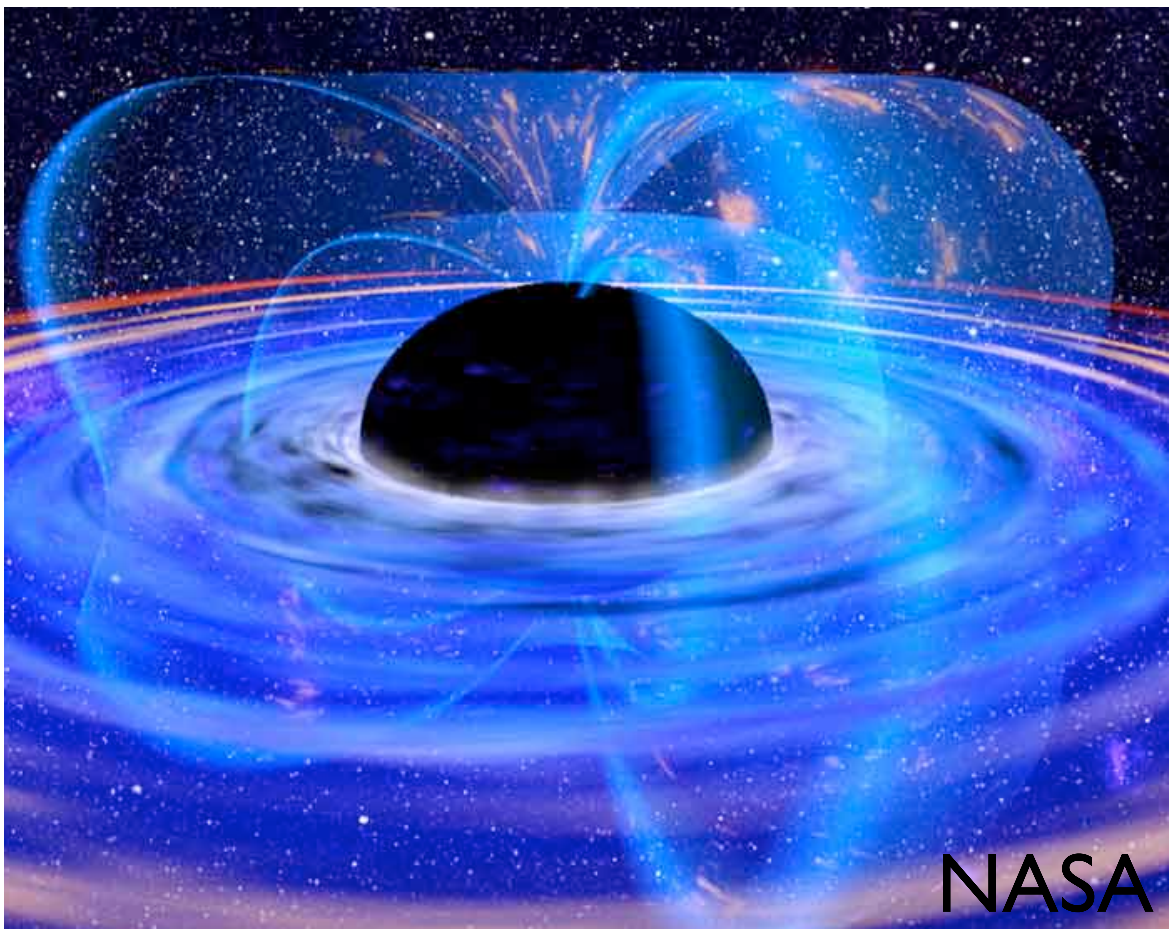
The Black Hole Information Paradox, and its resolution in string theory

Samir D. Mathur

The Ohio State University

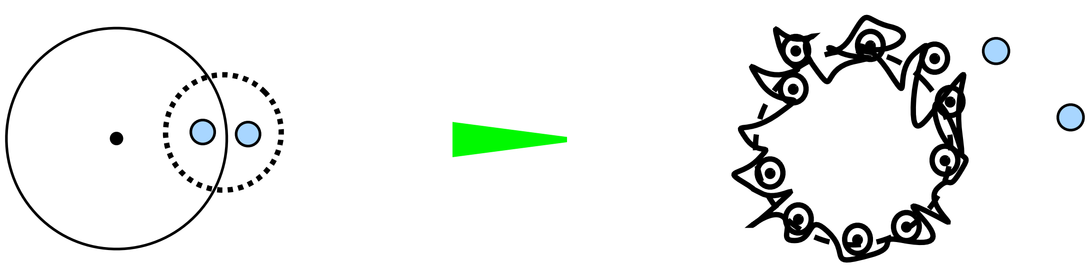


Hawking 1974:

General relativity predicts black holes

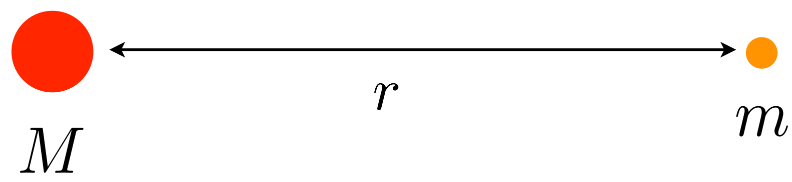
Quantum mechanics around black holes is INCONSISTENT

String theory: Quantum gravity effects alter the structure of the black hole radically ⟶ Fuzzballs



The paradox

Gravity is an attractive force



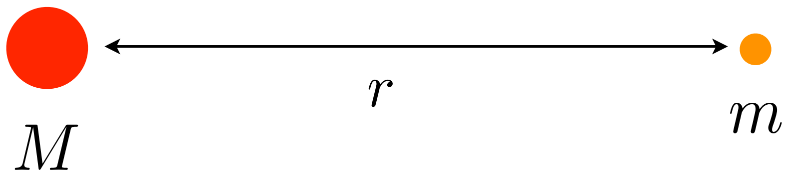
$$PE = -\frac{GMm}{r}$$

By itself, the small mass has an intrinsic energy

$$E = mc^2$$

When it is placed near the larger mass, what energy should we assign ?
Let us start with the Newtonian approximation ...

$$E = mc^2 - \frac{GMm}{r}$$

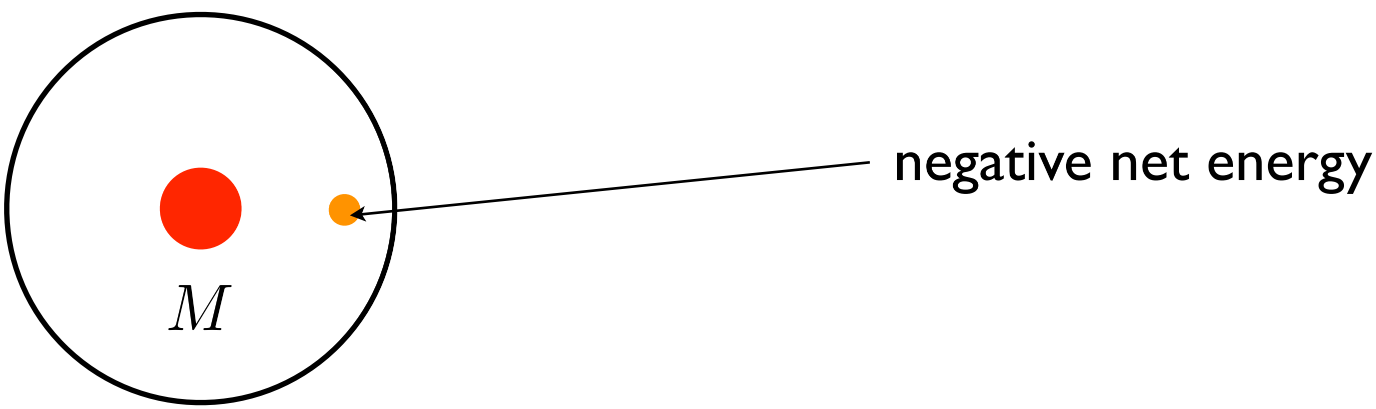


$$E = mc^2 - \frac{GMm}{r}$$

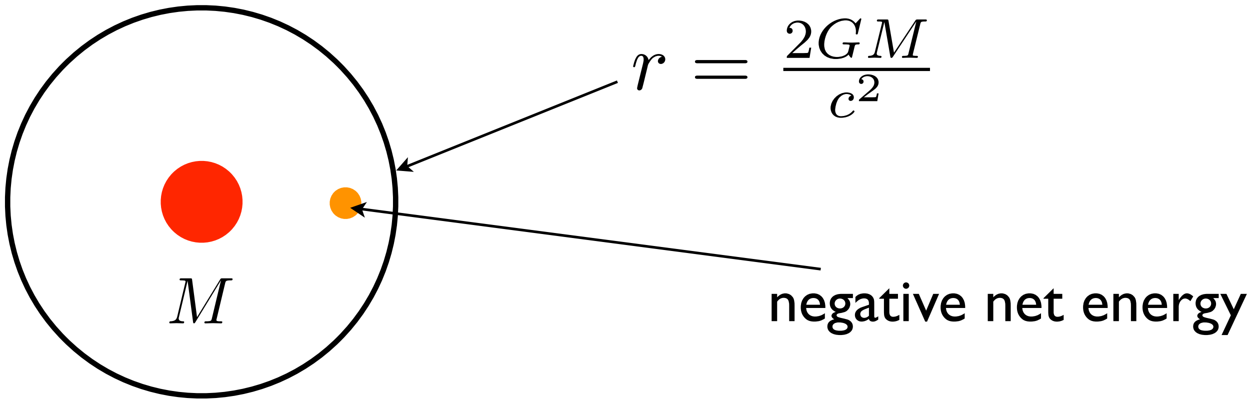
We see that the total energy of m becomes zero at

$$r = \frac{GM}{c^2}$$

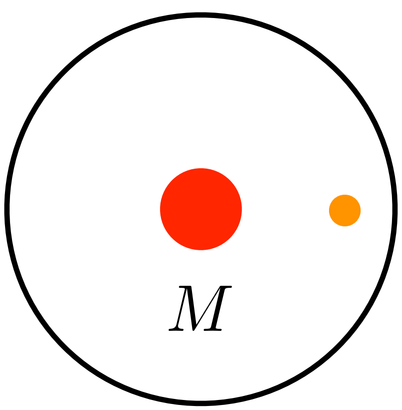
and for smaller r it is negative



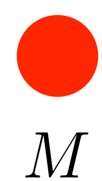
Doing this properly with general relativity does not change the answer much



