Gravitational waves from coalescing compact binaries: Source modelling and Astrophysics

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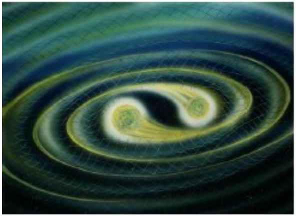
Coalescing compact binaries



Binary systems consisting of Neutron stars (NS) and/or Black Holes (BH).

Astrophysical Relevance

$\Rightarrow$ Most promising sources of GWs for ground-based as well as space based detectors.

$\Rightarrow$ Compact binaries consisting of NSs is believed to be one of the possible progenitors of short duration Gamma Ray Bursts (GRBs).

$\Rightarrow$ Galaxy mergers.

Three phases of binary evolution

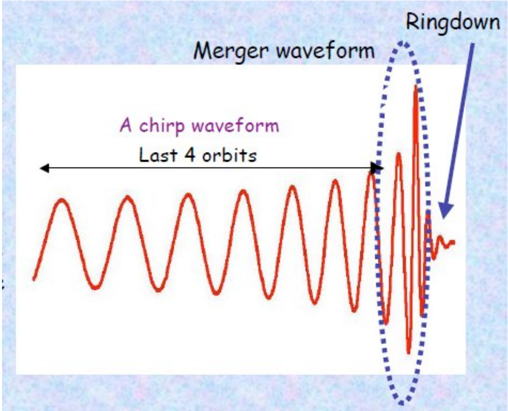


Figure courtesy: Clifford Will

Modelling the evolution of compact binaries

Three phases of evolution

- **Adiabatic inspiral**: Slow motion & Weak field regime–**Post-Newtonian (PN) Approximation** to General Relativity.
- **Merger**: Highly nonlinear phase of evolution–Numerical solutions of Einstein's equation–**Numerical Relativity**.
- **Ringdown**: Newly formed BH settles to a Kerr geometry by radiating away its asymmetries–**BH perturbation theory**.

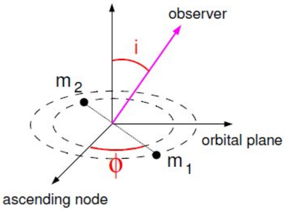
Precise prior waveform modelling $\Rightarrow$ Use of *matched filtering* technique in data analysis.

Improved source modelling $\Rightarrow$ Better astrophysics!!

Post-Newtonian modelling of binary Inspiral

[Review: L Blanchet,Living Rev. Relativity 9, (2006), 4.]

PN modelling of binary inspiral



- Adiabatic inspiral $\Rightarrow \frac{\dot{\omega}}{\omega^2} \ll 1 \Rightarrow T_{\rm GW} >> T_{\rm orb}$.
- Perturbative expansion of Gravitational Fields in powers of v/c (where v is typical velocity) $\Rightarrow$ PN approximation. **[$(v/c)^n \sim \frac{n}{2}$PN**].
- Domain of validity $r \ll \lambda$ (where λ is the characteristic Gravitational Wavelength of the source.)
- Also use other expansions post-Minkowskian (expansion in powers of G), multipole expansion of the gravitational field.

PN waveform

$$h(t) = A(t)e^{i\Phi(t)}$$

PN modelling: Challenges

Provide the maximum accurate waveforms including all possible physical effects.

- Max accuracy: Good enough for the matched filtering procedure to capture the signals without much loss of signal to noise ratio & Good enough to estimate the binary parameters with good accuracy.

  * Phase information more important than amplitude information for detection.
  * Amplitude modelling crucial for accurate parameter extraction.

- Physical effects: Effect of spins, orbital eccentricity, tidal effects $\cdots$.

