Brane world overview – some key issues

Soumitra SenGupta
Department of Theoretical Physics
IACS, Kolkata, India

IAGRG meeting, SINP, Kolkata, 29 January, 2009

Good Reviews

- Caba Csaki, hep-ph/0404096, hep-ph/0510275
- Roy Maartens, gr-qc/0312059
- Keith Dienes, 2002 Tasi Lectures
- S.SenGupta, arXive: 0812.1092

Collaborators

J.Mitra, D.Maity, S.Sur, A.Dey

R.Koley, S.Pal, S.Sen, A.Sinha

S.Das, D.Choudhury,S.Kar, P.Majumdar,B.Mukhopadhyaya, S.Sen

Few Questions

- Are there extra dimensions ?
- If yes, how many? Why are they invisible?
- Finely tuned cosmological constant? Dark energy?
- Is the universe torsion free?
- Finely tuned Higgs mass within Tev?
- Why SM fermions have different masses ?
- Are they confined to brane or can move into the extra dimension?
- New Physics beyond Tev scale? Supersymmetry ? Superparticles?
- What are the dark matter candidates?

CAN BRANEWORLD ANSWER THESE ?

What is a brane ?

A p-brane is a hypersurface with p - spatial dimension

A Point – 0-brane

A String – 1-brane

A 2-dim surface – 2-brane etc.

Features of p-brane

- In an N-dimensional spacetime p can be either of 0,1,2,3...(N-1)
- In general for $p < (N-1)$, the remaining dimensions are extra dimensions from brane point of view
- Lower dimensional branes are embedded in higher dimensional spacetime
- Various influences of these extra dimensions on the brane – <u>Braneworld Physics</u>
- We live in a 3-brane
- Are ther extra dimensions?
- What motivates us to think about such extra dimensions?

Motivation – String theory

String theory predicts the existence of extra spatial dimensions – 10 or 26?

Branes are natural in string theory

String equation of motion says – the ends of the open strings must satisfy either of the two boundary conditions :

Boundary condition

- Neumann condition – ends are dynamic

$$\partial_i X^\mu \quad = \quad 0 \quad (\mu = 1, ...p)$$

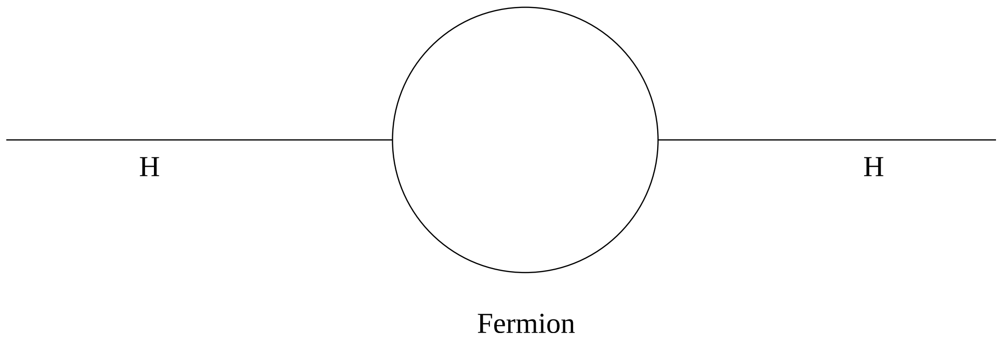
- Dirichlet condition – ends are fixed on some hypersurface

$$\delta X^\mu \quad = \quad 0 \quad (\nu = p + 1, ...D - 1)$$

p-Neumann conditions imply the ends can move on a p-dimensional hypersurface – called Dp-brane

Motivation – Elementary particle Theory

- Standard model has been extremely successful in explaining physical phenomena upto scale close to Tev
- The theory is expected to be valid upto Planck scale or scale of quantum gravity $M_P \sim 1/\sqrt{G} \sim 10^{19}$ Gev
- Large quadratic correction to the mass of the only scalar in the theory —- 'Higgs' is a serious problem



Hierarchy problem

Vast disparity between the weak and Planck scale – Gauge hierarchy problem

$$\delta m_H^2 \sim \Lambda^2$$

where Λ is the cutoff scale i.e.Planck scale or

To keep m_H within Tev, one needs extreme fine tuning $\longrightarrow$ unnatural ! Challenge for an otherwise sucessful standard model ?

Resolution through Supersmmetry

Supersymmetry removes this unnatural feature at the expense of bringing in the following problems :

- It incorporates a large number of (so far undetected) superpartners in the theory
- Absence of the superpartners implies that the supersymmetry, even if it is true, is a broken symmetry at the present energy scale
- Breakdown of supersymmetry generates large cosmological constant $\sim M_S^4$ which is not consistent with the present observed small value

Alternative path ?

- What will happen if the signature of supersymmetry is not found at Tev scale ?
- It may be there at high scale, as required in String theory, but <u>that will not resolve the hierarchy problem</u>
- If we do not want to subscribe to exotic ideas like landscape or anthropic principle in favour of fine tuning or unnaturalness – we will have to look for some alternative paths

Extra dimensional Theories – Braneworld

Braneworld models resolves the fine tuning problem by assuming the existence of extra spatial dimensions

Two important models

- Large compact extra dimension – Arkani-hamed, Dimopoulos,Dvali (ADD) Model
- Warped extra dimension – Randall-Sundrum (RS) Model

Origin in String theory?

