Black Holes and the Big Bang: Loop Quantum Gravity Perspectives

Parthasarathi Majumdar,

Saha Institute of Nuclear Physics, Kolkata, India

Sangam : Confluence of Cosmology and Gravitation

IAGRG 26

Harishchandra Research Institute, Allahabad, 19-21 January 2011

January 18, 2011

Why Quantum Gravity ?

Why Quantum Gravity ?

- Black hole entropy : does not exist classically

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : ‘big bang’ (past) and black hole/naked (future)**

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : 'big bang' (past) and black hole/naked (future)**

Analogy with incompleteness of Maxwell electrodynamics $\Rightarrow$ expect appropriate formulation of 'Quantum Spacetime Geometry' to resolve conundra

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : ‘big bang’ (past) and black hole/naked (future)**

Analogy with incompleteness of Maxwell electrodynamics $\Rightarrow$ expect appropriate formulation of ‘Quantum Spacetime Geometry’ to resolve conundra

No Complete Theory Yet of Quantum Sptm Geom !

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : ‘big bang’ (past) and black hole/naked (future)**

Analogy with incompleteness of Maxwell electrodynamics $\Rightarrow$ expect appropriate formulation of ‘Quantum Spacetime Geometry’ to resolve conundra

No Complete Theory Yet of Quantum Sptm Geom !

Survey **Loop Quantum Gravity** proposal on

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : 'big bang' (past) and black hole/naked (future)**

Analogy with incompleteness of Maxwell electrodynamics $\Rightarrow$ expect appropriate formulation of 'Quantum Spacetime Geometry' to resolve conundra

No Complete Theory Yet of Quantum Sptm Geom !

Survey **Loop Quantum Gravity** proposal on

- Ab initio understanding of black hole entropy $\longrightarrow$ area law $+$ signature corrections

Why Quantum Gravity ?

- **Black hole entropy : does not exist classically**
- **Inevitability of sptm singularities in cl GR : 'big bang' (past) and black hole/naked (future)**

Analogy with incompleteness of Maxwell electrodynamics $\Rightarrow$ expect appropriate formulation of 'Quantum Spacetime Geometry' to resolve conundra

No Complete Theory Yet of Quantum Sptm Geom !

Survey **Loop Quantum Gravity** proposal on

- Ab initio understanding of black hole entropy $\longrightarrow$ area law + signature corrections
- Resolution of Big Bang singularity (simple models)

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2}\,(k_B = 1)$$

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2}\,(k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \rightarrow$ quantum gravity

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2} \, (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \longrightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2}\,(k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33}cm \longrightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**

Need to go beyond classical GR - compulsion, not aesthetics

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2} \, (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \rightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**

Need to go beyond classical GR - compulsion, not aesthetics

Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2} \, (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33}cm \longrightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**
Need to go beyond classical GR - compulsion, not aesthetics
Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Two issues to be addressed:

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4 l_P^2} \; (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \longrightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**
Need to go beyond classical GR - compulsion, not aesthetics
Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Two issues to be addressed:

- **How is it that** $S_{bh} = S_{bh}(\mathcal{A}_{hor})$ **while** $S_{thermo} = S_{thermo}(vol)$ **?**

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2} \, (k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \rightarrow$ quantum gravity

$S_{bh} \propto l_P^{-2} \Rightarrow$ **non-perturbative**
Need to go beyond classical GR - compulsion, not aesthetics
Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Two issues to be addressed:

- **How is it that** $S_{bh} = S_{bh}(\mathcal{A}_{hor})$ **while** $S_{thermo} = S_{thermo}(vol)$ **?**
- **What degrees of freedom contribute to** S_{bh} **?**

Grav energy globally defined

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi} \int_{\mathcal{S}_\infty} d^2\sigma^{ab} \nabla_a K_b$$

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi} \int_{\mathcal{S}_\infty} d^2\sigma^{ab} \nabla_a K_b$$

Classically, bulk $\Rightarrow$ boundary entirely

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi}\int_{\mathcal{S}_\infty} d^2\sigma^{ab}\nabla_a K_b$$

Classically, bulk $\Rightarrow$ boundary entirely

Holography: 3 dim bulk info encoded on 2 dim bdy

QGR: $\exists$ indep qu fluct on bdy : $\mathcal{H} = \mathcal{H}_v \otimes \mathcal{H}_b$

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi}\int_{\mathcal{S}_\infty} d^2\sigma^{ab}\nabla_a K_b$$

Classically, bulk $\Rightarrow$ boundary entirely

Holography: 3 dim bulk info encoded on 2 dim bdy

QGR: $\exists$ indep qu fluct on bdy : $\mathcal{H} = \mathcal{H}_v \otimes \mathcal{H}_b$

$$|\Psi\rangle \;=\; \sum_{v,b} c_{vb} \underbrace{|\psi_v\rangle}_{blk} \underbrace{|\chi_b\rangle}_{bdy} \in \mathcal{H}_v \otimes \mathcal{H}_b$$

‘Quantum Einstein EQ’ (bulk)

$$\hat{H}_v \, |\psi_v\rangle \; = \; 0$$

‘Quantum Einstein EQ’ (bulk)

$$\hat{H}_v \, |\psi_v\rangle \;=\; 0$$

$$
\begin{aligned}
Z \;&=\; \sum_b \left(\sum_v |c_{vb}|^2 \| \, |\psi_v\rangle \, \|^2 \right) \langle \chi_b | \exp -\beta \hat{H}_{bdy} | \chi_b \rangle \\
&\equiv\; Z_{bdy}
\end{aligned}
$$

‘Quantum Einstein EQ’ (bulk)

$$\hat{H}_v \, |\psi_v\rangle \; = \; 0$$

$$Z \; = \; \sum_b \left(\sum_v |c_{vb}|^2 || \, |\psi_v\rangle \, ||^2 \right) \langle\chi_b| \exp -\beta \hat{H}_{bdy} |\chi_b\rangle$$
$$\equiv \; Z_{bdy}$$

Bulk states decouple! $\longrightarrow$ **Thermal holography !** (PM 2007, 2009)

‘Quantum Einstein EQ’ (bulk)

$$\hat{H}_v \, |\psi_v\rangle \; = \; 0$$

$$Z \; = \; \sum_b \left(\sum_v |c_{vb}|^2 || \, |\psi_v\rangle \, ||^2 \right) \langle\chi_b| \exp -\beta \hat{H}_{bdy} |\chi_b\rangle$$
$$\equiv \; Z_{bdy}$$

Bulk states decouple! $\longrightarrow$ **Thermal holography !** (PM 2007, 2009)

Weaker version of holography cf ‘Holographic Hypothesis’ ’t Hooft 1993; Susskind 1995

‘Quantum Einstein EQ’ (bulk)

$$\hat{H}_v \, |\psi_v\rangle \; = \; 0$$

$$Z \; = \; \sum_b \left(\sum_v |c_{vb}|^2 || \, |\psi_v\rangle \, ||^2 \right) \langle \chi_b | \exp - \beta \hat{H}_{bdy} | \chi_b \rangle$$
$$\equiv \; Z_{bdy}$$

Bulk states decouple! $\longrightarrow$ **Thermal holography !** (PM 2007, 2009)

Weaker version of holography cf ‘Holographic Hypothesis’ ’t Hooft 1993; Susskind 1995

Canonical Ensemble of (isolated) horizons (as sptm bdy) : States characterized by $A_n \sim n \, l_P^2$, $n \in$ **Z** (LQG)

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$\begin{aligned} Z(\beta) &= \sum_n g(M(A_n)) \exp -\beta M(A_n) \\ &\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor}) \end{aligned}$$

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$Z(\beta) = \sum_n g(M(A_n)) \exp -\beta M(A_n)$$
$$\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor})$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$\begin{aligned} Z(\beta) &= \sum_n g(M(A_n)) \exp -\beta M(A_n) \\ &\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor}) \end{aligned}$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Stable thermal equil

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$Z(\beta) = \sum_n g(M(A_n)) \exp -\beta M(A_n)$$
$$\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor})$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Stable thermal equil

$$\Rightarrow S_{can} > 0 \Rightarrow \Delta > 0$$

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$
\begin{aligned}
Z(\beta) &= \sum_n g(M(A_n)) \exp -\beta M(A_n) \\
&\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor})
\end{aligned}
$$

