Binary black hole waveforms in gravitational wave astrophysics

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LIGO-G1700951

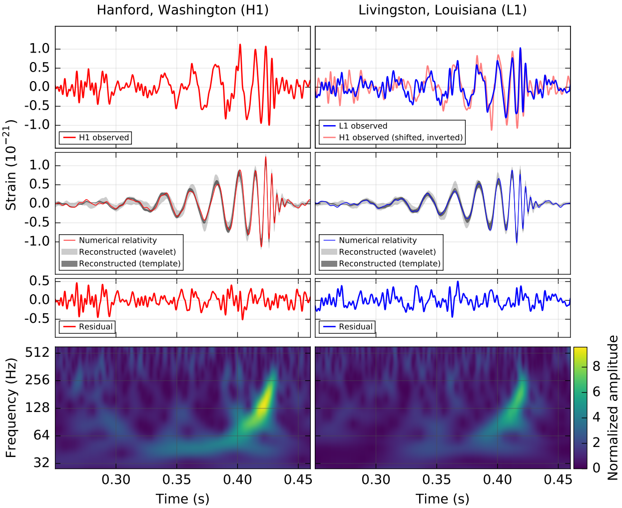
May 20, 2017

The first detection

(Phys. Rev. Lett. 116, 061102 (2016))

- A number of analysis pipelines analyze the data in real time
- One of these, looking for unmodeled bursts, reported an event at 10:50 am, CET on Sep. 14
- Reported to the collaboration at 11:30 am, CET
- Was clear very quickly that this was most likely not an injection, and most likely from a binary black hole system
- Subsequent analyses confirmed that this is indeed a true signal
- Results published on February 11 (PRL **116** 061102 (2016))

The first detection



The first detection

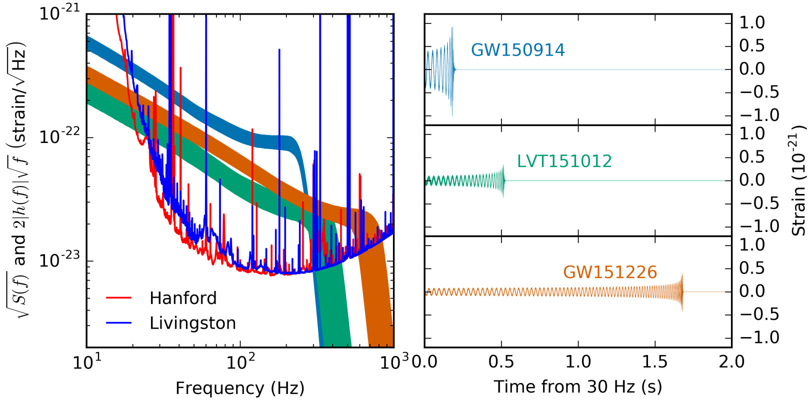
- Data has been band-passed between 50 and 350 Hz
- Strong spectral lines removed
- Signal arrived 6.9ms later at Hanford
- Hanford time series is shifted and inverted
- Comparison with numerical relativity simulation shown in solid line agrees with reconstructed waveform

The first observational run

- The first observational run (O1) of Advanced LIGO took place from Sep 12, 2015 to Jan 19, 2016.
- Total coincident time between H1 and L1 is 51.5 days
- After data-quality cuts, we are left with 46.1 days
- Detectors are being upgraded and the next run will begin later this year
- There are two BBH detections: GW150914 and GW151226 with significance better then 5σ
- There is a third, more marginal event, LVT151012 – an unambiguous detection is not claimed for this event

The three events

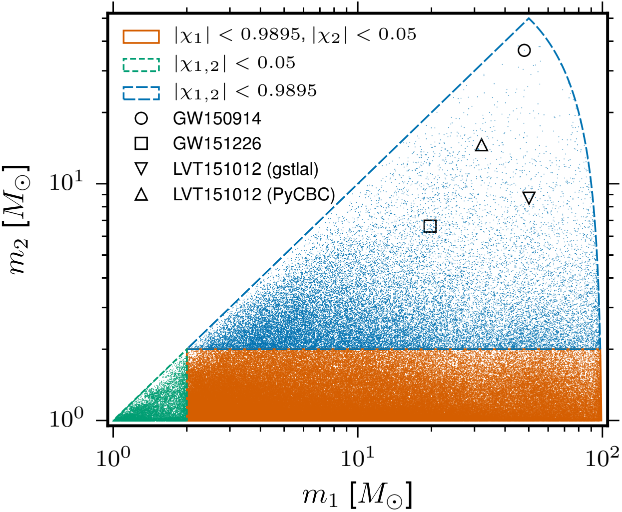
(arXiv:1606.04856)



Template bank used in the search

(arXiv:1606.04856)

