Late-time behaviour of perturbations on black holes

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with Sudipta Sarkar (IIT Gn) and Srijit Bhattacharjee (IIIT Allahabad)
and
Partha Paul and Bidisha Chakrabarty (IOP)

Before my talk, I want to say something about GRG

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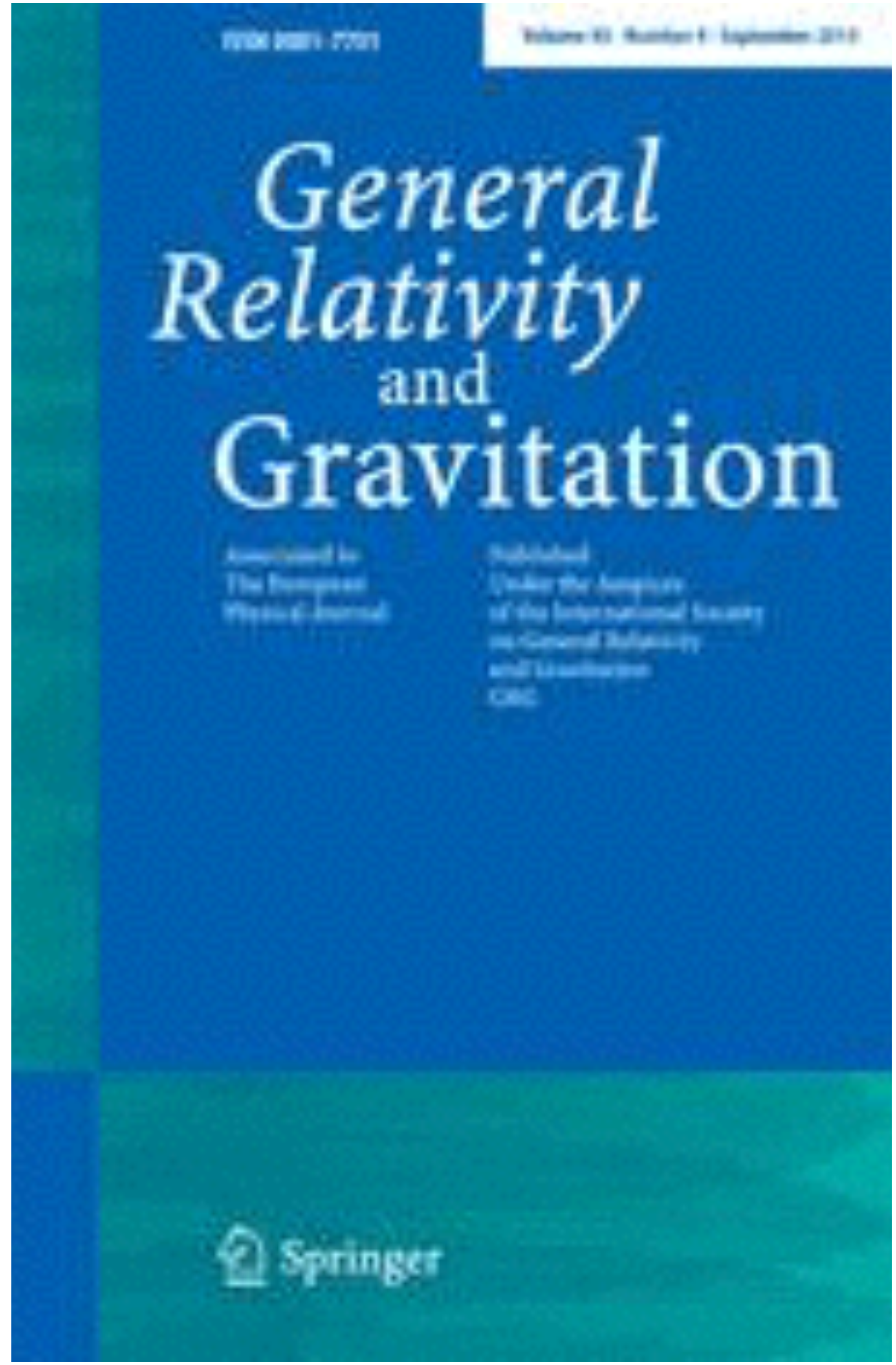

Topics Covered

Analytical general relativity, including its interface with geometrical analysis
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Relativistic astrophysics
Gravitational waves: data analysis, astrophysical sources and detector science
Extensions of general relativity
Supergravity
Gravitational aspects of string theory and its extensions
Quantum gravity: canonical approaches, in particular loop quantum gravity
Quantum gravity: path integral approaches, in particular spin foams, Regge calculus and dynamical triangulations
Non commutative geometry and gravitation
Experimental gravity, in particular tests of general relativity

GRG features

- Topical issues/collections, Invited reviews, Editor's choice articles, ***Golden oldies***

- Working on developing an interface with *Living Reviews in Relativity*

- Indian authors submit the most number of articles (20% of the total): 90 (2013), 98 (2014), 82 (2015), published are 14 (2013), 15 (2014), and 19 (2015). Our over-all acceptance rate is about 30%.

- Please talk to me if you would like to know more. We invite you to submit your most exciting work to GRG. We are also looking to qualified referees. We invite suggestions for exceptional authors or guest editors. **amitabh.virmani@gmail.com** (2017-2020).

Late-time behaviour of perturbations on black holes

- Classically, how waves decay on black holes?
- How do they accumulate near the inner horizon? and affect the internal structure of a black hole?
- Are the answers compatible with firewall/fuzzball proposals?
- Not all our ideas are fully distilled, nevertheless I will try to present a clear picture, with a review