<u>ADD model</u> – Toroidal compactification in a String model

<u>RS model</u> – warped throat geometry in a String model

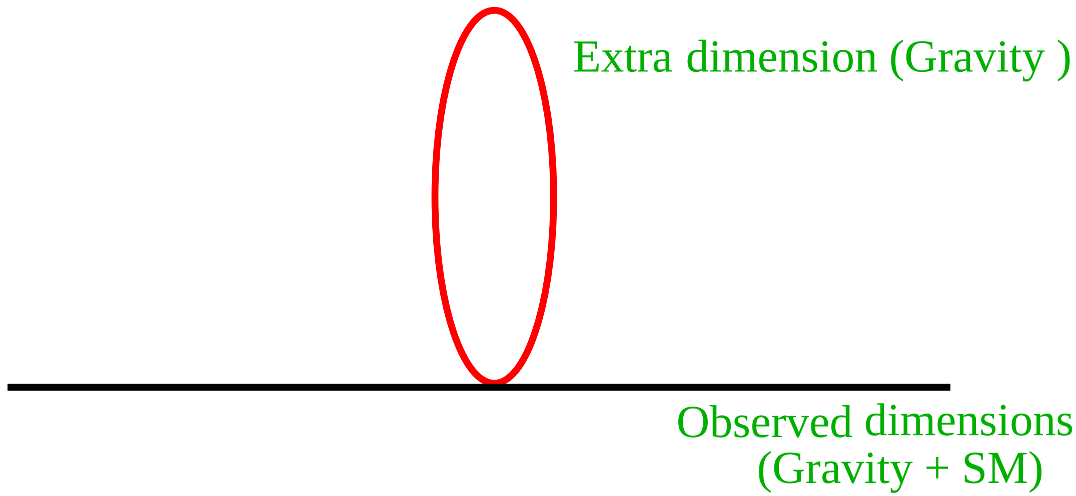
(Klebanov-Strassler geometry)

arge compact extra dimension – ADD mod

A new geometric approach to solve this fine tuning problem

$$\boxed{\text{How many spacetime dimensions does our universe have?}}$$

No.of Spacetime dimensions $\quad d = \underbrace{4}_{Observed} + \underbrace{(d-4)}_{Unobserved}$



Only gravity enters the extra dimension

Reduced Planck scale !

Newton's law of gravitation in extra dimension is

$$F = -\frac{G_{4+n} m_1 m_2}{r^{2+n}}$$

n is number of extra dimensions
G_{4+n} is the value of Newton's constant in higher dimension

If our microscope is not sufficient to look into the extra dimension of radius R, then we get back our inverse square law

$$\frac{G_{4+n}}{R^n} = G_4$$

Remember $M = \frac{1}{\sqrt{G}}$

Therefore extra dimension changes the quantum gravity scale or Planck energy as

$$M_5^{n+2} \sim \frac{M_4^2}{R^n}$$

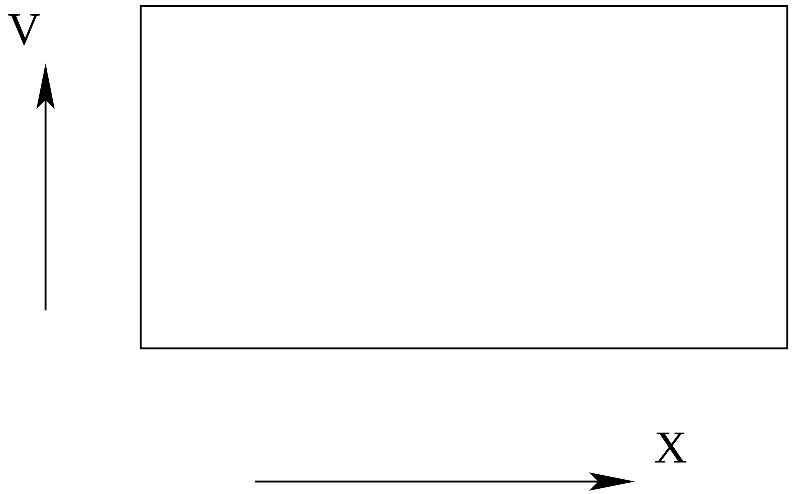
A large radius R pulls down the value of M_{4+n} to Tev !

Standard model is valid only upto energy $\Lambda = M_{4+n}$ (cut-off) which has the desired value $\sim$ Tev
Higgs mass correction is restricted to Tev only

So extra dimension makes a miracle and remove fine tuning problem !!

