Small Scale Structure of Spacetime

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29th IAGRG Meeting
Indian Institute of Technology Guwahati
May 18-20, 2017

The “mesoscopic” domain of spacetime

Fig: K. Kuchar, Phys. Rev. D. 22, 1285 (1980)

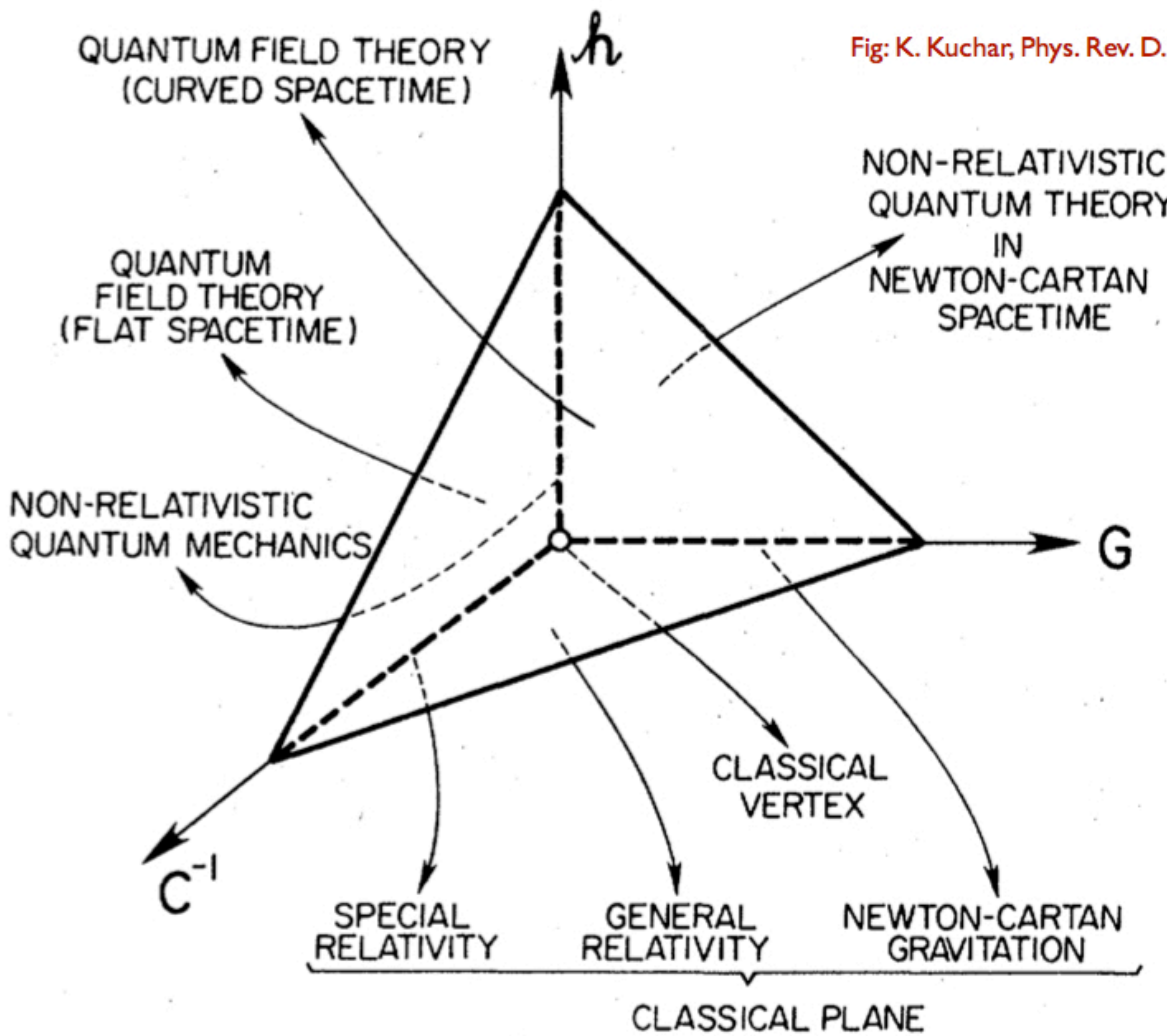


FIG. 1. Dimensional pyramid.

… and its implications for quantum gravity

spacetime can have "thermal" properties

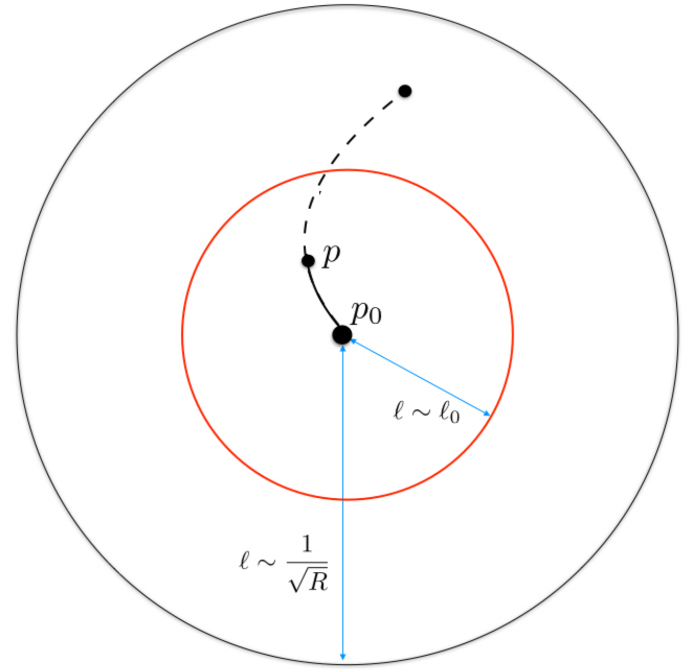
gravity affects the causal horizons of observers $\longrightarrow$ these horizons have thermal properties $\longrightarrow$ their displacements are given by laws of thermodynamics $\Rightarrow$ **hence the connection between gravitational dynamics and thermodynamics**

quantum+gravity effect imply existence of a minimal spacetime length, $\ell_0 \sim O(1)10^{-33}$ cm

QM requires concentration of high energy to probe small length scales $\longrightarrow$ GR predicts formation of BHs under such circumstances $\longrightarrow$ limits access to info behind their event horizons $\Rightarrow$ **"spacetime intervals can't be operationally def to precision better than ℓ_0"**

Small scale structure of spacetime

? *What is the best mathematical description of spacetime at small scales*



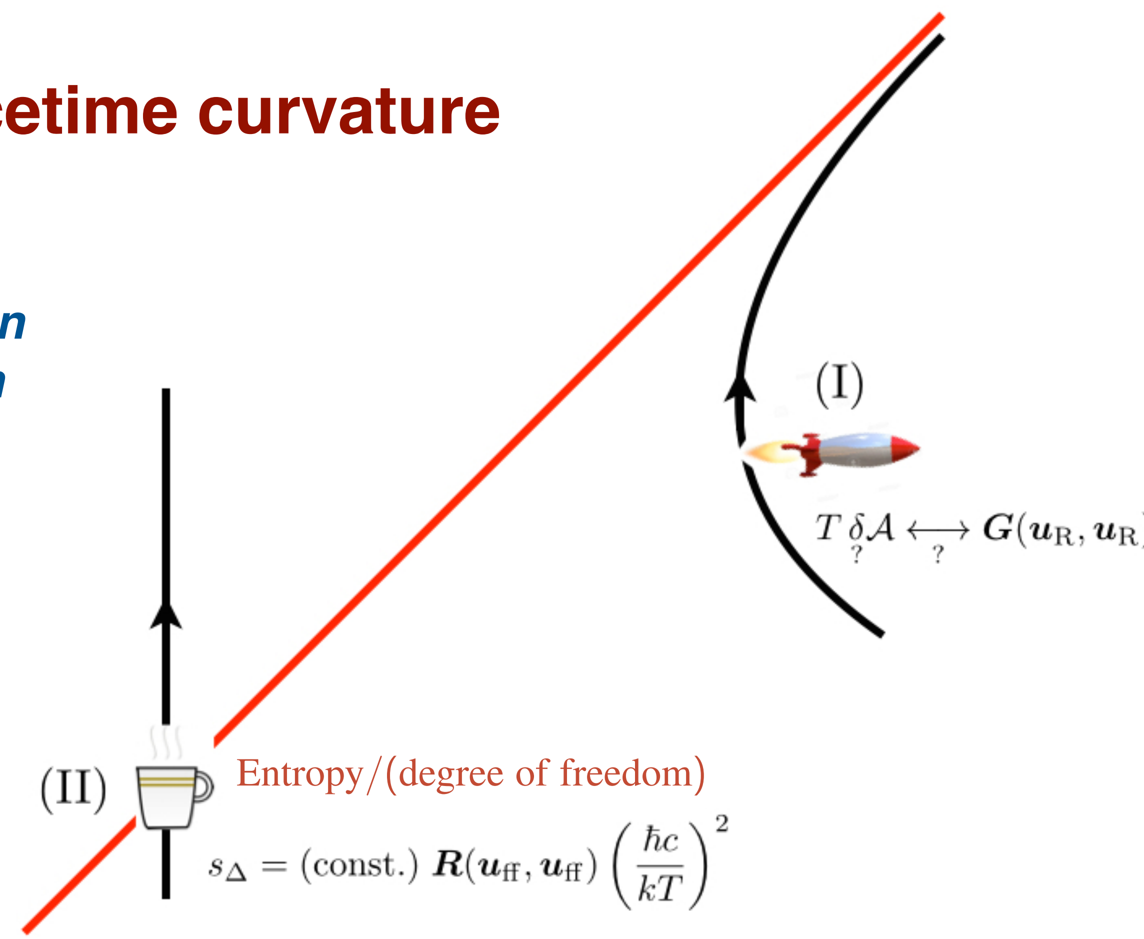
The qmetric $q_{ab}(p; p_0, \ell_0)$

$g_{ab}(p) \to q_{ab}(p; p_0, \ell_0)$

Thermal entropy and spacetime curvature

? *What are the effects of spacetime curvature on thermal properties of a freely falling quantum system*

? *What information about spacetime curvature can be obtained from thermal properties of acceleration horizons*



On the Hypotheses which lie at the Bases of Geometry
Bernhard Riemann, Gottingen lecture, 1854 (translated by W. Clifford)

Now it seems that the **empirical notions on which the metrical determinations of space are founded**, the notion of a solid body and of a ray of light, **cease to be valid for the infinitely small**. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small **do not conform to the hypotheses of geometry**; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena.