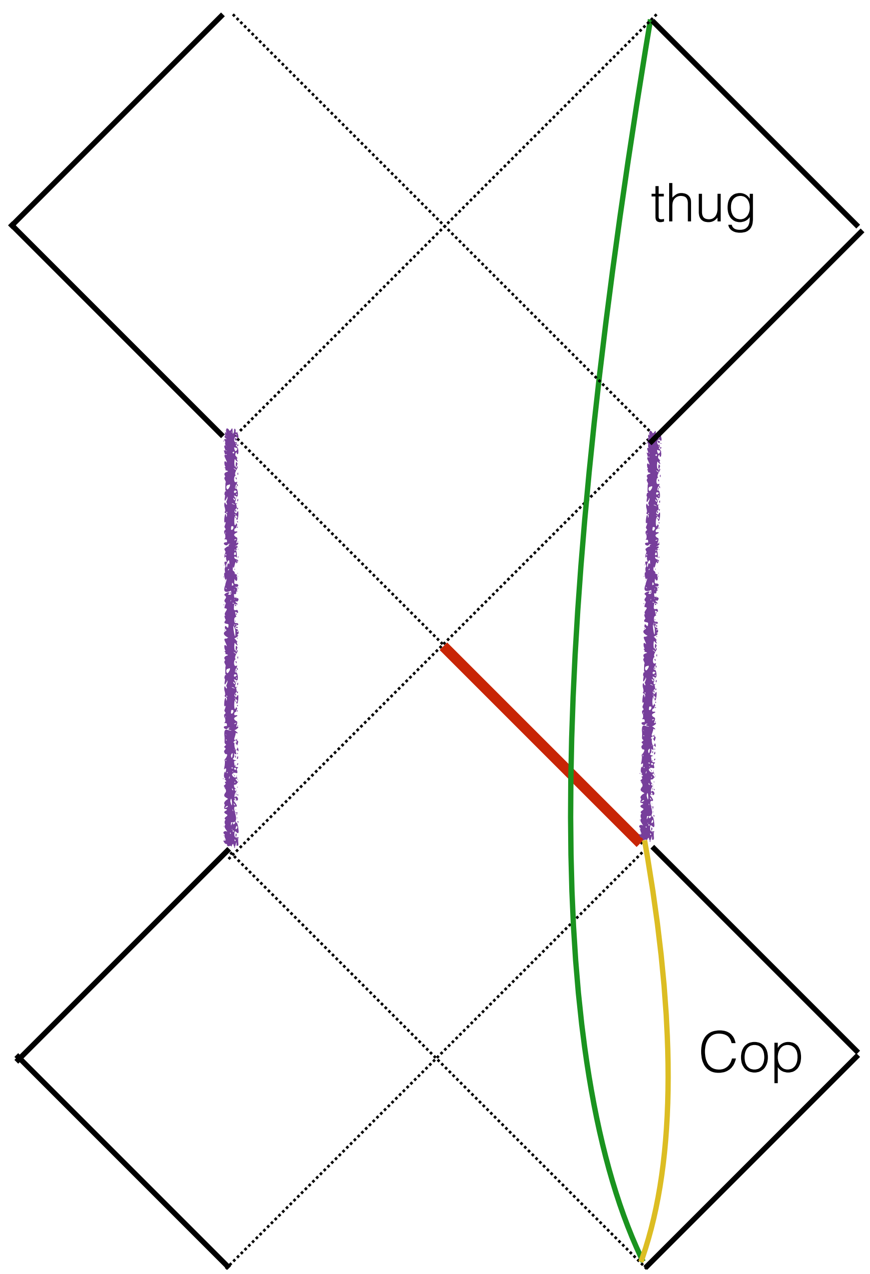
Outline

- Basics
- Price’s law
- Internal structure
- Our work
- Comments

Classical Internal Structure

- Classical internal structure of black holes is a topic much discussed in general relativity and science fiction alike

RN tunnel



Close the RN tunnel

Classical Internal Structure

- Classical internal structure of black holes is a topic much discussed in general relativity and science fiction
- We all believe that one shouldn't be able to travel from one universe to another by falling through a black hole

Collapse

- The central physical problem in general relativity is the study of collapse of self-gravitating systems
- This leads of holy grails of general relativity: formation of black holes, singularities, and Cauchy horizons
- This study in turn lead to the question of cosmic censorship, which perhaps is the central goal of mathematical general relativity

Ingredients

- Oppenheimer and Snyder in 1939 presented a detailed model of gravitational collapse with homogeneous dust
- It was later shown that this model has an instability, when introduced inhomogeneities — shell crossing singularities
- More robust investigation is necessary, this led to the birth of modern mathematical relativity

Progress

- Christodoulou, in 1960's studied Einstein-Scalar system in spherically symmetric setting and obtained mathematically precise results for gravitational collapse

- Oppenheimer Synder model was ODEs. Christodoulou's worked with PDE's. His work initiated the rigorous theory of nonlinear PDE's in GR

- Mihalis Dafermos and others have changed the perspective on these problems

This talk

- In this talk, I will not discuss any of the mathematics that these authors have introduced
- I will present a physics perspective on these problems (which is much older) and is currently revisited in a number of context
- There are issues of great interest to both physicists and mathematicians: Price’s law and the complete Penrose diagram for dynamical black holes

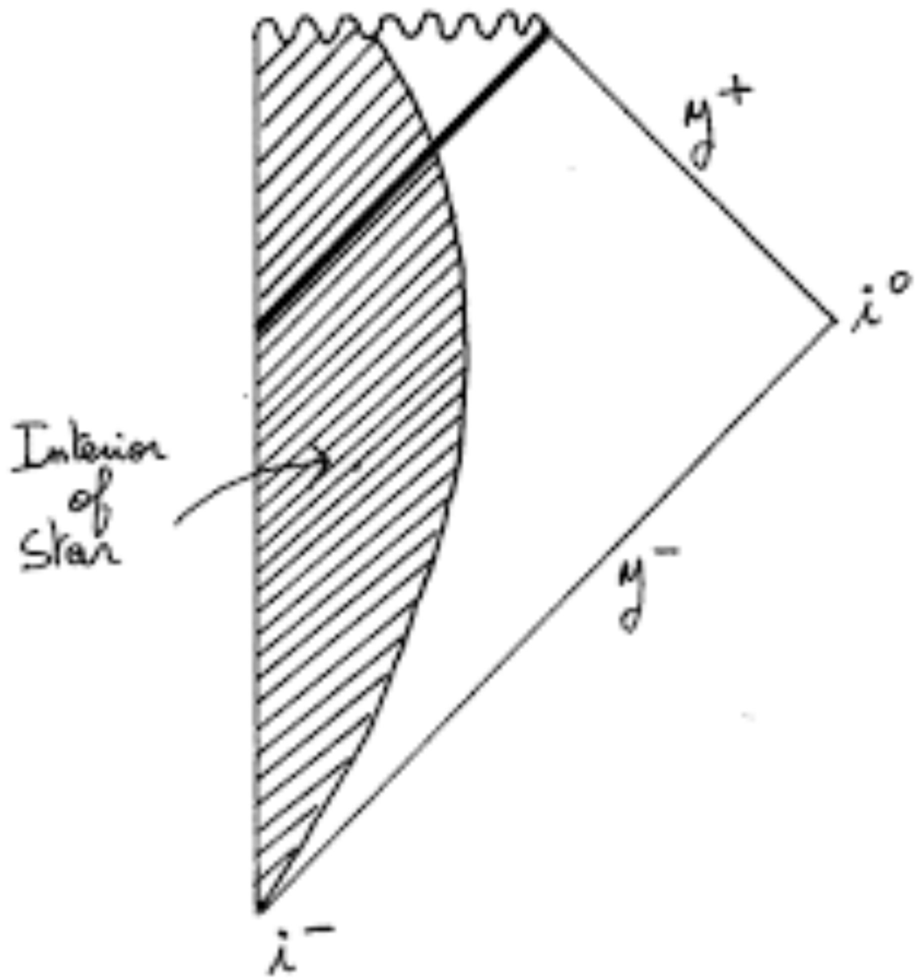
Part II

Price’s law

Price’s law

- Dynamical black holes typically approach a stationary solution at late times
- Non-stationary perturbations decay either by falling through the horizon or dispersing to infinity
- These decay properties are governed by the Price’s law

