Torsional Instantons In Quantum Gravity

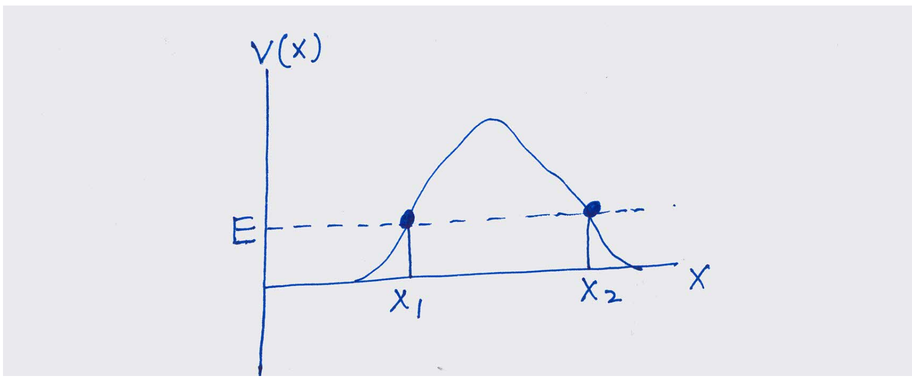
Sandipan Sengupta

IUCAA, Pune
INDIA

[References:
PRD90, 124081, R.Kaul and S.Sengupta, 2014;
arXiv 1501.00779, S.Sengupta, 2015]

Nonperturbative phenomena in QM

• In classical physics, a particle with total energy $E < V(x)$ cannot penetrate the potential barrier $V(x)$



• However, in quantum theory there is a small but finite amplitude of finding it inside the barrier (tunneling)

• Tunneling amplitude is given by the WKB formula: $T(E) = e^{-\frac{1}{\hbar}\int_{x_1}^{x_2} dx\ [V(x)-E]^{\frac{1}{2}}}$

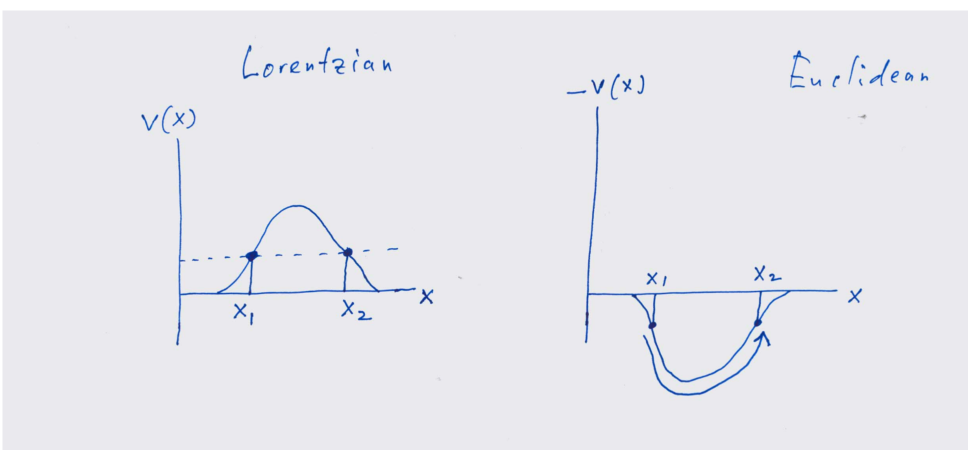
• This is a nonperturbative quantum effect, since the amplitude does not have a good power series expansion in $\hbar$; Cannot be captured by perturbation theory

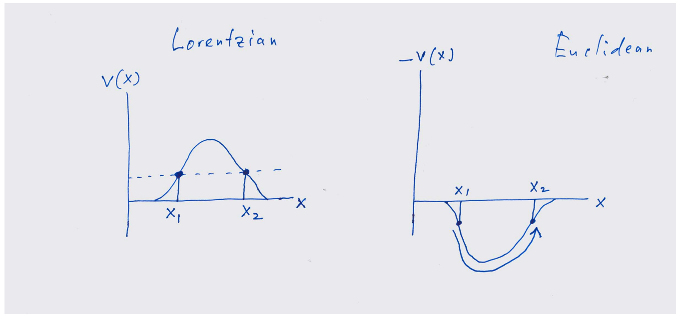
Nonperturbative effects≡ Instanton physics

- There is an alternative but more efficient way of analysing such nonperturbative processes- Instanton methods