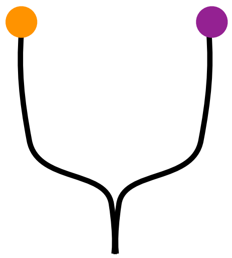
So we see that

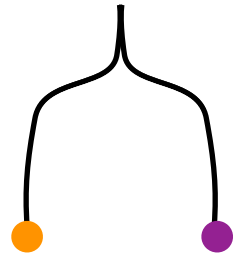


has less energy than



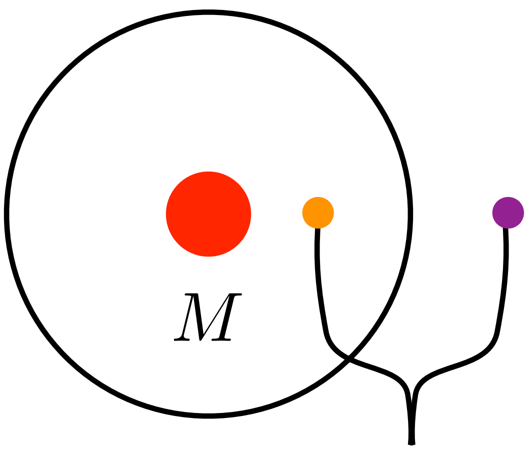
In quantum mechanics, the vacuum can have fluctuations which produce a particle-antiparticle pair





$$\Delta E \, \Delta t \sim \hbar$$

But if a fluctuation happens near the horizon, the particles do not have to re-annihilate



$$\Delta E = 0 \quad \rightarrow \quad \Delta t = \infty$$

Thus the negative energy particle gets automatically placed in the correct position inside the horizon

The outer particle drifts off to infinity as ‘Hawking radiation’

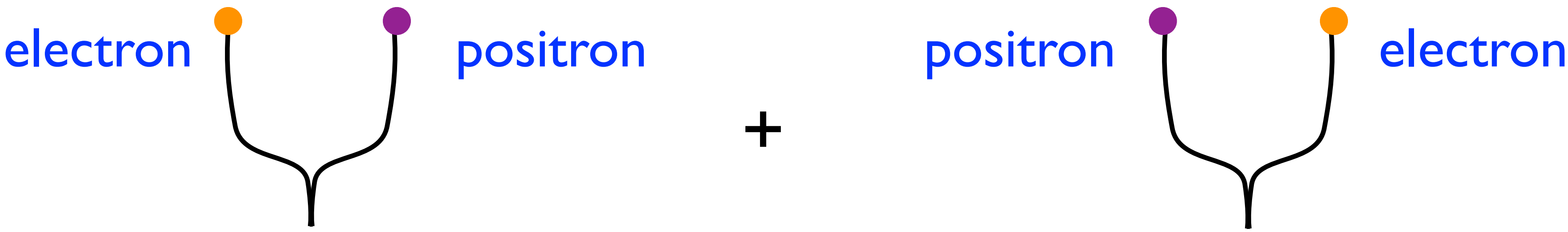
The mass of the hole has gone down, so the horizon shrinks slightly

The process repeats, and another particle pair is produced

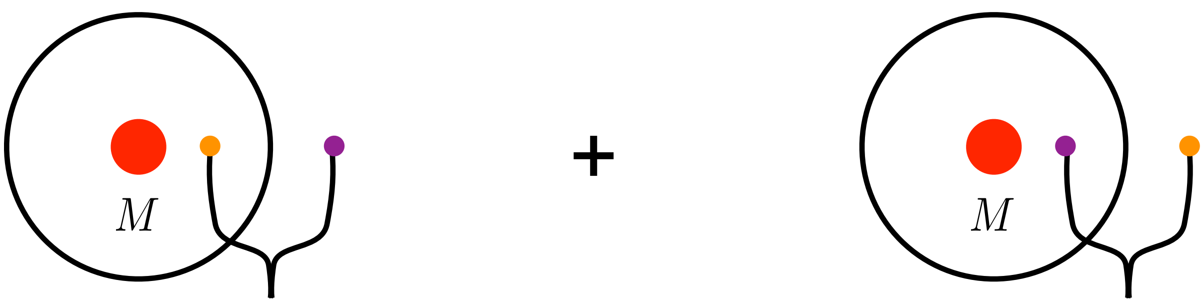
The energy of the hole is now in the radiation

A massless (or planck mass) remnant is left

The crucial issue now has to do with ‘entanglement’



Vacuum fluctuations typically produce entangled states ...

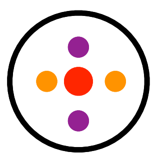


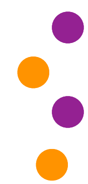
So the state of the radiation is entangled with the state of the remnant

The amount of this entanglement is very large ...

If N particles are emitted, then there are 2^N possible arrangements

We can call an electron a 0 and a positron a 1





	111111	000000
	....	....
+	010011	101100
+	000000	111111

(a) Information loss: The evaporation goes on till the remnant has zero mass. At this point the remnant simply vanishes

vacuum

000000

....

101100

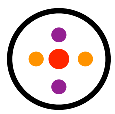
111111

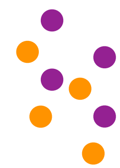
The radiation is entangled, but there is nothing that it is entangled WITH

The radiation cannot be assigned ANY quantum state ... it can only be described by a density matrix ... this is a violation of quantum mechanics(Hawking 1974)

(b) We assume the evaporation stops when we get to a planck sized remnant.

The remnant must have at least 2^N internal states

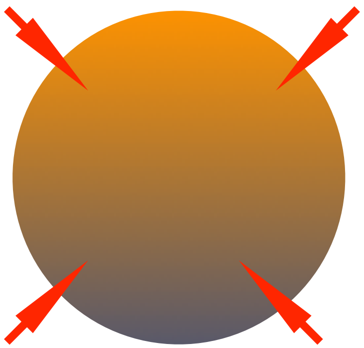


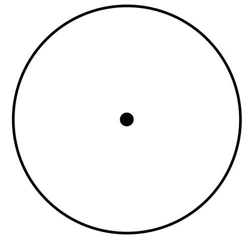


	111111	000000
	....	....
+	010011	101100
+	000000	111111

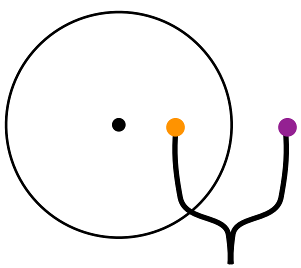
But how can we hold an unbounded number of states in planck volume with energy limited by planck mass?

The black hole information paradox

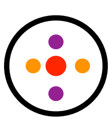


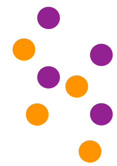


General Relativity:
Black holes form



Quantum mechanics:
entangled pairs are created





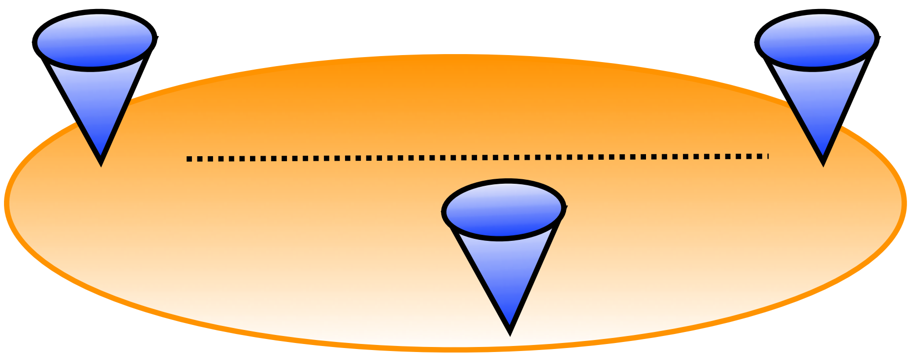
There is a problem near the
endpoint of evaporation

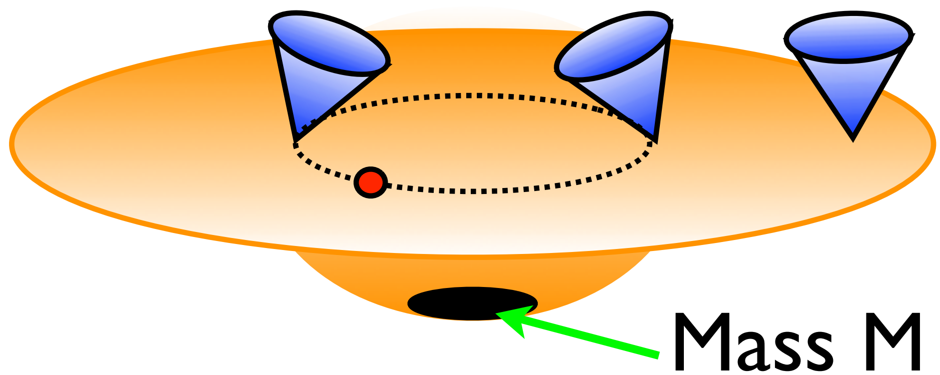
Can we imagine a different structure for the black hole?