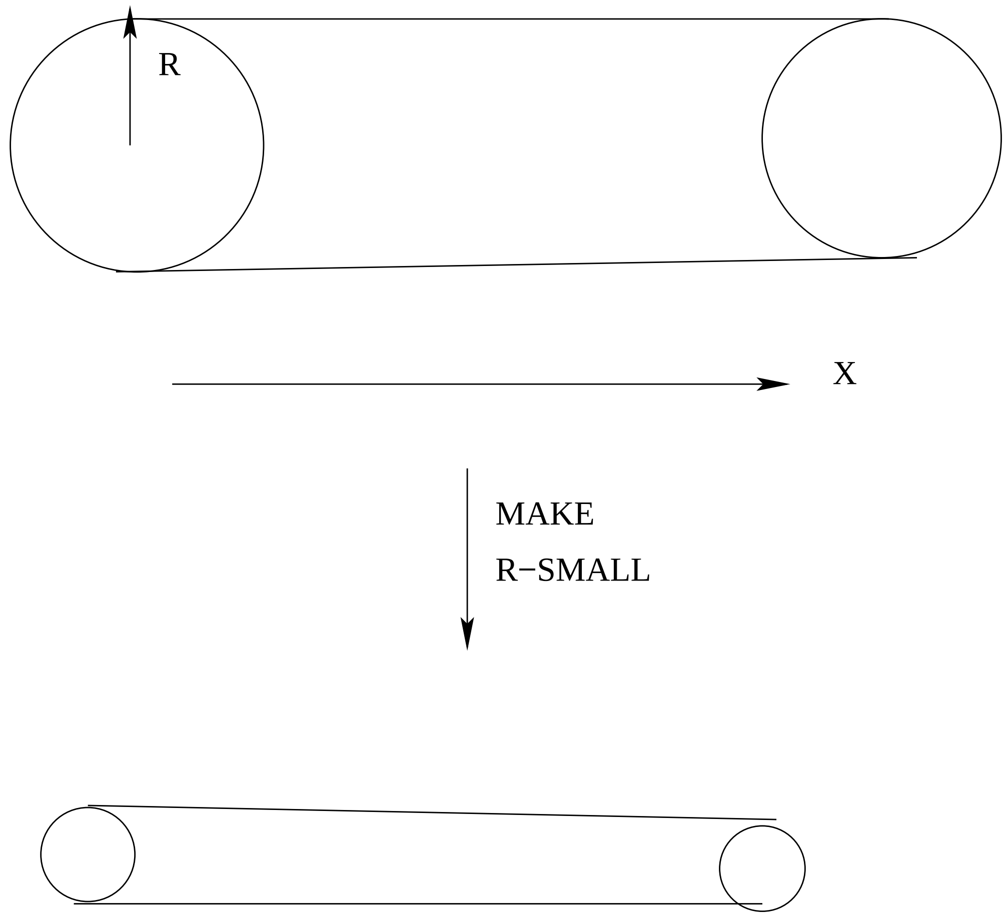
Why Extra dimensions are invisible?

Consider for simplicity a two dimensional surface (x-v)



Suppose at a certain epoch the dimension v curled up in a circle – at 'compactification scale' M_c

Curled up dimension



If radius R is smaller than microscopic resolution of an experimentalist sitting on the X-axis, then it is effectively an one-dimensional world.
– compactification of extra dimension

There may be many such extra compact dimensions

We can not probe these compact dimensions easily – why?

- Either R is large, but allows only gravitational interaction to enter which can resolve only upto a micrometer
- Or 'R' is smaller than the present microscopic resolution for any interarction —- 10^{-19}m

Signature of ADD model – Massive Tower

Consider any free particle wavefunction on the two dimensional plane,

$$\psi(x, v) = e^{i(xp_x + vp_v)},$$

If the particle is massless then it's energy is,

$$E^2 = p_x^2 + p_v^2, (c = 1)$$

As v is a periodic coordinate with period $2\pi R$, therefore

$$\psi(x, v) = \psi(x, v + 2\pi R)$$

This implies,

$$p_v = \frac{2\pi n}{R}$$

Therefore,

$$E^2 = p_x^2 + 4\pi^2 n^2/R^2, (n = 0, 1, 2, ...)$$

After compactification , the effective theory in one dimension has a ’tower’ of massive particles, called Kaluza-Klein (KK) tower

Larger the value of ’R’, smaller is the mass of the KK tower and we may be able to excite these states at lower energy accelerators.

For d compact extra dimensions, we have d integers n_i, where

$$n_1^2 + n_2^2 + \ldots\ldots\ldots\ldots = n^2$$

A large number of degenerate states
These interact with our standard model particles i.e. quarks, leptons etc. and give new phenomena!!

Can we see them at LHC? – A big question

Hierarchy in a new guise ?

We started with only two scales –

M_{ew} and M_P

But we have introduce a new heirarchial length scale through the large radius R

How shall we stabilize such an intermediate scale R^{-1}?