- Although there are no classical trajectories in real time which can penetrate a barrier, such trajectories can exist in Euclidean theory (imaginary time)

- In Euclidean time $\tau = it$, $L_E = \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + V(x)$: motion of a particle in a potential $-V(x)$





• Such pseudoparticle solutions of classical Euclidean EOM are known as instantons; these have finite action

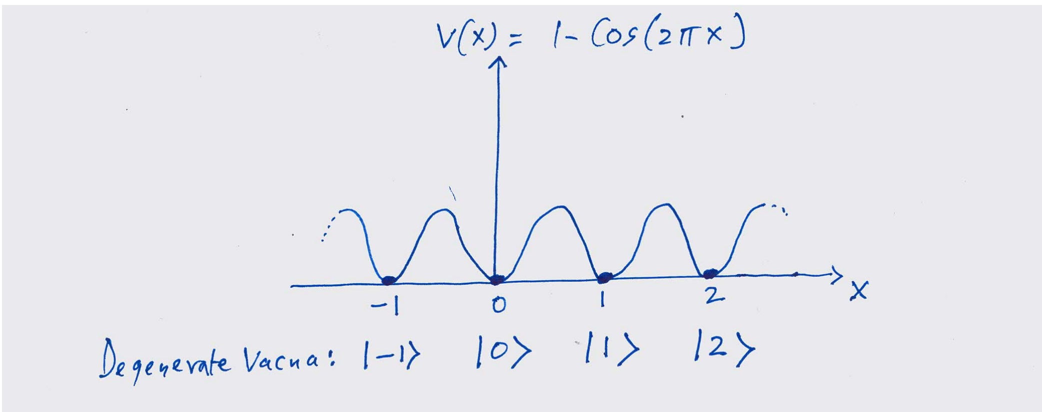
• The tunneling amplitude then just becomes the Euclidean path integral for instantons:
$\langle x_2|e^{-HT}|x_1\rangle = \int_1^2 Dx\ e^{-\frac{S_E(x)}{\hbar}}$

• Instanton solutions, being the stationary points of the action, lead to the most dominant contribution to this Euclidean path integral; reproduces the WKB formula exactly!

• Thus, although instantons are particles in imaginary time (Euclidean theory), they describe real quantum effects (of Lorentzian theory)

Electrons on a lattice $\equiv$ Instanton physics

- Particles moving in a periodic potential:
$L = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 - [1 - cos(2\pi x)]$



- There are infinite no. of (classical) minimum energy states for which $x = n$; Each of these minima should correspond to a quantum ground state $|n\rangle$

- However, due to tunneling between these states, the true quantum vacua are not $|n\rangle$, but a superposition of all these degenerate states: $|\theta\rangle = \sum_n e^{in\theta}\,|n\rangle$ (Bloch wavefunctions)

Interpretation of the θ parameter

- The energy of this nonperturbative vacuum has a band structure parametrised by the 'quantum' coupling constant θ: $E = \frac{1}{2}\hbar\omega + E_\theta$

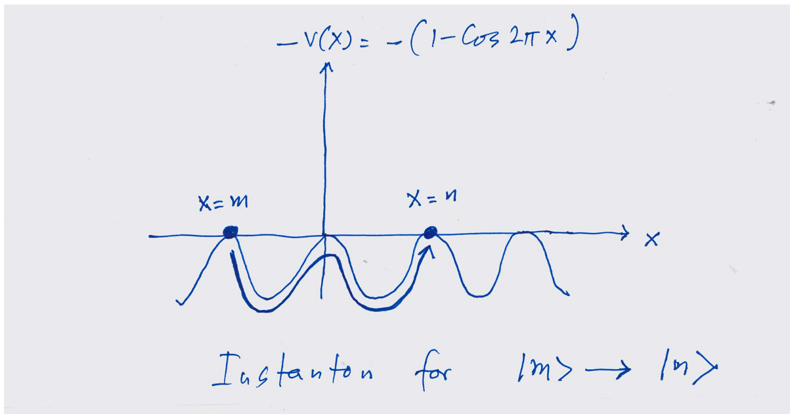
- The true ground state and its vacuum energy as above can be found by instanton methods

- How to understand the origin of θ within the Lagrangian formulation?