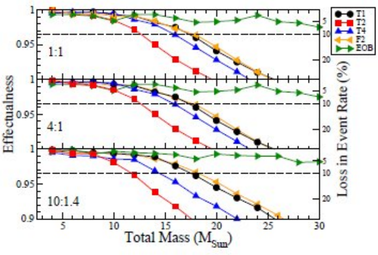
Two kinds of errors

Statistical errors due to the detector noise + Systematic errors due to inaccurate model of the waveform thats used. [Cutler & Vallisneri, 2008]

Characterizing detection & parameter estimation efficiency of PN waveforms

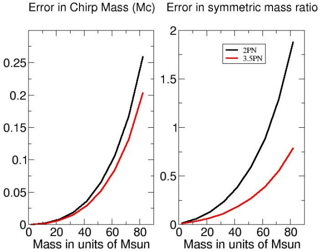
Works of Damour, Iyer & Sathyaprakash (98-2004), Cutler & Vallisneri 08, Buonanno, Iyer, Pan, Oschner& Sathyaprakash 08, KGA, Iyer, Sathyaprakash & Sundararajan 2005, · · ·

Detection



Buonanno, Iyer, Ochsner, Pan & Sathyaprakash, 2008

Parameter Estimation



KGA, Iyer, Sathyaprakash & Sundararajan, 2005

A 3PN waveform would be good enough for the detection & parameter estimation of signals in the Advanced interferometers.

Nonspinning circular orbit binaries

Progresses till last decade:

Phasing:

PN Order	Obtained by
0PN	Peters & Mathews (1963)
1PN	Blanchet & Schaefer (1992)
1.5PN	Blanchet & Schaefer; Wiseman (1993)
2PN	Blanchet, Damour & Iyer, Will & Wiseman (1995)
2.5PN	Blanchet (1996)

Amplitude:

PN Order	Obtained by
0PN	Peters & Mathews (1963)
1PN	Blanchet & Schaefer (1992)
1.5PN	Blanchet, Damour & Iyer, Will & Wiseman (1995)
2PN	Blanchet, Iyer, Will & Wiseman (1996)

Progresses made in the last Decade

Phasing

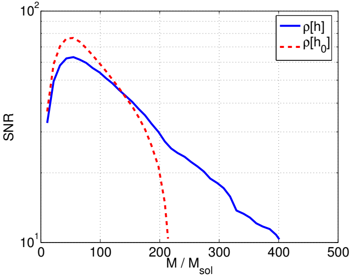
- Blanchet, Faye, Damour, Iyer *et al* in a series of papers derived the energy flux and the complete phasing formula up to 3.5PN order beyond the quadrupole order [BDEI, 2004] for nonspinning compact binaries
- This accuracy is believed to be adequate for the data analysis of Advanced GW interferometers, both for detection & parameter estimation.

Amplitude

- The amplitude of the GW waveform/polarization was computed at 2.5PN [KGA, Blanchet, Iyer, Qusailah, 2004] and 3PN [Blanchet, Faye, Iyer & Sinha, 2008] for nonspinning binaries.

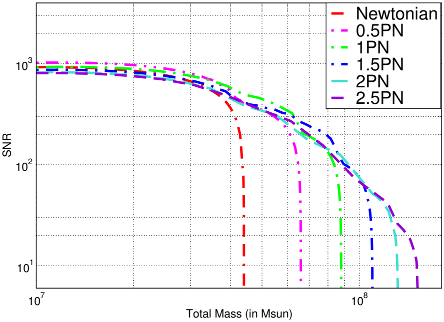
Why bother about Amplitude: SNRs

AdvLIGO:



[Van Den Broeck & Sengupta, 2006]

LISA:

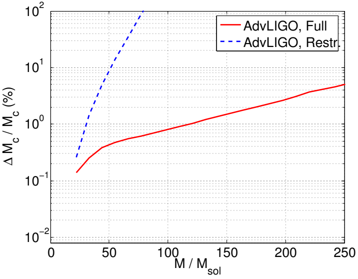


[KGA, Iyer, Sathyaprakash& Sinha, 2007]

Inclusion of PN terms in the amplitude significantly enhances the "mass reach" of GW interferometers.

