g. the size of the AdS) as well as electric fields e_i and magnetic charges p_i.

- Define

$$f(u, e, p) = \int_{Hor} \mathcal{L}$$

- Extremise with respect to the u_A: $\frac{\partial f}{\partial u_A} = 0$ and define $q_i = \frac{\partial f}{\partial e_i}$.

- The Wald entropy for this extremal black hole is the Legendre transform with respect to the electric fields.

$$S(p, q) = 2\pi \left(e_i \frac{\partial f}{\partial e_i} - f \right).$$

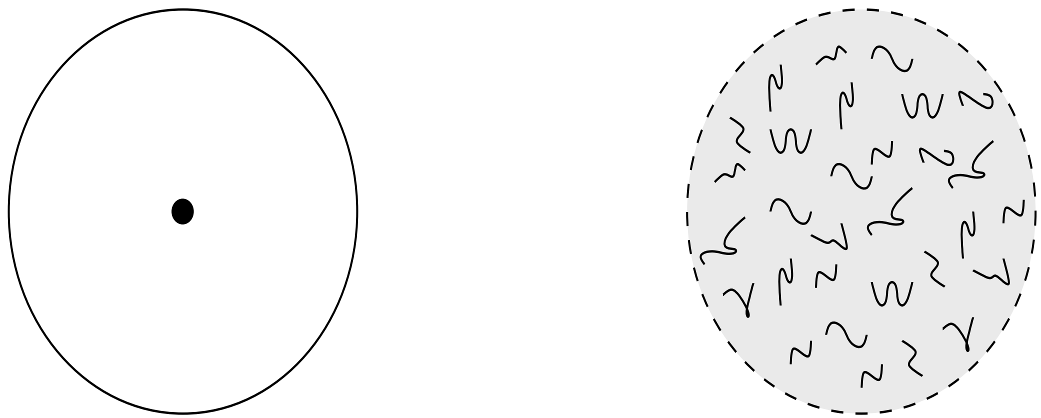
3 The Fuzzball Proposal

The Fuzzball Proposal

Given that the dual field theory gives a detailed accounting of the microstates of black holes, can ask:

What do the *individual* black hole microstates look like from the point of view of the gravity description?

Mathur's Conjecture:



1(a) 1(b)

The Fuzzball Proposal *continued*

- A typical microstate of a black hole has a size $N^{\alpha} l_p$ (with $N \gg 1$).
- This is comparable to the horizon size of the original geometry.
- The geometry corresponding to the typical microstate has no singularity *and no horizon*. and can moreover be described in terms of a gravity solution.
- Thus the correct description of black holes is in terms of these geometries which differ significantly from the "naive" singular geometry at the horizon scale.
- The presence of the horizon is from some kind of "coarse graining".
- This would be underlying a resolution of the information loss paradox.

Use the simple two and three charge extremal black holes as a laboratory for these claims.

The Fuzzball Proposal *continued*

The two charge 5d black hole ("D1-D5 system") corresponds to a ten dimensional solution on a T^4. The black hole arises from wrapping a black-string on a circle in one of the transverse six dimensions.

$$ds^2 = \frac{1}{\sqrt{(1+\frac{Q_1}{r^2})(1+\frac{Q_5}{r^2})}}[-dt^2+dy^2]+\sqrt{(1+\frac{Q_1}{r^2})(1+\frac{Q_5}{r^2})}dx_i dx_i+\sqrt{\frac{(1+\frac{Q_1}{r^2})}{(1+\frac{Q_5}{r^2})}}dz_a dz_a.$$

- The horizon coincides with the singularity at $r=0$. The Bekenstein-Hawking entropy is zero.
- The near horizon geometry is (locally) $AdS_3 \times S^3 \times T^4$.
- Microscopically (from the dual 2d CFT), one finds an entropy $S_{micro} \propto \sqrt{Q_1 Q_5}$. (Can be understood precisely as arising from the higher derivative corrections of the previous section.)
- What are the geometric duals to these microstates?

The Fuzzball Proposal *continued*

There is a continuous family of solutions carrying the same charges

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a.$$

where H^{-1}, K, A_i are harmonic functions in the x_i parametrised by a continuous curve $\vec{x} = \vec{F}(s)$.

- Non-spherically symmetric solution of generic size $(Q_1 Q_5)^{\frac{1}{6}} l_p$ (for $Q_1, Q_5 \gg 1$).
- Regular everywhere; singularity capped off. Horizon absent.
- Solutions differ only at horizon scale.
- Quantizing the space of these solutions gives $e^{S_{micro}}$ states as expected.
- Many features can be understood in terms of "fractionation" of underlying string microstates.

Though somewhat special, this example lends support to Mathur's conjecture.