Hierarchy problem comes back in a new guise

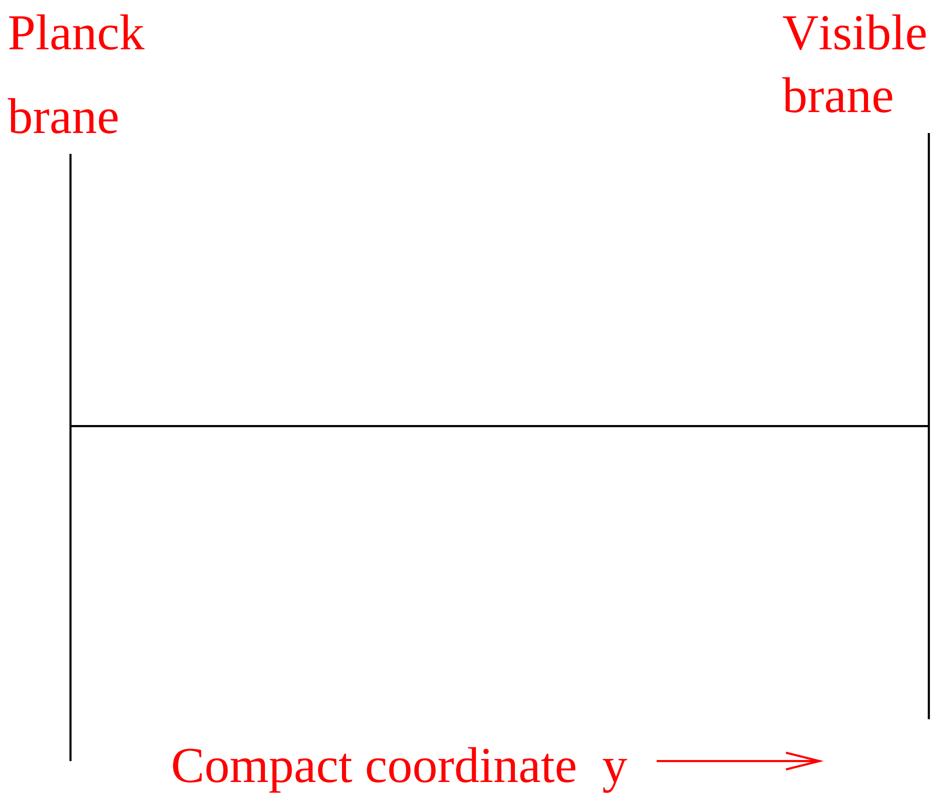
Warped geometry – Randall Sundrum mod

The Einstein action in 5 dimensional ADS_5 space:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g_5} \left[\mathcal{R}_d - \Lambda\right]$$

Compactify the extra coordinate $y = r\phi$ on a S_1/Z_2 orbifold (identify ϕ to $-\phi$ i.e lower semi-circle to upper semi circle)

Place two 3-branes at the two orbifold fixed points $\phi = 0, \pi$, where r is the radius of S_1



The Z_2 orbifolded coordinate $y = r\phi$ with $0 \leq \phi \leq \pi$ and r is the radius of the S_1

Action ($M_{Pl(5)} \equiv M$):

$$
\begin{aligned}
S &= S_{Gravity} + S_{vis} + S_{hid}
\end{aligned}
$$

where,

$$
\begin{aligned}
S_{Gravity} &= \int d^4x \; r \; d\phi \sqrt{-G} \, [2M^3 R - \underbrace{\Lambda}_{5-dim}] \\
S_{vis} &= \int d^4x \sqrt{-g_{vis}} \, [L_{vis} - V_{vis}] \\
S_{hid} &= \int d^4x \sqrt{-g_{hid}} \, [L_{hid} - V_{hid}]
\end{aligned}
$$

Metric ansatz:

$$ds^2 = e^{-A}\,\eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\phi^2 \;\leftarrow\; \text{extra dim}$$

Computing the warp factor $A(y)$

Warp factor and the brane tensions are found by solving the 5 dimensional Einstein's equation with orbifolded boundary conditions

$$A = 2kr\phi$$

$$V_{hid} = -V_{vis} = 24M^3k \qquad \left[k^2 = \frac{-\Lambda}{24M^3}\right]$$

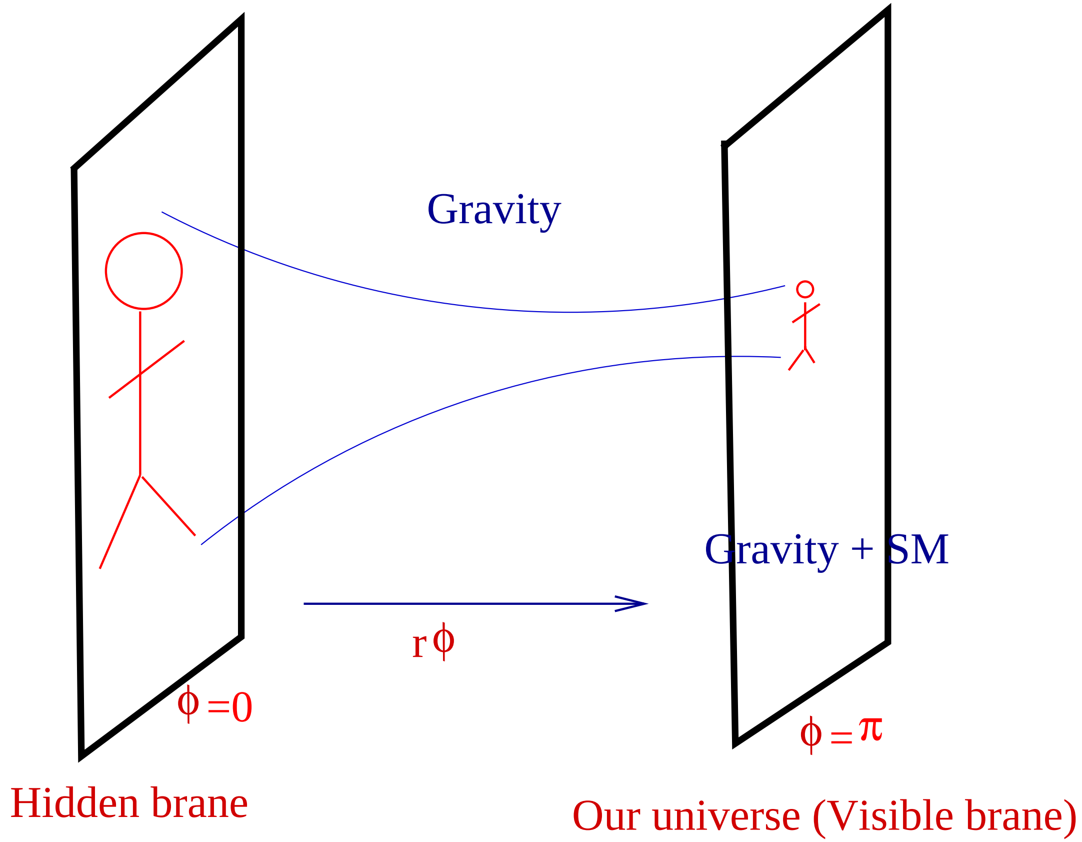
Warping

$$\left(\frac{m_H}{m_0}\right)^2 \;=\; e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \approx (10^{-16})^2$$