Why bother about amplitude: Parameter estimation

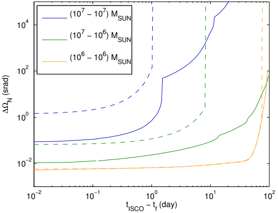
- Improves accuracy of mass estimation of the compact binary using Advanced ground-based detectors.



[Van Den Broeck & Sengupta, 2006]

- Improves mass estimation, angular resolution & Luminosity distance estimation of LISA. Will enable LISA to do high precision cosmology!

  [KGA, Iyer, Sathyaprakash, Sinha, Van Den Broeck, 2007, Trias & Sintes, 2008, Porter & Cornish, 2009]

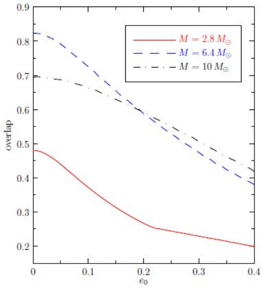


[Trias & Sintes, 2008]

Modelling binaries in elliptical orbits

Motivation

- There may be astrophysical processes which may counter the GW radiation driven circularization (e.g: Kozai mechanism).
- If there is a residual eccentricity, circular orbit templates will not be efficient in detecting elliptic binaries [Martel & Poisson 2000, Tessmer& Gopakumar, 2008, Brown & Zimmerman, 2009]



[Brown & Zimmerman, 2009]

Breakthroughs in modelling elliptic orbit binaries

State of Art

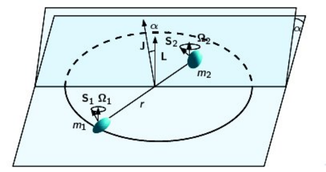
- Gopakumar & Iyer calculated energy flux, angular momentum flux and polarizations from elliptic binaries up to 2PN order[Gopakumar & Iyer, 1997, 2002].
- Damour, Gopakumar & Iyer discusses construction of 2.5PN templates for eccentric orbit binaries using "method of variation of constants" [Damour, Gopakumar & Iyer 2004].
- KGA, Blanchet, Iyer, Qusailah & Sinha computes 3PN energy and angular momentum flux from elliptical orbit binaries[KGA, Blanchet, Iyer &Qusailah 2007a, 2007b; KGA, Blanchet, Iyer & Sinha, 2008].
- Construction of Fourier domain Analytical waveforms for elliptical orbit binaries[Yunes, KGA, Berti, Will, 2009, Tessmer & Schäfer, 2009, 20010, Favata (Unpublished)]
- 3PN Polarizations for elliptic orbit binaries [Work in progress][Mishra, KGA, Iyer].

Spin & precession

Many astrophysical binaries may have spin. No reliable estimation.

Precession

- Spins are aligned along the orbital angular momentum $\Rightarrow$ NO precession.
- If the spins are not aligned along the orbital angular momentum vector $\Rightarrow$ Spins precess around the total angular momentum axis.



[Figure courtesy: Guillaume Faye]

The resultant waveform has more complex structure with the spin induced modulations.

Progresses in modelling spinning binaries

State of Art

- Kidder, Will, Wiseman, Cutler, Thorne $\cdots$: 2PN phasing & 1.5PN h_{ij} (1993-1995).
- Blanchet, Buonanno & Faye, 2006: 2.5PN phasing.
- KGA, Buonanno, Faye & Oschsner, 2008: Complete 1.5PN amplitude for precessing binaries & 2PN amplitude for non-precessing binaries.

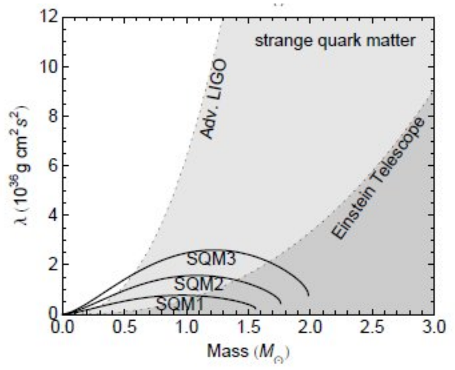
More accuracy required!