- In fact, computing expectation values of observables in $|\theta\rangle$-vacuum is equivalent to using an eff. Lagrangian $L_{eff} = \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + V(x) + i\theta\frac{dx}{d\tau}$ in the path-integral

- The total derivative term, and hence θ, do not affect the classical dynamics; However, they do affect the quantum dynamics

Topological interpretation of θ



• Integral of this term gives the instanton number of an instanton tunnelling between two degenerate minima $x_i(\tau_i) = m$ and $x(\tau_f) = n$:
$x_f - x_i = \int_i^f d\tau \, \frac{dx(\tau)}{d\tau} = n - m$ (integer-valued)

• The initial and final points $x_i(\tau_i) = n$ and $x_f(\tau_f) = m$ have the same (minimum) energy and can be identified as the same physical points

• Thus, the τ (and x) space effectively becomes a circle

Instanton effect $\equiv$ Topological density

• Hence, $\int_i^f d\tau \, \frac{dx(\tau)}{d\tau} = n - m$ describes the number of times x circle wraps around the τ circle- a topological property (integer valued winding number for the map $S^1 \to S^1$)

• Hence, the total divergence $\frac{dx(\tau)}{d\tau}$ in $L_{eff} = L_0 + i\theta \frac{dx(\tau)}{d\tau}$ is a topological derivative, and θ has a topological origin

• In general, the existence of instanton effects necessarily implies that the corresponding Lagrangian is ambiguous upto the addition of topological densities $L = L_0 + i\theta L_{top}$

• The topological parameter θ appears as an additional coupling constant, to be determined from experiments

Instanton physics and quantum gravity

- Instanton methods can be particularly useful for gravity theory in four dimensions

- Gravity as a perturbative quantum theory does not make much sense; However, instantons do not care about perturbative physics

- In other words, gravitational instanton effects can capture important nonperturbative features of quantum gravity which are otherwise difficult to unravel

Gravitational Instantons

- In four dimensions, gravity theory can be reformulated as a SO(3,1) gauge theory when written in terms of tetrad (e_μ^I) and spin-connection (ω_μ^{IJ}) fields (SO(4) in Euclidean)

- In terms of these, there are three possible topological densities $I_E,\ I_P,\ I_{NY}$:

Euler: $\epsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}R_{\mu\nu}{}^{IJ}(\omega)R_{\alpha\beta}{}^{KL}(\omega)$

Pontryagin: $\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu}{}^{IJ}(\omega)R_{\alpha\beta IJ}(\omega)$

Nieh-Yan: $\epsilon^{\mu\nu\alpha\beta}\left[e_\mu^I e_\nu^J R_{\alpha\beta IJ}(\omega) - 2\left(D_\mu e_\nu^I\right)\left(D_\alpha e_{\beta I}\right)\right]$

- Most general gravity Lagrangian (with or without matter):

$L = L_0 + \phi I_E + \theta I_P + \eta I_{NY}$

- Instantons carrying nontrivial Euler and Pontryagin topological charges are well-known

- But so far, no known example of a Nieh-Yan instanton (one that solves Euclidean EOM and carries a NY charge)

Nieh-Yan instantons

$L = L_0 + ... + \eta I_{NY}$

- $I_{NY} = d(e \wedge T) = 0$ if torsion T is zero

- Thus, NY instantons can necessarily live only in a first order gravity theory (with or without matter) where torsion does not vanish

- Is there a first order theory of gravity which has NY instantons?

- Do they lead to (observable) nonperturbative effects in quantum gravity?

Gravity coupled to axionic gravity

- Consider a first order gravity theory coupled to axion (antisymmetric tensor gauge field $B_{\mu\nu}$) since such (bosonic) matter can induce torsion in the first order theory

- Euclidean Lagrangian density:

$$L(e,\omega,B) = -\frac{1}{2\kappa^2} e e^\mu_I e^\nu_J R_{\mu\nu}^{\ \ IJ} + \beta e H^{\mu\nu\alpha} H_{\mu\nu\alpha} + \frac{1}{2\kappa} e H^{\mu\nu\alpha} e^I_\mu D_\nu e_{\alpha I}$$

where $H_{\mu\nu\alpha} = \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu} + \partial_\alpha B_{\mu\nu} = \partial_{[\mu} B_{\nu\alpha]}$