The question of the **validity of the hypotheses of geometry in the infinitely small** is bound up with the question of the ground of the metric relations of space . . .

The answer to these questions can only be got by starting from the conception of phenomena which has hitherto been justified by experience, and which Newton assumed as a foundation, and **by making in this conception the successive changes required by facts which it cannot explain** . . .

This leads us into the domain of another science, of physic, into which the object of this work does not allow us to go today.

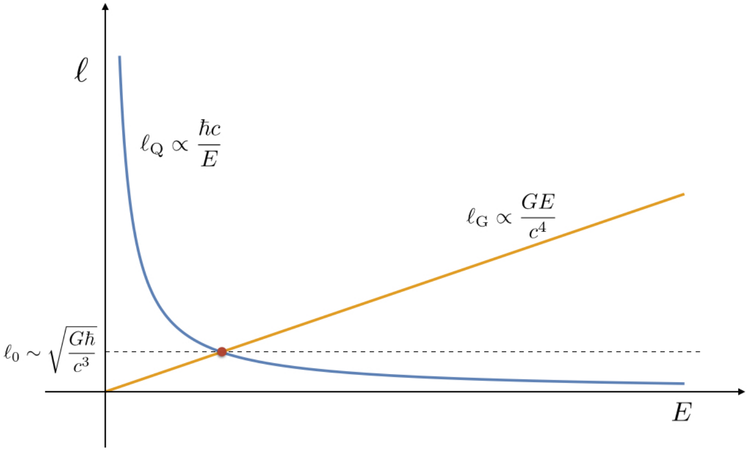
References

- Minimal Length and Small Scale Structure of Spacetime
  Dawood Kothawala (PRD, 2013) [arXiv:1307.5618]

- Grin of the Cheshire cat: Entropy density of spacetime as a relic from QG [arXiv:1405.4967]
- Entropy density of space-time from zero point length [arXiv:1408.3963]
  Dawood Kothawala, T. Padmanabhan (PRD, 2014; PLB, 2014)

- Small scale structure of spacetime: van Vleck determinant and equi-geodesic surfaces
  J. Stargen, **Dawood Kothawala** (PRD, 2015) [arXiv:1503.03793]

- Renormalized spacetime is two-dimensional at the Planck scale
  T. Padmanabhan, S. Chakraborty, **Dawood Kothawala** (GRG Letts., 2015) [arXiv:1503.03793]



QM+SR+GR $\Longrightarrow$ spacetime intervals can not be operationally def to an accuracy better than $\ell_0 \sim 10^{-33}$ cm

The minimal length scale

- various arguments based on principles of QM and GR suggest the existence of a minimal spacetime length $L_0 = \mu\sqrt{G\hbar/c^3}$, $\mu = O(1)$

  Review: L.Garay, gr-qc/9403008

- there is considerable evidence that such a length might manifest itself in a Lorentz invariant manner via modification of geodesic distance $\sigma^2(p, P)$

- the short distance behavior of the propagator gets modified as

$$G(p, P) \equiv \frac{1}{\sigma^2} \to \frac{1}{\sigma^2 + \ell_0^2}$$

B. S. DeWitt (1964, 1981): graviton exchange between scalar particles; non-local effective action based on $\sigma^2(p, P)$

M. R. Brown (1981, 1984): non-analytic structure of "effective metric" at small scales

Narlikar & Padmanabhan (1985): quantum conformal fluctuations;
Padmanabhan (1997): path-integral duality

Ohanian (1997, 1999): path integral average over gravitational field $\boldsymbol{h} \equiv \boldsymbol{g} - \boldsymbol{\eta}$

I. Agullo, J. Salas, G. Olmo, Parker (2008): deformation of two-point functions and trans-planckian effects

AND MANY OTHERS . . .

The World Function

geodesic intervals as more fundamental than the metric

- *the key input:*

  quantum gravitational fluctuations modify the geodesic distance $d(p,P) = \sqrt{\epsilon\sigma^2}$ between the spacetime events p and P in the background metric g_{ab} such that

$$\left\langle \sigma^2(p,P|(g \circ h)) \right\rangle = \sigma^2(p,P|g) + \epsilon L_0^2$$

  where

$$\begin{aligned} \sigma^2(p,P) &= (\lambda_P - \lambda_p) \int_{\mathcal{C}} g_{ab} t^a t^b \quad \stackrel{\text{def}}{:=} \quad 2\Omega(p,P) \\ &= -(t-T)^2 + (\boldsymbol{x} - \boldsymbol{X})^2 \qquad \text{flat spacetime} \end{aligned}$$

- to describe a space(time) in which $\lim_{p \to P} d(P,p) \neq 0$ calls for a non-trivial modification of conventional description of spacetime geometry

  this seems to first have been noticed in A. March, Z. Phys. 104, 93 (1936)

- indeed, almost all information about spacetime geometry can be encoded in the coincidence limit (denoted below by "[...]") of covariant derivatives of $\Omega(p, P)$

$$g_{a'b'} = g_{ab} = \left[\nabla_a \nabla_b \Omega(x, x')\right] = \left[\nabla_{a'} \nabla_{b'} \Omega(x, x')\right]$$

$$R_{a'(c'd')b'} = (3/2) \left[\nabla_a \nabla_b \nabla_c \nabla_d \Omega(x, x')\right]$$

- for a smooth differentiable manifold, the metric near any event has a Taylor exp

$$g_{ab}(x; X) = \eta_{ab}(X) - \frac{1}{3} R_{acbd}(X)\ (x - X)^c (x - X)^d + \text{higher order terms}$$

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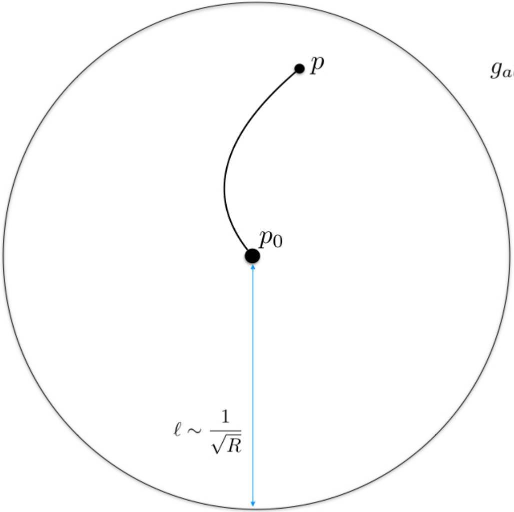
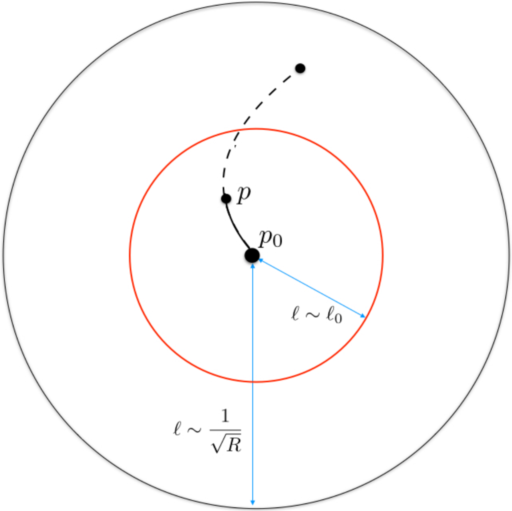
$$g_{ab}(x; X) = \underset{?}{\eta_{ab}(X)} - \frac{1}{3} \underset{?}{\boldsymbol{R}_{acbd}(X)} \, (x - X)^c (x - X)^d + \text{higher order terms}$$



$$g_{ab}(p) = \eta_{ab}(p_0) + O(R\,x^2)$$



$$g_{ab}(p) \to q_{ab}(p; p_0, \ell_0)$$

- Our key inputs would be much less restrictive and/or specialized:

Q1: geodesic distances have a Lorentz invariant lower bound.

$$\begin{aligned} \sigma^2 &\rightarrow \mathcal{S}_{\ell_0}\left[\sigma^2\right] \\ \mathcal{S}_{\ell_0}[0] &= \ell_0^2 \end{aligned}$$

Q2: the modified d'Alembartian $\widetilde{{}_{p_0}\Box_p}$ yields the following modification for the two point functions $G(p, p_0)$ of fields in *all maximally symmetric spacetimes*:

$$G\left[\sigma^2\right] \rightarrow \widetilde{G}\left[\sigma^2\right] = G\left[\mathcal{S}_{\ell_0}\left[\sigma^2\right]\right]$$