Tidal effects in binaries involving Neutron Stars

Tidal effects play an important role in the modelling of NSs as opposed to BHs which can very well be approximated to be point particles.

- Formally tidal effects are 5PN order or higher effect. But towards to end stages, their numerical value will be comparable to the highest point particle PN terms computed $\Rightarrow$ Visible signature.
- Recent works to incorporate the finite size PN effects in the binary modelling.
  - Flanagan & Hinderer, 2008: Leading order tidal effect & its astrophysical implications.
  - Vines, Flanagan & Hinderer, 2010: 1PN correction to the leading order tidal effect term.
  - Recent works also by Damour, Nagar, Binnington, Poisson $\cdots$.

Detecting the tidal effects in NS binaries with GW observations



[Hinderer, Lackey, Lang & Read, 2010.]

- Tidal deformability parameter λ measures NS's quadrupole response to companion's tidal field.
- Value of λ spans an order of magnitude depending on the EOSs considered.
- AdvLIGO will see tidal effects from unusually stiff EOS, where as 3rd generation Einstein Telescope would see clean tidal signatures.

Numerical Relativity

[Review: Centrella, Baker, Kelly & Van Meter, Rev. Mod. Phys. **82**, 3069 (2010)]

Binary BH simulations

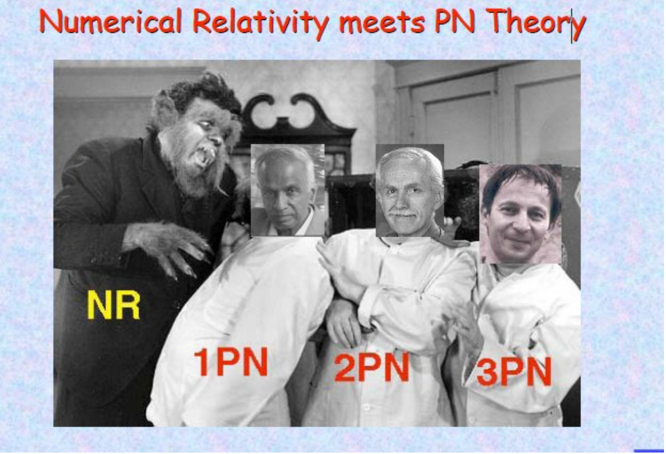
The biggest breakthrough in binary modelling in the last decade!

[Pretorius 2005, Jena Univ group, UTB group, Goddard Group, Caltech & Cornell group ·]

Very wide consequences in

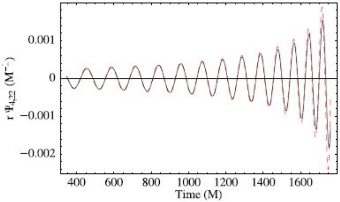
- Understanding the highly nonlinear aspects of GR.
- Excellent agreement with PN waveforms in the inspiral regime.
- Analytical/semi-analytical methods to produce gravitational waveforms which can be used extensively in data analysis pipelines.
- GW data analysis strategies incorporating merger + ringdown phase.
- Detailed understanding of various binary configurations: eccentricity, spins.
- Astrophysics
  - Kick velocity of the merger remnant and astrophysical consequences.
  - Final spin of the newly formed BH.