Canon entropy

$$
S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta
$$

Stable thermal equil

$$
\Rightarrow S_{can} > 0 \Rightarrow \Delta > 0
$$

Criterion for **Thermal Stability** PM 2007

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$Z(\beta) = \sum_n g(M(A_n)) \exp -\beta M(A_n)$$
$$\simeq \exp [S(A_{hor}) - \beta M(A_{hor})] \cdot \Delta^{-1/2}(A_{hor})$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Stable thermal equil

$$\Rightarrow S_{can} > 0 \Rightarrow \Delta > 0$$

Criterion for **Thermal Stability** PM 2007

$$\frac{M(A_{hor})}{M_P} > \frac{S(A_{hor})}{k_B}$$

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$Z(\beta) = \sum_n g(M(A_n)) \exp -\beta M(A_n)$$
$$\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor})$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Stable thermal equil

$$\Rightarrow S_{can} > 0 \Rightarrow \Delta > 0$$

Criterion for **Thermal Stability** PM 2007

$$\frac{M(A_{hor})}{M_P} > \frac{S(A_{hor})}{k_B}$$

Classical geom not used in derivation : QG origin

Hor partition fct ($G = c = k_B = 1$) Das, Bhaduri, PM 2001; Chatterjee, PM 2003, 2005

$$Z(\beta) = \sum_n g(M(A_n)) \exp -\beta M(A_n)$$
$$\simeq \exp\left[S(A_{hor}) - \beta M(A_{hor})\right] \cdot \Delta^{-1/2}(A_{hor})$$

Canon entropy

$$S_{can}(A_{hor}) = S(A_{hor}) + \frac{1}{2} \log \Delta$$

Stable thermal equil

$$\Rightarrow S_{can} > 0 \Rightarrow \Delta > 0$$

Criterion for **Thermal Stability** PM 2007

$$\frac{M(A_{hor})}{M_P} > \frac{S(A_{hor})}{k_B}$$

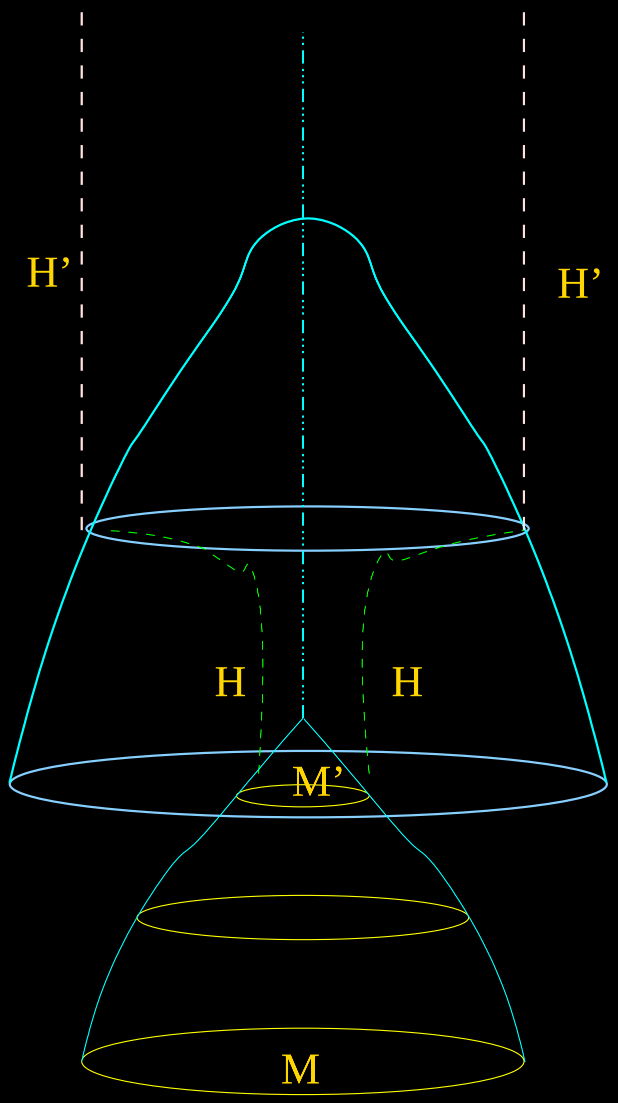
Classical geom not used in derivation : QG origin

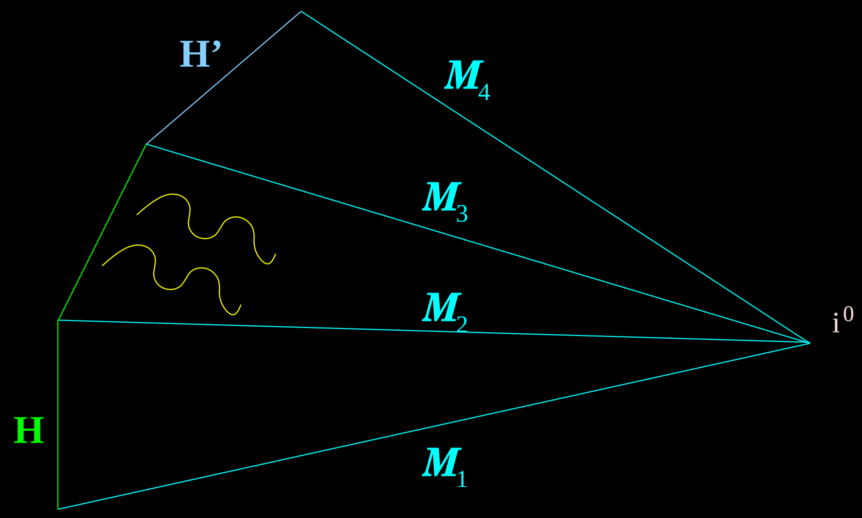
But $S(A_{hor}) = ?$

- Equil (Isolated) horizon ? Ashtekar et. al. 1997-2000

- Equil (Isolated) horizon ? Ashtekar et. al. 1997-2000
- Horizon deg of freedom & dynamics ? Ashtekar et. al. 1997-2000; Basu, Kaul, PM 2009,; Kaul, PM 2010; Basu, Chatterjee, Ghosh 2010; Engel et. al. 2009-10

- Equil (Isolated) horizon ? Ashtekar et. al. 1997-2000
- Horizon deg of freedom & dynamics ? Ashtekar et. al. 1997-2000; Basu, Kaul, PM 2009,; Kaul, PM 2010; Basu, Chatterjee, Ghosh 2010; Engel et. al. 2009-10
- Counting of horizon states ? Ashtekar et. al. 1997,2000; Kaul, PM 1998,2000; Das, Kaul, PM 2001


H’
H’
H
H
M’
M


H’
M_4
M_3
M_2
i^0
H
M_1

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph
- Compute pull back of cross-product of tetrads to sph fol of horizon

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph
- Compute pull back of cross-product of tetrads to sph fol of horizon
- Result:

$$\frac{k}{2\pi}\mathbf{F}_{ab}(\mathbf{A}) = -\Sigma_{ab}$$

where, $k = \# \cdot (A_{hor}/l_P^2)$, $k >> 1$

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph
- Compute pull back of cross-product of tetrads to sph fol of horizon
- Result:

$$\frac{k}{2\pi}\mathbf{F}_{ab}(\mathbf{A}) = -\Sigma_{ab}$$

where, $k = \# \cdot (A_{hor}/l_P^2)$, $k >> 1$

- $SU(2)$ Chern Simons gauge theory EoM

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph
- Compute pull back of cross-product of tetrads to sph fol of horizon
- Result:

$$\frac{k}{2\pi}\mathbf{F}_{ab}(\mathbf{A}) = -\Sigma_{ab}$$

  where, $k = \# \cdot (A_{hor}/l_P^2)$, $k >> 1$
- $SU(2)$ Chern Simons gauge theory EoM
- Can gauge fix to $U(1)$ CS with extra conditions on sources $\Rightarrow$ $SU(2)$ dynamics Basu, Kaul, PM 2009; Kaul, PM 2010

- Start w/ Schwarzschild metric : choose a tetrad basis and compute spin connection coeff and curvature comp Kaul, PM 2010
- Define Barbero-Immirzi $SU(2)$ connection
- Pull back to horizon (sph fol) and compute curvature on sph
- Compute pull back of cross-product of tetrads to sph fol of horizon
- Result:

$$\frac{k}{2\pi}\mathbf{F}_{ab}(\mathbf{A}) = -\Sigma_{ab}$$

where, $k = \# \cdot (A_{hor}/l_P^2)$, $k >> 1$

- $SU(2)$ Chern Simons gauge theory EoM
- Can gauge fix to $U(1)$ CS with extra conditions on sources $\Rightarrow$ $SU(2)$ dynamics Basu, Kaul, PM 2009; Kaul, PM 2010
- **Gravity-gauge theory (topol) link derived**