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Since the leading form of the two point function is given by

$$G(p, p_0) := \frac{\sqrt{\Delta}}{(\sigma^2)^{\frac{D-2}{2}}} \times (1 + \text{subdominant terms})$$

we expect Δ *and* σ^2 *to play a key role in our analysis*

The “qmetric”

- to implement **Q1**, we use the defining Hamilton-Jacobi eq for the world fn

$$g^{ab}\,\partial_a\Omega\,\partial_b\Omega = 2\Omega \quad\longrightarrow\quad q^{ab}\partial_a\mathcal{S}_{\ell_0}\partial_b\mathcal{S}_{\ell_0} = 4\mathcal{S}_{\ell_0}$$

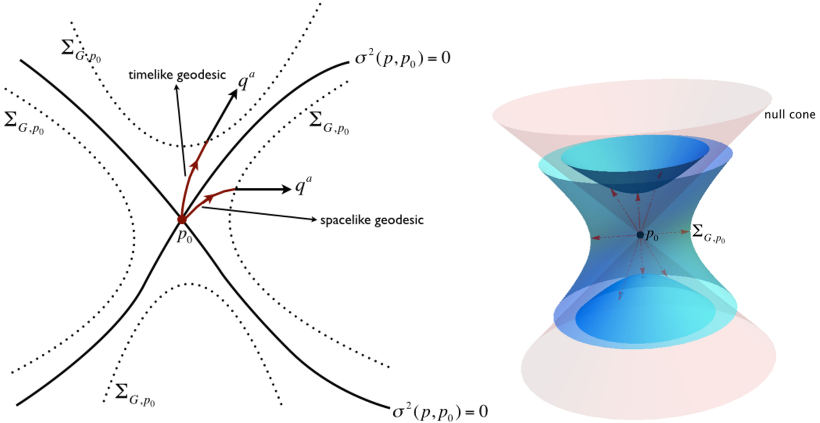
- to implement **Q2**, we use $\widetilde{\Box}$ and following identities satisfied by the VVD

$$\Delta(p,p_0) = \frac{1}{\sqrt{|g(p)|}\sqrt{|g(p_0)|}}\det\left\{\overset{(p)}{\nabla}_a\overset{(p_0)}{\nabla}_b\,\Omega(p,p_0)\right\}$$

$$I1: \qquad \nabla_{\boldsymbol{q}}\ln\Delta = \frac{D_1}{\sqrt{\epsilon\sigma^2}} - K$$

$$I2: \qquad \nabla^2_{\boldsymbol{q}}\ln\Delta = -\frac{D_1}{\epsilon\sigma^2} + K^2_{ab} + R_{ab}q^a q^b$$

$$\text{for max. symm. spaces}: \Delta^{-1/(D-1)} = \left\{\frac{\sin\theta}{\theta},\ 1,\ \frac{\sinh(|\sigma|/a)}{|\sigma|/a}\right\}$$



(a) Equi-geodesic surfaces σ_G attached to an event p_0 in an arbitrary curved spacetime.

(b) σ_G in Minkowski spacetime.

Figure: The geodesic structure of spacetime.

The “qmetric”

- the final result turns out to be

$$\boldsymbol{q} = \frac{\mathcal{S}_{\ell_0}}{\sigma^2}\left(\frac{\Delta}{\Delta_{\mathcal{S}}}\right)^{+\frac{2}{D_1}}\boldsymbol{g} + \epsilon\left\{\frac{\sigma^2\mathcal{S}'^2_{\ell_0}}{\mathcal{S}_{\ell_0}} - \frac{\mathcal{S}_{\ell_0}}{\sigma^2}\left(\frac{\Delta}{\Delta_{\mathcal{S}}}\right)^{+\frac{2}{D_1}}\right\}\boldsymbol{t}\otimes\boldsymbol{t}$$

which is:

- a non-local bi-tensor, disformally coupled to g_{ab}
- is singular in the limit $\sigma^2 \to 0$
- $\lim_{\ell_0\to 0}\boldsymbol{q} = \boldsymbol{g}$

- given the singular behaviour of $\boldsymbol{q}$, it is unclear whether local scalars constructed out of q_{ab} reduce, in the limit $\ell_0 \to 0$, to their corresponding form in $\boldsymbol{g}$; for e.g.,

$$\left[\widetilde{\mathbf{Ric}}\right](p_0) \stackrel{?}{=} \mathbf{Ric}(p_0) + \text{terms of order } \ell_0$$

- i will now discuss the structure of the gravitational lagrangian

$$(16\pi L_p^2)\, S_{\rm grav} = \int_{\mathcal{V}} R[g] \;+\; 2\int_{\partial\mathcal{V}} K[h]$$

and show that

  - $\lim_{\ell_0 \to 0} \lim_{\sigma^2 \to 0} \widetilde{\mathbf{Ric}}(p, p_0) \neq \mathbf{Ric}(p_0)$
  - the surface term $K\sqrt{h}$ is finite in the limit $\sigma^2 \to 0$, and yields entropy density of spacetime with a zero-point term
  - the key results are independent of precise form of $\mathcal{S}_{\ell_0}(\sigma^2)$

    . . . *hence, presumably also of the exact details of QG*

$\ell_0 \to 0$ with $\sigma^2 \neq 0$ (no surprises here!)	Strategy of this work	$p \to P$ with $\ell_0 \neq 0$ (leads to entropy density of emergent gravity paradigm)
$\sigma^2 \to \sigma^2(P, p)$	start with the geodesic interval $\sigma^2(p, P)$ for a metric g_{ab}	$\sigma^2 \to 0$
$\sigma^2_{(q)} \to \sigma^2(P, p)$	incorporate zero-point length via $\sigma^2_{(q)}(P, p, \ell_0^2) = \sigma^2(P, p) + \ell_0^2$	$\sigma^2_{(q)} \to \ell_0^2$
$q_{ab}(P, p, \ell_0^2) \to g_{ab}(P)$	find the "qmetric" $q_{ab}(P, p, \ell_0^2)$ associated with $\sigma^2_{(q)}(P, p, \ell_0^2)$	diverges as $(\ell_0^2/\sigma^2)\|_{\sigma \to 0}$
$R(P, p, \ell_0^2) \to R(P)$	compute the Ricci biscalar $R(P, p, \ell_0^2)$ for the $q_{ab}(P, p, \ell_0^2)$	

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$R(P, p, \ell_0^2) \to R(P)$	compute the Ricci biscalar $R(P, p, \ell_0^2)$ for the $q_{ab}(P, p, \ell_0^2)$	$R(P, p, \ell_0^2) \to \mathcal{S}_g + O(\ell_0)$

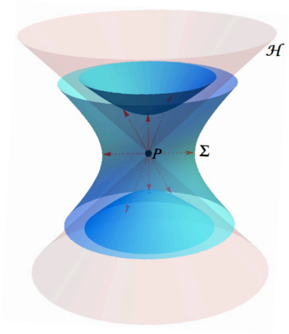
The **exact** form of the Ricci scalar can be written in a compact form in terms of geometric quantities associated with $\sigma^2 = \text{const surface}, \ \Sigma$

$$\widetilde{\textbf{Ric}}(p, p_0) = \left[\frac{\sigma^2}{\mathcal{S}_{\ell_0}} \zeta^{-2/D_1} \, \mathcal{R}_{\Sigma_{G,p_0}} - \frac{D_1 D_2}{\mathcal{S}_{\ell_0}} + 4(D+1)(\ln \Delta_{\mathcal{S}})^{\bullet}\right]$$

$$- \frac{\mathcal{S}_{\ell_0}}{\lambda^2 \mathcal{S}'^2_{\ell_0}} \left\{K_{ab}K^{ab} - \frac{1}{D_1} K^2\right\} + 4\mathcal{S}_{\ell_0} \left\{-\frac{D}{D_1} \left[(\ln \Delta_{\mathcal{S}})^{\bullet}\right]^2 + 2(\ln \Delta_{\mathcal{S}})^{\bullet\bullet}\right\}$$

where

$$\zeta = \Delta/\Delta_{\mathcal{S}}$$
$$(\ln \Delta_{\mathcal{S}})^{\bullet} = \mathrm{d} \ln \Delta_{\mathcal{S}} / \mathrm{d}\mathcal{S}_{\ell_0}$$
$$(\ln \Delta_{\mathcal{S}})^{\bullet\bullet} = \mathrm{d}(\ln \Delta_{\mathcal{S}})^{\bullet} / \mathrm{d}\mathcal{S}_{\ell_0}$$



$$K_{ab} = \frac{1}{\lambda}h_{ab} - \frac{1}{3}\lambda\mathcal{S}_{ab} + \frac{1}{12}\lambda^2\nabla_{\boldsymbol{t}}\mathcal{S}_{ab} - \frac{1}{60}\lambda^3F_{ab} + O(\lambda^4)$$

$$K = \frac{D_1}{\lambda} - \frac{1}{3}\lambda\mathcal{S} + \frac{1}{12}\lambda^2\nabla_{\boldsymbol{t}}\mathcal{S} - \frac{1}{60}\lambda^3F + O(\lambda^4)$$

$$R_\Sigma = \frac{\epsilon D_1 D_2}{\lambda^2} + R - \frac{2\epsilon(D+1)}{3}\mathcal{S} + O(\lambda)$$

where: $\mathcal{S}_{ab} = R_{aibj}t^i t^j, \quad \mathcal{S} = R_{ab}t^a t^b$

$$F_{ab} = \nabla^2_{\boldsymbol{t}}\mathcal{S}_{ab} + (4/3)\mathcal{S}_{ak}\mathcal{S}^k{}_b; \quad F = F_{ab}g^{ab}$$

using the above, a lengthy calculation yields the desired local object defined by:

$$\left[\widetilde{\textbf{Ric}}\right](p_0) = \lim_{p\to p_0}\widetilde{\textbf{Ric}}(p, p_0)$$