$$\Rightarrow kr \;=\; \frac{16}{\pi}\ln(10) = 11.6279\ldots \quad \leftarrow \text{RS value}$$

with
$k \sim M_P$ and $r \sim l_P$

So hierarchy problem is resolved without introducing any new scale


Gravity
Gravity + SM
rϕ
ϕ=0
ϕ=π
Hidden brane
Our universe (Visible brane)

Outcome

A large hierarchy emerges naturally from a small conformal factor

Signature of RS model –Massive towers

Like ADD model we have

Bulk graviton KK modes of mass at Tev range

Massless graviton mode couples to standard model fields at the brane as $\sim 1/M_P$

Massive graviton modes couple $\sim e^{kr\pi}/M_P \sim Tev^{-1}$

Detectable signatures at LHC?

RS model implies torsion free universe ?

B.Mukhopadhyaya, S.Sen, S.SenGupta, Phys. Rev. Lett.(89)12

Phys.Rev.D 76:121501, (2007), Phys. Rev. D (70) 066009 (200

The connection

$$\bar{\Gamma}^K_{NL} = \Gamma^K{}_{NL} - \frac{1}{M^{\frac{3}{2}}} H_{NL}{}^K$$

Action

$$\mathcal{S}_G = \int d^4x \int d\phi \sqrt{-G}\, 2[M^3 R(G) - H_{MNL} H^{MNL}]$$

According to String theory – $H_{MNL} = \partial_{[M} B_{NL]}$
Kaluza-Klein decomposition :

$$B_{\mu\nu}(x,\phi) = \sum_{n=0}^{\infty} B^n_{\mu\nu}(x) \frac{\chi^n(\phi)}{\sqrt{r_c}}$$

The zero mode is suppressed on our brane

$$\chi^0 = \sqrt{kr_c} e^{-kr_c\pi}$$

No such suppression for graviton zero mode

<u>Warped geometry makes our universe torsion-free !</u>

Some features of RS model

- Negative tension Visible brane to describe our Universe. Such negative tension branes are unstable
- The effctive visible 3-brane being flat has zero cosmological constant which is not consistent with its presently observed small value.
- Problem of stabilizing the brane separation modulus r

RS - II model

- In this model, one of the brane is taken to infinity

The gravitational potential on the other brane turns out to be,

$$V(r) \;=\; \frac{Gm_1m_2}{r}(1 + 1/r^2k^2)$$

- The correction term appears from the continuum KK modes of graviton
- Bulk scalar field induces inflation on the brane – Interesting cosmology
- But does not solve the hierarchy problem

Generalization of RS model

S.Das, D.Maity, S.SenGupta, JHEP 0805:042, 2008

R.Koley, J.Mitra, S.SenGupta, To appear in Europhys.Lett.

- Non-zero cosmological constant on the visible 3-brane
- Positive tension Tev brane when a large hierarchy exists between the two branes
- Inspiration of looking for positive tension branes comes from string theory where we have positive tension D-branes to construct low energy phenomenology

Earlier work

In the original RS scenario, the visible 3-brane being flat has zero cosmological constant.

$$ds^2 = e^{-2kry}\eta_{\mu\nu}dx^\mu dx^\nu + r^2dy^2$$

Such model was generalized to Ricci flat spaces :

$$R_{\mu\nu} = 0$$

The warp factor turned out to be the same as obtained by RS

See Chamblin, Hawking, Real : Phys.Rev.D, 61,065007 (2000)

Generalization to ADS and DS spaces

We start with the metric :

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r^2 dy^2 \ .$$

and the action:

$$S = \int d^5x \sqrt{-G}(M^3\mathcal{R} - \Lambda) + \int d^4x \sqrt{-g_i}\mathcal{V}_i$$

Λ is the bulk cosmological constant, $\mathcal{R}$ is the bulk (5-dimensional) Ricci scalar and $\mathcal{V}_i$ is the tension of the i^{th} brane

Equation of Motion

$$^4G_{\mu\nu} = -\Omega g_{\mu\nu} \qquad ,$$

$$e^{-2A} \left[-6A'^2 + 3A'' - \frac{\Lambda}{2M^3} \right] = \Omega \, .$$

Ω is the four dimensional cosmological constant on the $3 - brane$.

On simplification we obtain,

$$6A'^2 = -\frac{\Lambda}{2M^3} + 2\Omega e^{2A}$$

$$3A'' = \Omega e^{2A} \, .$$

This corresponds to a constant curvature brane spacetime, as opposed to a Ricci flat spacetime.

$\Omega > 0$ and $\Omega < 0$, $g_{\mu\nu}$ may correspond to dS-Schwarzschild and AdS-Schwarzschild spacetimes respectively. Bulk is Ads i.e. $\Lambda < 0$,

Negative Ω – ADS case

Define the parameter $\omega^2 \equiv -\Omega/3k^2 \geq 0$, The solution for the

warp factor,

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c\,1} + ky\right)$$

Note that the above solution is an exact solution for the warp factor in presence of Ω.