Loop Quantum Gravity : background-indep, non-perturbative

Loop Quantum Gravity : background-indep, non-perturbative

$SL(2,C)$ Variables : $e_a^I \,, \omega_a^{IJ} \rightarrow \Sigma_{ab}^{IJ} \equiv e_{[a}^I e_{b]}^J \,,\; R_{ab}^{IJ} \equiv \partial_{[a}\omega_{b]}^{IJ} + \omega_{[a}^{IK}\omega_{b]K}^{J}$

Loop Quantum Gravity : background-indep, non-perturbative

$SL(2,C)$ Variables : e_a^I , $\omega_a^{IJ} \longrightarrow \Sigma_{ab}^{IJ} \equiv e_{[a}^I e_{b]}^J$, $R_{ab}^{IJ} \equiv \partial_{[a}\omega_{b]}^{IJ} + \omega_{[a}^{IK}\omega_{b]K}^{J}$

$$\begin{aligned} S[e,\omega] &= S_{HP} + S_{NY} \\ S_{HP} &= -\frac{1}{16\pi G}\int_{\mathcal{M}} e\,\Sigma_{IJ}^{ab}\,R_{ab}^{IJ} \\ S_{NY} &= \frac{1}{2\gamma}\int_{\mathcal{M}} \epsilon^{abcd}\left[\left(D_a e_b^I\right)\left(D_c e_{dI}\right) - \frac{1}{2}\Sigma_{ab}^{IJ}\,R_{cdIJ}\right] \end{aligned}$$

Loop Quantum Gravity : background-indep, non-perturbative

$SL(2,C)$ Variables : $e_a^I\,,\omega_a^{IJ} \longrightarrow \Sigma_{ab}^{IJ} \equiv e_{[a}^I e_{b]}^J\,,\ R_{ab}^{IJ} \equiv \partial_{[a}\omega_{b]}^{IJ} + \omega_{[a}^{IK}\omega_{b]K}^J$

$$\begin{aligned} S[e,\omega] &= S_{HP} + S_{NY} \\ S_{HP} &= -\frac{1}{16\pi G}\int_{\mathcal{M}} e\,\Sigma_{IJ}^{ab}\,R_{ab}^{IJ} \\ S_{NY} &= \frac{1}{2\gamma}\int_{\mathcal{M}} \epsilon^{abcd}\left[(D_a e_b^I)\,(D_c e_{dI}) - \frac{1}{2}\Sigma_{ab}^{IJ}\,R_{cdIJ}\right] \end{aligned}$$

S_{NY} is topological, γ (Barbero-Immirzi) resembles θ_{YM}

Loop Quantum Gravity : background-indep, non-perturbative

$SL(2,C)$ Variables : $e^I_a \,, \omega^{IJ}_a \longrightarrow \Sigma^{IJ}_{ab} \equiv e^I_{[a} e^J_{b]} \,,\ R^{IJ}_{ab} \equiv \partial_{[a}\omega^{IJ}_{b]} + \omega^{IK}_{[a}\omega^J_{b]K}$

$$\begin{aligned} S[e,\omega] &= S_{HP} + S_{NY} \\ S_{HP} &= -\frac{1}{16\pi G}\int_{\mathcal{M}} e\,\Sigma^{ab}_{IJ}\, R^{IJ}_{ab} \\ S_{NY} &= \frac{1}{2\gamma}\int_{\mathcal{M}} \epsilon^{abcd}\left[\left(D_a e^I_b\right)\left(D_c e_{dI}\right) - \frac{1}{2}\Sigma^{IJ}_{ab}\, R_{cdIJ}\right] \end{aligned}$$

S_{NY} is topological, γ (Barbero-Immirzi) resembles θ_{YM}

Hamiltonian formulation (Date, Kaul, Sengupta 2008) : On spatial slice M_t local Lorentz boosts gauge fixed to give Barbero-Immirzi $SU(2)$ gauge theories : $A^{(\gamma)I}_a \equiv \gamma\omega^I_a + K^I_a \,,\ E^a_I : |\det\, q| q^{ab} = E^a_I E^b_J \delta^{IJ}$

Loop Quantum Gravity : background-indep, non-perturbative

$SL(2,C)$ Variables : $e_a^I \, , \omega_a^{IJ} \to \Sigma_{ab}^{IJ} \equiv e_{[a}^I e_{b]}^J \, , \; R_{ab}^{IJ} \equiv \partial_{[a}\omega_{b]}^{IJ} + \omega_{[a}^{IK}\omega_{b]K}^{J}$

$$\begin{aligned} S[e,\omega] &= S_{HP} + S_{NY} \\ S_{HP} &= -\frac{1}{16\pi G}\int_{\mathcal{M}} e\,\Sigma_{IJ}^{ab}\,R_{ab}^{IJ} \\ S_{NY} &= \frac{1}{2\gamma}\int_{\mathcal{M}} \epsilon^{abcd}\left[\,(D_a e_b^I)\,(D_c e_{dI}) - \frac{1}{2}\Sigma_{ab}^{IJ}\,R_{cdIJ}\right] \end{aligned}$$

S_{NY} is topological, γ (Barbero-Immirzi) resembles θ_{YM}

Hamiltonian formulation (Date, Kaul, Sengupta 2008) : On spatial slice M_t local Lorentz boosts gauge fixed to give Barbero-Immirzi $SU(2)$ gauge theories : $A_a^{(\gamma)I} \equiv \gamma\omega_a^I + K_a^I \, , \; E_I^a : |\det\, q|q^{ab} = E_I^a E_J^b \delta^{IJ}$

$$\left[A_a^I(x), E_J^b(y)\right]_{PB} = 8\pi G\gamma\,\delta_a^b\,\delta_J^I\,\delta^{(3)}(x,y)$$

First class constraints

First class constraints

$$G_I(\Lambda) \;=\; \int_{\mathbf{M}_t} \Lambda^I \; D_a(A) \; E^a_I \quad \text{Gauss}$$

$$C_a(N^a) \;=\; \int_{\mathbf{M}_t} N^a \; E^b_I \; F^I_{ab} \quad \text{Diffeo}$$

$$C(N) \;=\; \frac{1}{16\pi G\gamma} \int_{\mathbf{M}_t} N \; \frac{E^a_I E^b_J}{|E|^{1/2}} \; \left[\frac{1}{2}\epsilon^{IJK} F_{abK}(A) - (1+\gamma^{-2}) K^I_a K^J_b\right] \text{ Hamil}$$

First class constraints

$$G_I(\Lambda) = \int_{\mathbf{M}_t} \Lambda^I \, D_a(A) \, E^a_I \quad \text{Gauss}$$

$$C_a(N^a) = \int_{\mathbf{M}_t} N^a \, E^b_I \, F^I_{ab} \quad \text{Diffeo}$$

$$C(N) = \frac{1}{16\pi G\gamma} \int_{\mathbf{M}_t} N \, \frac{E^a_I E^b_J}{|E|^{1/2}} \left[\frac{1}{2}\epsilon^{IJK} F_{abK}(A) - (1+\gamma^{-2}) K^I_a K^J_b\right] \text{Hamil}$$

Global variables :

First class constraints

$$G_I(\Lambda) = \int_{\mathbf{M}_t} \Lambda^I \, D_a(A) \, E^a_I \quad \text{Gauss}$$

$$C_a(N^a) = \int_{\mathbf{M}_t} N^a \, E^b_I \, F^I_{ab} \quad \text{Diffeo}$$

$$C(N) = \frac{1}{16\pi G\gamma} \int_{\mathbf{M}_t} N \, \frac{E^a_I E^b_J}{|E|^{1/2}} \left[\frac{1}{2}\epsilon^{IJK} F_{abK}(A) - (1+\gamma^{-2}) K^I_a K^J_b\right] \text{ Hamil}$$

Global variables :

$$\text{holonomies } \; h_C(A) \equiv \mathcal{P} \, \exp \int_C A$$

$$\text{fluxes } \; \mathcal{E}[S, f] \equiv \int_S E^a_I \, \epsilon_{abc} \, dx^a \wedge dx^b \, f^I$$

First class constraints

$$G_I(\Lambda) = \int_{\mathbf{M}_t} \Lambda^I \, D_a(A) \, E^a_I \quad \text{Gauss}$$

$$C_a(N^a) = \int_{\mathbf{M}_t} N^a \, E^b_I \, F^I_{ab} \quad \text{Diffeo}$$

$$C(N) = \frac{1}{16\pi G\gamma} \int_{\mathbf{M}_t} N \, \frac{E^a_I E^b_J}{|E|^{1/2}} \left[\frac{1}{2}\epsilon^{IJK} F_{abK}(A) - (1+\gamma^{-2}) K^I_a K^J_b\right] \text{Hamil}$$