Numerical relativity takes on Post-Newtonian theory



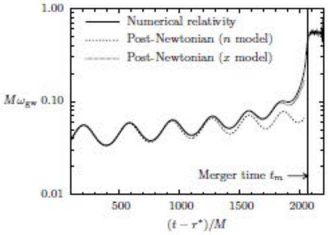
[Picture courtesy Cliff Will]

Nonspinning, circular:



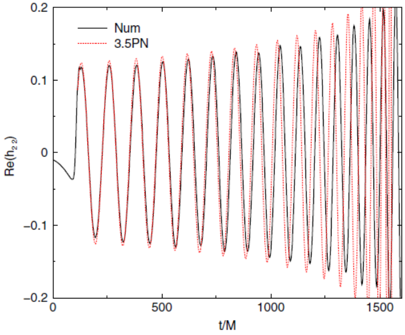
[Hannam et al, 2007]

Nonspinning eccentric:



[Hinder, 2007]

Spin with precession:

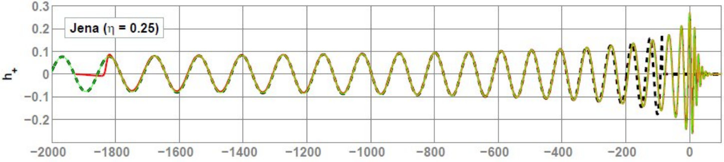


[Campanelli et al, 2007]

Excellent agreement for all cases

Analytical parametrizations of NR waveforms

Time domain: PN smoothly stitched to NR waveform



Ajith et al 2007

- Stitch the PN waveform with the NR waveform.
- Take the FFT of the "stitched waveform".
- Fit it with a function of the form below and estimate various fitting coefficients in the amplitude and the phasing.

Effective waveform

$$\tilde{h}(f) = \frac{C(\mathcal{M},\eta)}{D_L}\, A_{\rm eff}(f)\, e^{i\,\Psi_{\rm eff}(f)}$$

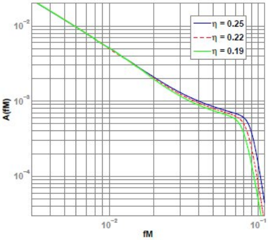
Fourier Domain Phenomenological Waveform

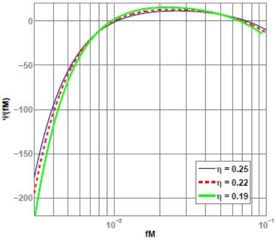
$$h(f) = A_{\rm eff}(f)e^{i\,\Psi_{\rm eff}}.$$

$$A_{\rm eff}(f) \equiv C \begin{cases} (f/f_{\rm merger})^{-7/6} & \text{if } f < f_{\rm merger} \\ (f/f_{\rm merger})^{-2/3} & \text{if } \mathrm{f_{merger}} \leq \mathrm{f} < \mathrm{f_{ring}} \\ w\,\mathcal{L}(f, f_{\rm ring}, \sigma) & \text{if } \mathrm{f_{ring}} \leq \mathrm{f} < \mathrm{f_{cut}} \end{cases}$$

Fourier Domain:

[Ajith et al, 2007, 2008, 2009]





Effective One-body Formalism

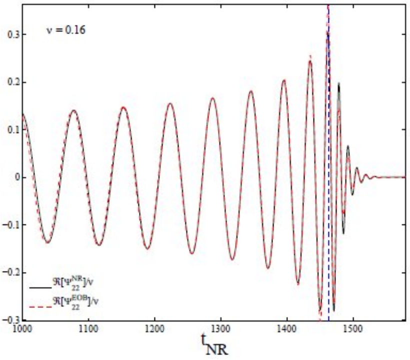
[Buonanno & Damour 1999].

EOB overview

- Two body problem as effective one body moving in an effective potential. Hamiltonian GR.
- Aims to cover the non-perturbative regime which PN approximation cannot access. ($\Rightarrow$ Resummation methods.)
- Goal is to obtain accurate analytical description of motion and radiation reaction including all three phases of the binary evolution.
- **EOB had predicted the smooth transition from inspiral to merger and the shape of the waveform well before the NR results came!!**
- But to have very precise agreement with NR waveforms, they have to adjust the "flexibility parameters" of the theory.