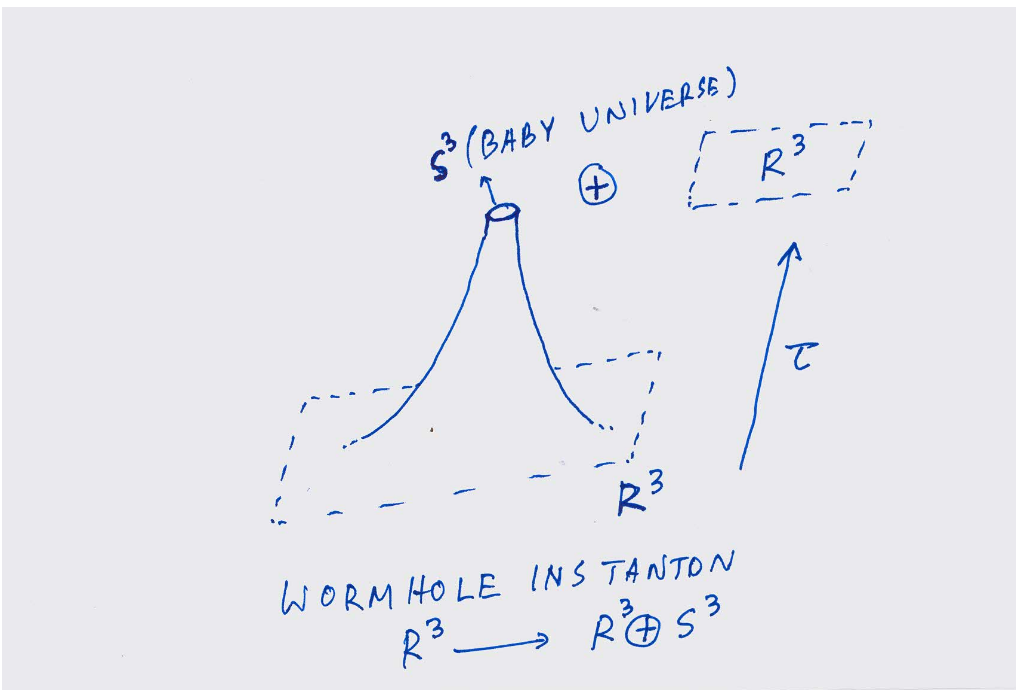
- The last term introduces a nonvanishing torsion (ω EOM): $T_{\mu\nu}^{\ \ I} := D_{[\mu} e^I_{\nu]} = \kappa H_{\mu\nu\alpha} e^{\alpha I}$

- Take a spherically sym. (Euclidean) metric $ds^2 = d\tau^2 + a^2(\tau)\, d\Omega^2(\chi,\theta,\phi)$ and solve the EOMs

Giddings-Strominger instanton

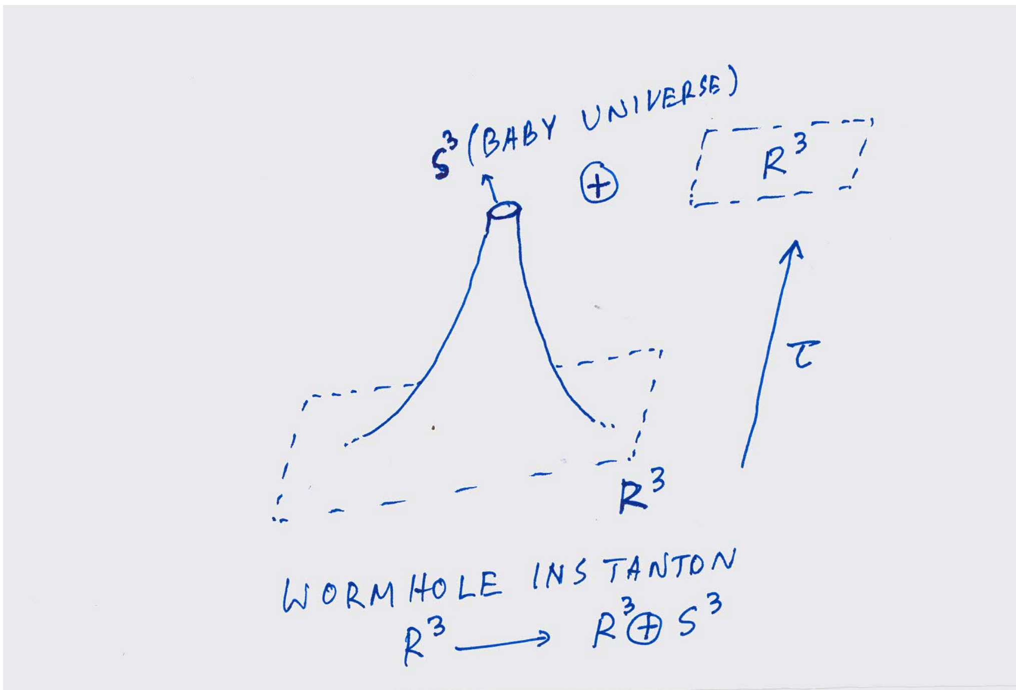
- The EOM for $a(\tau)$ becomes: $\dot{a}^2(\tau) = 1 - \frac{a_0^4}{a^4(\tau)}$