Global variables :

$$\text{holonomies} \;\; h_C(A) \equiv \mathcal{P} \; \exp \int_C A$$

$$\text{fluxes} \;\; \mathcal{E}[S, f] \equiv \int_S E^a_I \, \epsilon_{abc} \, dx^a \wedge dx^b f^I$$

Cylindrical functionals: $\psi[A] = \psi \; \left(h_{C_1}(A), h_{C_2}(A), ...h_{C_n}(A)\right)$

Algebra of observables :

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\; X[S,r]\,\psi \\ &\ldots \end{aligned}$$

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\; X[S,r]\,\psi \\ &\dots \end{aligned}$$

$X[S,r] \rightarrow$ derivation on $\{\psi\}$

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\, X[S,r]\,\psi \\ &\dots \end{aligned}$$

$X[S,r] \longrightarrow$ derivation on $\{\psi\}$

Convenient basis : Generalized spin networks

Algebra of observables :

$$
\begin{aligned}
\psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\
[\,\psi, E[S,r]\,] &= 8\pi G\gamma\; X[S,r]\,\psi \\
&\dots
\end{aligned}
$$

$X[S,r] \rightarrow$ derivation on $\{\psi\}$

Convenient basis : Generalized spin networks

Edges : oriented curves embedded in M_t carrying $SU(2)$ spin $j = 0, 1/2, 1, 3/2, \dots$

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\, X[S,r]\,\psi \\ &\dots \end{aligned}$$

$X[S,r] \longrightarrow$ derivation on $\{\psi\}$

Convenient basis : Generalized spin networks

Edges : oriented curves embedded in M_t carrying $SU(2)$ spin $j = 0, 1/2, 1, 3/2, \ldots$

Vertices : invariant $SU(2)$ tensors constructed out of spins on edges entering or leaving vertex

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\, X[S,r]\,\psi \\ &\dots \end{aligned}$$

$X[S,r] \longrightarrow$ derivation on $\{\psi\}$

Convenient basis : Generalized spin networks

Edges : oriented curves embedded in M_t carrying $SU(2)$ spin $j = 0, 1/2, 1, 3/2, \dots$

Vertices : invariant $SU(2)$ tensors constructed out of spins on edges entering or leaving vertex

Graph g consisting of l edges with spins $j_1, \dots, j_l$ and v vertices $\longrightarrow \psi = \prod_{i\in\{l\}} h_i(A) \cdot \prod_{k\in\{v\}} \mathcal{T}_k$

Algebra of observables :

$$\begin{aligned} \psi_1 \cdot \psi_2\,[A] &= \psi_1\,[A]\,\psi_2\,[A] \\ [\,\psi, E[S,r]\,] &= 8\pi G\gamma\, X[S,r]\,\psi \\ &\dots \end{aligned}$$

$X[S,r] \rightarrow$ derivation on $\{\psi\}$

Convenient basis : Generalized spin networks

Edges : oriented curves embedded in M_t carrying $SU(2)$ spin $j = 0, 1/2, 1, 3/2, \dots$

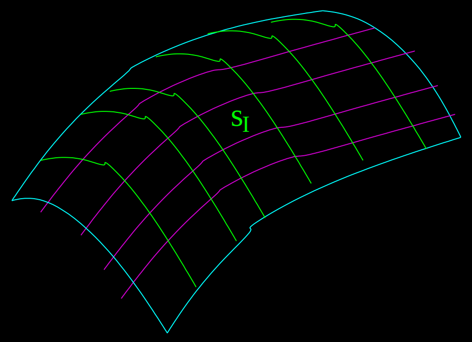
Vertices : invariant $SU(2)$ tensors constructed out of spins on edges entering or leaving vertex

Graph g consisting of l edges with spins $j_1, \dots, j_l$ and v vertices $\rightarrow \psi = \prod_{i\in\{l\}} h_i(A) \cdot \prod_{k\in\{v\}} \mathcal{T}_k$

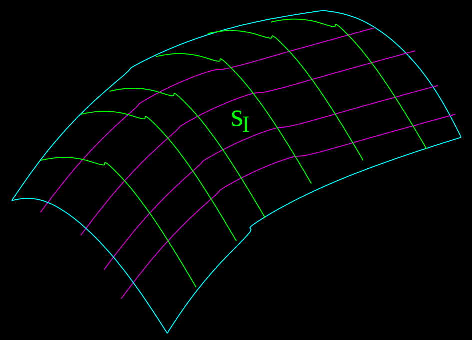
A given $\psi \rightarrow$ linear combination of spinnet graphs

Spin network : Quantum Space

Area operator (also volume, length) have bded, discrete spectrum

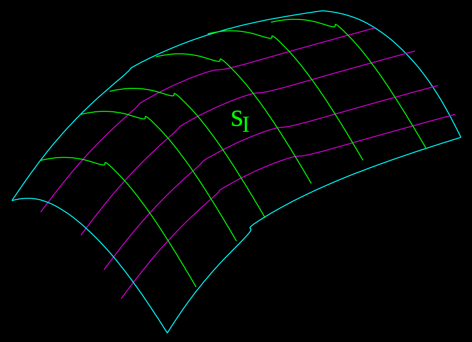


Area operator (also volume, length) have bded, discrete spectrum



$$\hat{\mathcal{A}}_S \equiv \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2g(\hat{E})]$$

Area operator (also volume, length) have bded, discrete spectrum

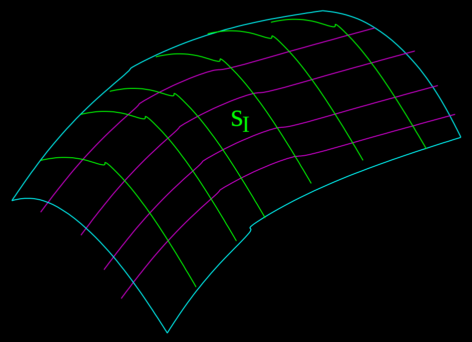


$$\hat{A}_S \equiv \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2g(\hat{E})]$$

$$a(j_1, \ldots, j_N) = \frac{1}{4}\gamma l_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}$$

$$\lim_{N\to\infty} a(j_1, \ldots . j_N) \leq \mathcal{A}_{cl} + O(l_P^2)$$

Area operator (also volume, length) have bded, discrete spectrum



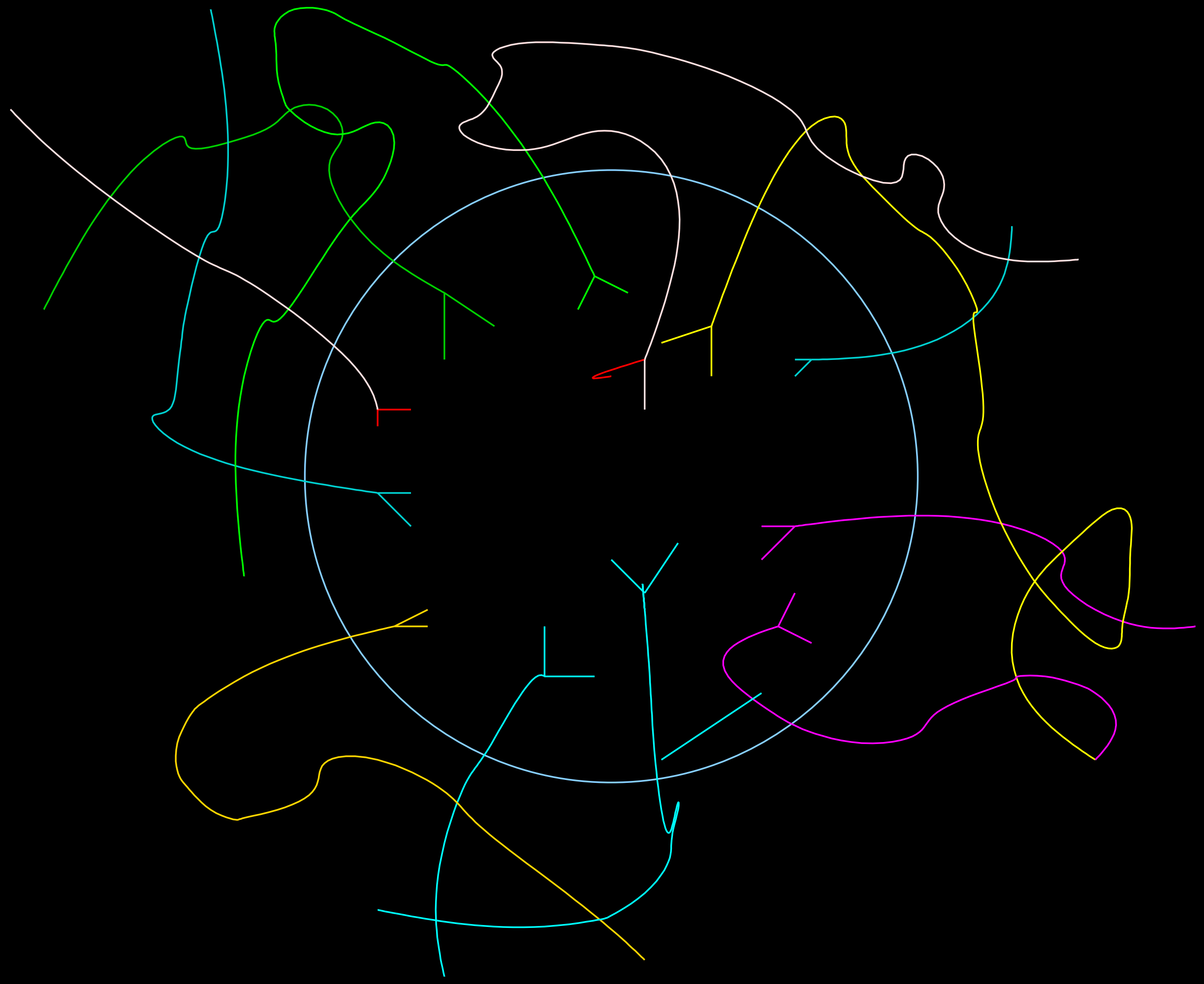
$$\hat{A}_S \equiv \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2 g(\hat{E})]$$