- the final result is

$$\left[\widetilde{\mathbf{Ric}}\right](p_0) = \lim_{p \to p_0} \widetilde{\mathbf{Ric}}(p, p_0)$$

$$= \underbrace{\alpha \left[R_{ab} t^a t^b\right]_{p_0}}_{O(1) \text{ term}} - \frac{\ell_0^2}{15} \underbrace{\left[\frac{1}{3}\mathcal{S}_{ab}\mathcal{S}^{ab} + \frac{3}{2}\ddot{\mathcal{S}} + \frac{5}{3}\mathcal{S}^2\right]_{p_0}}_{O({\ell_0}^2) \text{ term}}$$

the limit, therefore, is non-trivial

although $q_{ab} \to g_{ab}$ when $\ell_0 = 0$, $R_{(q)} \neq R_{(g)}$ in the same limit!

the "zero-point length" leaves it's vestige ... like the Grin of the Cheshire cat!

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most importantly, the leading term above is precisely the entropy density which arises as Noether charge of diff-invariance, and is prominent in emergent gravity paradigm!

Summary

$$\frac{1}{16\pi L_P^2}\,[\widetilde{\mathbf{Ric}}](p_0) = \alpha\,\mathcal{S}_g \;-\; \frac{\mu^2}{240\pi}\,\left[\frac{1}{3}\mathcal{S}_{ab}\mathcal{S}^{ab} + \frac{3}{2}\ddot{\mathcal{S}} + \frac{5}{3}\mathcal{S}^2\right]$$

$$\frac{1}{8\pi L_P^2}\,\lim_{\lambda\to 0}\left[K\sqrt{h}\right]_q = \mathcal{S}_0 - \frac{\mu^4}{24\pi}\mathcal{S}_g$$

- $\mu = \ell_0/L_p = O(1)$
- $\mathcal{S}_{ab} = R_{aibj}t^i t^j$
- $\mathcal{S}_0 = (3/8\pi)\,\mu^2$
- $\mathcal{S}_g = R_{ab}t^a t^b$

Some important comments on the result

- note that there are no terms of the form $1/\ell_0^2$ in curvature; these can, however, appear in regions where the Riemann tensor diverges

- the coincidence limit of Ricci scalar is finite although for the metric it is not

- the above points are a consequence of some miraculous cancellations in the intermediate steps, that happen solely because of the differential geometric properties of the Synge world fn bi-scalar and the vanVleck determinant, and remove terms like $1/\sigma^2$ and $1/\ell_0^2$; this in turn is a consequence of the disformal structure of the qmetric

  such terms do appear, for e.g., in quantised conformal fluctuations

Disformal vs. Conformal transforms

[DK, arxiv:1406.2672]

$$\mathbf{Ric}\left[F^2\boldsymbol{g} - \epsilon\alpha^{-1}\Theta\, \boldsymbol{t}\otimes\boldsymbol{t}\right] = (1+\Theta)\ \mathbf{Ric}\left[F^2\boldsymbol{g}\right] - \ \Theta\ \left(R_\Sigma + 2\epsilon\boldsymbol{\nabla}\cdot\boldsymbol{a}\right)_{F^2\boldsymbol{h}} + \ \epsilon\dot{\Theta}\ F^{-1}K_{\Sigma,F^2\boldsymbol{h}}$$

$$(\Theta = \alpha F^2 - 1)$$

CONFORMAL	DISFORMAL
$\overset{\star}{g}_{ab} = F^2 g_{ab}$	$\overset{\star}{g}_{ab} = F^2 g_{ab} - \epsilon\left(F^2 - F^{-2}\right) t_a t_b$
$\overset{\star}{h}_{ab} = F^2 h_{ab}$	$\overset{\star}{h}_{ab} = F^2 h_{ab}$
$\overset{\star}{K}_{ab} = FK_{ab} + (\nabla_{\boldsymbol{t}} F)\, h_{ab}$	$\overset{\star}{K}_{ab} = F^3 K_{ab} + \left(F^2 \nabla_{\boldsymbol{t}} F\right) h_{ab}$
$\mathrm{Tr}\overset{\star}{K}_{ab} = F^{-1}\mathrm{Tr}K + D_1 F^{-2}\nabla_{\boldsymbol{t}} F$	$\mathrm{Tr}\overset{\star}{K}_{ab} = F\,\mathrm{Tr}K + D_1 \nabla_{\boldsymbol{t}} F$

Relics of the space-time minimal length

- our result suggests that non-local and non-analytic effects of a minimal length might leave residues which are independent of ℓ_0
- in our case, such effects leave their imprints in the form of
  - $\mathcal{S}_0 = 3\mu^2/8\pi$ zero-point entropy density of space-time
  - $\mathcal{S}_g = R_{ab}t^a t^b$ gravitational entropy density of space-time
- such quantum relics are not unfamiliar in physics; e.g.
  - effects of Lorentz violating regulators at higher energies can generically get dragged to lower energies due to radiative corrections, leaving $O(1)$ residual effects [Collins et. al., Polchinski]
  - conformal anomaly; $D \to 4$ limit in dimensional regularization [see, for e.g., Birrell & Davies]
  - non-relativistic relic of the $O(1/c)$ expansion of the relativistic point particle wave fn [Padmanabhan et. al.]
  - nonlocal quantum residue of discreteness of the causal set type [Sorkin]

Volume and Area of spacetime regions bounded by equi-geodesic surfaces

- it is also interesting to calculate the volume bounded by an equi-geodesic surface corresponding to an event p_0, and the surface are:

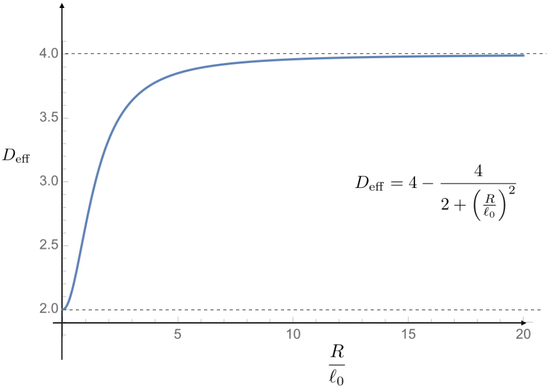
$$\sqrt{q} = \sigma \left(\sigma^2 + \ell_0^2\right) \left[1 - \frac{1}{6} \mathcal{S}_{\boldsymbol{g}} \left(\sigma^2 + \ell_0^2\right)\right] \sqrt{h_\Omega}$$

$$\sqrt{h} = \left(\sigma^2 + \ell_0^2\right)^{3/2} \left[1 - \frac{1}{6} \mathcal{S}_{\boldsymbol{g}} \left(\sigma^2 + \ell_0^2\right)\right] \sqrt{h_\Omega}$$

In the limit $\sigma \to 0$: volume $\to 0$ while area remains finite! *Holography*??

- define an "effective" *geometric* dimension based on equi-geod surfaces of size R

$$D_{\mathrm{eff}} = D + \frac{\mathrm{d}}{\mathrm{d} \ln R} \left\{ \ln \left(\frac{V_D(R, \ell_0)}{V_D(R, \ell_0 = 0)} \right) \right\}$$



a simple calculation then shows that

$$D_{\mathrm{eff}} \to 4 \qquad (R \gg \ell_0)$$
$$D_{\mathrm{eff}} \to 2 \qquad (R \ll \ell_0)$$

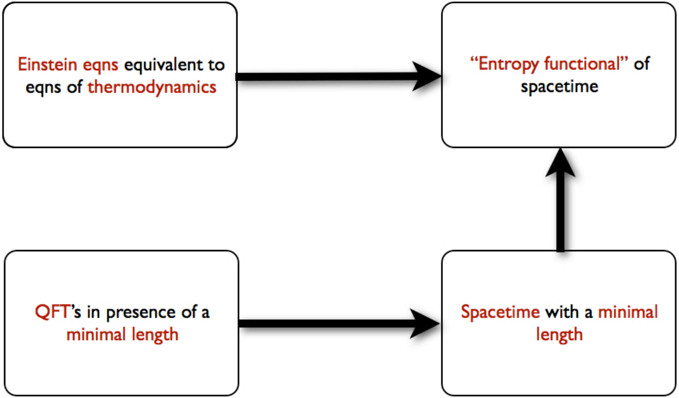
See [Carlip 1009.1136] for relevance of such a dimensional reduction in QG

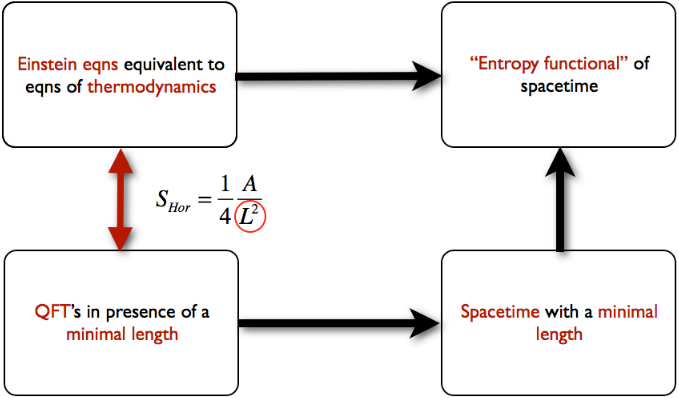
Implications for space-time thermodynamics

The above analysis also leads to an unexpected and non-trivial connection between

small scale struct of spacetime $\Longleftrightarrow$ spacetime thermod and gravity

- the object $\mathcal{S}_g = R_{ab} t^a t^b$ is well-known to be connected with the Noether charge of Diff inv, and hence to entropy of local Rindler horizons; this in turn has been used to derive gravitational dynamics from space-time thermodynamics [Jacobson, Padmanabhan]
- our result for modified Ricci scalar and surface term in gravitational action suggests that even classically, the correct variational principle for gravity must be based on $R_{ab} t^a t^b$ rather than the conventional Einstein-Hilbert lagrangian R
- such a thermodynamic variational principle for gravity based on an entropy functional is already known in which the d.o.f varied are arb. normalised vectors t^a [Padmanabhan et.al.]