General relativity

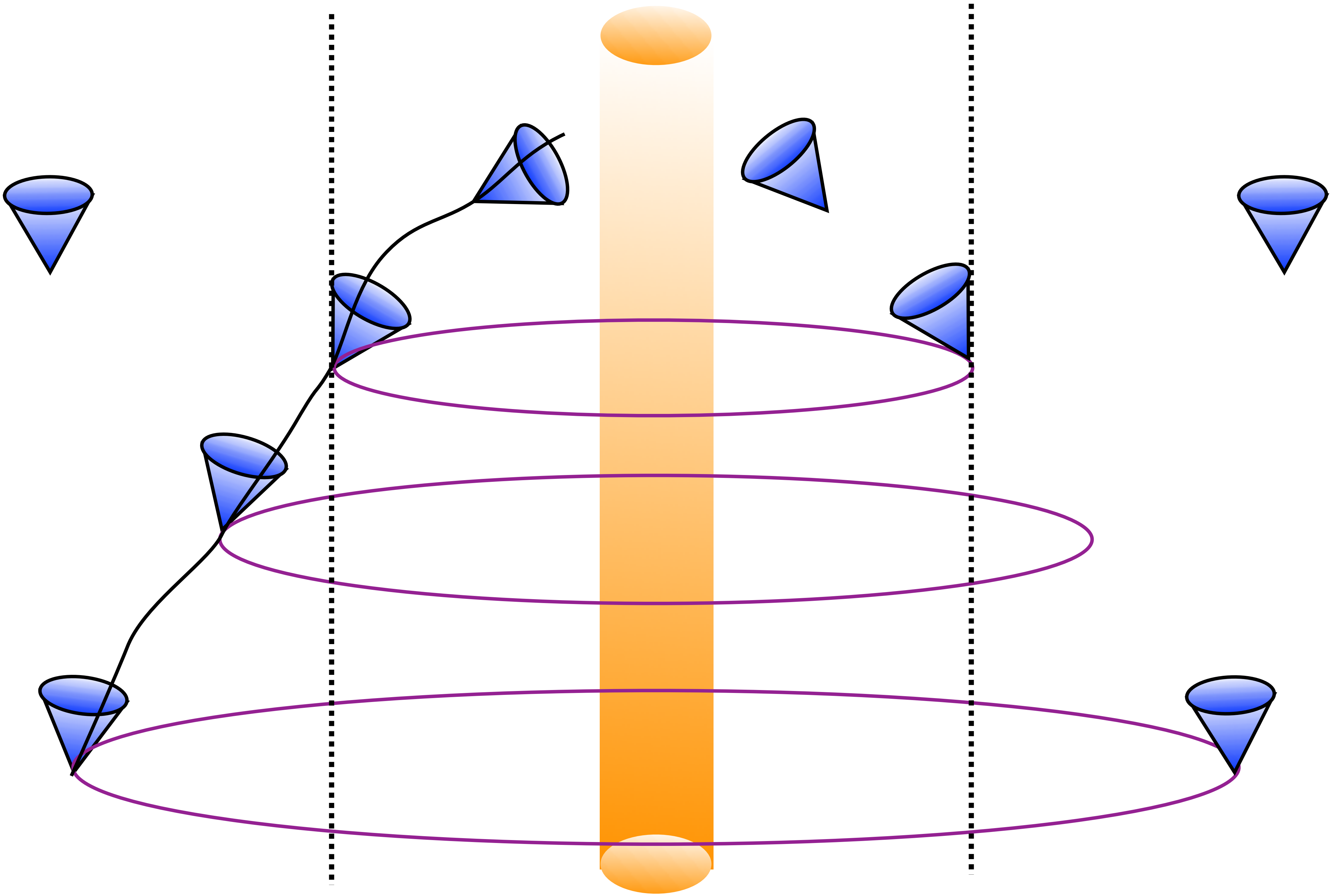
Mass curves spacetime

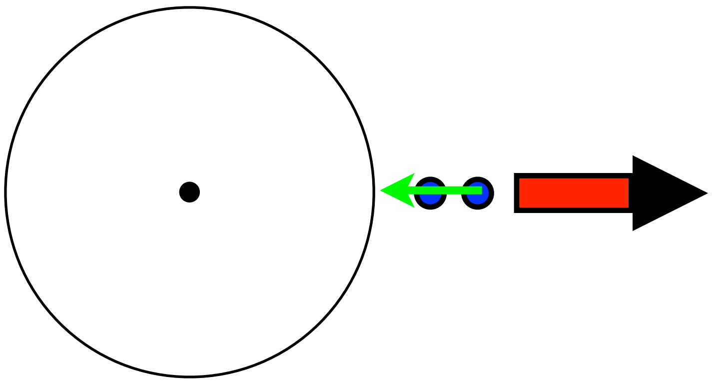
All the 'force' of gravity is encoded in this curvature of spacetime

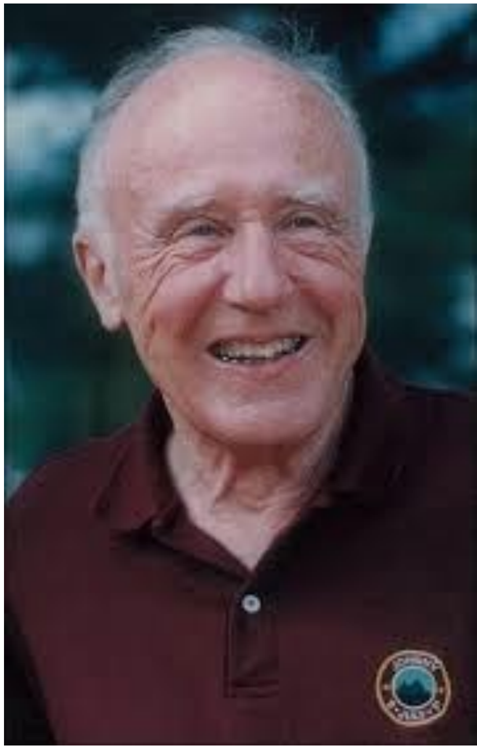




The Black Hole





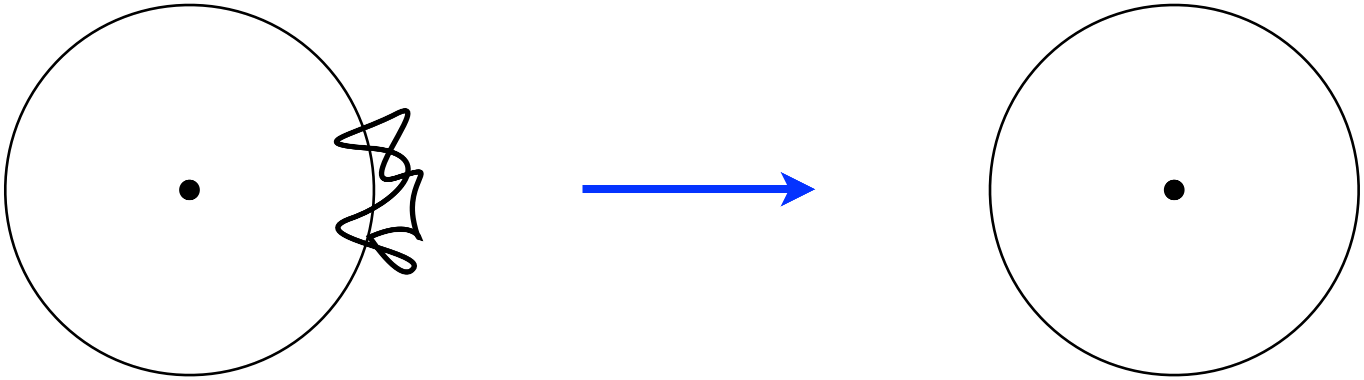


It is very hard to stay near the horizon:
any structure there falls in

The black hole then reduces back to its standard shape:

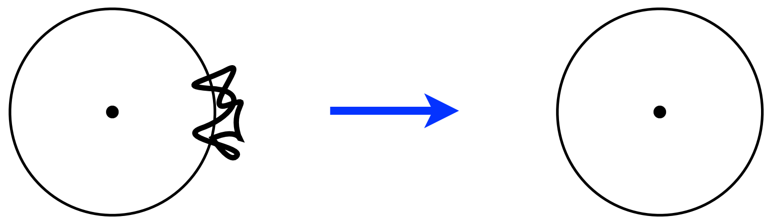
"Black holes have no hair"

If you place a string near the horizon, it will fall in,
so just having string theory does not solve anything

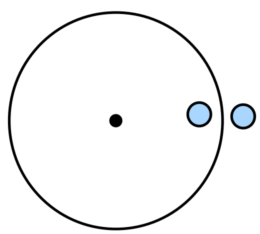


So the information paradox is a combination of two observations:

(1) The no-hair 'theorems' tell us the black hole tends to quickly settle down to a state where the region around the horizon is vacuum



(2) The vacuum creates entangled pairs by the Hawking process



But we will now see that in string theory there is indeed a way that the no hair 'theorem' gets bypassed ...

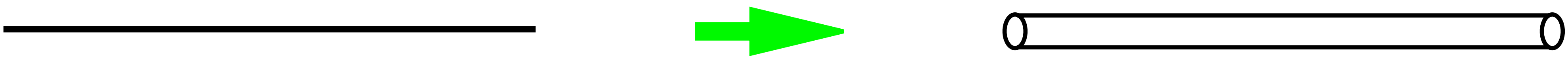
Fuzzballs

Avery, Balasubramanian, Bena, Carson, Chowdhury, de Boer, Gimon, Giusto, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Niehoff, Park, Peet, Potvin, Puhm,Ross, Ruef, Saxena, Simon, Skenderis, Srivastava,Taylor,Turton,Vasilakis,Warner ...