$$a(j_1, \ldots, j_N) = \frac{1}{4}\gamma l_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}$$

$$\lim_{N \to \infty} a(j_1, \ldots j_N) \leq \mathcal{A}_{cl} + O(l_P^2)$$

Equispaced $\forall j_p = 1/2$

‘Quantum’ Isolated Horizon → effective description (Ashtekar, Baez, Corichi, Krasnov 1997)



Need to compute $S_{IH} = \log \dim \mathcal{H}_{CS+ptsources(j_1,\ldots j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Need to compute $S_{IH} = \log\dim\mathcal{H}_{CS+ptsources(j_1,...j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Witten (1986) : $\dim\mathcal{H}_{CS} = \#conf\ blocks\ of\ SU(2)_k\, WZW\ (CFT_2)$ on punctured S^2

Need to compute $S_{IH} = \log \dim \mathcal{H}_{CS+ptsources(j_1,...j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Witten (1986) : $\dim \mathcal{H}_{CS} = \#conf\ blocks\ of\ SU(2)_k\, WZW\ (CFT_2)$ on punctured S^2

4 dim gravity $\rightarrow$ 2 dim CFT link

Need to compute $S_{IH} = \log \dim \mathcal{H}_{CS+ptsources(j_1,...j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Witten (1986) : $\dim \mathcal{H}_{CS} = \#conf\ blocks\ of\ SU(2)_k\ WZW\ (CFT_2)$ on punctured S^2

4 dim gravity $\rightarrow$ 2 dim CFT link

$\Rightarrow$ (Kaul, PM 1998)

$$
\begin{aligned}
\dim\ \mathcal{H}_{CS+(j_1,...,j_n)} &= \prod_{p=1}^{n} \sum_{m_p=-j_p}^{j_p} [\delta_{m_1+\cdots+m_n,0} \\
&- \frac{1}{2}\delta_{m_1+\cdots+m_n,-1} \\
&- \frac{1}{2}\delta_{m_1+\cdots+m_n,1}]
\end{aligned}
$$

If $j_p = \frac{1}{2} \, \forall \, p = 1, \ldots, n$

If $j_p = \frac{1}{2} \; \forall \; p = 1, \ldots, n$

$$S_{mc} = S_{IH} = \underbrace{\frac{A_{IH}}{4l_P^2}}_{\text{(Ashtekar et. al. 1997)}} \underbrace{- \frac{3}{2} \log \left(\frac{A_{IH}}{4l_P^2} \right) + const. + O(A_{IH}^{-1})}_{\text{(Kaul,PM 2000)}}$$

If $j_p = \frac{1}{2} \, \forall \, p = 1, \ldots, n$

$$S_{mc} = S_{IH} = \underbrace{\frac{A_{IH}}{4 l_P^2}}_{\text{(Ashtekar et. al. 1997)}}$$

$$- \underbrace{\frac{3}{2} \log \left(\frac{A_{IH}}{4 l_P^2} \right) + const. + O(A_{IH}^{-1})}_{\text{(Kaul,PM 2000)}}$$

Infinite series of corrections to semicl BHAL : characteristic signature of LQG

If $j_p = \frac{1}{2} \, \forall \, p = 1, \ldots, n$

$$S_{mc} = S_{IH} = \underbrace{\frac{A_{IH}}{4l_P^2}}_{\text{(Ashtekar et. al. 1997)}} \underbrace{- \frac{3}{2} \log\left(\frac{A_{IH}}{4l_P^2}\right) + const. + O(A_{IH}^{-1})}_{\text{(Kaul,PM 2000)}}$$

Infinite series of corrections to semicl BHAL : characteristic signature of LQG

Corollary :

$$\beta = \beta_{Haw}\left(1 + \frac{6l_P^2}{A_{hor}} + \ldots\right)$$

Grav Hamiltonian operator

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Symmetry Reduction Bojowald 2001; rev. Date 2010

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Symmetry Reduction Bojowald 2001; rev. Date 2010

Given $\mathcal{H}$ and symmetry group $\mathcal{G}$, $\mathcal{H}/\mathcal{G} \longrightarrow$ space of orbits. Restrict to $\mathcal{H}_{inv} \subset \mathcal{H}/\mathcal{G} \longrightarrow$ space of trivial orbits

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Symmetry Reduction Bojowald 2001; rev. Date 2010

Given $\mathcal{H}$ and symmetry group $\mathcal{G}$, $\mathcal{H}/\mathcal{G}$ $\longrightarrow$ space of orbits. Restrict to $\mathcal{H}_{inv} \subset \mathcal{H}/\mathcal{G} \longrightarrow$ space of trivial orbits

Strategy : **Quantization after reduction :** Reduce to $\Gamma_{inv} \subset \Gamma/\mathcal{G}$ and then quantize

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Symmetry Reduction Bojowald 2001; rev. Date 2010

Given $\mathcal{H}$ and symmetry group $\mathcal{G}$, $\mathcal{H}/\mathcal{G} \longrightarrow$ space of orbits. Restrict to $\mathcal{H}_{inv} \subset \mathcal{H}/\mathcal{G} \longrightarrow$ space of trivial orbits

Strategy : **Quantization after reduction :** Reduce to $\Gamma_{inv} \subset \Gamma/\mathcal{G}$ and then quantize

Example: sph symm models : $ds^2 = d\tau^2 - f^2(\tau,R)dR^2 - r^2(\tau,R)\,(d\theta^2 + \sin^2\theta\, d\phi^2) \longrightarrow f(\tau,R)$, $r(\tau,R)$ only dynamical dof

Mini-superspace : homogeneous and isotropic spacetimes (Bianchi cosmologies) described by finitely many dof

Grav Hamiltonian operator

$$\hat{C}(N) = \sum_{v,IJK} N(v)\epsilon^{IJK}\, tr\left[h_{v,I}\, h_{v+I,J}\, h^{-1}_{v+J,I}\, h^{-1}_{v,J}\, h_{v,K}\, [h^{-1}_{v,K}, \hat{V}]\right]$$

Symmetry Reduction Bojowald 2001; rev. Date 2010

Given $\mathcal{H}$ and symmetry group $\mathcal{G}$, $\mathcal{H}/\mathcal{G} \longrightarrow$ space of orbits. Restrict to $\mathcal{H}_{inv} \subset \mathcal{H}/\mathcal{G} \longrightarrow$ space of trivial orbits

Strategy : **Quantization after reduction :** Reduce to $\Gamma_{inv} \subset \Gamma/\mathcal{G}$ and then quantize

Example: sph symm models : $ds^2 = d\tau^2 - f^2(\tau, R)dR^2 - r^2(\tau, R)\,(d\theta^2 + \sin^2\theta\, d\phi^2) \longrightarrow f(\tau, R)$, $r(\tau, R)$ only dynamical dof

Mini-superspace : homogeneous and isotropic spacetimes (Bianchi cosmologies) described by finitely many dof $\longrightarrow$ quantum mech system Bojowald 2001