- This is the same eqn as that describing a Giddings-Strominger wormhole (1988) (found in a second order theory of axionic gravity)



- $a(\tau)$ can start (at distant past) at a very large value and end at a minimum size a_0

Giddings-Strominger instanton



• Each such instanton creates a baby universe (S^3) which carries an axion charge $Q = \int_{S_3} d^3x\ \epsilon^{abc} H_{abc}$

• What is the topological index (instanton number?

GS Wormhole $\equiv$ Torsional instanton

- The GS wormholes have a nonvanishing torsion $D_{[\mu}e^I_{\nu]} = \kappa H_{\mu\nu\alpha}e^{\alpha I})$

- The only torsional topological index is Nieh-Yan number:

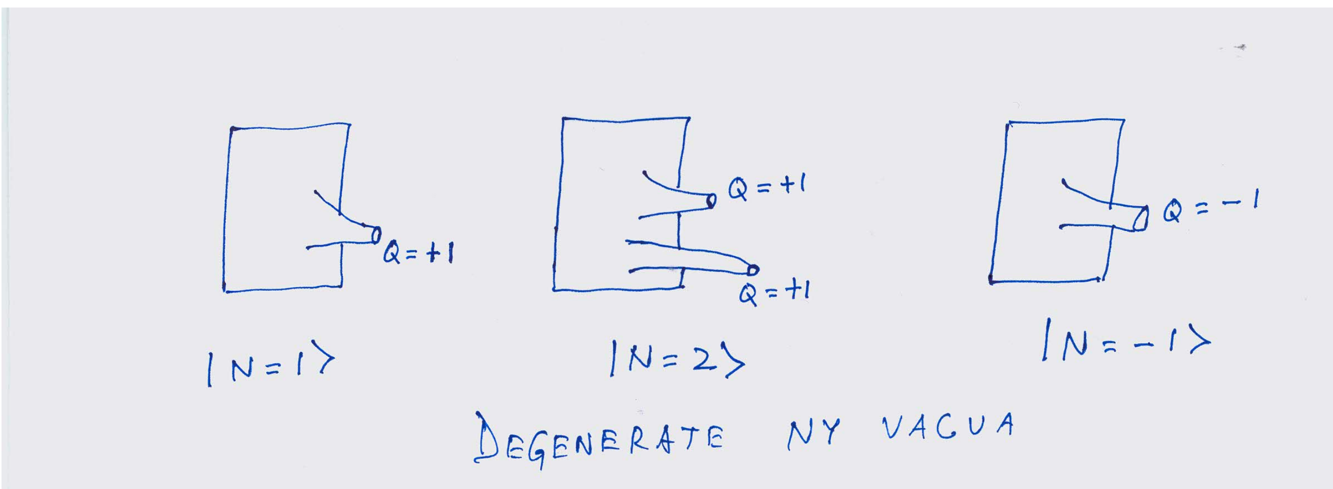
$$
\begin{aligned}
N_{NY} &= -\frac{1}{\pi^2\kappa^2}\int_{M^4} d^4x\ \partial_\mu\left[\epsilon^{\mu\nu\alpha\beta}e_{\nu I}D_\alpha(\omega)e^I_\beta\right] \\
&= \frac{1}{2\pi^2\kappa}\int_{S^3} d^3x\ [\epsilon^{abc}H_{abc}] \ = \ Q
\end{aligned}
$$

- Nieh-Yan no. of the GS instanton is nonvanishing and is exactly equal to the axion charge carried by the corresponding baby universe!

- GS wormholes are the NY instantons one has been looking for!