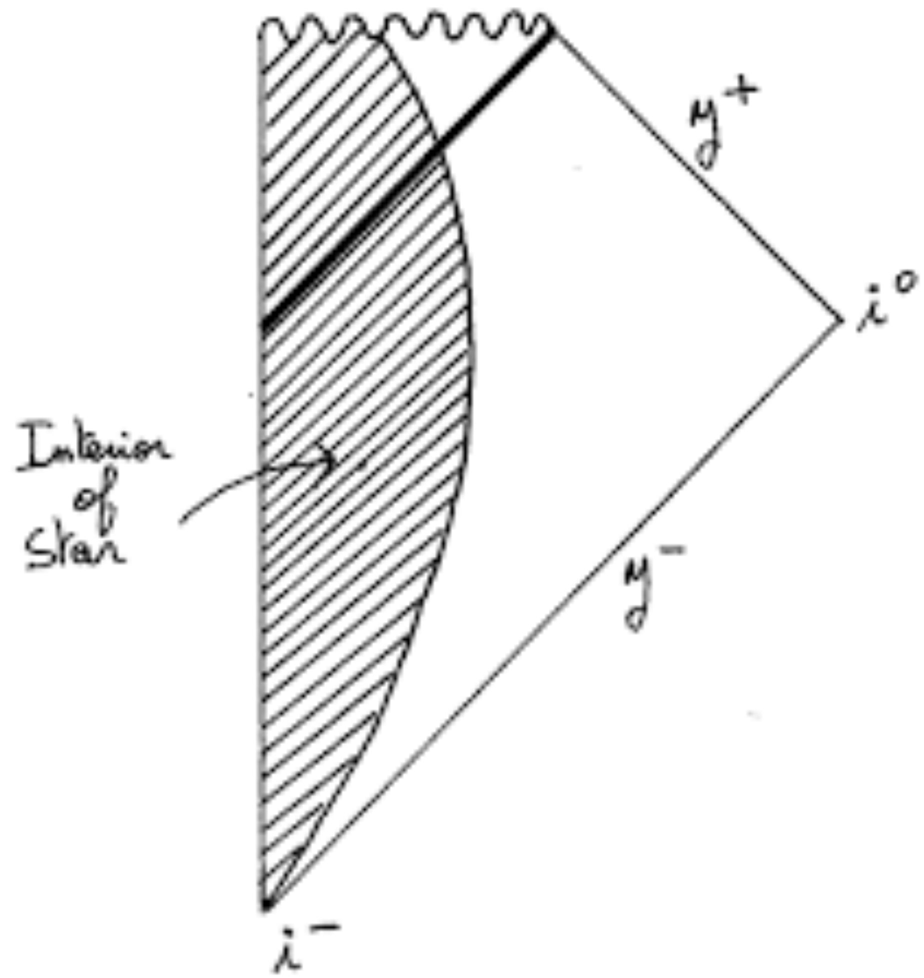
Price’s law: The set-up

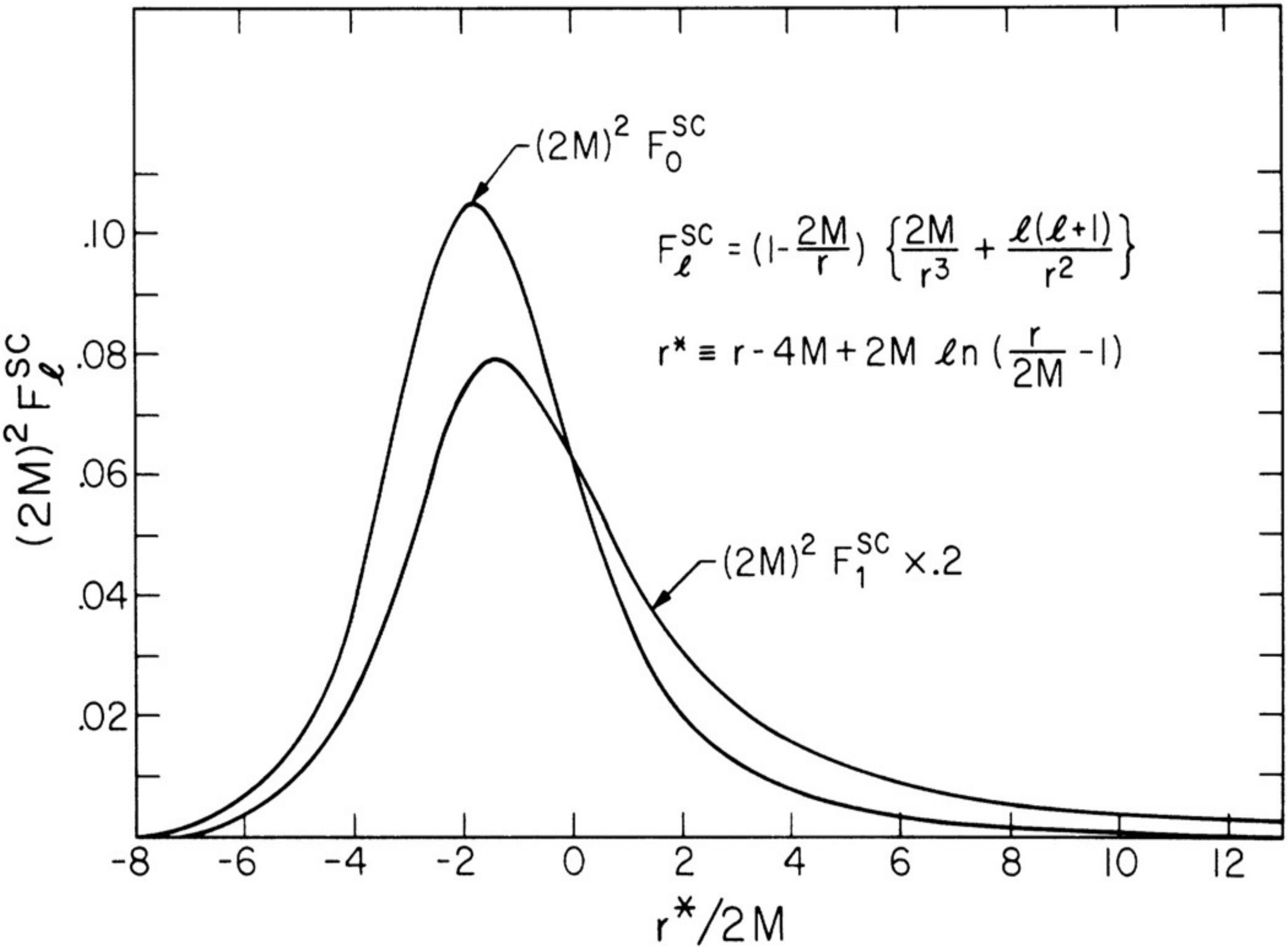


- Consider the star at r = 4 M
- Imagine the particles of the star as a source of (probe) scalar field
- Let the star collapse, and ask how the (probe) scalar field decays at late time
- These tails are governed by Price’s law

Price's law: The set-up

- The probe scalar has to decay, due to no hair theorems — no stationary scalar configuration outside the black hole
- Price (1972) did a detailed analysis of this question, and found power law tails at late times t
- His analysis was soon generalised. Power law tails were also found in u and v coordinates