Symmetry reduction on cl phase space $\Gamma = \{A_a^I\ ,\ E_I^a\}$

Symmetry reduction on cl phase space $\Gamma = \{A_a^I \, , \; E_I^a\}$

$$A_a^I(t,x) \;=\; \Phi_i^I(t)\, \omega_a^i(x)$$

$$E_I^a(t,x) \;=\; p_I^i(t)\, \sqrt{g_0(x)}X_i^a(x)$$

Symmetry reduction on cl phase space $\Gamma = \{A_a^I\,,\ E_I^a\}$

$$A_a^I(t,x) \;=\; \Phi_i^I(t)\,\omega_a^i(x)$$
$$E_I^a(t,x) \;=\; p_I^i(t)\,\sqrt{g_0(x)}X_i^a(x)$$

$\omega^i \rightarrow$ Maurer-Cartan 1-forms on $\mathrm{M}_t \equiv \mathcal{G}^*$ and $X_i \rightarrow$ dual; $g_0 \equiv det(g_{ij})$

Symmetry reduction on cl phase space $\Gamma = \{A_a^I\ ,\ E_I^a\}$

$$A_a^I(t,x) \ = \ \Phi_i^I(t)\,\omega_a^i(x)$$
$$E_I^a(t,x) \ = \ p_I^i(t)\,\sqrt{g_0(x)}X_i^a(x)$$

$\omega^i \longrightarrow$ Maurer-Cartan 1-forms on $\mathrm{M}_t \equiv \mathcal{G}^*$ and $X_i \longrightarrow$ dual; $g_0 \equiv det(g_{ij})$

Isotropy $\Rightarrow$

Symmetry reduction on cl phase space $\Gamma = \{A_a^I\ ,\ E_I^a\}$

$$A_a^I(t,x) \;=\; \Phi_i^I(t)\, \omega_a^i(x)$$
$$E_I^a(t,x) \;=\; p_I^i(t)\, \sqrt{g_0(x)} X_i^a(x)$$

$\omega^i \rightarrow$ Maurer-Cartan 1-forms on $\mathbf{M}_t \equiv \mathcal{G}^*$ and $X_i \rightarrow$ dual; $g_0 \equiv det(g_{ij})$

Isotropy $\Rightarrow$

$$\Phi_i^I(t) \;=\; c(t)\, \Lambda_i^I$$
$$p_I^i(t) \;=\; p(t)\, \Lambda_I^i$$
$$[c\ ,\ p]_{PB} \;=\; \frac{8\pi\gamma G}{3}$$

Symmetry reduction on cl phase space $\Gamma = \{A_a^I\ ,\ E_I^a\}$

$$A_a^I(t,x) \;=\; \Phi_i^I(t)\,\omega_a^i(x)$$
$$E_I^a(t,x) \;=\; p_I^i(t)\,\sqrt{g_0(x)}X_i^a(x)$$

$\omega^i \longrightarrow$ Maurer-Cartan 1-forms on $\mathbf{M}_t \equiv \mathcal{G}^*$ and $X_i \longrightarrow$ dual; $g_0 \equiv det(g_{ij})$

Isotropy $\Rightarrow$

$$\Phi_i^I(t) \;=\; c(t)\,\Lambda_i^I$$
$$p_I^i(t) \;=\; p(t)\,\Lambda_I^i$$
$$[c\ ,\ p]_{PB} \;=\; \frac{8\pi\gamma G}{3}$$

Diagonal: g_{ij} diagonal $\Rightarrow \Phi_i^I = c_i\,\Lambda_i^I\ ,\ p_I^i = p^i\,\Lambda_I^i\ (ns)$

Symmetry reduction on cl phase space $\Gamma = \{A^I_a \,,\; E^a_I\}$

$$A^I_a(t,x) \;=\; \Phi^I_i(t)\,\omega^i_a(x)$$
$$E^a_I(t,x) \;=\; p^i_I(t)\,\sqrt{g_0(x)}X^a_i(x)$$

$\omega^i \rightarrow$ Maurer-Cartan 1-forms on $\mathbf{M}_t \equiv \mathcal{G}^*$ and $X_i \rightarrow$ dual; $g_0 \equiv det(g_{ij})$

Isotropy $\Rightarrow$

$$\Phi^I_i(t) \;=\; c(t)\,\Lambda^I_i$$
$$p^i_I(t) \;=\; p(t)\,\Lambda^i_I$$
$$[c\,,\;p]_{PB} \;=\; \frac{8\pi\gamma G}{3}$$

Diagonal: g_{ij} diagonal $\Rightarrow \Phi^I_i = c_i\,\Lambda^I_i\,,\;p^i_I = p^i\,\Lambda^i_I\;(ns)$

$$[c_i\,,\;p^j]_{PB} = \tfrac{8}{3}\pi G\gamma\,\delta^j_i$$

Singularity Resolution

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Connection and triads: $c = \gamma \dot{a}\ ,\ |p| = a^2$

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Connection and triads: $c = \gamma \dot{a}$, $|p| = a^2$

$$H_{cl} = -\frac{3}{8\pi G}\left(\gamma^{-2}\, c^2 \sqrt{|p|}\right) + \frac{1}{2}|p|^{-3/2}\, p_\phi^2$$

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Connection and triads: $c = \gamma \dot{a}$, $|p| = a^2$

$$H_{cl} = -\frac{3}{8\pi G} \left(\gamma^{-2} \, c^2 \, \sqrt{|p|} \right) + \frac{1}{2} |p|^{-3/2} \, p_\phi^2$$

Solve $H_{cl} = 0$ for c in terms of p and p_ϕ and then solve Hamilton's equations for $\phi(t)$, $p(t)$, $p_\phi(t)$

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Connection and triads: $c = \gamma \dot{a}$, $|p| = a^2$

$$H_{cl} = -\frac{3}{8\pi G} \left(\gamma^{-2} \, c^2 \, \sqrt{|p|} \right) + \frac{1}{2} |p|^{-3/2} \, p_\phi^2$$

Solve $H_{cl} = 0$ for c in terms of p and p_ϕ and then solve Hamilton's equations for $\phi(t)$, $p(t)$, $p_\phi(t)$

$$p(\phi) \; = \; p(\phi^*) \; \exp \pm \left(\frac{8\pi G \gamma}{3} \right)^{1/2} \; (\phi - \phi^*)$$

$$p_\phi(\phi) \; = \; const.$$

where, $\dot{\phi}(t) > 0 \Rightarrow \phi \rightarrow$ emergent time

Singularity Resolution Lectures by G. Date 2010

Strategy : Couple (spatially flat) FRW backgd to massless scalar field $\phi \rightarrow$ 'emergent' time and study classical and quantal dynamics

Connection and triads: $c = \gamma \dot{a}$, $|p| = a^2$

$$H_{cl} = -\frac{3}{8\pi G} \left(\gamma^{-2} \, c^2 \, \sqrt{|p|} \right) + \frac{1}{2} |p|^{-3/2} \, p_\phi^2$$

Solve $H_{cl} = 0$ for c in terms of p and p_ϕ and then solve Hamilton's equations for $\phi(t)$, $p(t)$, $p_\phi(t)$

$$\begin{aligned} p(\phi) &= p(\phi^*) \, \exp \pm \left(\frac{8\pi G \gamma}{3} \right)^{1/2} \, (\phi - \phi^*) \\ p_\phi(\phi) &= const. \end{aligned}$$

where, $\dot{\phi}(t) > 0 \Rightarrow \phi \rightarrow$ emergent time

As $\phi \rightarrow \pm\infty$, $p \rightarrow 0$, $\mathcal{E} \equiv p^{-6} \, p_\phi^2 / 2 \rightarrow \infty \Rightarrow$ Big Bang singularity !