[Original work by Buonanno & Damour, 1999. Also Jaronowski, Schäfer, Chen $\cdots$. Recent extensions by Nagar, Pan, $\cdots$.]

EOBNR



[Damour, Nagar et al 2008]

Efforts to extend EOB to capture features of simulations with mass asymmetry & incorporate tidal effects are going on by various groups!

Mixed binary simulations

Motivation

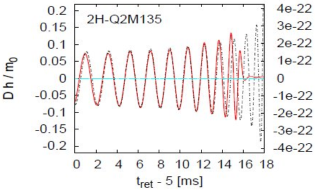
- NS-BH systems are among the most promising GW sources which may also be visible as short duration GRBs $\Rightarrow$ Cosmological implications.
- The detailed nature of the EOS of the NS and other features of NS will show up very close to the merger of the binary.
- Important step towards Binary NS simulations.

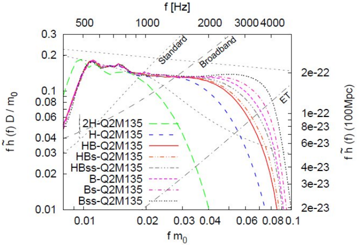
Important features of the NS-BH simulations

- For large mass ratio Q($= \frac{m_{\rm BH}}{m_{\rm NS}}$), no tidal disruption and NS is simply swallowed by the BH. Waveform similar to Binary BH waveforms.
- For $Q \leq 2$, NS may be tidally disrupted by the BH before being swallowed. Amplitudes of merger and ringdown waveforms are very low and the waveform is characterized by inspiral waveform followed by quick damping.
- Tidal disruption suppresses the 'kick' velocity of the binary significantly.

[Shibata et al 2009]

Typical time & Frequency domain waveforms. [Kyutoku et al, 2010]





- $f_{\rm cut}$ (above which GW amplitude damps fast) depends on compactness ($\mathcal{C} = \frac{GM_{\rm NS}}{Rc^2}$) of the NS & EOS.
- Mass of the disk formed after merger also depends on compactness & EOS similarly.
- The final spin of the BH formed seems to depend more on the mass ratio than the EOS of the NS.

Astrophysics

Constraining Short GRB progenitor

- GW carries signatures of coalescence dynamics, formation process of a disk (around the formed BH), EOS of NSs, masses & mass ratio.
- **Formation mechanism of ShGRB $\Rightarrow$ BH and a surrounding disk/torus. GW observations can constrain the mass of the disk above which this mechanism can act as a GRB central engine** [Kiuchi et al, 2010]**.**

GW Cosmography

- With advanced GW detectors several of such NS-NS, NS-BH binaries would be observed. Hope is that a good fraction of them will also have EM counter parts (such as GRBs).
- GW observation provides luminosity distance D_L, EM observations would provide redshift z.
- With many sources, one can practically make a Hubble diagram ($z - D_L$ curve) and estimate various cosmological parameters including dark energy parameter. [Schutz, 1986; Holz & Hughes, 2005, Sathyaprakash et al, 2009; Cutler & Holz 2009.]

Collaborations

- **NINJA**: Numerical INJections Analysis is a collaboration between the data analysis community & the numerical relativity community with the goal of using the NR waveforms efficiently for data analysis purposes
- **NR-AR**: Numerical Relativity-Analytical Relativity is the close collaboration between Numerical Relativity and Analytic Relativity communities.

Summary and Conclusion

- Tremendous progress in modelling compact binaries over the last decade.
- Accuracy of the existing waveforms seems good enough for the data analysis of second generation detectors for many physical scenarios (spin is an exception).

Future

- PN modelling of spinning binaries.
- Efficient analytical parametrizations of NR waveforms for different configurations.
- Modelling binaries with other theories of gravity (Scalar-tensor theory, Chern-Simon theory etc) and obtain good accuracy waveforms.
- Black hole NR simulations for asymmetric binaries & Simulations of binaries involving NS as a component.
- Self-force problem for extreme mass ratio inspirals.