Interior of Star
$\mathcal{I}^+$
i^0
$\mathcal{I}^-$
i^-



$(2M)^2 F_0^{SC}$
$F_\ell^{SC} = (1 - \frac{2M}{r}) \left\{ \frac{2M}{r^3} + \frac{\ell(\ell+1)}{r^2} \right\}$
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$(2M)^2 F_1^{SC} \times .2$
$(2M)^2 F_\ell^{SC}$
.10
.08
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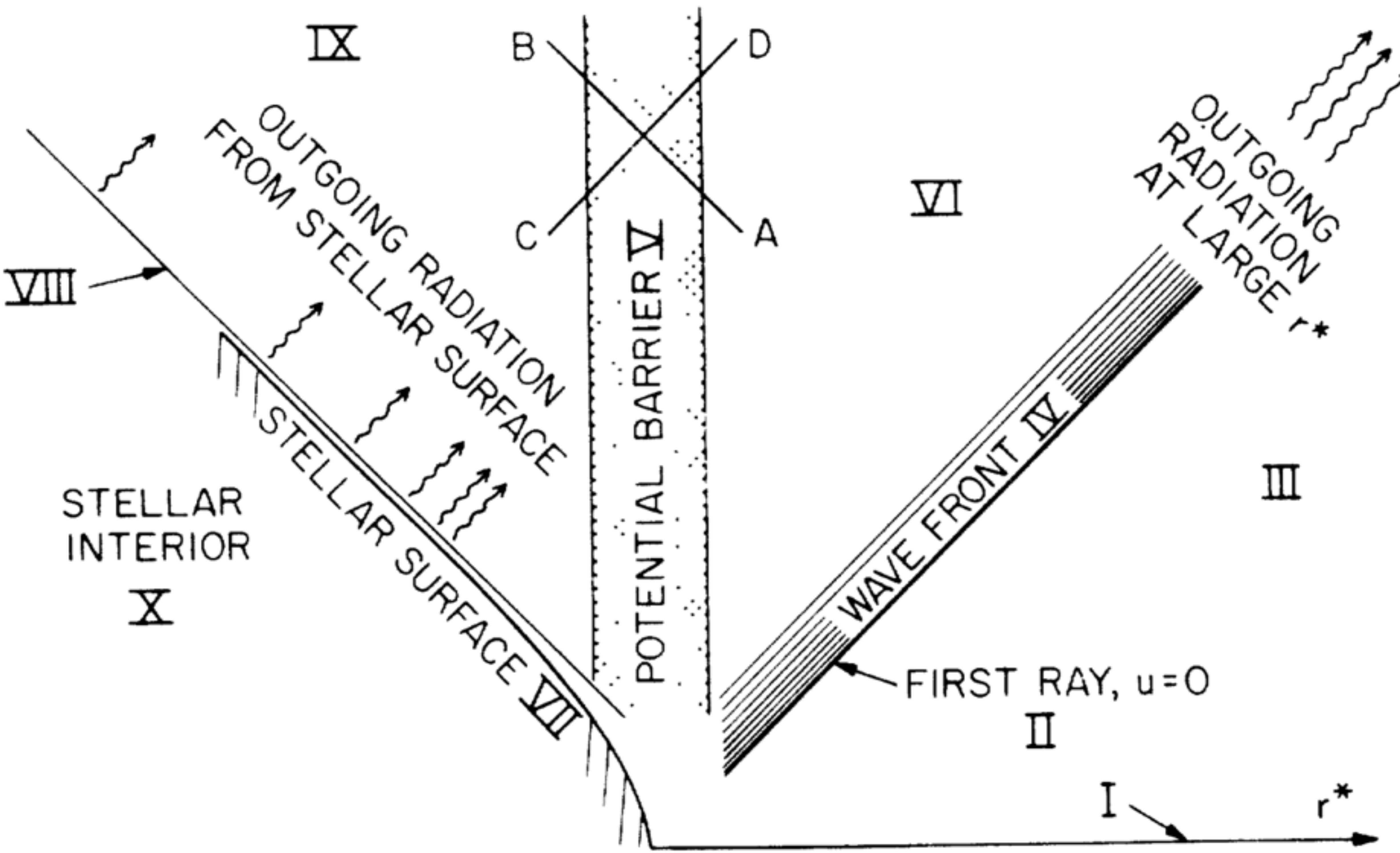
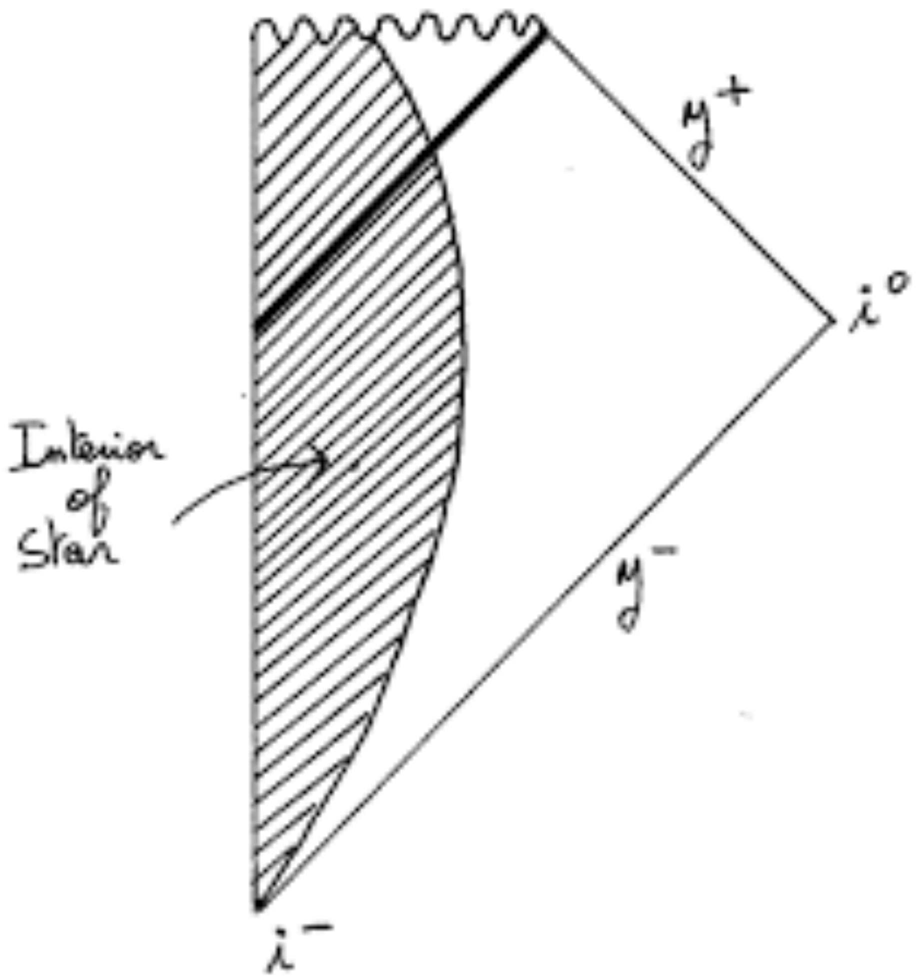
12
$r^*/2M$

Price's law set-up



Results



$$\phi(t, \text{fixed } r) \sim \frac{1}{t^{2l+3}}$$

$$\phi(u, \text{fixed } r) \sim \frac{1}{u^{l+2}}$$

$$\phi(v, \text{fixed } r) \sim \frac{1}{v^{2l+3}}$$

Late time behaviour of waves propagating on Schwarzschild is a power law tail.