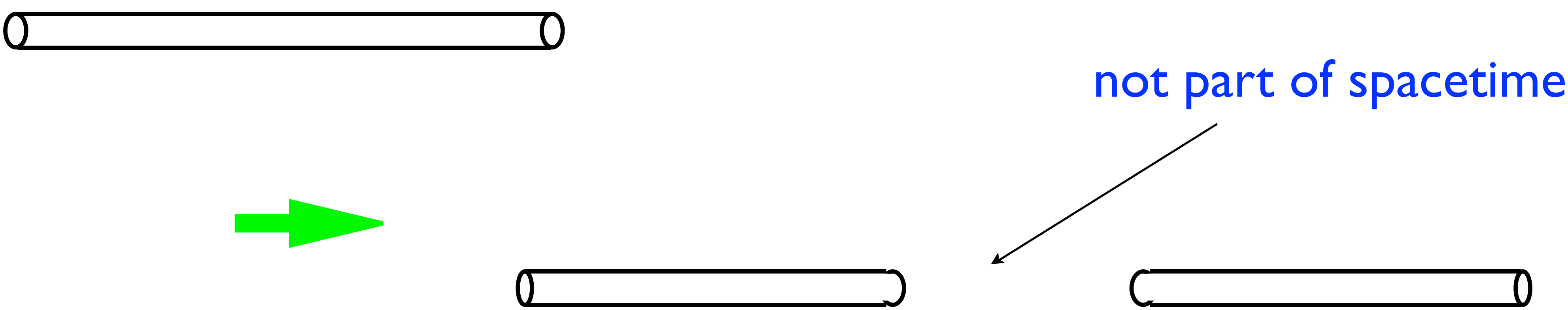
First consider a rough analogy …

Witten 1982: ‘Bubble of nothing’

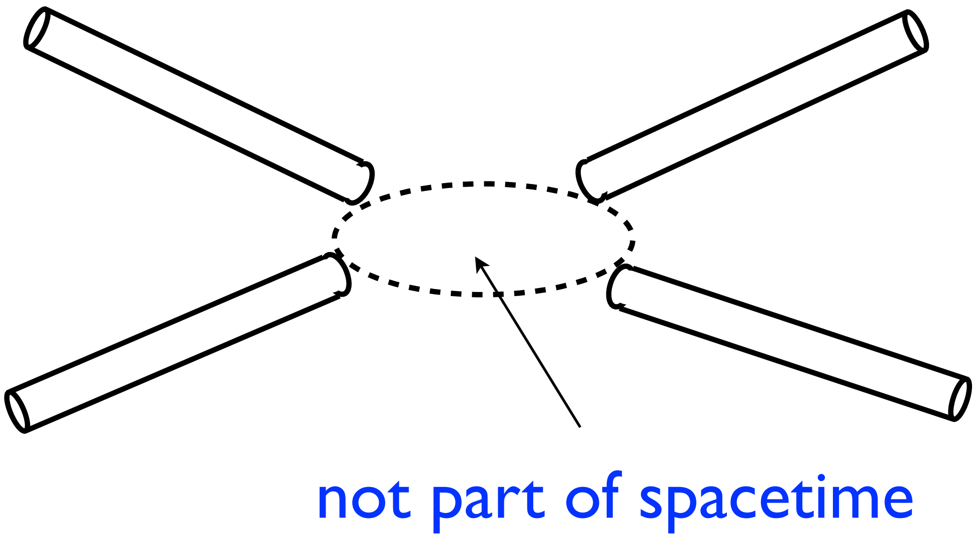
Consider Minkowski space with an extra compact circle



This space-time is unstable to tunneling into a ‘bubble of nothing’



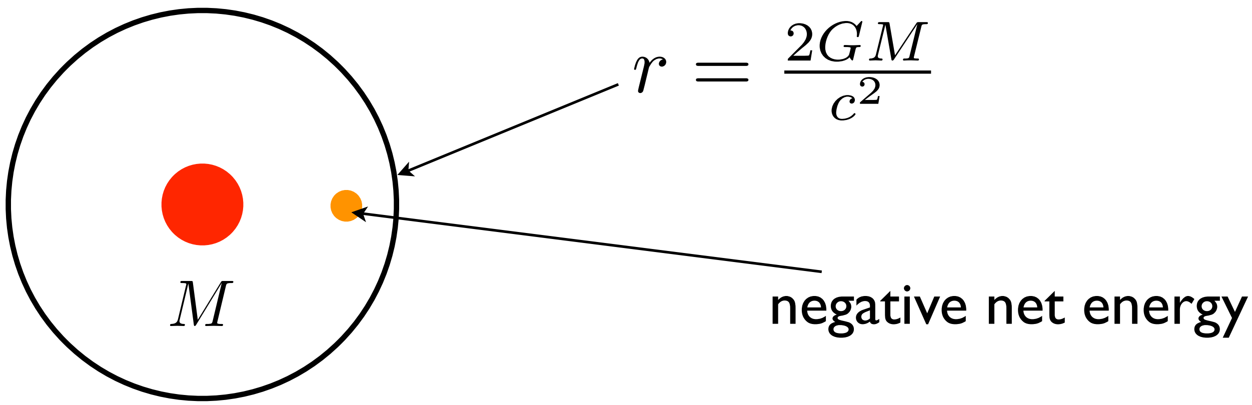
In more dimensions :



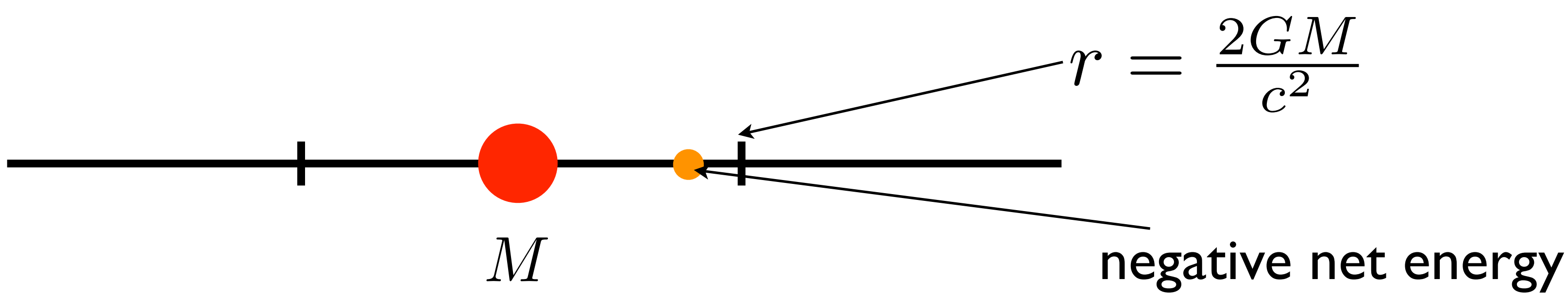
People did not worry about this instability too much, since it turns out that fermions cannot live on this new topology without having a singularity in their wave function …

But now consider the black hole …

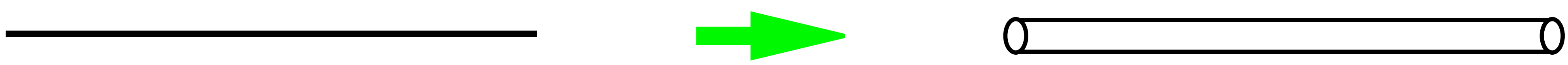
We live in 3 space and 1 time dimension. Recall the black hole ...



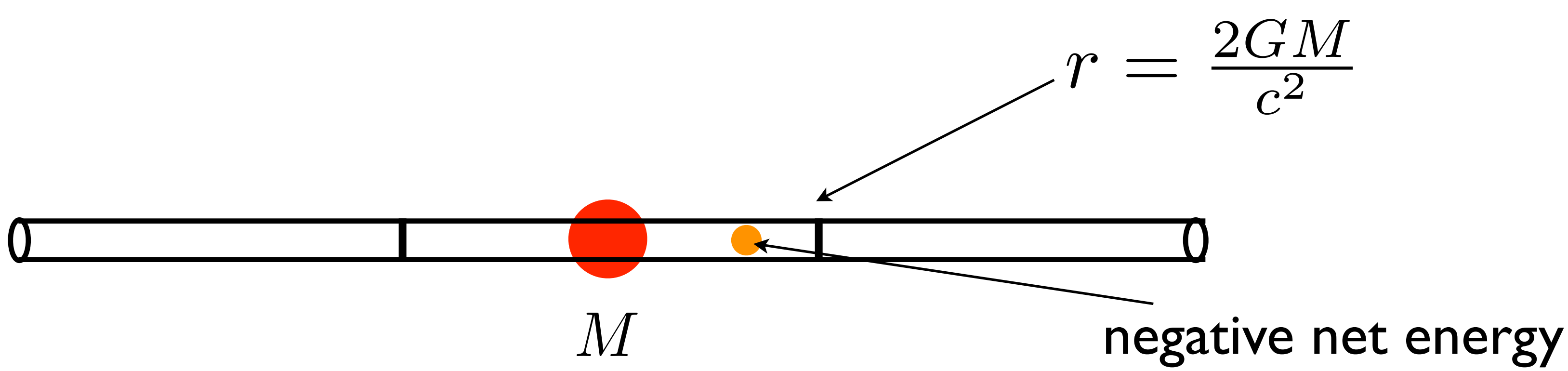
Let us draw just one space direction for simplicity



Now suppose there was an extra dimension (e.g., string theory has 6 extra dimensions)