The Fuzzball Proposal *continued*

The three charge black hole ("D1-D5-P" system) is a nontrivial solution with a macroscopic size horizon. More rigorous testing ground for these ideas.

Can one find geometries corresponding to the $e^{S_{Bek}}$ microstates which exhibit the above features?

- A large class of regular solutions with no horizon found which carry the right charges.
- However, generic solution not yet constructed.

Thus question is not definitively settled though there is encouraging support.

- Will be interesting to see if these ideas also can be tested in non-extremal cases.
- Notion of "fractionation" gives an intuitive picture of why microstates swell up to horizon size and may be valid in general case.

4 The Hydrodynamics of Black Holes

The Hydrodynamics of Black Holes

- (3+1) dimensional (super) Yang-Mills theories provide a microscopic description for asymptotically AdS_5 geometries.

- High temperature behaviour of gauge theories captures physics of AdS Schwarzschild black holes

$$ds^2 = -(1 - \frac{M}{r^2} + \frac{r^2}{L^2})^{-1} dt^2 + (1 - \frac{M}{r^2} + \frac{r^2}{L^2}) + r^2 d\Omega_3^2.$$

- Entropy of strongly coupled gauge theory *is* the Black hole entropy.

$$S_{Bek} = \frac{A_H}{4G} = S_{YM}(\lambda = \infty) = \frac{3}{4} S_{YM}(\lambda = 0).$$

- When microscopic description exists, can go beyond equilibrium statistical mechanics.

The Hydrodynamics of Black Holes *continued*

- When deviations from equilibrium are small (long wavelength fluctuations), can use a hydrodynamic description.
- Hydrodynamic behaviour captured by various transport coefficients such as shear and bulk viscosities (η, ζ resp.).
- Defined by means of constitutive relations for the stress tensor:

$$T_{ij} = P\delta_{ij} - \frac{\eta}{\rho + P}\left(\partial_i T_{0j} + \partial_j T_{0i} - \frac{2}{3}\delta_{ij}\partial_k T_{0k}\right) - \frac{\zeta}{\rho + P}\delta_{ij}\partial_k T_{0k}.$$

(ρ, P are the equiibrium energy and pressure densities.)

- Equivalently, from zero frequency poles of finite temperature *equilibrium* correlation functions (Kubo relations)

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] \rangle.$$

The Hydrodynamics of Black Holes *continued*

- Hydrodynamic regime of strongly interacting regime of Yang-Mills theory of interest in understanding Quark-Gluon plasma.
- Use dictionary between correlators of Yang-Mills and gravitational observables to compute Stress tensor correlators at strong gauge coupling[a]

$$\eta(\lambda = \infty) = \frac{\sigma_{abs}(0)}{16\pi G}.$$

where $\sigma_{abs}(0) = A_H$ is the graviton absorption cross section of the black hole.

- Therefore a (universal) prediction for the dimensionless ratio to the entropy density is

$$\frac{\eta}{s}(\lambda = \infty) = \frac{1}{4\pi} = \frac{\hbar}{4\pi k_B}.$$

- Extremely small value which is exceeded by all known matter. (Perturbative Yang-Mills calculation gives $\eta(\lambda \ll 1) \propto \frac{1}{\lambda^2 \ln \frac{1}{\lambda}}$.)

[a] Kovtun, Son, Starinets

The Hydrodynamics of Black Holes *continued*

Similar predictions for other transport coefficients at strong coupling. For example $\zeta = 0$.

Can we understand this hydrodynamic behaviour on the gravity side?

- Old work on membrane paradigm by Price and Thorne etc.,: analogies with fluid behaviour (Raychaudhuri equation), attributed viscosity and other transport coefficients to the stretched horizon membrane.
- Remarkably $(\frac{\eta}{s})_{memb} = \frac{1}{4\pi}$! (For usual Schwarzschild black holes as well).
- However, $\zeta_{memb} < 0$ – unphysical.
- Therefore membrane paradigm very suggestive of the picture that the microscopics suggests. Needs revisiting. Membrane stress tensor needs better definition.

The Hydrodynamics of Black Holes *continued*

- QUESTION: Can one understand the universality of $\frac{\eta}{s}$ in Einstein gravity?

- QUESTION: Can one derive a Wald like formula for the viscosity in the presence of higher derivative corrections?

- QUESTION: Can one find a Boltzmann kinetic theory description in the gravitational description?

5 In Summary

Concluding remarks

- Gauge/string duality provides an excellent laboratory for probing different aspects of quantum gravity exhibited by a microscopic description.
- Microscopic description is consistent with gravity expectations for corrections to the Bekenstein entropy.
- Perhaps the picture of the black hole geometry drastically modified (at horizon scale) in a microscopic description.
- Hydrodynamic description of the horizon points to going beyond equilibrium thermodynamics and interesting new physics.

The end