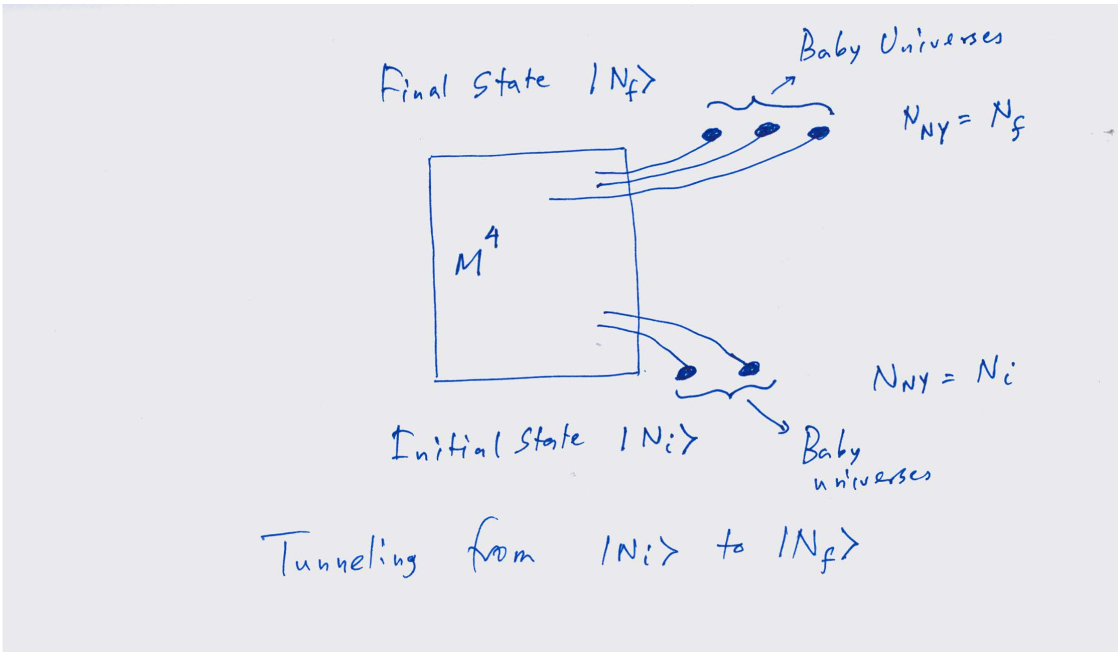
Tunneling: Nonperturbative effects

- Naively, there can be infinite number of ground states with different Nieh-Yan numbers N (baby universes): degenerate perturbative vacua $|N\rangle$



- However, NY instantons induce tunneling between these states and break this degeneracy

η Vacuum in quantum gravity



• As in the PP problem, the true vacuum of quantum gravity is a nonperturbative one: $|\eta\rangle = \sum_N e^{i\eta N} |N\rangle$, characterised by a new coupling constant η

• The Hamiltonian or the 'vacuum energy density' receives a modification of the size:

$$\rho_\eta = -2e^{-S_{inst}} K cos\eta$$

[See PRD 2014, R. Kaul and S.S for details]

η as a vacuum angle in quantum gravity

• The effective Euclidean Lagrangian which captures this nonperturbative vacuum structure parametrised by η is:

$$L = L_0 + i\eta I_{NY}$$

• Such a Lagrangian has been considered earlier in the context of canonical gravity and its quantization (e.g.Loop gravity), where the topological parameter multiplying the NY density in the Lagrangian is known as the Barbero-Immirzi parameter

• Thus, the vacuum angle η in the Giddings-Strominger theory can be identified precisely with the BI topological parameter; and is an exact analogue of the θ parameter of PP or QCD

To summarize..

- There are torsional instanton configurations (GS wormholes) in first order gravity, whose instanton number is the Nieh-Yan topological number

- Tunneling effects induced by these lead to a nonperturbative ground state $|\eta\rangle$ in quantum gravity

- This vacuum is parametrised by a P and T odd 'quantum' coupling constant, the Barbero-Immirzi parameter η; has to be determined from experiments

Phemenological Implications of $|\eta\rangle$ vacuum

- Such torsional instanton effects could be particularly relevant in the context of various parity violating phenomena in particle physics and cosmology

- The electric dipole moment of neutron depends on the θ-angle of QCD; Does it also depend on the gravitational η-angle?

- Can the vanishingly small energy density of the η vacuum provide a possible solution to the cosmological constant problem?
[arXiv:1501.00779, S.Sengupta, 2015]

- One can also look for parity violating signatures in CMB power spectrum and study the relevance of η dependent effects

- Such effects, if observationally relevant, might bring quantum gravity to the realm of experimental tests

THANK YOU

I express my deep gratitude to Ghanashyam Date, Sayan Kar, Romesh Kaul and Madhavan Varadarajan for many sessions of enlightening discussions during various stages of this work. I am indebted to the members of the TP group at RRI for their help and encouragement throughout my stint as a postdoc, during which most of this work was done.