Parameter space is 4 dimensional $(m_1, m_2, \chi_1, \chi_2)$



The three events

Event	GW150914	GW151226	LVT151012
ρ	23.7	13.0	9.7
Significance	$> 5.3\,\sigma$	$> 5.3\,\sigma$	1.7σ
$m_1^{\rm source}/{\rm M}_\odot$	$36.2^{+5.2}_{-3.8}$	$14.2^{+8.3}_{-3.7}$	23^{+18}_{-6}
$m_2^{\rm source}/{\rm M}_\odot$	$29.1^{+3.7}_{-4.4}$	$7.5^{+2.3}_{-2.3}$	13^{+4}_{-5}
$\mathcal{M}^{\rm source}/{\rm M}_\odot$	$28.1^{+1.8}_{-1.5}$	$8.9^{+0.3}_{-0.3}$	$15.1^{+1.4}_{-1.1}$
$M^{\rm source}/{\rm M}_\odot$	$65.3^{+4.1}_{-3.4}$	$21.8^{+5.9}_{-1.7}$	37^{+13}_{-4}
$\chi_{\rm eff}$	$-0.06^{+0.14}_{-0.14}$	$0.21^{+0.20}_{-0.10}$	$0.0^{+0.3}_{-0.2}$
$M_{\rm f}^{\rm source}/{\rm M}_\odot$	$62.3^{+3.7}_{-3.1}$	$20.8^{+6.1}_{-1.7}$	35^{+14}_{-4}
$a_{\rm f}$	$0.68^{+0.05}_{-0.06}$	$0.74^{+0.06}_{-0.06}$	$0.66^{+0.09}_{-0.10}$
z	$0.09^{+0.03}_{-0.04}$	$0.09^{+0.03}_{-0.04}$	$0.20^{+0.09}_{-0.09}$

What is all this good for?

- Wide range of issues in stellar evolution, astrophysics and cosmology. Example: what is the distribution of masses and spins in BBH mergers as a function of redshift? We make certain assumptions now, but better to just be able to measure it!
- Will allow us to test astrophysical scenarios for forming stellar mass BHs (e.g binary evolution vs. CHE?)
- Fundamental physics and deviations from gemeral relativity – deviations can be quite small and results possibly won't be accepted unless waveforms are sufficiently accurate

Different goals have different waveform accuracy and modeling requirements

A back of the envelope analysis with the simplest waveform

- Basic question: why is this a BBH system?
- The signal frequency increases from 35 to 150 Hz over about 8 cycles: a binary system is a plausible explanation (maximum orbital frequency is then 75Hz)
- At leading order, frequency evolution of a binary system follows:

$$\dot{f} = \frac{96\pi^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}$$

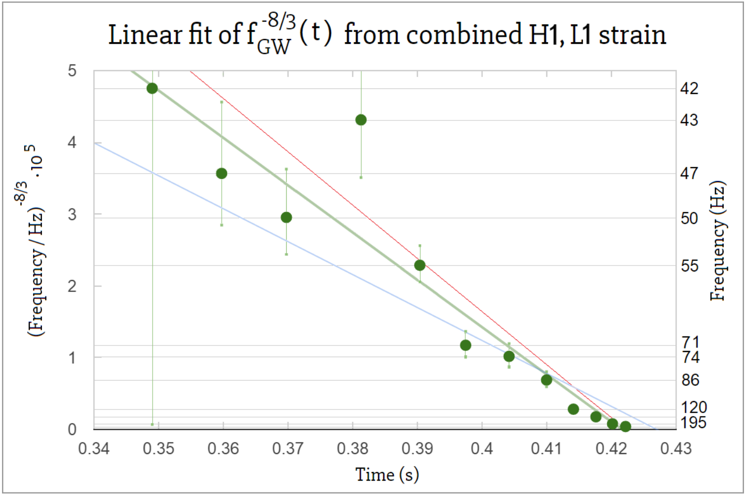
  where $\mathcal{M}$ is the chirp mass: $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$
- Alternatively, we can integrate this:

$$f^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t)$$

- The chirp mass turns to be about $\mathcal{M} \approx 30 M_\odot$

Why is this a binary black hole system?

(arXiv:1608.01940)



(Green: best fit, Blue: $\mathcal{M} = 30M_\odot$, Red: $\mathcal{M} = 40M_\odot$)

Why is this a binary black hole system?