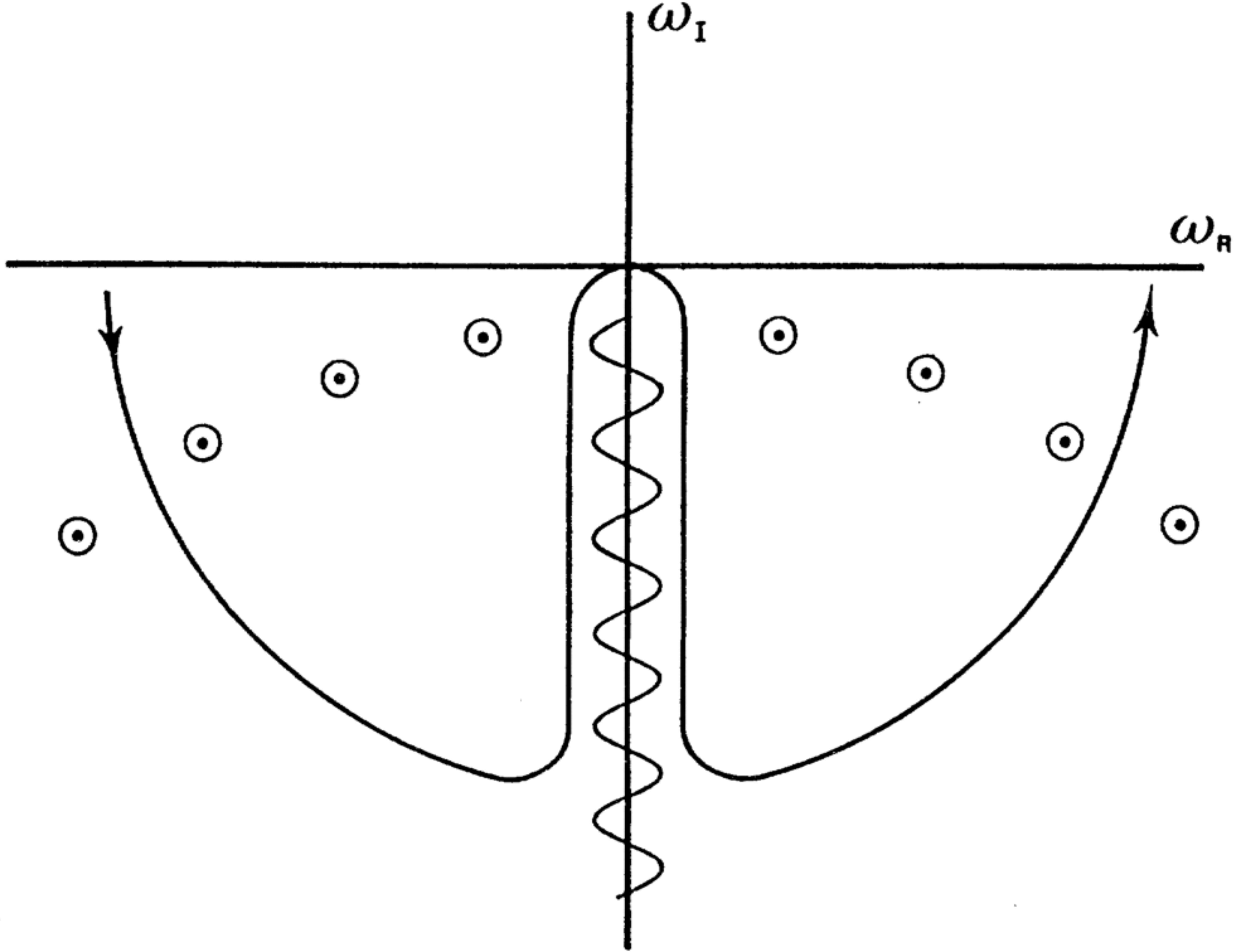
Green's function approach

- The Price's law is perhaps most conveniently reproduced from a Green's function analysis

- The propagation equations results in Klein-Gordon equation with an effective potential

$$[\partial_t^2 - \partial_x^2 + V(x)]\phi(x,t) = 0$$

- The effective potential describes the scattering of $\phi(x,t)$ with the background curvature.


ω_I
ω_R

Three distinct contributions

- Prompt contribution: disturbances simply propagate, contour contribution
- Quasinormal ringing: superposition of modes with time dependence $e^{i\omega t}$, poles
- Late time tails: after exponential decay, there are late time tails, branch cut

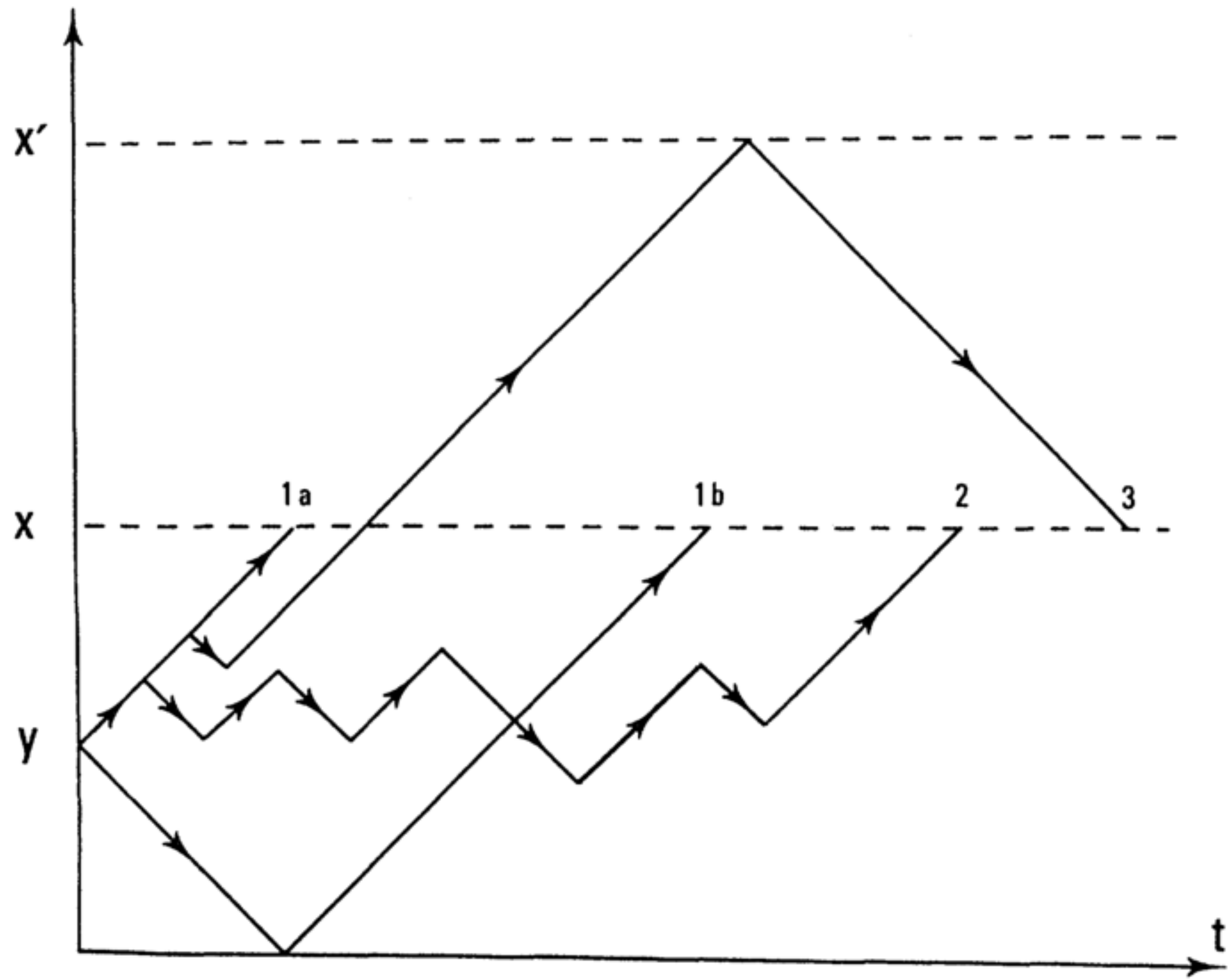
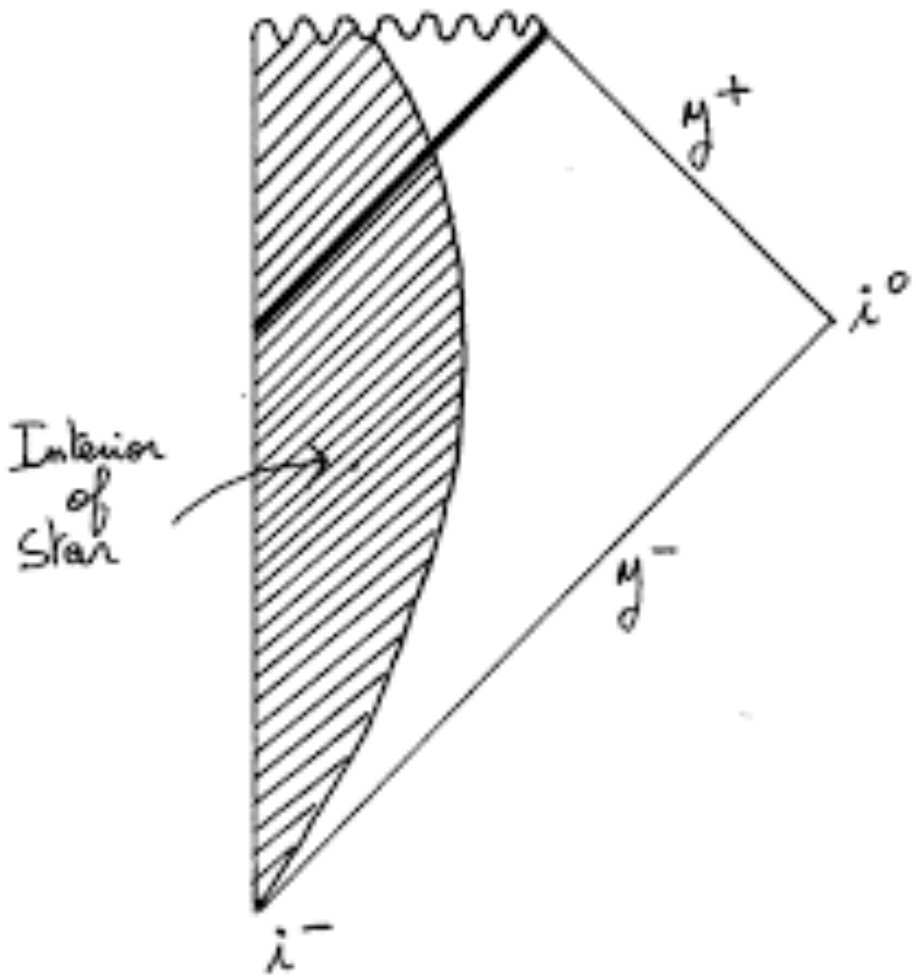