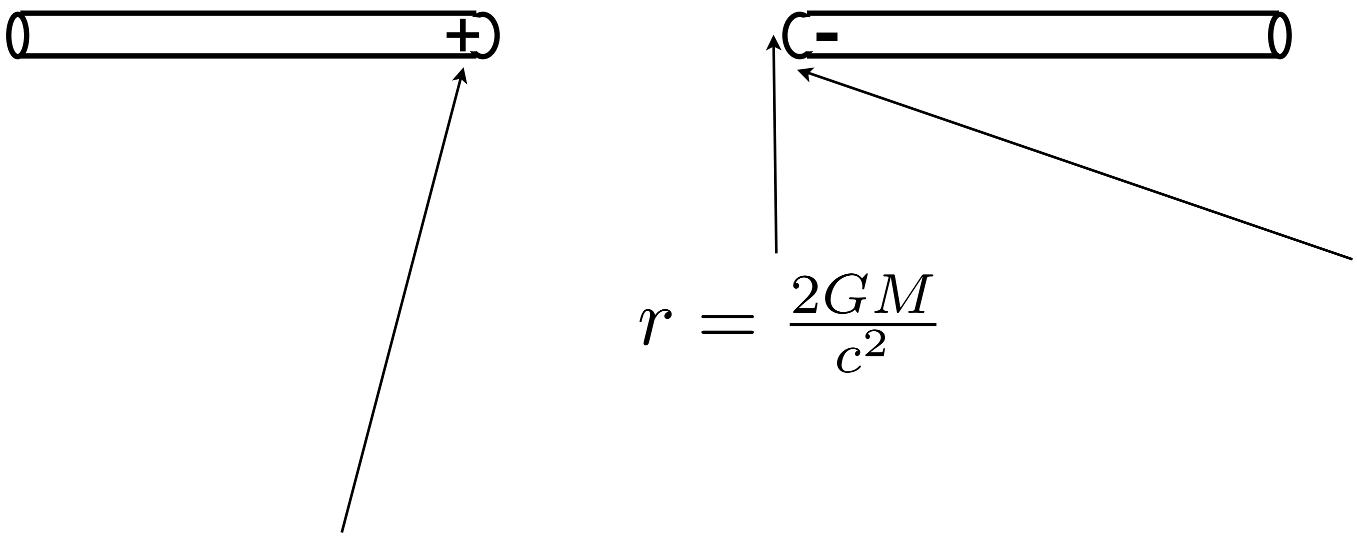


People have thought of extra dimensions for a long time, but they seemed to have no particular significance for the black hole problem



But there is a completely different structure possible with compact dimensions ...

No place to put particles with net negative energy



$$r = \frac{2GM}{c^2}$$

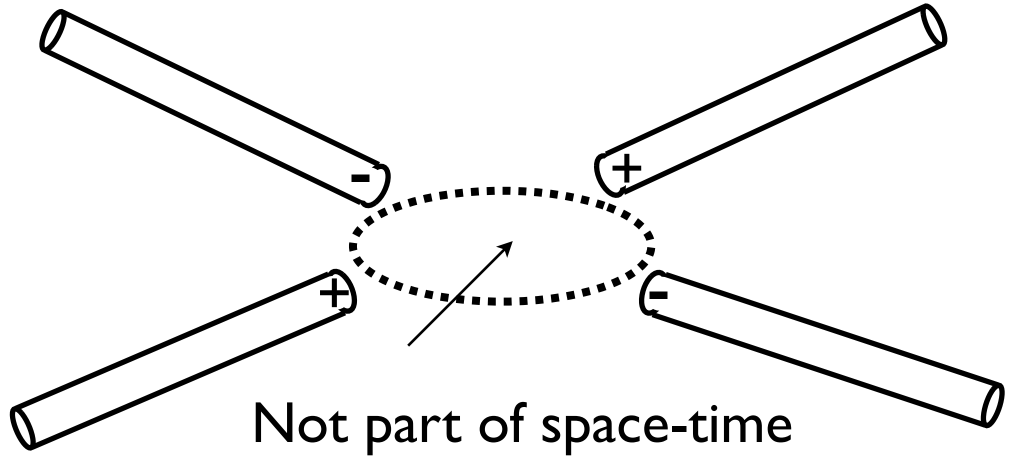
The mass M is captured by the energy in the curved manifold

There is an extra 'twist' in the space-time which makes it consistent to have both boson and fermion wave functions

(Kaluza Klein monopoles and anti-monopoles)

1-dimension

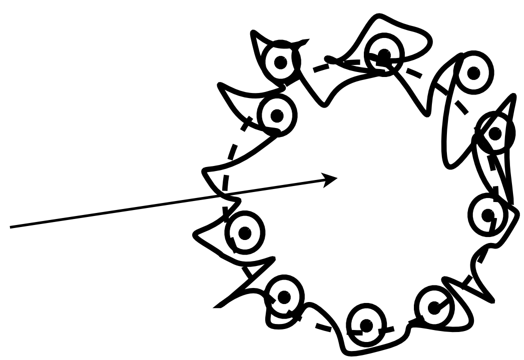
In more dimensions :



Not part of space-time
(no horizon forms)

We will draw only the structure near the horizon :

not part of
spacetime



"Fuzzball"

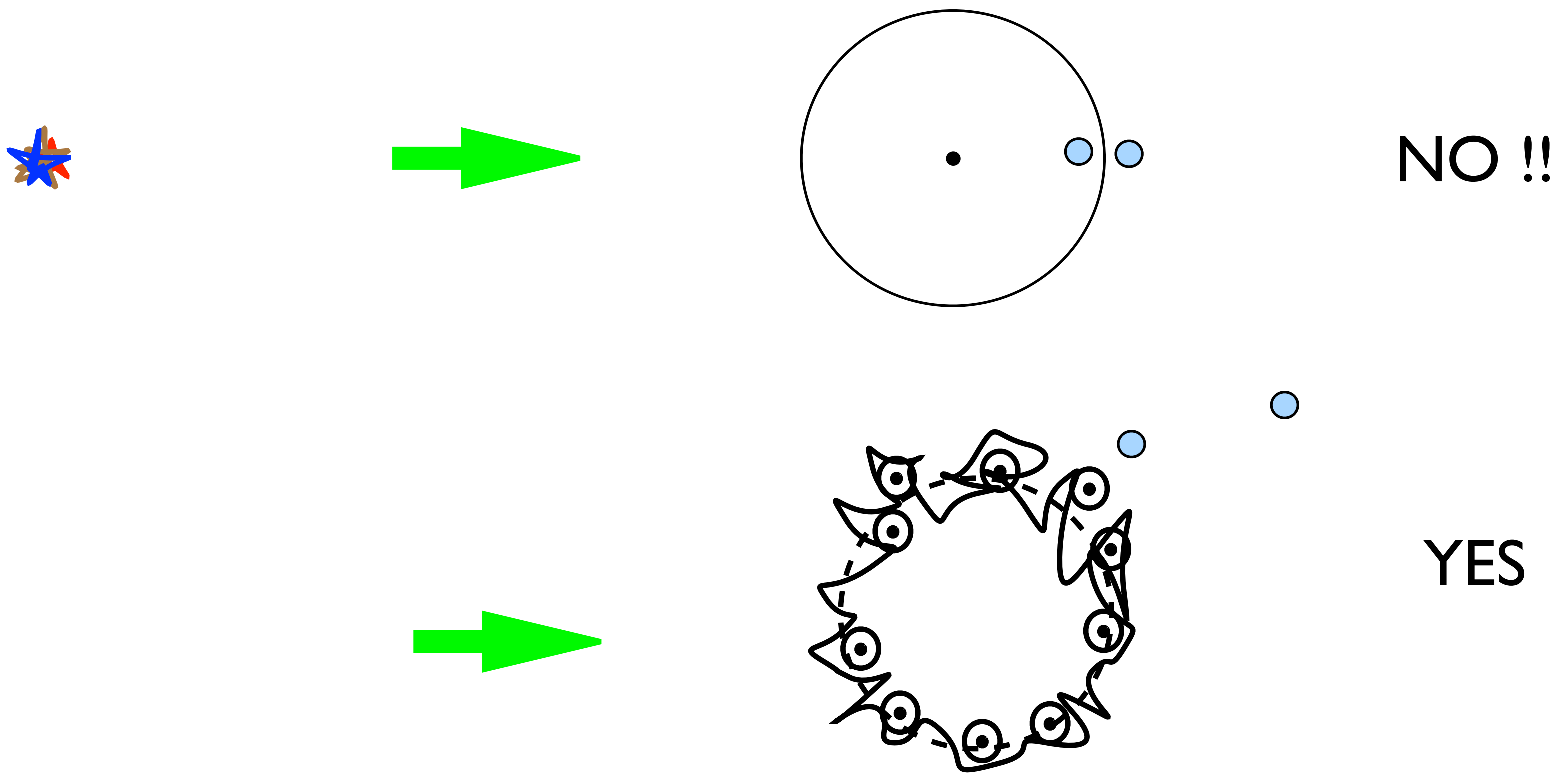
Nothing can fall into the hole because spacetime ends just outside the horizon

In string theory, the gravitational coupling is a tunable parameter

If we make the coupling small, then we just get free strings, branes etc.