- If the masses were equal, the total mass M is

$$M = m_1 + m_2 = 2 \times 2^{1/5}\mathcal{M} \approx 70 M_{\odot}$$

- At an orbital frequency of 75 Hz, Kepler's third law leads to a separation of

$$R = \left(\frac{GM}{\omega_{orb}^2}\right)^{1/3} \approx 350\,\mathrm{km}$$

- The two objects need to fit within this orbit, i.e. $R_1 + R_2 < R$
- If the two objects were non-spinning Schwarzschild black holes, then their Schwarzschild radius would be 103 km so,

$$R_1 + R_2 \approx 206\,\mathrm{km} < R$$

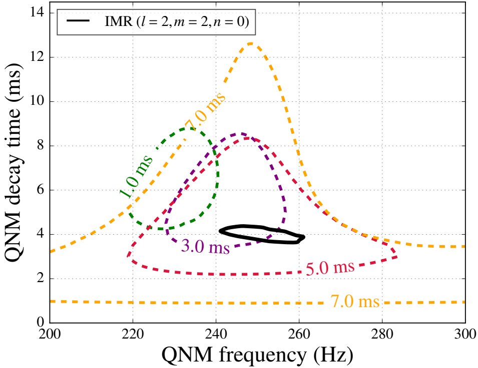
Why is this a binary black hole system?

- No other plausible alternative stellar models predict stars which would be so massive and so compact.
- Argument can be easily extended for non-equal masses, eccentricity and spins
- This is not a proof because it does not include non-linearities of GR and does not use coordinate independent quantities, and it does not replace more detailed analysis – it does not work for weaker detections
- However, it is a useful consistency check – we would have been worried if this had not given the approximate answer!

Is the final black hole a Kerr black hole?

- The ringdown waveform is a superposition of damped sinusoids determined by three quantum numbers (n, ℓ, m)
- Assuming the final BH to be Kerr, the frequencies and damping times are determined by the mass and spin of the final black hole
- Can we determine the final BH parameters based just on the ringdown? Can we check whether it is a Kerr BH?
- The complete waveform would have this information and would give smaller error bars
- However, fewer assumptions $\implies$ stronger test

Is the final black hole a Kerr black hole?



Key unknown: at what point can the final black hole be described as a perturbation of Kerr? We can hope to answer this question observationally! Testing Kerr nature requires more than 1 mode (Dreyer et al, 2004)

Numerical Relativity

- The most complete analysis requires complete waveforms, i.e. including inspiral, merger and ringdown
- This requires us to model the coalescence phase – the best we can do currently is with numerical simulations
- We want to solve the Einstein field equations for black holes with some initial configuration configuration $(d_0, m_{1,2}, \mathbf{P}_{1,2}, \mathbf{S}_{1,2} \ldots)$
- Specify initial data (q_{ab}, K_{ab}) satisfying the constraint equations on a spatial hypersurface

$$\widetilde{R} + K^2 - K_{ab}K^{ab} = 0$$

$$\widetilde{\nabla}_a K^{ab} - q^{ab}\widetilde{\nabla}_a K = 0$$

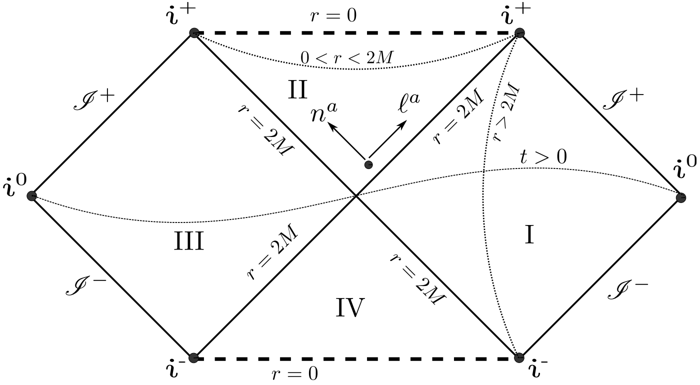
Numerical Relativity

- Simplest initial data is the Brill-Lindquist solution $K_{ab} = 0$, $q_{ab} = \psi^4 \delta_{ab}$
- The constraint reduces to the Laplace equation for ψ and a suitable solution is

$$\psi = 1 + \frac{m_1}{2r_1} + \frac{m_2}{2r_2}$$

  with $r_{1,2}$ being the distances from the two "punctures" – head-on collision between two black holes
- Easy to generalize to include arbitrary linear momenta and spins (with K_{ab} non-zero)
- It is now well understood how to set up initial data, evolve it, extract waveforms, locate black holes and measure their parameters
- Black hole parameters measured on margnally trapped surfaces – typically with surface integrals

Numerical Relativity



The “Phenom” models

- Two approaches used so far: “Phenom” and “EOBNR”
- 8 intrinsic physical parameters of the system: $m_{1,2}, \mathbf{S}_{1,2}$
- Extrinsic parameters: $D, \mathbf{n}, \psi, \iota$
- $M = m_1 + m_2$, $q = m_2/m_1$, $\eta = m_1 m_2/M^2$, $\chi = S/m^2$
- Searches assume aligned spins, i.e. $\mathbf{S}_{1,2}$ aligned with $\mathbf{L}$
- Spins combined into a single “effective spin” parameter [Khan et al 2015, Puerrer et al 2016]

$$\chi_{eff} = \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2} - \frac{38\eta}{113}(\chi_1 + \chi_2)$$