FIG. 3. A spacetime diagram illustrating heuristically the different contributions G_L, G_Q, and G_P (see text for definition) to the Green's function.

Price’s law, summary

- Power law tails, describing asymptotic tails in the exterior region
- Probe scalar — linear problem — spectral methods
- In the Einstein Maxwell Scalar field system — non-linear problem — new math methods — Dafermos & Rodnianski, 2003

Results: Power law tails



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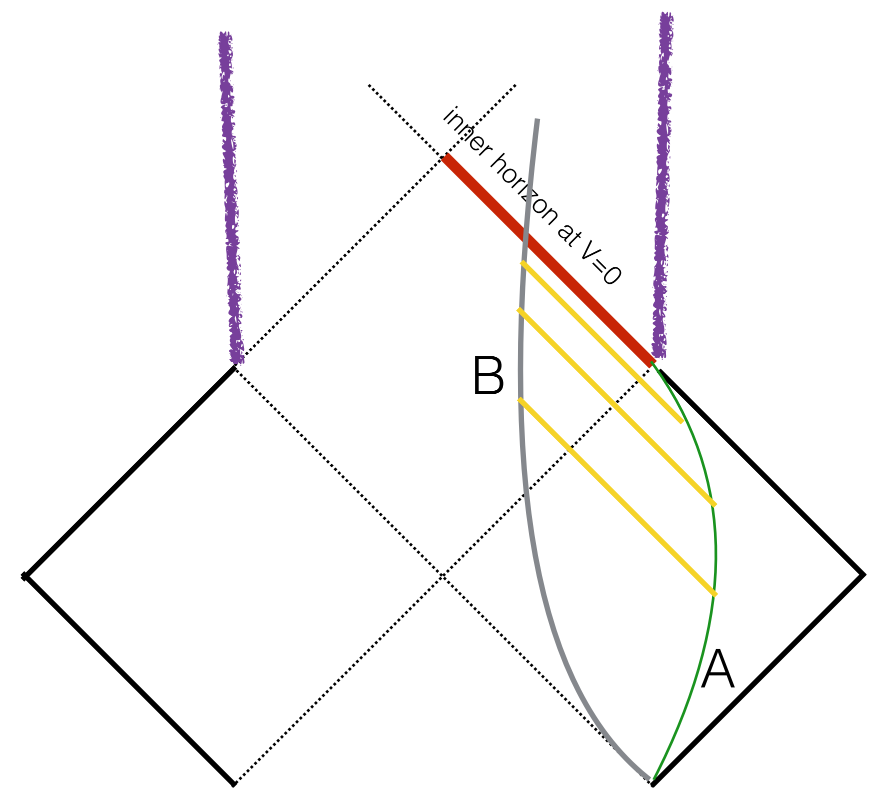
Part III

Internal Structure (Penrose diagram)

Internal structure of black holes: Penrose

- Two observers in RN black hole
- Observer A remains outside
- Observer B falls and crosses the inner horizon
- Observer A has infinite time to send signals, but B has finite time to collect
- Infinite blue shift — “blue wall”

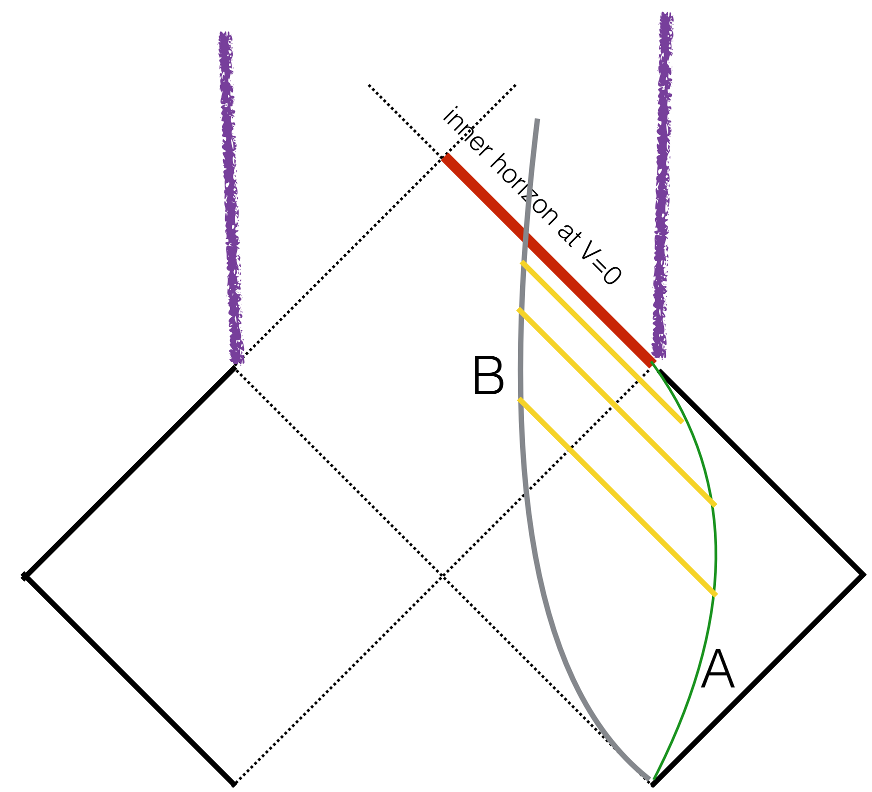


Probe analysis

- Using a probe analysis this “blue wall” can be confirmed, Chandrasekhar et al
- In AdS/CFT context this calculation has also been done by several authors, Balasubramanian et al, Rozali et al
- How to take into account back reaction? Can we find metric near the inner horizon and analyse the physics of an in-falling observer.

