On Gravitational Dynamics

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NFL : Newton's First Law

Uniform Motion $\equiv$ No Motion

Absence of Force

- Space is homogeneous

$$x \leftrightarrow y$$

- Time is homogeneous

why not $x \leftrightarrow t$?

Match D : $x \leftrightarrow ct$

c : Universal constant velocity

- New Mechanics $\rightarrow SR$

- Spacetime

NFL : No Force! Motion is geodesic of homogeneous spacetime

NSL : $m\overrightarrow{\ddot{x}} = \overrightarrow{F}$

Galileo's Experiment :

Gravity : $m_i\overrightarrow{\ddot{x}} = m_g \nabla\phi$

$m_i = m_g$ why?

No Force $\xrightarrow{\text{Dual}}$	Universal Force
	Links to All
	Present everywhere
	& Always
Universal property	Universal Property
homogeneous	inhomogeneous
Flat Minkowski	curved spacetime
$R_{abcd} = 0$	$R_{abcd} \neq 0$

R_{abcd} to determine gravitational dynamics

No need for PE & $m_i = m_g$.

Bianchi identity

Wheeler : Boundary of boundary is zero

$$D^2 = 0 = \nabla \times \nabla\phi = \nabla \cdot \nabla \times \bar{A}$$

Trace is vacuous

Tensor Field : g_{ab}

$$A^b R^a{}_{bcd} = A^a{}_{;cd} - A^a{}_{;dc}$$

$R_{ab[cd;e]} = 0$

Now trace is non-vacuous.

$\Rightarrow G^{ab}_{\;;b} = 0$

$G_{ab} = KT_{ab} + \Lambda g_{ab}$

Einstein's equation follows from curvature

Λ : New constant

• Constant curvature of homogeneous & isotropic spacetime – Absence of force

• Signature of spacetime being dynamic not fixed

Einstein's equation also follows from

$$\delta \int R_{ab} g^{ab} \sqrt{-g} d^4 x$$
$$= \int [R_{ab} \sqrt{-g} \delta g^{ab} + R \delta \sqrt{-g} + (\delta R_{ab}) g^{ab} \sqrt{-g}] d_x^4$$
$$= \int (R_{ab} - \tfrac{1}{2} R) \sqrt{-g} d_x^4 + \int \delta R_{ab} = 0$$

Variation of dynamic part, R_{ab} makes no contribution.

Higher order in curvatures.

Find an analogue of R_{abcd} satisfying Bianchi identity.

$$\mathcal{R}_{abcd} = R_{abmn} R_{cd}^{\ \ mn} + \alpha R_{[a^m R_b]mcd} + \beta R R_{abcd}$$

Bianchi : $\mathcal{R}_{ab[cd;e]}$ to yield a divergence free tensor requires

$$\alpha = 4, \;\; \beta = 1$$

Guass -Bonnet term.

$$\mathcal{R}^{cd}_{\;\;[cd;e]} = \frac{1}{2}\mathcal{R}_{,e} \neq 0$$

$$\mathcal{R}^{cd}_{\;\;[cd;e]} - \frac{1}{2}\mathcal{R}_{,e} = -H_{e;c}^{\;\;c} = 0$$

This suggests

$$F_{abcd} = \mathcal{R}_{abcd} - \gamma\mathcal{R}(g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$F^{cd}_{\;\;[cd;e]} = H_{e;c}^{\;\;c} = 0$$

This requires

$$\mathcal{R}_{abcd} = Q_{ab}{}^{mn} R_{mncd}$$

$$Q^{ab}{}_{cd} = \delta^{aba_1b_1\cdots a_nb_n}_{cdc_1d_1\cdots c_nd_n} R_{a_1b_1}{}^{c_1d_1} \cdots R_{a_nb_n}{}^{c_nd_n}.$$

For quadratic GB

$$Q_{abcd} = R_{abcd} - 2R_{a[c}g_{d]b} + 2R_{b[c}g_{d]a} + Rg_{a[c}g_{d]b}.$$

In general

$$F_{abcd} = \mathcal{R}_{abcd} - \frac{n-1}{n(d-1)(d-2)}\mathcal{R}(g_{ac}g_{bd} - g_{ad}g_{bc})$$

n : order of polynomial

d : dimension of spacetime

$$n(F_{ab} - \frac{1}{2}F g_{ab}) = H_{ab}$$

$$H_{a;b}^{\ \ b} = 0.$$

$$n = 1,\ F_{ab} = R_{ab},\ G_{ab} = H_{ab}$$

Theorem : *The second order quasi-linear differential operator as a second rank divergence free tensor in the equation of motion for gravitation could always be derived from the trace of the Bianchi derivative of the fourth rank tensor, F_{abcd}, which is a homegeneous polynomial in curvatures. The trace of the curvature poynomial is proportional to the corresponding term in the Lovelock action and corresponding to each term in the Lovelock Lagrangian, there exists a fourth rank tensor which is a new characterization of the Lovelock Lagrangian.*

Lovelock Polynomial

- quasi-linearity
- Metric and Palitini Variation

  $\rightarrow$ the same equation.

- Bianchi identity $\rightarrow$ Equation.

New identification of Lovelock gravity

For $d > 4$, H_{ab} is as natural as G_{ab} and hence must be included.

Equation to follow from F_{abcd} $\rightarrow$ potential for H_{ab}

GB :

1. One loop (string) correction
2. Higher D
3. High Energy

CG $\rightarrow$ GB / Lovelock $\rightarrow$ QG

GB : Intermediatory Limit

QG / High Energy $\rightarrow$ Higher D?

Higher D

- Isometric Flat Space Embedding

- Self interaction iteration

First : $(\partial)^2$

Second : $(\partial)^4$: R^2_{abcd}

$\Rightarrow$ GB $\rightarrow$ Relevant only in $D > 4$.

Higher D required.

How to see higher D?

Matter Probes confirmed to 4 - D

Only gravity in higher D

Purely gravitational experiment?

Signature in sub mm / cosmology?

Search is on.