If we make the coupling large, we expected a black hole …

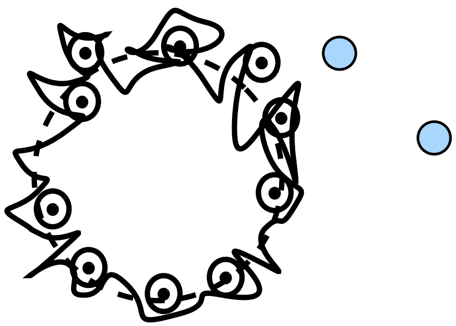
But in all the cases that have been worked out, we always get a fuzzball



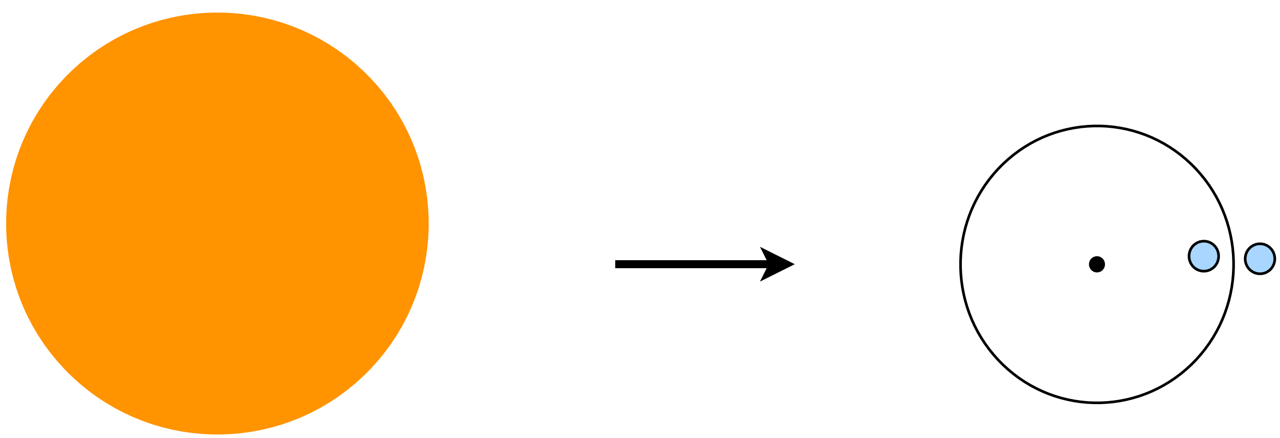
This resolves the black hole information paradox …

How does the classical expectation get violated so dramatically ?

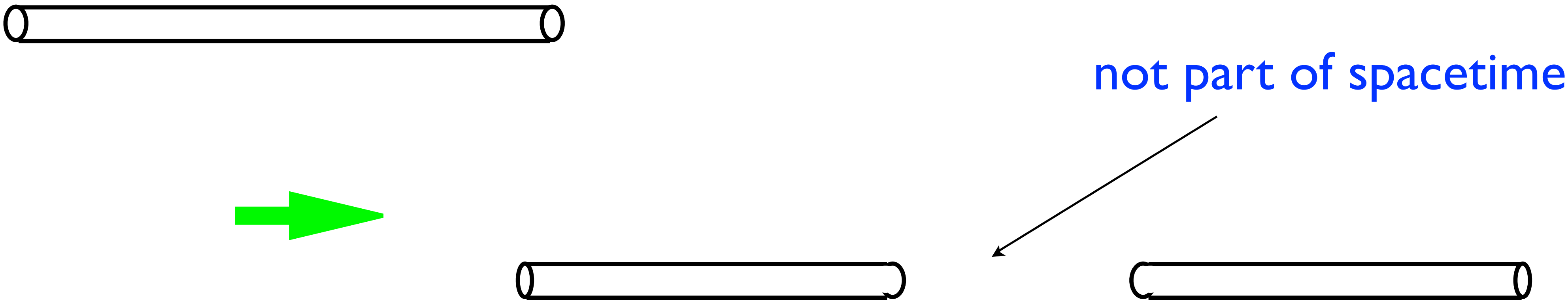
The fuzzball construction seems to be the only correct solution to the paradox ...



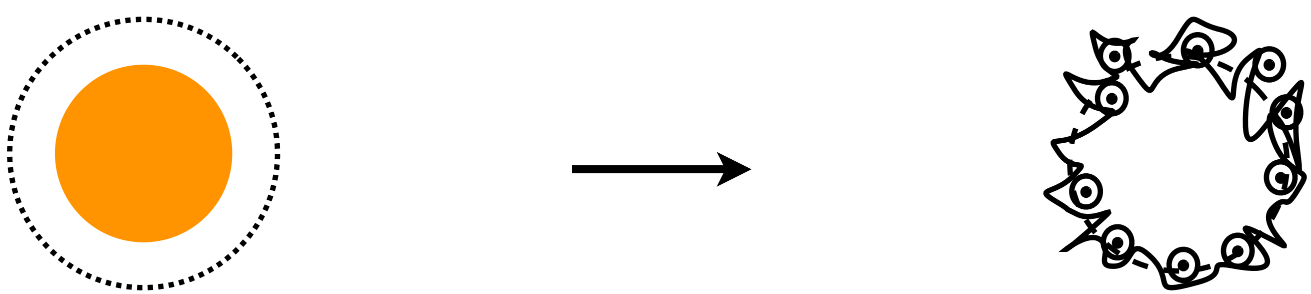
But if a star collapses, then the physics looks quite classical, and so one seems to make the usual black hole with a smooth horizon ...

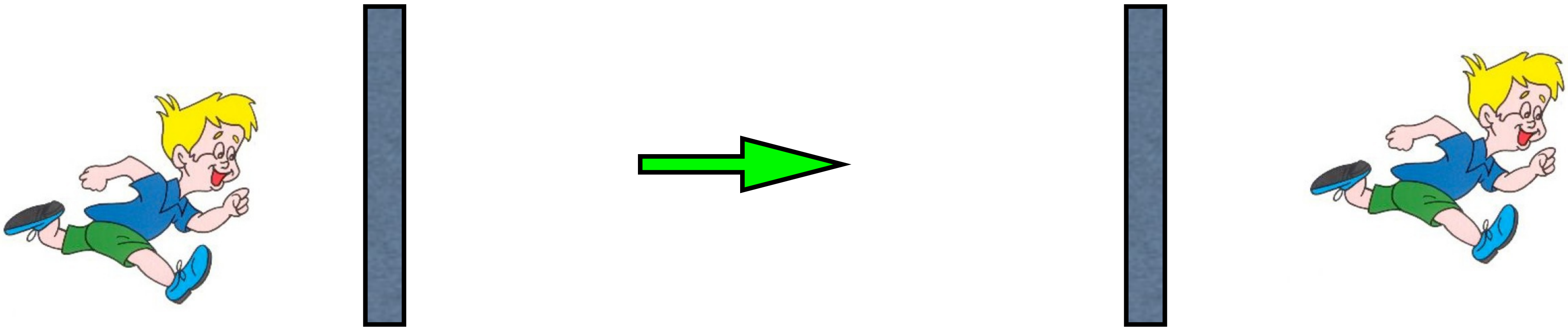


Recall Witten's 'bubble of nothing' where Minkowski space tunnels into a new topology …



It turns out that a collapsing shell can tunnel into a fuzzball state …





There is always a small probability that an object can tunnel ...

But this probability is usually ignorable for a macroscopic object ...

Is there something special about a black hole ?

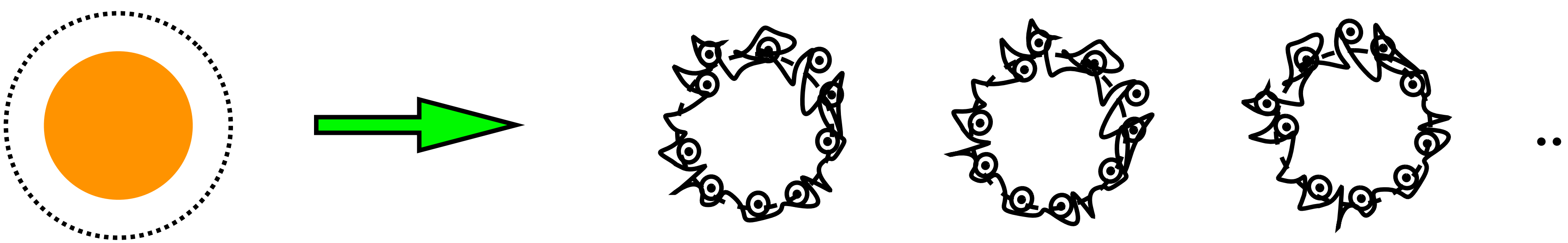
In 1972, Bekenstein taught us that black holes have an entropy

$$S = \frac{c^3}{\hbar}\frac{A}{4G} \sim \frac{A}{l_p^2}$$

This means that a solar mass black hole has $\sim 10^{10^{144}}$ states

This is far larger than the number of states of normal matter with the same energy

We must multiply the (small) amplitude of tunneling by the (large) number of fuzzball states that we can tunnel to …

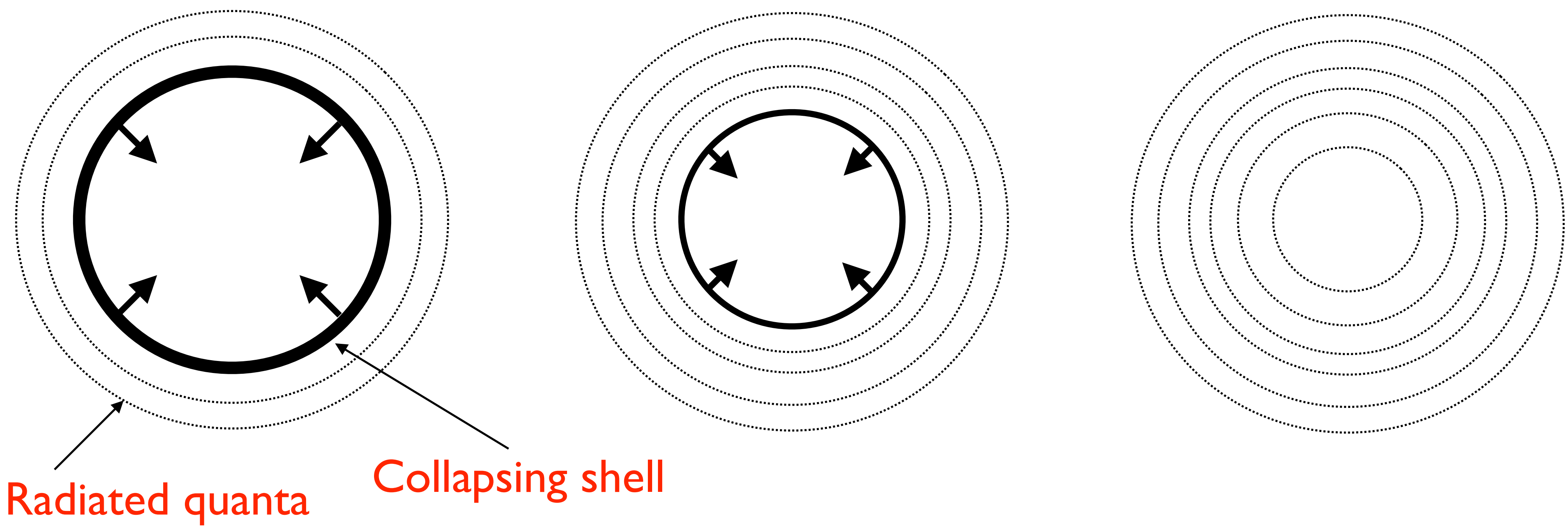


First consider a toy model:

Consider a shell made of an ingoing gravitational wave with energy M

Let there be a large number N of massless scalars in the theory :

$$N \gg (M/m_p)^2$$



The shell evaporates to a collection of scalar quanta, without forming a horizon

We do not have a large number of massless scalars in string theory …

But we have $e^{S_{bek}}$ massive fuzzball states

Emission rate without back reaction:

$$\Gamma \sim e^{-\frac{\omega}{T}} \sim e^{-8\pi G M \omega}$$

Emission rate for a spherical shell with back reaction (Kraus-Wiczek-Parikh)

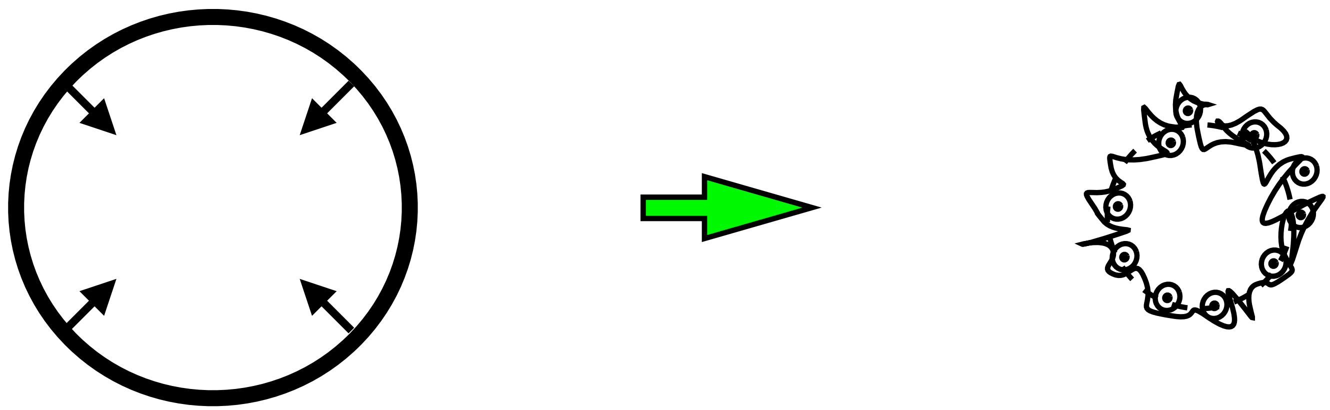
$$\Gamma \sim e^{-8\pi G (M - \frac{\omega}{2})\omega}$$

This is a special case of the general expression

$$\Gamma \sim e^{-\Delta S_{bek}} \sim e^{S_{bek}(M-\omega) - S_{bek}(M)}$$

This expression works for both massless and massive shells, and can be used for any ω

We set $\omega = M$



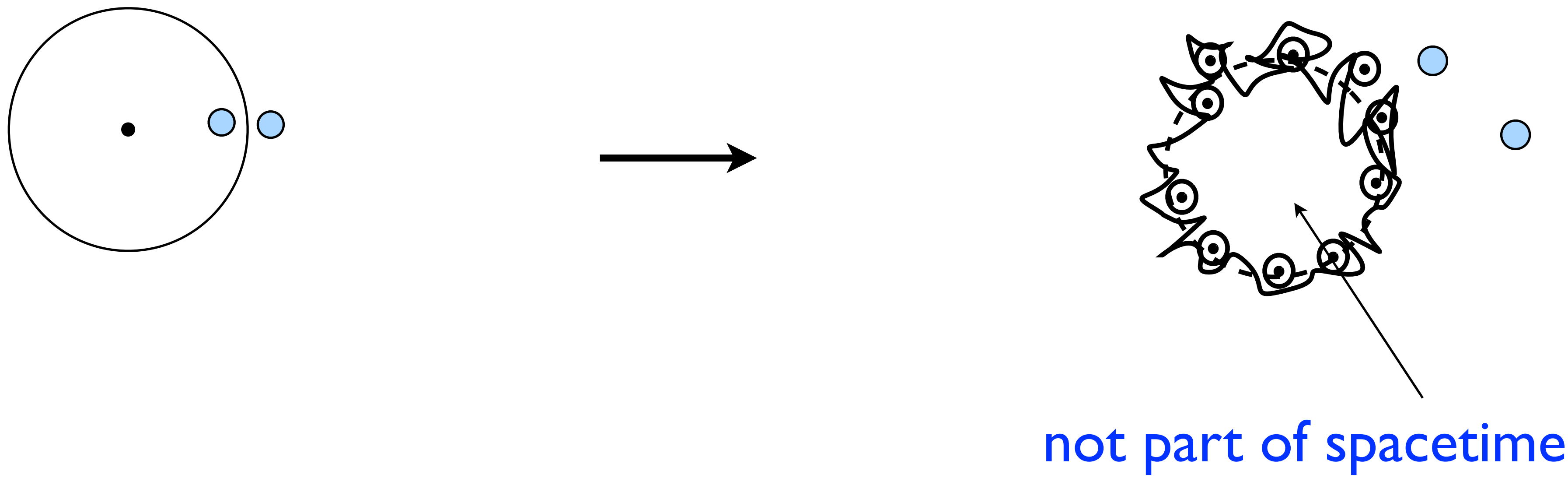
$$\Gamma_{species} \sim e^{-S_{bek}(M)}$$

Overall rate of tunneling into fuzzballs :

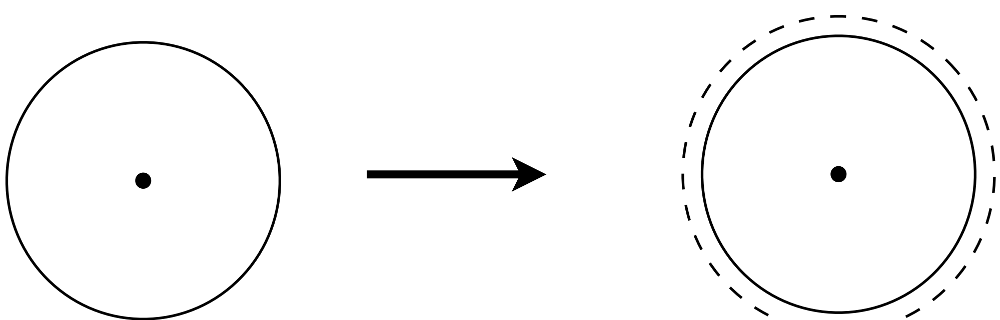
$$\Gamma \sim f\, e^{-S_{bek}(M)} \sim e^{S_{bek}} e^{-S_{bek}} \sim 1$$

Thus in a time of order unity the collapsing shell transitions to a linear combination of fuzzball states

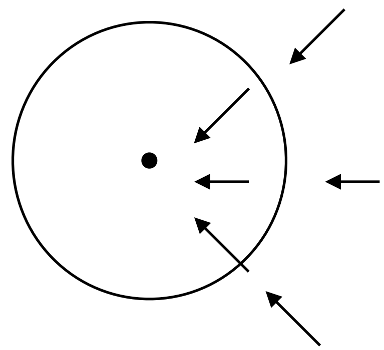
What happens when you fall onto the fuzzball?

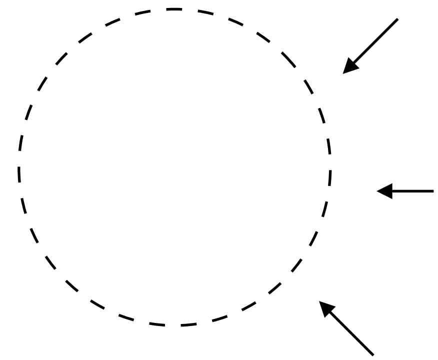


The membrane paradigm (Thorne, Price, Macdonald 1988)



For the physics outside the hole, we can place an IMAGINARY membrane just outside the horizon, with appropriate boundary conditions