The RS solution $A = ky$ is recovered in the limit $\omega \to 0$.

The brane tensions are:

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_1^2}e^{2kr\pi} - 1}{\frac{\omega^2}{c_1^2}e^{2kr\pi} + 1}\right]$$

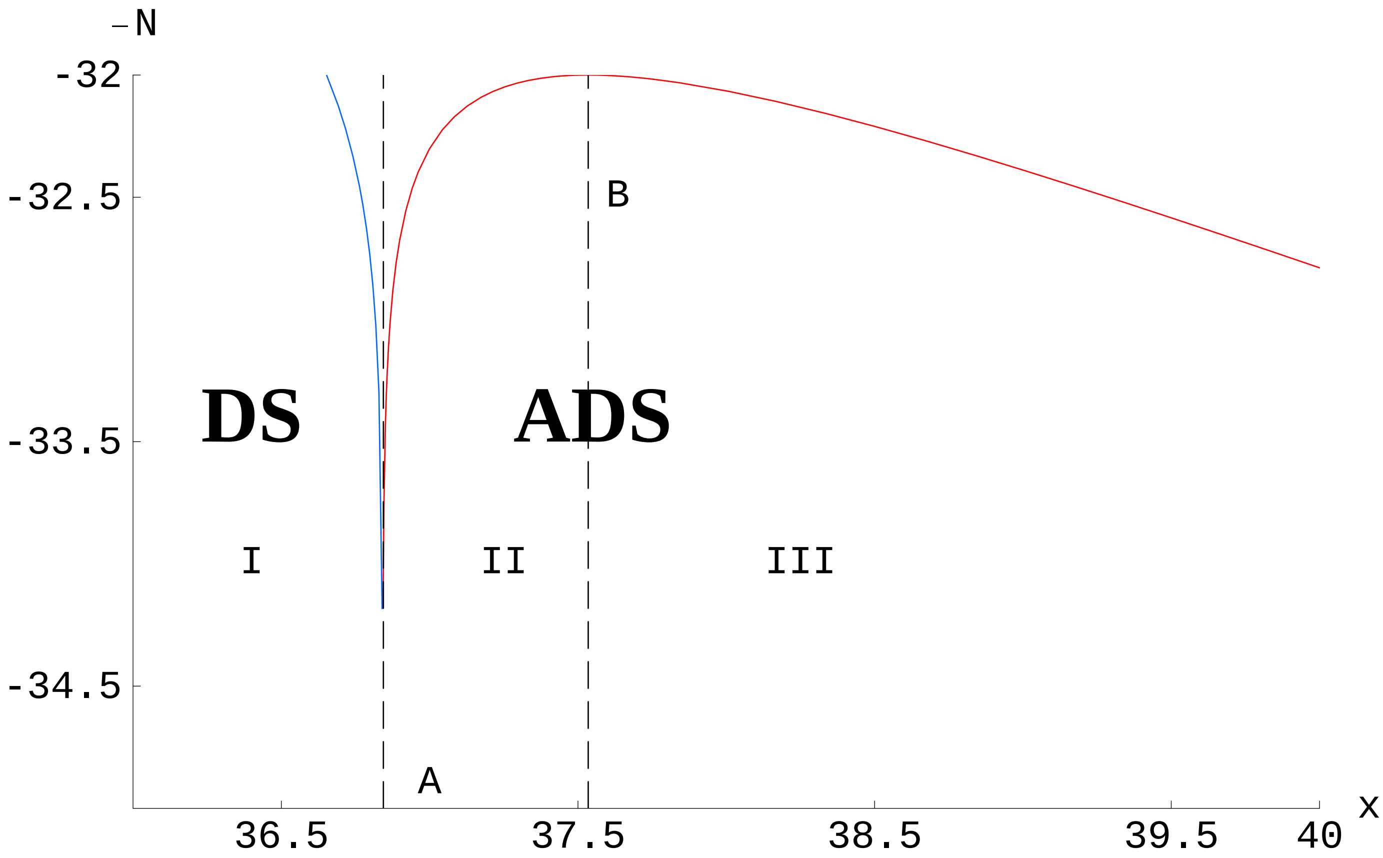
$$\mathcal{V}_{hid} = 12M^3k\left[\frac{1 - \frac{\omega^2}{c_1^2}}{1 + \frac{\omega^2}{c_1^2}}\right]$$

Normalizing the warp factor to unity at the orbifold fixed point $y = 0$, we get:

$$c_1 = 1 + \sqrt{1 - \omega^2} \; .$$

To solve the hierarchy problem, we equate the warp factor at $y = r\pi$ to the ratio of the Higgs to the Planck mass:

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + kr\pi\right) = 10^{-16} \; .$$



−N
-32
-32.5
-33.5
-34.5
B
DS
ADS
I
II
III
A
x
36.5
37.5
38.5
39.5
40

Positive Ω – DS case

Consider$\Omega > 0$

The warp factor is now given by:

$$e^{-A} = \omega \sinh\left(\ln\frac{c_2}{\omega} - ky\right) \ ,$$

where now $\omega^2 \equiv \Omega/3k^2$ and $c_2 = 1 + \sqrt{1+\omega^2}$.