Backreaction

- Naturally it is a difficult problem
- One can at best make models
- And confirm these models in detailed numerical simulations
- If the model conforms to numerical simulations, one uses the model to extract as much physics as possible



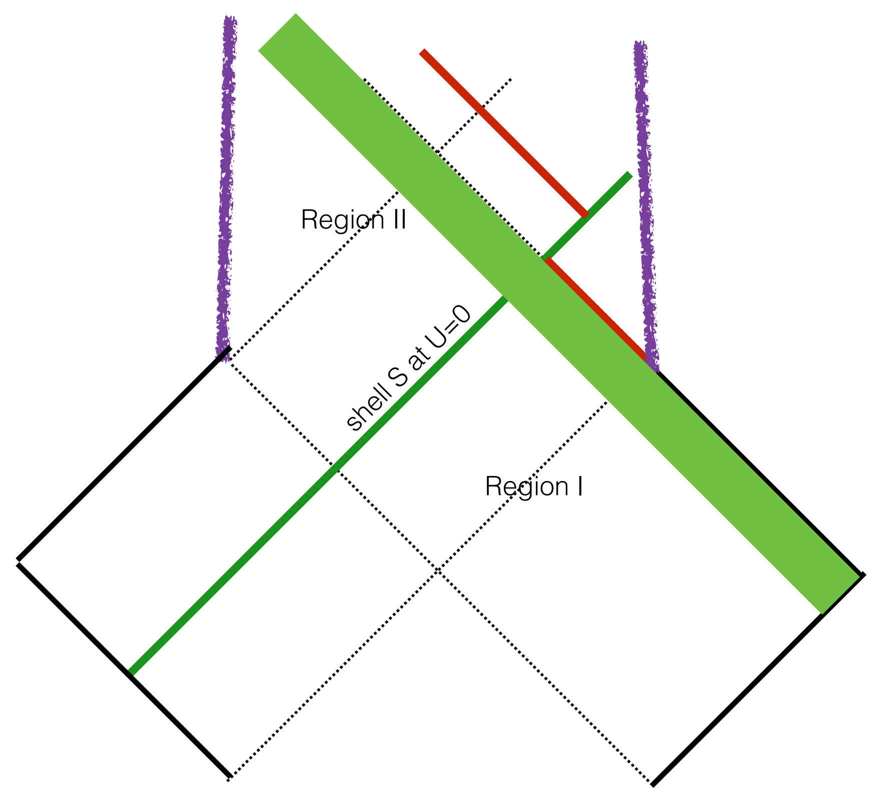
Models for Backreaction

- The Poisson Israel model — “Mass-inflation model” is the most studied model in this regard

- Ori presented a simpler version of the Poisson Israel model where all physics remains the same, and has the added advantage that metric can be determined everywhere

- I will present the Ori model, then its generalisation to AdS

Ori model



Ori model

- Think of this can two charged Vaidya solutions glued across shell S
- From the gluing one can determine the behaviour of the mass function from one side to the other
- On the outside take the ingoing Vaidya solution as dictated by Price's law

Ori model: results

- Effective mass parameter diverges at the inner horizon, the famous mass-inflation phenomenon

- Metric smooth near the inner horizon, certain curvature components, i.e., tidal forces, diverge

- Tidal distortion (geodesic deviation) across the mass-inflation singularity finite

- Thus, weak singularity at the inner horizon

Our work
Ori model in AdS

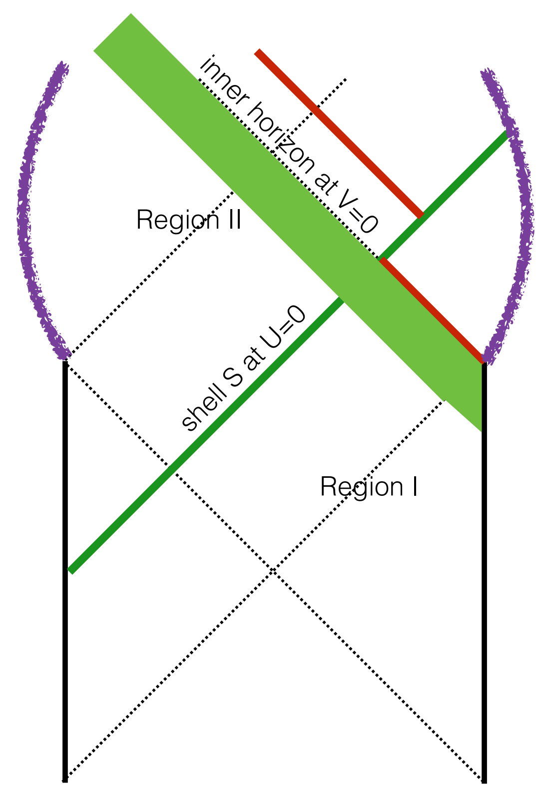
AdS

- Since the work of Horowitz and Hubeny it is known that such power law tails are not there in AdS
- So, how to model the infalling radiation. This point did not allow to make progress for a long time.

Ori model in AdS, our work

- Since no power law tails in AdS, first guess would be to take the lowest quasi normal mode exponential tail in our model
- However, the imaginary part of the lowest quasinormal mode in global AdS is shown to go to zero Festuccia & Liu; Dias, Marolf, Horowitz, Santos
- Two Cambridge mathematicians have computed the tails in AdS, logarithmic tails Holzegel and Smulevici

Ori model in AdS, our work



with Srijit Bhattacharjee and Sudipta Sakar

Ori model in AdS: results

- Qualitatively the same results, some details are different
- Effective mass parameter diverges at the inner horizon, the famous mass-inflation phenomenon
- Metric smooth near the inner horizon, certain curvature components, i.e., tidal forces, diverge
- Tidal distortion (geodesic deviation) across the mass-inflation singularity finite
- Thus, weak singularity at the inner horizon