Einstein eqns equivalent to eqns of thermodynamics
"Entropy functional" of spacetime
QFT's in presence of a minimal length
Spacetime with a minimal length


Einstein eqns equivalent to eqns of thermodynamics
"Entropy functional" of spacetime
$S_{Hor} = \frac{1}{4}\frac{A}{L^2}$
QFT's in presence of a minimal length
Spacetime with a minimal length

Thermal entropy and spacetime curvature

***Motivation*:** There are several peculiar effects that arise in the study of statistical mechanics of self-gravitating systems, such as *negative specific heat*, deriving mostly from the fact that gravity couples to *everything* and operates *unshielded* with an *infinite range*.

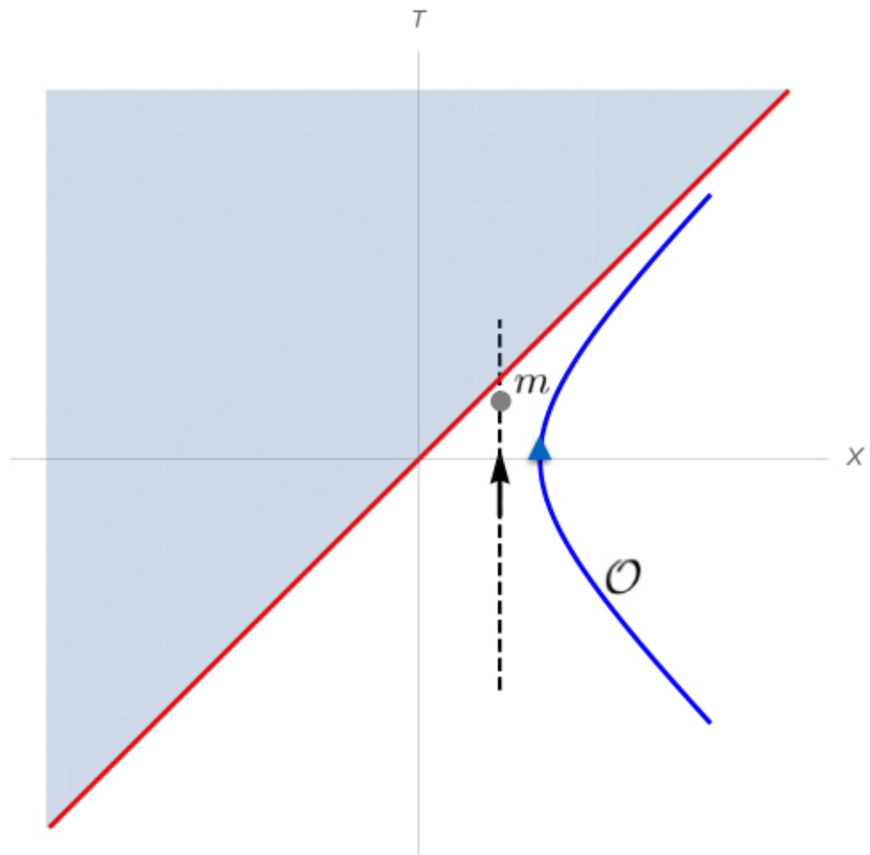
D. Lynden-Bell, R. Wood, MNRAS 138, 495 (1968)
T. Padmanabhan, Phys. Rep.188, 285 (1990)

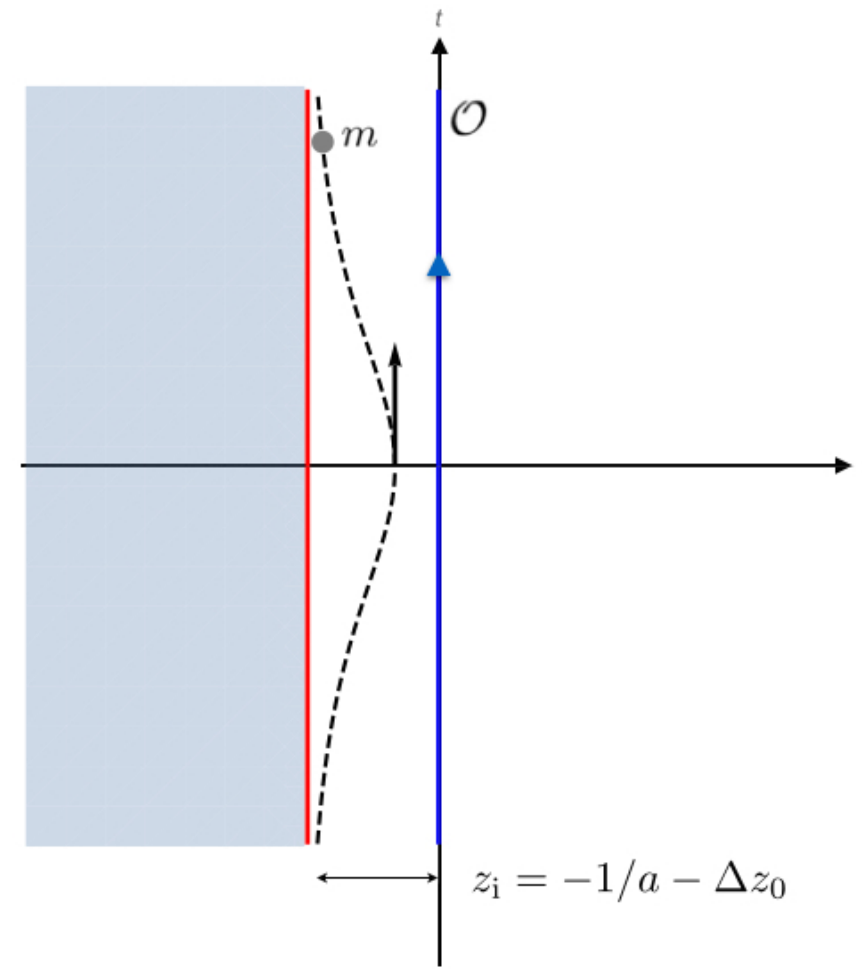
A black hole magnifies the quantum effects in it's vicinity, revealing a gamut of exotic features, the most famous being its thermal attributes.

This ***gravity ↔ quantum ↔ thermodynamics*** connection has been gaining increasing attention in recent years due to it's potential relevance for our understanding of gravity, and perhaps spacetime itself, at a fundamental level.

Thermal entropy and spacetime curvature

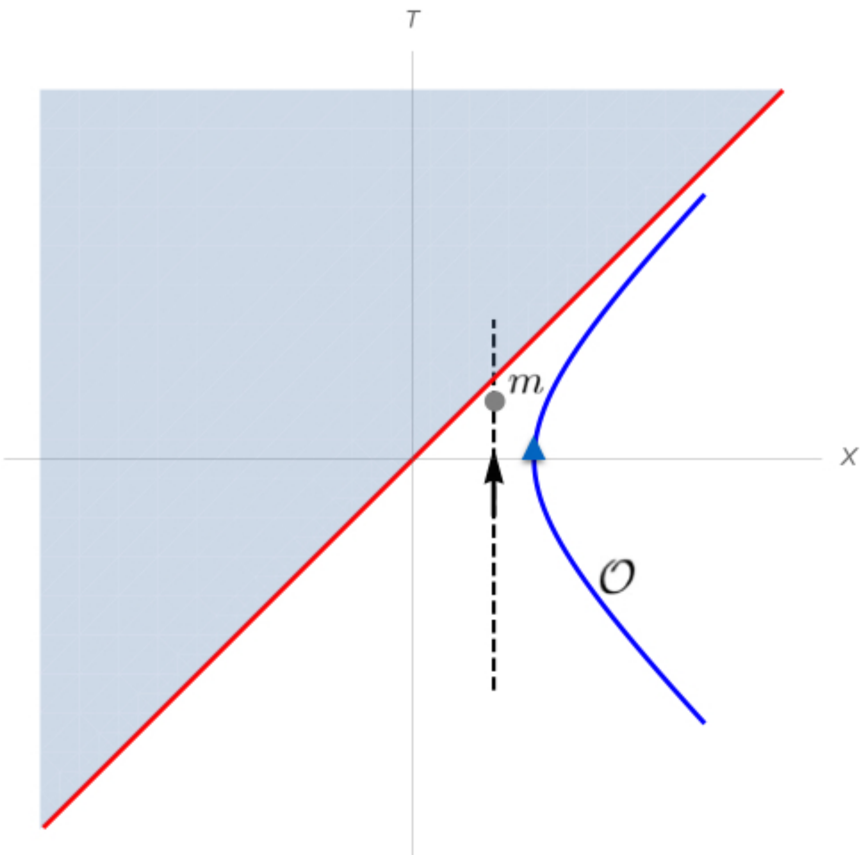
? *What information about spacetime curvature can be obtained from thermal properties of acceleration horizons*