(additional rescaling to ensure $-1 < \chi_{eff} < 1$)

The "Phenom" models

- First Phenom model developed soon after successful BBH simulations: Ajith et al 2007
- Most recent aligned spin model: Khan et al 2015
- Main idea is to introduce phenomenological parameters λ which are convenient to model the waveform and fit with NR results
- Need a mapping between λ and physical parameters – the latest Phenom model has 17 phenomenological parameters which are mapped to χ_{eff}, η

The “Phenom” models

- Use PN based ansatz in inspiral regime
- Merger uses fits inspired by NR
- Ringdown is of course a damped sinusoid with parameters from NR fits
- “Target” waveforms are hybrids of NR + PN/EOB
- Modeling is in frequency domain – efficient for searches
- Analytic expressions in the end – efficient for searches

The "EOBNR" models

- Developed initially by Damour & Buonanno in 1998
- Most recent update: Bohe et al 2017
- Main idea is to replace the real binary system by an "effective" model of a test particle orbiting a deformed Schwarzschild/Kerr black hole
- For non-spinning system use a deformed Schwarzschild effective metric

$$ds^2_{eff} = -A(r)dt^2 + \frac{D(r)}{A(r)}dr^2 + r^2 d\Omega^2$$

- $A(r)$ and $D(r)$ chosen to get correct energies [Buonanno & Damour 1997]

$$A = 1 - \frac{2M}{r} + 2\eta\frac{M^3}{r^3} + \cdots$$

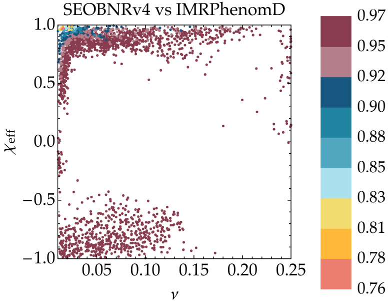
$$D = 1 - 6\eta\frac{M^2}{r^2} + \cdots$$

The “EOBNR” models

- Attractive idea but deformation of Kerr is not a solution of any field equations
- Additional “phenomenological” parameters are introduced in effective metric for the merger which is calibrated by NR simulations
- Uses both spins instead of a single spin parameter
- Ringdown attached at peak with correction for non-adiabatic evolution – calibrated with NR
- Requires numerical solution of ODEs – not efficient for searches
- Speed up for searches – reduced order modeling []
- For calibration uses both results from NR and extreme mass-ratio systems

The “EOBNR” models

[Bohe et al 2017]



- $4M_\odot < M < 100M_\odot$, O1 noise curve starting at 25Hz
- Only 2.1% of the points have effectualness less than 0.97
- The latest Phenom and EOB models agree well for aligned spin systems
- Disagreements only for high mass ratios and large spins

Including precession

- Consider now generic orientations of $\mathbf{S}_{1,2}$
- Total angular momentum: $\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2$
- New effect in general relativity: $\mathbf{L}$ and $\mathbf{S}_{1,2}$ precess around $\mathbf{J}$
- Direction of $\mathbf{J}$ fixed to a good approximation
- Special case: if $\mathbf{J}$ vanishes at some point in the evolution – transitional precession
- Searches including precession are much more expensive [Indik et al 2017, Harry et al 2015]
- Precessing models used mostly for parameter estimation

Including precession

- Very good approximation in inspiral phase: precessing waveform obtained by applying time dependent rotations on non-precessing waveform [Schmidt et al 2012]
- The rotations track the precession angles through the PN evolution and, if we start with the $\ell = m = 2$ mode, we will end up with all $\ell = 2$ modes
- The precession angles are assumed to carry through the merger – approximation
- Same procedure applied to EOBNR – apply rotations corresponding to a prcessing EOB model to “twist-up” the non-precessing EOBNR
- Precession IMR models not calibrated with NR simulations

This was just the beginning....

- The second observational run is ongoing
- Eventually we hope to see enough events that we can meaningfully distinguish between different astrophysical stellar evolution scenarios
- We hope to see events involving neutron stars (BNS or NS-BH systems) and the associated electromagnetic counterparts
- Binary systems are not the only ones we hope to see. Some other possibilities are: continuous waves (CW) emitted by rapidly rotating neutron stars, supernovae, evidence of a stochastic background...
- BBH modeling: better understanding of precession effects in merger and higher modes
- Better accuracy required, especially for events with SNR > 25