Part IV

Comments

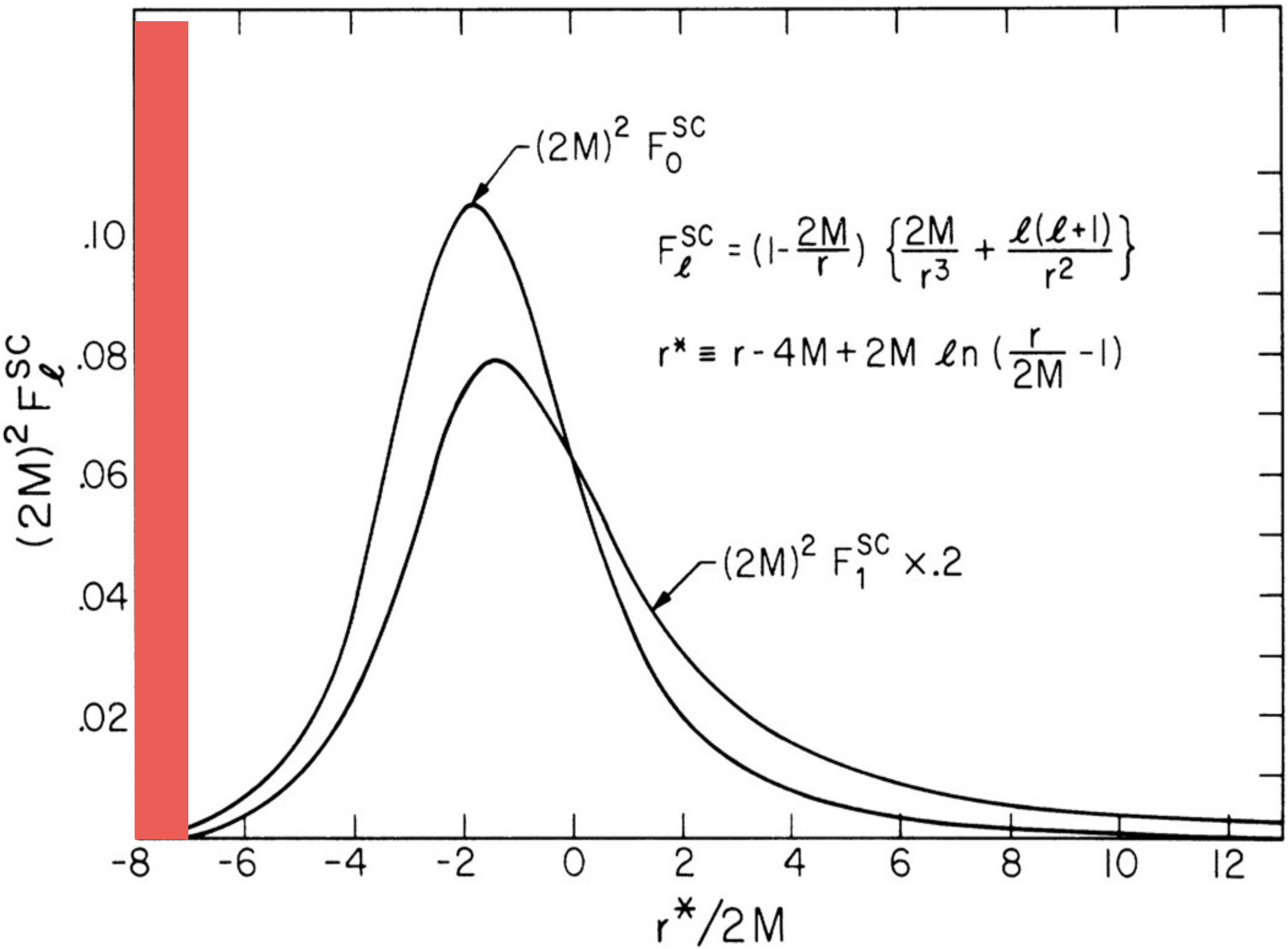
More on Price’s law

- In a recent paper Eperon, Reall, Santos, have pointed that waves on fuzzballs decay qualitatively differently than on black holes

- For the derivation of Price’s law, the absorptive boundary conditions and the nature of the potential are the only two ingredients

Comments

- As per the fuzzball thinking, there are large number of modes at the bottom of the throat that get excited to absorb any extra energy
- A probe calculation on a particular fuzzball should not be taken too far. As soon as any excitation is added at the bottom of the throat, new modes will get excited



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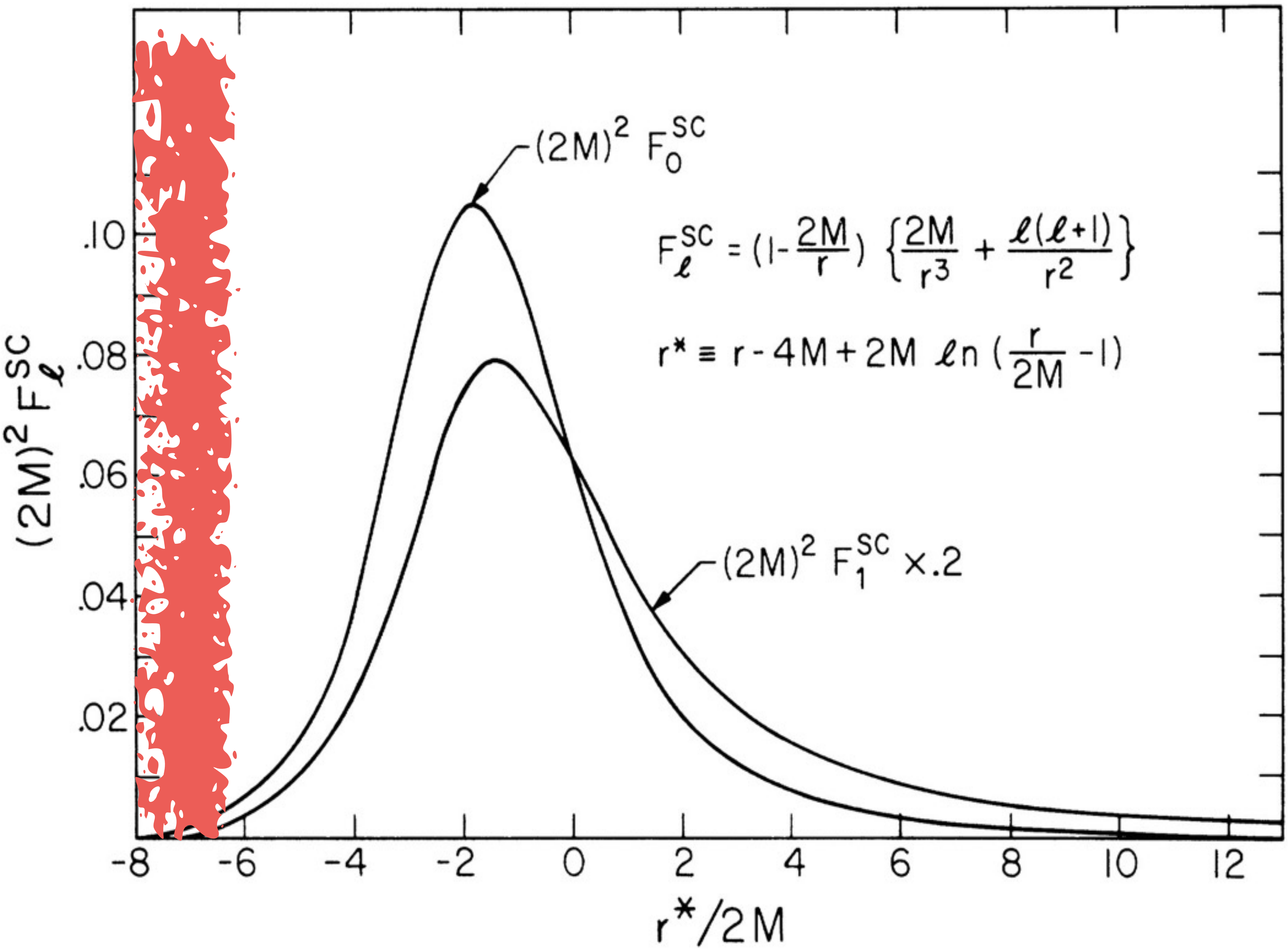

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Challenge: Price's law from the D1-D5 CFT?

- Can we recover the Price's law from a CFT calculation? We are hopeful. For extremal black holes, throat contribution is more important than asymptotic contribution.

- Das-Mathur were able to recover grey-body factors and other features. Can the same logic be pushed for decay of waves?

- We are puzzled by these questions, particularly in the light of Eperon, Reall, Santos paper. We are exploring.