Loop Quantization

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c\ ,\ \ \lambda \in \mathbf{R}$

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c\ ,\ \ \lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c\ ,\ \lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

$$\hat{p}|\mu\rangle \ = \ \frac{1}{6}\,\gamma\, l_P^2\, \mu|\mu\rangle\ ,\ \langle\mu|\mu'\rangle = \delta_{\mu,\mu'}$$

$$\hat{h}_\nu|\mu\rangle \ \equiv \ \widehat{\exp i\mu c}|\mu\rangle = |\mu+\nu\rangle\ ,\ \mu,\nu \in \mathbf{R}$$

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c\ ,\ \ \lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

$$\hat{p}|\mu\rangle \ = \ \frac{1}{6}\,\gamma\, l_P^2\, \mu|\mu\rangle\ ,\ \ \langle\mu|\mu'\rangle = \delta_{\mu,\mu'}$$

$$\hat{h}_\nu|\mu\rangle \ \equiv \ \widehat{\exp i\mu c}|\mu\rangle = |\mu+\nu\rangle\ ,\ \ \mu,\nu \in \mathbf{R}$$

- $\hat{p}$ has discrete spectrum; volume $\hat{V} \equiv |\hat{p}|^{3/2}$

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c$, $\lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

$$\hat{p}|\mu\rangle \;=\; \frac{1}{6}\,\gamma\, l_P^2\, \mu|\mu\rangle\ ,\ \langle\mu|\mu'\rangle = \delta_{\mu,\mu'}$$

$$\hat{h}_\nu|\mu\rangle \;\equiv\; \widehat{\exp i\mu c}|\mu\rangle = |\mu+\nu\rangle\ ,\ \mu,\nu \in \mathbf{R}$$

- $\hat{p}$ has discrete spectrum; volume $\hat{V} \equiv |\hat{p}|^{3/2}$
- Matrix elements of $\hat{h}_\nu$ not continuous in $\nu \Rightarrow \hat{c} \longrightarrow$ cannot be defined

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c$, $\lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

$$\hat{p}|\mu\rangle = \frac{1}{6}\gamma\, l_P^2\, \mu|\mu\rangle\ ,\ \langle\mu|\mu'\rangle = \delta_{\mu,\mu'}$$
$$\hat{h}_\nu|\mu\rangle \equiv \exp\hat{i\mu c}|\mu\rangle = |\mu+\nu\rangle\ ,\ \mu,\nu \in \mathbf{R}$$

- $\hat{p}$ has discrete spectrum; volume $\hat{V} \equiv |\hat{p}|^{3/2}$
- Matrix elements of $\hat{h}_\nu$ not continuous in $\nu \Rightarrow \hat{c} \longrightarrow$ cannot be defined
- $\hat{p}^{-1}$ or its positive powers cannot exist since spectrum of $\hat{p}$ includes 0

Loop Quantization

Algebra of observables $\longrightarrow$ algebra of $\exp i\lambda c$, $\lambda \in \mathbf{R}$

Explicit holonomy-flux rep.

$$\hat{p}|\mu\rangle = \frac{1}{6}\gamma\, l_P^2\, \mu|\mu\rangle \, , \; \langle\mu|\mu'\rangle = \delta_{\mu,\mu'}$$

$$\hat{h}_\nu|\mu\rangle \equiv \exp\widehat{i\mu c}|\mu\rangle = |\mu+\nu\rangle \, , \; \mu,\nu \in \mathbf{R}$$

- $\hat{p}$ has discrete spectrum; volume $\hat{V} \equiv |\hat{p}|^{3/2}$
- Matrix elements of $\hat{h}_\nu$ not continuous in $\nu \Rightarrow \hat{c} \longrightarrow$ cannot be defined
- $\hat{p}^{-1}$ or its positive powers cannot exist since spectrum of $\hat{p}$ includes 0

Need ‘regularized’ $\hat{p}$ using $h_j(c) \equiv \exp \mu_0 c \Lambda^I \tau_I$, $2j \in \mathbf{Z}$

$$
\begin{aligned}
\widehat{|p|^{-1}_{j,l}}|\mu\rangle &= \left(\frac{1}{3}j\mu_o\gamma l_P^2\right)^{-1}\left[F_l(q)\right]^{1/(l-1)}|\mu\rangle\ , \\
q &\equiv \frac{\mu}{2\mu_0 j} \equiv \frac{p}{2jp_0}\ , \ l \in (0,1) \\
F_l(q >> 1) &\approx q^{l-1} \\
F_l(q \approx 0) &\approx \frac{3q}{l+1}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{|p|^{-1}_{j,l}}|\mu\rangle &= \left(\frac{1}{3}j\mu_o\gamma l_P^2\right)^{-1}[F_l(q)]^{1/(l-1)}|\mu\rangle\ , \\
q &\equiv \frac{\mu}{2\mu_0 j} \equiv \frac{p}{2jp_0}\ ,\ l \in (0,1) \\
F_l(q >> 1) &\approx q^{l-1} \\
F_l(q \approx 0) &\approx \frac{3q}{l+1}
\end{aligned}
$$

Eigenvalues bounded above $\Rightarrow$ matter densities remain bounded over classical singularity !

$$
\begin{aligned}
\widehat{|p|^{-1}_{j,l}}|\mu\rangle &= \left(\frac{1}{3} j\mu_o\gamma l_P^2\right)^{-1} [F_l(q)]^{1/(l-1)}|\mu\rangle \ , \\
q &\equiv \frac{\mu}{2\mu_0 j} \equiv \frac{p}{2jp_0} \ , \ l \in (0,1) \\
F_l(q >> 1) &\approx q^{l-1} \\
F_l(q \approx 0) &\approx \frac{3q}{l+1}
\end{aligned}
$$

Eigenvalues bounded above $\Rightarrow$ matter densities remain bounded over classical singularity !

$\Rightarrow$ singularity-causing $\sim |p|^{-3/2}$ term in Hamiltonian remains finite $\rightarrow$ seed for singularity-resolution

Quantum gravitational Hamiltonian

Quantum gravitational Hamiltonian

$$\begin{aligned}\hat{H}_{grav,sym}|\mu\rangle &= \frac{3}{\mu_0\gamma l_P^2}\left(|V_{\mu+3\mu_0}-V_{\mu+\mu_0}||\mu+4\mu_0\rangle\right.\\ &+ |V_{\mu-\mu_0}-V_{\mu-3\mu_0}||\mu-4\mu_0\rangle\\ &- \left.\left[|V_{\mu+3\mu_0}-V_{\mu+\mu_0}|+|V_{\mu-\mu_0}-V_{\mu-3\mu_0}|\right]|\mu\rangle\right)\end{aligned}$$

Quantum gravitational Hamiltonian

$$\begin{aligned}\hat{H}_{grav,sym}|\mu\rangle \;=\; & \frac{3}{\mu_0\gamma l_P^2}\left(|V_{\mu+3\mu_0} - V_{\mu+\mu_0}||\mu+4\mu_0\rangle\right. \\ & + \; |V_{\mu-\mu_0} - V_{\mu-3\mu_0}||\mu-4\mu_0\rangle \\ & - \; \left.\left[|V_{\mu+3\mu_0} - V_{\mu+\mu_0}| + |V_{\mu-\mu_0} - V_{\mu-3\mu_0}|\right]|\mu\rangle\right)\end{aligned}$$

$V_\mu \equiv (\frac{1}{6}\gamma l_P^2\,|\mu|)^{3/2} \rightarrow$ eigenvalues of $\hat{V}$

Quantum gravitational Hamiltonian

$$\hat{H}_{grav,sym}|\mu\rangle = \frac{3}{\mu_0\gamma l_P^2}\Big(|V_{\mu+3\mu_0} - V_{\mu+\mu_0}||\mu + 4\mu_0\rangle$$
$$+ |V_{\mu-\mu_0} - V_{\mu-3\mu_0}||\mu - 4\mu_0\rangle$$
$$- \left[|V_{\mu+3\mu_0} - V_{\mu+\mu_0}| + |V_{\mu-\mu_0} - V_{\mu-3\mu_0}|\right]|\mu\rangle\Big)$$

$V_\mu \equiv (\frac{1}{6}\gamma l_P^2\, |\mu|)^{3/2} \longrightarrow$ eigenvalues of $\hat{V}$

Wavefunction $|\Psi\rangle = \sum_\mu \psi(\phi,\mu)|\mu\rangle$ gives the Wheeler-Dewitt equation (with symmetric factor-ordering)

Quantum gravitational Hamiltonian

$$\begin{aligned}\hat{H}_{grav,sym}|\mu\rangle \;&=\; \frac{3}{\mu_0\gamma l_P^2}\left(|V_{\mu+3\mu_0} - V_{\mu+\mu_0}||\mu+4\mu_0\rangle\right.\\ &+\; |V_{\mu-\mu_0} - V_{\mu-3\mu_0}||\mu-4\mu_0\rangle \\ &-\; \left.\left[|V_{\mu+3\mu_0} - V_{\mu+\mu_0}| + |V_{\mu-\mu_0} - V_{\mu-3\mu_0}|\right]|\mu\rangle\right)\end{aligned}$$

$V_\mu \equiv (\frac{1}{6}\gamma l_P^2\,|\mu|)^{3/2} \to$ eigenvalues of $\hat{V}$