Expressions for the brane tensions:

$$\mathcal{V}_{vis} = 12M^3k\left[\frac{\frac{\omega^2}{c_2^2}e^{2kr\pi}+1}{\frac{\omega^2}{c_2^2}e^{2kr\pi}-1}\right]$$

$$\mathcal{V}_{hid} = 12M^3k\left[\frac{1+\frac{\omega^2}{c_2^2}}{1-\frac{\omega^2}{c_2^2}}\right]$$

Findings

- Exact solutions for the warp factors are determined for both DS and ADS cases which can resolve the gauge hierarchy problem
- Both positive and negative brane tension for the Tev brane are now admissible
- The magnitude of the negative induced cosmological constant on the 3-brane has an upper bound $\sim 10^{-32}$ in Planck unit.
- The hierarchy problem is resolved

New anthropic principle?

The fine tuning problem in connection with the Higgs mass and the cosmological fine tuning problem are intimately related and one implies the other!

Modulus stabilization

Two branes with gravity in the bulk is unstable

Question

How to stabilize the modulus r ?

Goldberger Wise Mechanism

- Include a massive scalar field in the bulk of five dimensional gravitational theory which depends only on the extra coordinate

$$S_{scalar} = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} \sqrt{G}(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2)$$

$$S_{boundary} = -\int \sqrt{-g_p}\lambda_p(\Phi^2 - v_p)^2 - \int d^4x\sqrt{-g_s}\lambda_s(\Phi^2 - v_s)^2$$

- Solve the equation of motion for the scalar field with appropriate boundary condition

$$\Phi(\phi) = e^{2\sigma}[Ae^{\nu\sigma} + Be^{-\nu\sigma}]$$

where $\nu = \sqrt{4 + m^2/k^2}$.

- Plug in the solution in the action and integrate out the extra coordinate to find an effective potential for the modulus r

$$V_\Phi(r) = k(\nu+2)A^2(e^{2\nu kr\pi}-1) + k(\nu-2)B^2(1-e^{-2\nu kr\pi}) +$$

$$\lambda_s e^{-4kr\pi}[\Phi(\pi)^2 - v_s^2]^2 + \lambda_p[\Phi(0)^2 - v_p^2]$$

- Minimize the potential to find the stable value of the modulus r
- It turned out to be $\sim M_P^{-1}$ – as desired
- Crucial assumption is:
  <u>backreaction of the scalar on the metric is negligible</u>

Backreaction

- Effect of back-reaction of the scalar field on the metric was found for some classes of scalar potntial
- Is r stable against the correction due to backreaction?

- **We re-examine the stability issue in the back-reacted RS model when the scalar field is introduced in the bulk**

D. Maity, S.SenGupta, S.Sur, Phys.Lett.B643 348 (2006)

and

D. Maity, S.SenGupta, S. Sur, To appear in Class.quantum.Gra

A.Dey, D.Maity, S.SenGupta, Phys.Rev.D75:107901,2007

Europhys. Lett. 83: 51002, 2008

We begin with a very general action :

$$S = \int d^5x \left[-M^3 R + F(\phi, X) - V(\phi)\right]$$
$$\int d^4x \, dy \, \sqrt{-g_a}\,\delta(y - y_a)\lambda_a(\phi)$$

where, $X = \partial_A \phi \partial^A \phi$

Take the line element in the form

$$ds^2 \; = \; e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu \; - \; dy^2,$$

The field equations in the bulk turn out to be

$$\begin{aligned}
F_X \phi'' - 2F_{XX}\, {\phi'}^2 \phi'' \; &= \; 4F_X \phi' A' - \frac{\partial F_X}{\partial \phi} {\phi'}^2 - \frac{1}{2}\left(\frac{\partial F}{\partial \phi} - \frac{\partial V}{\partial \phi}\right) \\
{A'}^2 \; &= \; 4CF_X\, {\phi'}^2 \; + \; 2C\left[F(X, \phi) \; - \; V(\phi)\right] \\
A'' \; &= \; 8CF_X\, {\phi'}^2 \; + \; 4C \sum_a \lambda_a(\phi) \delta(y - y_a)
\end{aligned}$$

where $C \; = \; \frac{1}{24M^3}$

We consider two examples :

1) Along with higher derivative term ,we take a scalar potential inspired from supergravity models:

$$V = \frac{1}{16}\left[\frac{\partial W}{\partial \phi}\right]^2 - 2CW^2$$

with

$$W = A - B\phi^2$$

A, B are parameters of the superpotential W

2) $F(X,\phi) = -f(\phi)\sqrt{1-X}$ and $V(\phi) = 0$
which corresponds to a tachyon like scalar field in the bulk

Our result

- Higher derivative terms stabilizes the modulus
- Stable values of the modulus depend on the choice of the parameters of the potential
- For appropriate choice the stable value of r can produce Planck to Tev scale warping

More warped dimensions

Six dimensional warped space-time

D.Choudhury, S.SenGupta, Phys.Rev.D76:064030,2007

R.Koley, J.Mitra, S.SenGupta, Phys.Rev.D78: 045005, 2008

We consider a doubly warped space-time :