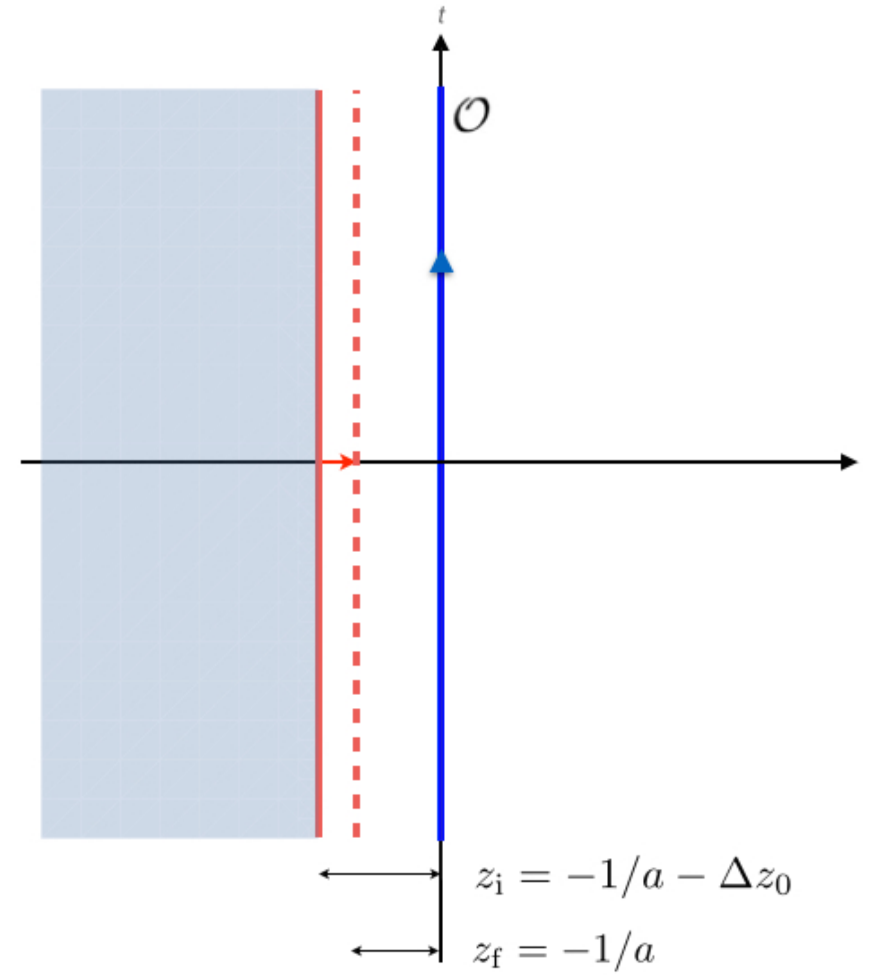


Thermal entropy and spacetime curvature

? ***What information about spacetime curvature can be obtained from thermal properties of acceleration horizons***



$$T\,\underset{?}{\delta\mathcal{A}} \underset{?}{\longleftrightarrow} \boldsymbol{G}(\boldsymbol{u}_{\mathrm{R}}, \boldsymbol{u}_{\mathrm{R}})$$



$$T_U\ \delta_{z_0}\left(\frac{1}{4}\mathscr{A}\right) = \frac{\eta}{8\pi}\int_{x^3=z_0} R^{3\mathbb{A}}_{\ 3\mathbb{A}}\ \varepsilon\ \mathrm{d}^2x_\perp$$

$$T_U\ \delta_\varepsilon S = -\eta\left[\left(\int_{\mathscr{H}} T^0_0\mathrm{d}^2x_\perp\right)\ \Delta z_0 + \frac{\hat{\chi}}{2}\left(\frac{\Delta z_0}{2}\right)\right]$$

$$T\,\delta S = \bar{F}\,\Delta z_0 + \mathrm{d}E_g$$

Thermal entropy and spacetime curvature

? ***What are the effects of spacetime curvature on thermal properties of a freely falling quantum system***

Consider the following result for ***canonical partition function*** for a ***box of ideal gas*** and a collection of ***harmonic oscillators***

DK, Phys. Lett. B 720, 410 (2013)
DK, Proceedings of this conference

$$\lim_{\text{large } T} \Delta Z\left(\beta, R_{abcd}, \{\varkappa\}\right) = (\text{const.}) R_{00} \Lambda^2 + \{\varkappa\} \text{ dependent terms}$$

which leads to the following ***conjecture*** for ***thermal entropy / (degree of freedom)***

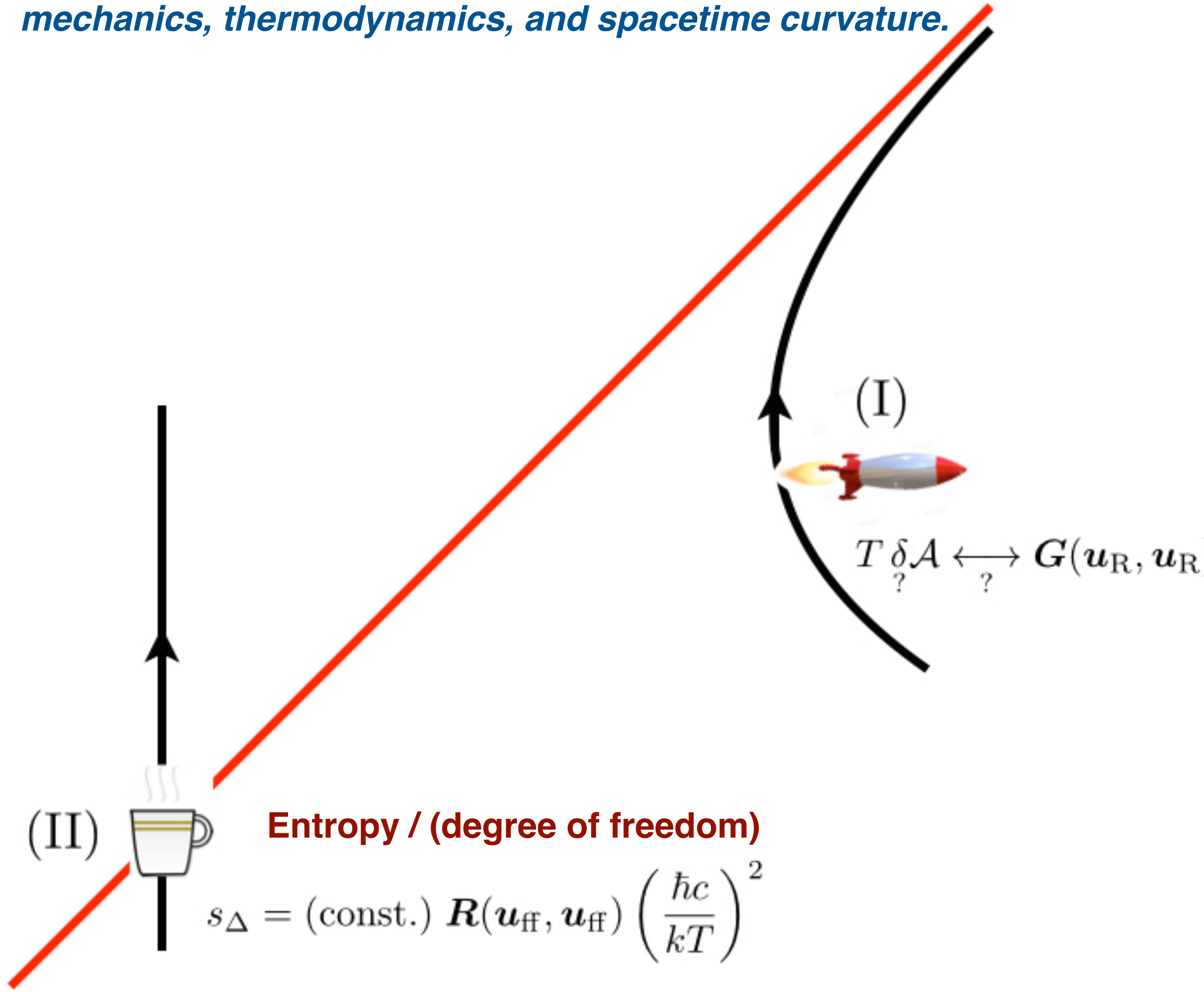
Entropy of a system at temperature T generically acquires a system independent contribution in a curved spacetime characterized by the dimensionless quantity