Wavefunction $|\Psi\rangle \;=\; \sum_\mu \psi(\phi,\mu)|\mu\rangle$ gives the Wheeler-Dewitt equation (with symmetric factor-ordering)

$$\begin{aligned}f_+(\mu)\Psi(\phi,\mu+4\mu_0) + f_0(\mu)\Psi(\phi,\mu) \;&+\; f_-(\mu)\Psi(\phi,\mu-4\mu_0) \\ &=\; -\frac{16}{3}\pi G\gamma^3\mu_0^3 l_P^2\, H_{mat}(\mu)\Psi(\phi,\mu)\end{aligned}$$

Quantum gravitational Hamiltonian

$$\begin{aligned}\hat{H}_{grav,sym}|\mu\rangle &= \frac{3}{\mu_0\gamma l_P^2}\left(|V_{\mu+3\mu_0} - V_{\mu+\mu_0}||\mu+4\mu_0\rangle\right.\\ &+ |V_{\mu-\mu_0} - V_{\mu-3\mu_0}||\mu-4\mu_0\rangle\\ &- \left.\left[|V_{\mu+3\mu_0} - V_{\mu+\mu_0}| + |V_{\mu-\mu_0} - V_{\mu-3\mu_0}|\right]|\mu\rangle\right)\end{aligned}$$

$V_\mu \equiv (\frac{1}{6}\gamma l_P^2\,|\mu|)^{3/2} \rightarrow$ eigenvalues of $\hat{V}$

Wavefunction $|\Psi\rangle = \sum_\mu \psi(\phi,\mu)|\mu\rangle$ gives the Wheeler-Dewitt equation (with symmetric factor-ordering)

$$\begin{aligned}f_+(\mu)\Psi(\phi,\mu+4\mu_0) + f_0(\mu)\Psi(\phi,\mu) &+ f_-(\mu)\Psi(\phi,\mu-4\mu_0)\\ &= -\frac{16}{3}\pi G\gamma^3\mu_0^3 l_P^2\, H_{mat}(\mu)\Psi(\phi,\mu)\end{aligned}$$

2nd order Difference eq on lattice $\mathcal{L} \equiv \mu = \mu' + 4\mu_0 n\ ,\ n \in \mathbf{Z}$

Effective Hamiltonian

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\hat{p^{-1}}$

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\hat{p^{-1}}$

Modified FRW : take $p >> p_0$

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\hat{p^{-1}}$

Modified FRW : take $p >> p_0$

$$\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \equiv \rho_{eff} = \rho_{cl}\left(1 - \frac{8\pi G\gamma\mu_0^2}{3}\, p\, \rho_{cl}\right)$$

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\widehat{p^{-1}}$

Modified FRW : take $p >> p_0$

$$\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \equiv \rho_{eff} = \rho_{cl}\left(1 - \frac{8\pi G\gamma\mu_0^2}{3}\, p\, \rho_{cl}\right)$$

where $p = \frac{1}{4}a^2$, $\rho_{cl} \equiv H_{mat}\, |p|^{-3/2}$

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\hat{p^{-1}}$

Modified FRW : take $p >> p_0$

$$\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \equiv \rho_{eff} = \rho_{cl}\left(1 - \frac{8\pi G \gamma \mu_0^2}{3}\, p\, \rho_{cl}\right)$$

where $p = \frac{1}{4}a^2$, $\rho_{cl} \equiv H_{mat}\, |p|^{-3/2}$

Solution has a bounce in the region of classical singularity !

Effective Hamiltonian

WDW differential eq : assume $\Psi(\phi, \mu)$ is slowly varying wrt μ

WKB approximation for $\mu >> \mu_0 \Rightarrow H_{eff} = \tilde{H}_{grav} + \tilde{H}_{mat}$ where, $\tilde{H}_{grav}$ has received corrections due to use loop quantization using holonomies, and $\tilde{H}_{mat}$ has received corrections due to use of regularized $\widehat{p^{-1}}$

Modified FRW : take $p >> p_0$

$$\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \equiv \rho_{eff} = \rho_{cl}\left(1 - \frac{8\pi G\gamma\mu_0^2}{3}\, p\, \rho_{cl}\right)$$

where $p = \frac{1}{4}a^2$, $\rho_{cl} \equiv H_{mat}\, |p|^{-3/2}$

Solution has a bounce in the region of classical singularity !

Is the universe before the ‘Big Bounce’ identical to the present universe ?

Summary : Black hole entropy

Summary : Black hole entropy

- Weaker version of holography derived from QGR, albeit heuristic

Summary : Black hole entropy

- Weaker version of holography derived from QGR, albeit heuristic
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not

Summary : Black hole entropy

- Weaker version of holography derived from QGR, albeit heuristic
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not
- Gravity-Gauge theory link explicit : $SU(2)$ CS Topol gauge theory on IH

Summary : Black hole entropy

- Weaker version of holography derived from QGR, albeit heuristic
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not
- Gravity-Gauge theory link explicit : $SU(2)$ CS Topol gauge theory on IH
- Microcan bh entropy understood for macro bhs; BH area law receives infinite series of finite corrections (signature)

Summary : Black hole entropy

- Weaker version of holography derived from QGR, albeit heuristic
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not
- Gravity-Gauge theory link explicit : $SU(2)$ CS Topol gauge theory on IH
- Microcan bh entropy understood for macro bhs; BH area law receives infinite series of finite corrections (signature)
- Bekenstein entropy bound tightened due to LQG corrections

Summary : Singularity Resolution

Summary : Singularity Resolution

- FRW in terms of holonomy/flux $\Rightarrow$ big bang singularity moved away from boundary (cf ADM big bang associated with 'beginning' requiring boundary cond)

Summary : Singularity Resolution

- FRW in terms of holonomy/flux $\Rightarrow$ big bang singularity moved away from boundary (cf ADM big bang associated with ‘beginning’ requiring boundary cond)
- Holonomy corrections adequate to resolve big bang through Wheeler De Witt Difference Eq; but inverse volume corrections also do the job

Summary : Singularity Resolution

- FRW in terms of holonomy/flux $\Rightarrow$ big bang singularity moved away from boundary (cf ADM big bang associated with ‘beginning’ requiring boundary cond)
- Holonomy corrections adequate to resolve big bang through Wheeler De Witt Difference Eq; but inverse volume corrections also do the job
- Big Bang $\longrightarrow$ ‘Big Bounce’ although not clear if earlier universe is identical to ours

Summary : Singularity Resolution

- FRW in terms of holonomy/flux $\Rightarrow$ big bang singularity moved away from boundary (cf ADM big bang associated with 'beginning' requiring boundary cond)
- Holonomy corrections adequate to resolve big bang through Wheeler De Witt Difference Eq; but inverse volume corrections also do the job
- Big Bang $\longrightarrow$ 'Big Bounce' although not clear if earlier universe is identical to ours
- Natural prediction of an inflationary phase Ashtekar et. al. 2009; Bojowald et. al. 2009

Pending Issues

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse

  Basu, Chakraborty, PM in prep.

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse

  Basu, Chakraborty, PM in prep.

- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse

  Basu, Chakraborty, PM in prep.

- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ? CGHS: Ashtekar, Varadarajan 2007, ...

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse

  Basu, Chakraborty, PM in prep.

- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ? CGHS: Ashtekar, Varadarajan 2007, ...
- Resolution of black hole singularity : midi-superspace ? Ashtekar, Bojowald 2003; Pullin 2008; Modesto et. al. 2009, ...

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse
  Basu, Chakraborty, PM in prep.
- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ? CGHS: Ashtekar, Varadarajan 2007, ...
- Resolution of black hole singularity : midi-superspace ? Ashtekar, Bojowald 2003; Pullin 2008; Modesto et. al. 2009, ...
- Relation between LQC and LQG ?

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse Basu, Chakraborty, PM in prep.
- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ? CGHS: Ashtekar, Varadarajan 2007, ...
- Resolution of black hole singularity : midi-superspace ? Ashtekar, Bojowald 2003; Pullin 2008; Modesto et. al. 2009, ...
- Relation between LQC and LQG ?
- Relation with pheno data : CMB fluctuations, non-Gaussianity, ...

Pending Issues

- Beyond effective quantum horizon : horizon formation during collapse
  Basu, Chakraborty, PM in prep.
- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ? CGHS: Ashtekar, Varadarajan 2007, ...
- Resolution of black hole singularity : midi-superspace ? Ashtekar, Bojowald 2003; Pullin 2008; Modesto et. al. 2009, ...
- Relation between LQC and LQG ?
- Relation with pheno data : CMB fluctuations, non-Gaussianity, ...
- Dark matter and dark energy within LQC ?