$M^{1,5} \rightarrow [M^{1,3} \times S^1/Z_2] \times S^1/Z_2$

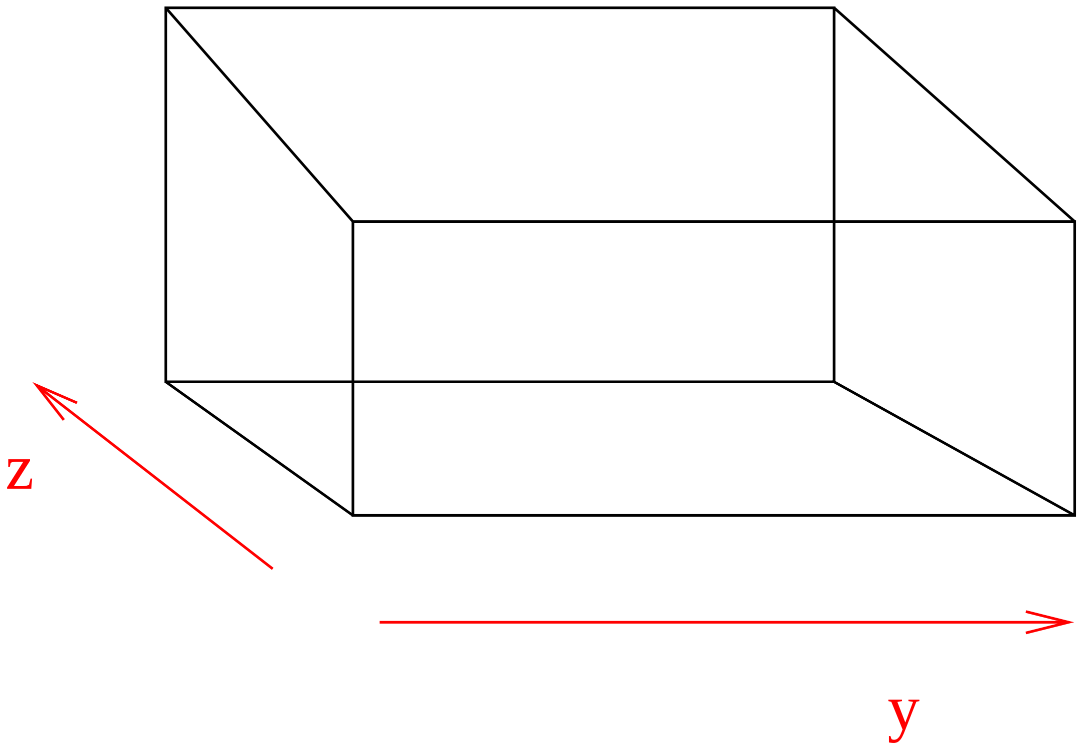
Four 4-branes are placed at the orbifold fixed points:

. Four 3-branes appear at the four intersection region of these 4-branes

The bulk is ADS with a negative cosmological constant Λ.

We thus have a brane-box like space-time

Brane-box



y and z are the compact coordinates

The six dimensional warped metric ansatz:

$$ds^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2$$

The total bulk-brane action is thus given by,

$$
\begin{aligned}
S &= S_6 + S_5 + S_4 \\
S_6 &= \int d^4x\, dy\, dz\, \sqrt{-g_6}\; (R_6 - \Lambda) \\
S_5 &= \int d^4x\, dy\, dz\; [V_1\, \delta(y) + V_2\, \delta(y-\pi)] \\
&+ \int d^4x\, dy\, dz\; [V_3\, \delta(z) + V_4\, \delta(z-\pi)] \\
S_4 &= \int d^4x \sqrt{-g_{vis}} [\mathcal{L} - \hat{V}]
\end{aligned}
$$

The solutions for the warp factord are:

$$\begin{aligned} a(y) &= \exp(-c\,y) \\ b(z) &= \frac{\cosh(k\,z)}{\cosh(k\,\pi)} \end{aligned}$$

Minimum warping at the 3-brane located at $y = 0, z = \pi$.
Maximum warping at the 3-brane located at $y = \pi, z = 0$.
Here

$$\begin{aligned} c &\equiv \frac{R_y\,k}{r_z\,\cosh(k\,\pi)} \\ k &\equiv r_z\sqrt{\frac{-\Lambda}{10\,M^4}} \end{aligned}$$

The $4+1$ brane tensions become coordinate dependent and are given as,

$$
\begin{aligned}
V_1(z) &= -V_2(z) = 8M^2\sqrt{\frac{-\Lambda}{10}}sech(kz) \\
V_3(y) &= 0 \\
V_4(y) &= -\frac{8M^4k}{r_z}\tanh(k\pi)
\end{aligned}
$$

the 3 branes appear at the intersection of the various 4 branes.

Planck scale mass m_0 is warped to

$$m = m_0 \, \frac{r_z \, c}{R_y \, k} \, \exp(-\pi \, c) = m_0 \, \frac{\exp(-\pi \, c)}{\cosh(k \, \pi)}$$

Substantial warping in the z-direction implies small warping along y

$$c \equiv \frac{R_y\, k}{r_z\, \cosh(k\,\pi)}$$

Out of four branes two have scale close to Tev, other two close to Planck scale

More warped directions give more clustering –

Fermion mass splitting

Conclusion

- Braneworld with large extra dimensions introduces new hierarchy
- RS -I model resolves this
- Generalized RSI model – non-flat brane with positive tension
  Interesting new Phenomenology/Cosmology/Black holes
- More than one warped dimensions – Fermion mass splitting in SM
  Interesting Phenomenology/Cosmology because of more moduli fields
- Fermion localization
- Effects of Radion

Can our collider or other gravitational experiments find a <u>conclusive signature</u> for braneworld/extra-dimension ?

and

UNVEIL AN UNEXPLORED WORLD!