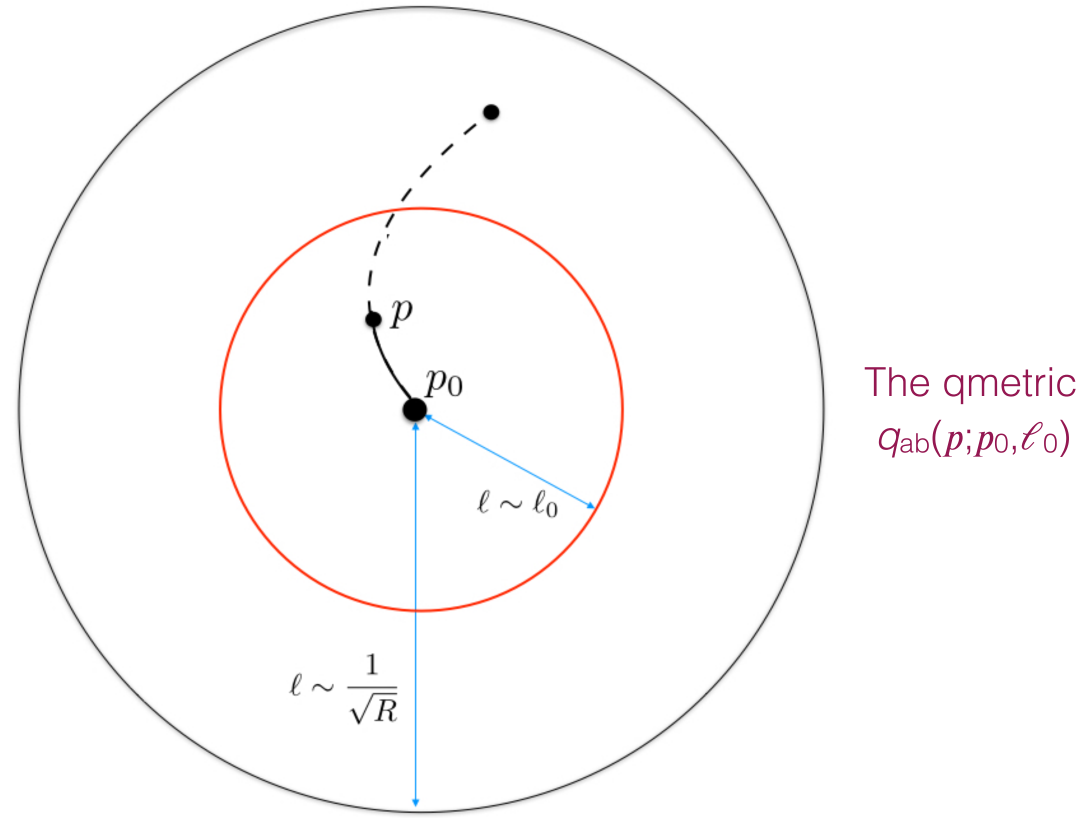
$$\Delta = \boldsymbol{R}(\boldsymbol{u}_{\rm ff}, \boldsymbol{u}_{\rm ff}) \left(\hbar c / kT\right)^2$$

at sufficiently large temperatures T.

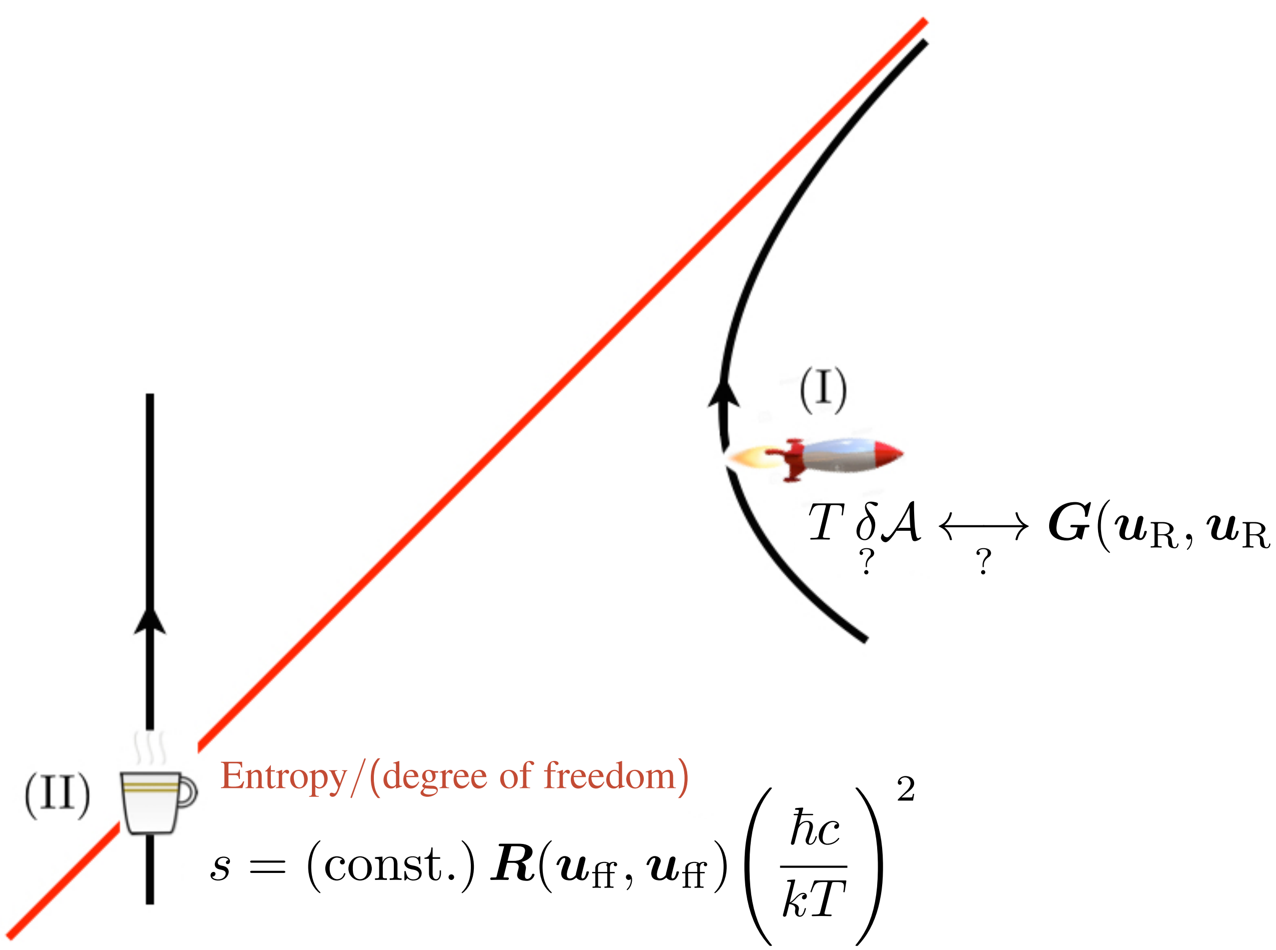
Thermal entropy and spacetime curvature

A combined description of a freely falling thermal system and the thermodynamics associated with an accelerated observer can yield physically useful insights into interplay between quantum mechanics, thermodynamics, and spacetime curvature.

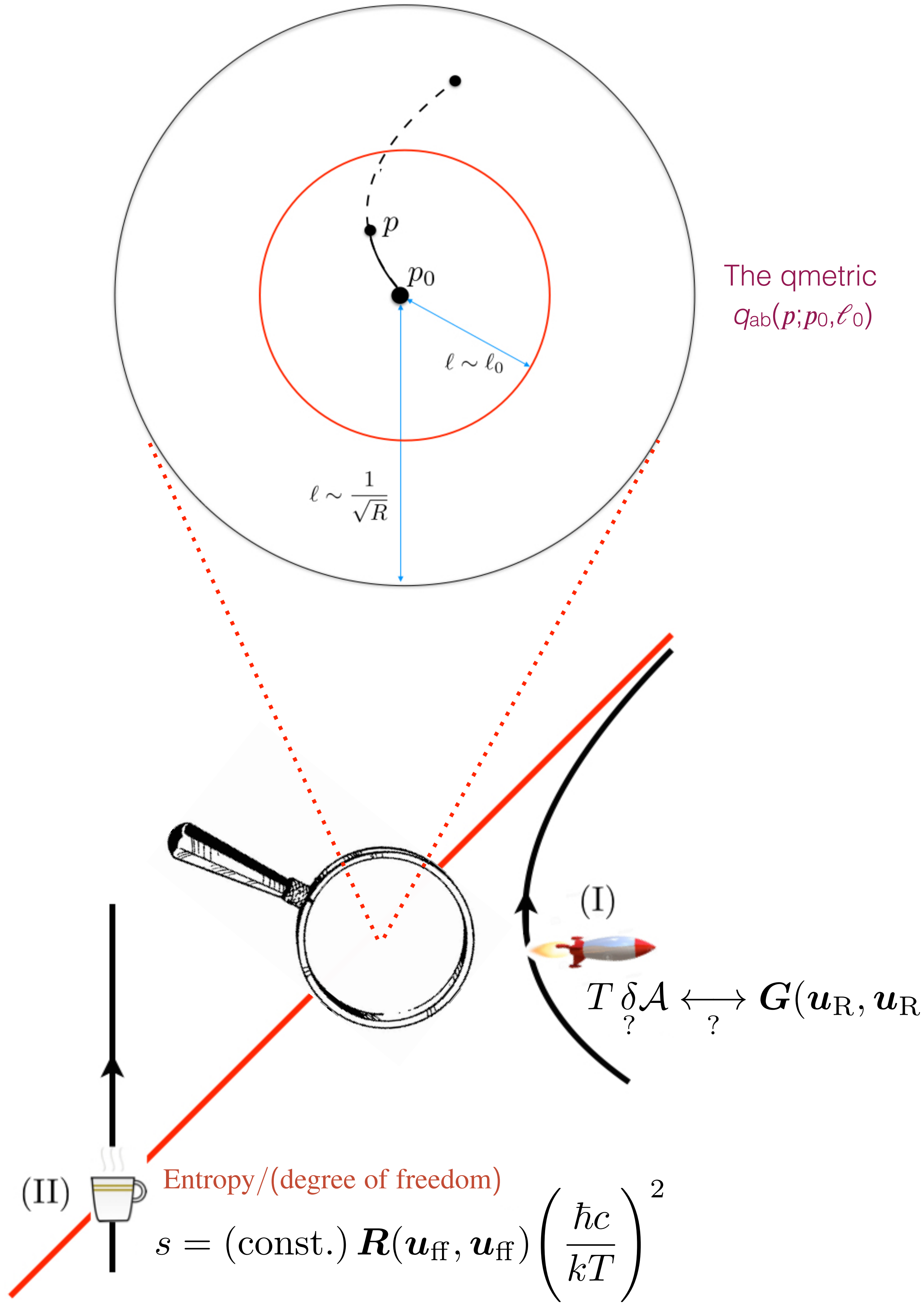




p
p_0
$\ell \sim \ell_0$
$\ell \sim \frac{1}{\sqrt{R}}$
The qmetric
$q_{ab}(p;p_0,\ell_0)$