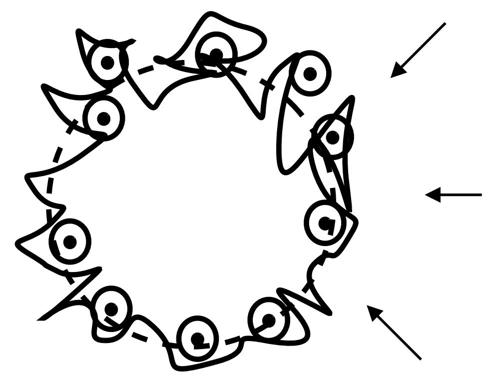




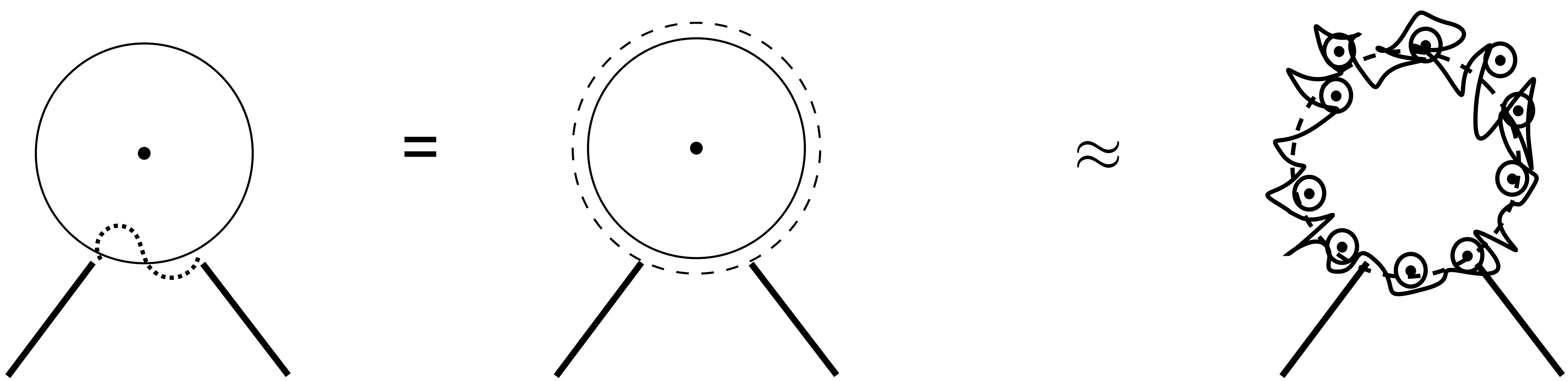
Electromagnetic fields live on both sides of the horizon

The fields outside can be obtained by deleting the interior, and assigning an appropriate conductivity to the membrane

But now we have a REAL 'membrane' just outside the horizon, where space-time ends …

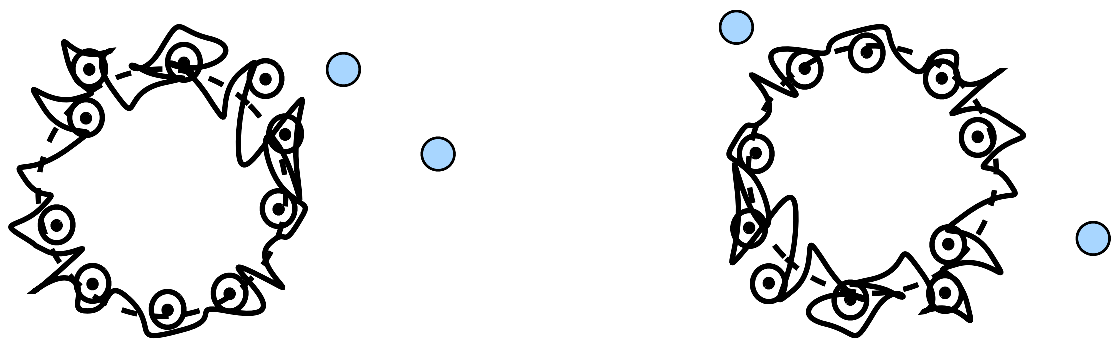


Conjecture of fuzzball complementarity: The surface of the fuzzball behaves APPROXIMATELY like the membrane of the membrane paradigm

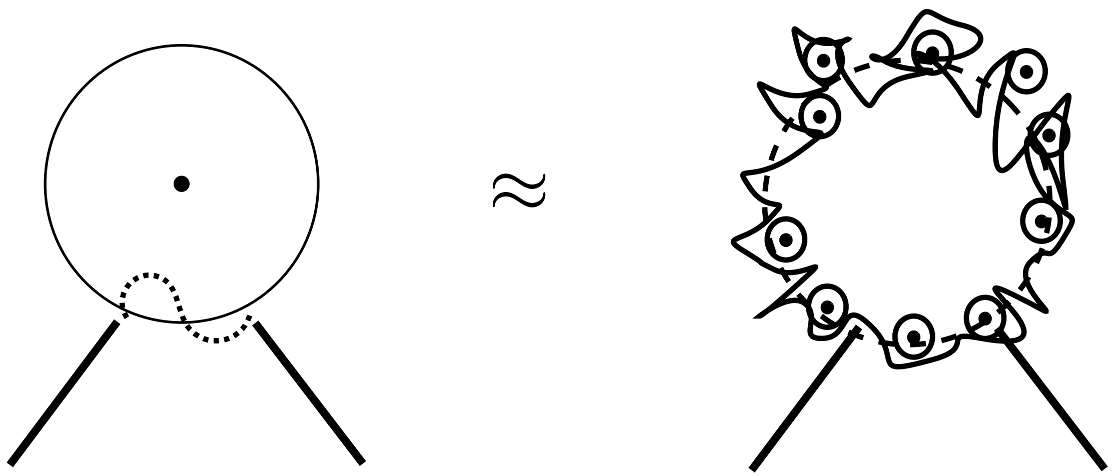


How will we tell the difference between the fuzzball and the black hole?

The word 'APPROXIMATION" is crucial here …

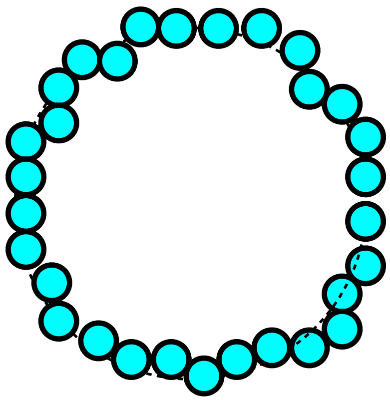
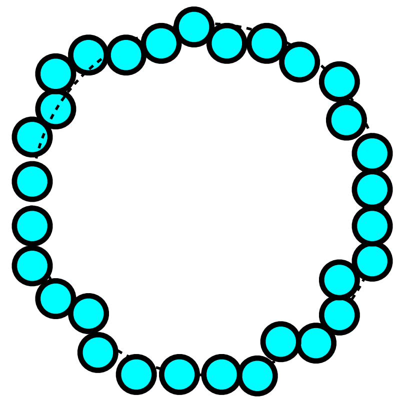


Different fuzzballs radiate low energy (E~T) Hawking quanta differently

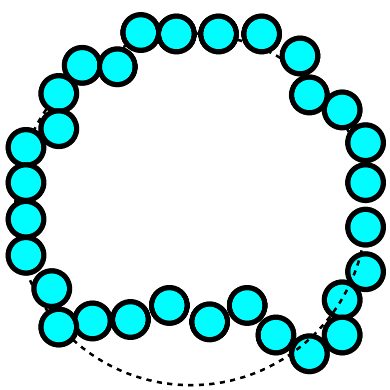
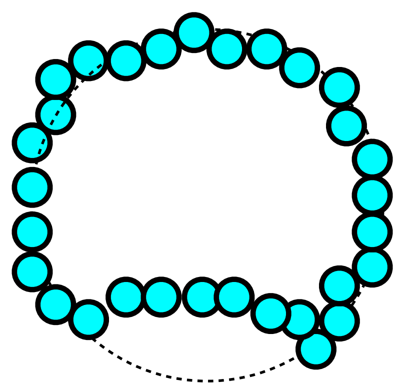


But when impacted by high energy quanta (E>>T) they may all behave approximately the same ..

Analogy

Under a scanning tunneling microscope, the surfaces of these water drops will look different

Under a large deformation, they follow the same hydrodynamic equation

In the fuzzball, freely falling objects will typically impact the surface with E>>T, so we will get the analog of 'hydrodynamic excitations'

Conjecture of Fuzzball complementarity :

Under the impact of freely falling particles
(E>>T) the fuzzball surface undergoes ‘hydrodynamic oscillations’
that are approximately independent of the state of the fuzzball

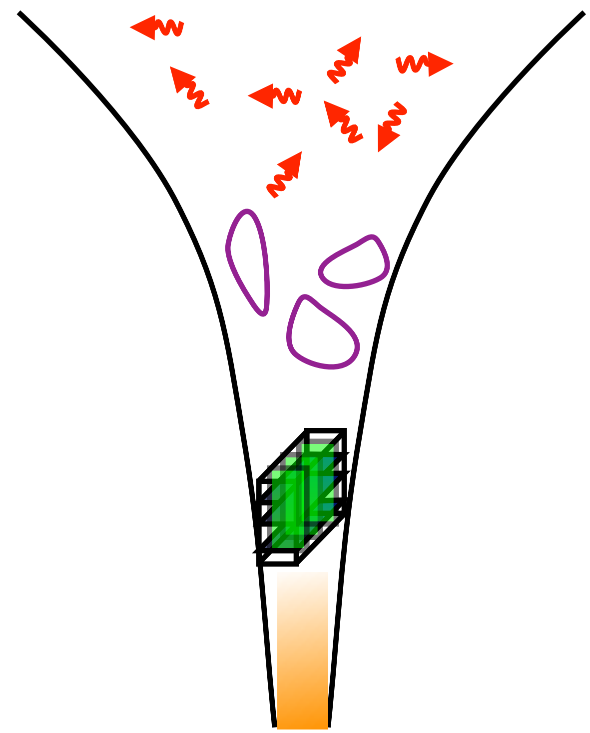
In this approximation, these oscillations encode the same dynamics
as the dynamics of free fall into the traditional hole

Summary

Looking forward:

Singularity of black hole is resolved by tunneling into fuzzballs

What about the singularity of the early Universe ??



Many results suggest a universal formula
for the entropy density of the early Universe:

$$s \sim \sqrt{\frac{\rho}{G}}$$

An interesting new set of ideas emerge from this …