(I)
$T\,\delta\mathcal{A} \underset{?}{\longleftrightarrow} \boldsymbol{G}(\boldsymbol{u}_{\rm R},\boldsymbol{u}_{\rm R})$
(II)
Entropy/(degree of freedom)
$s = (\text{const.})\,\boldsymbol{R}(\boldsymbol{u}_{\rm ff},\boldsymbol{u}_{\rm ff})\left(\frac{\hbar c}{kT}\right)^2$



p
p_0
The qmetric
$q_{ab}(p;p_0,\ell_0)$
$\ell \sim \ell_0$
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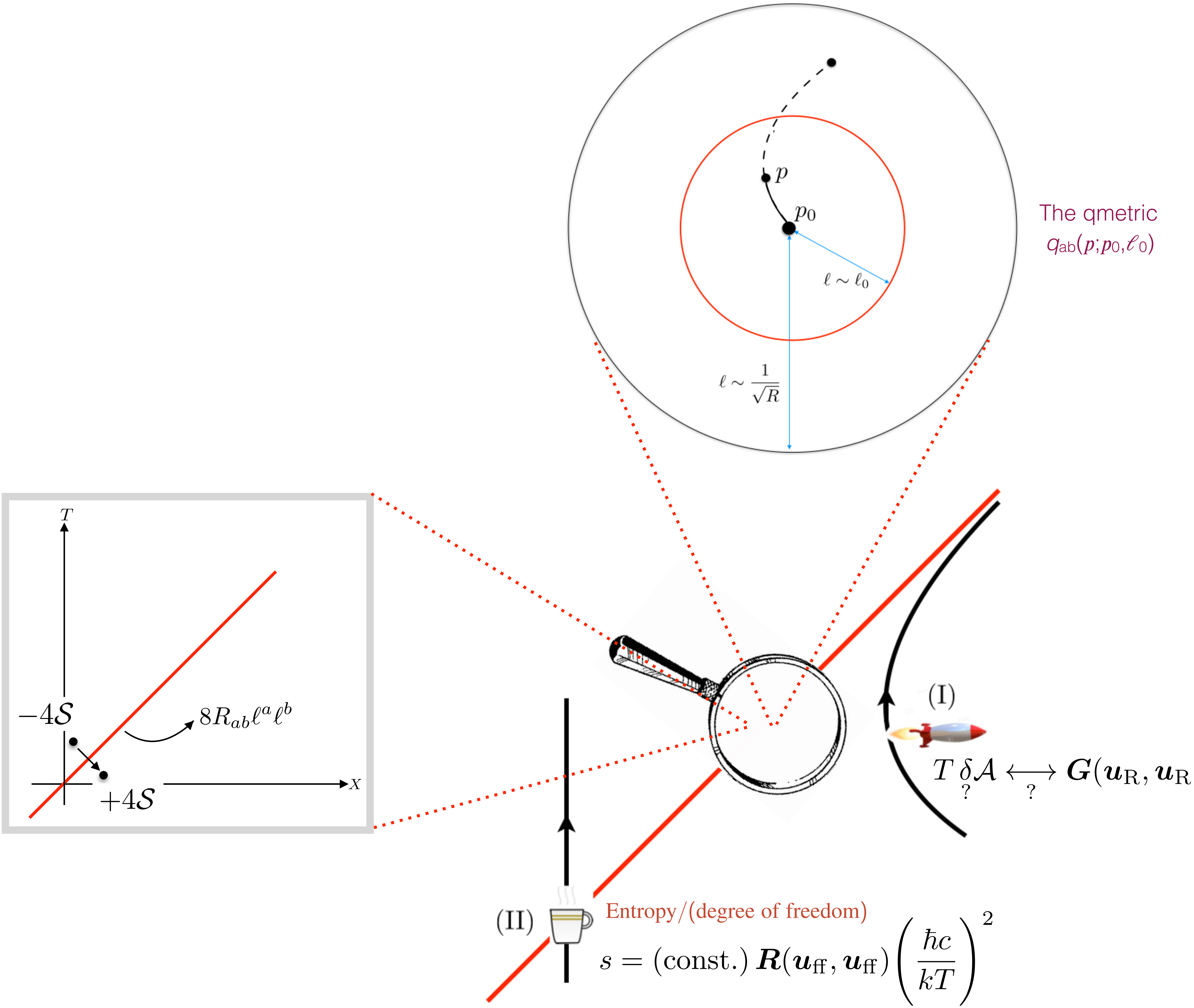


The qmetric
$q_{ab}(p;p_0,\ell_0)$
p
p_0
$\ell \sim \ell_0$
$\ell \sim \frac{1}{\sqrt{R}}$
T
X
$-4\mathcal{S}$
$+4\mathcal{S}$
$8R_{ab}\ell^a\ell^b$
(I)
$T\,\underset{?}{\delta}\mathcal{A} \underset{?}{\longleftrightarrow} \boldsymbol{G}(\boldsymbol{u}_{\rm R},\boldsymbol{u}_{\rm R})$
(II)
Entropy/(degree of freedom)
$s = (\text{const.})\, \boldsymbol{R}(\boldsymbol{u}_{\rm ff},\boldsymbol{u}_{\rm ff}) \left(\frac{\hbar c}{kT}\right)^2$

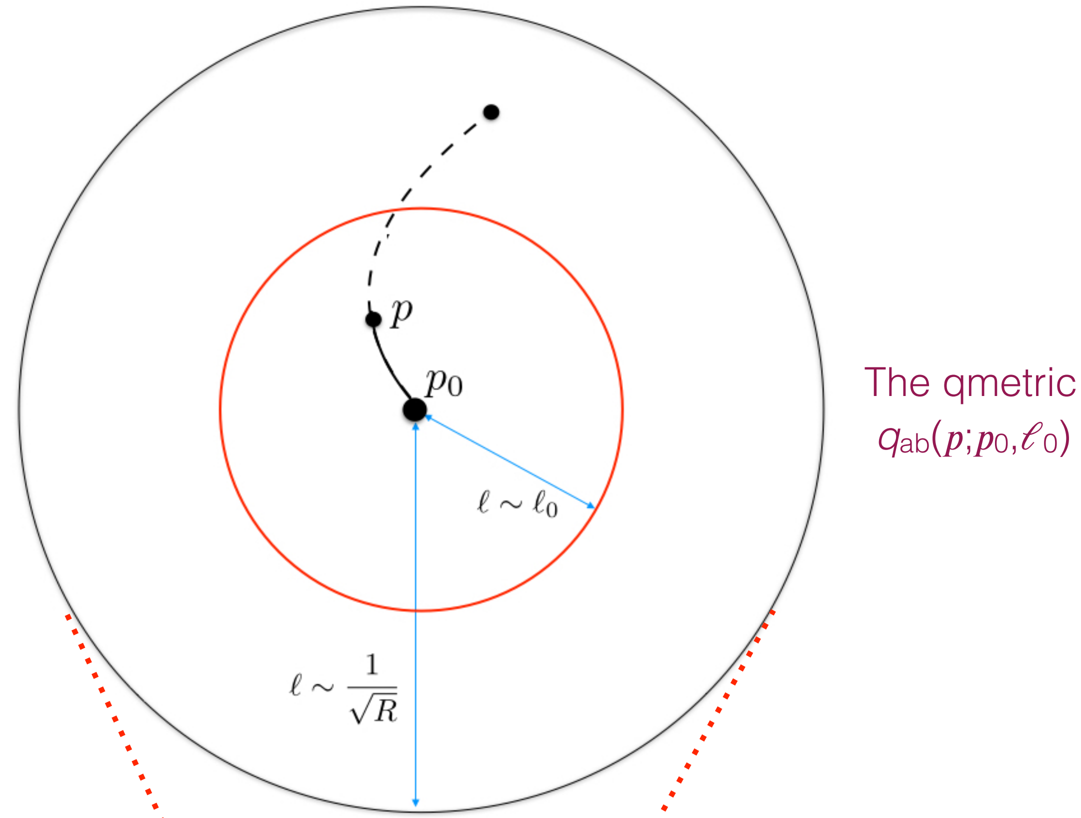
$\Lambda \ell_0^2$

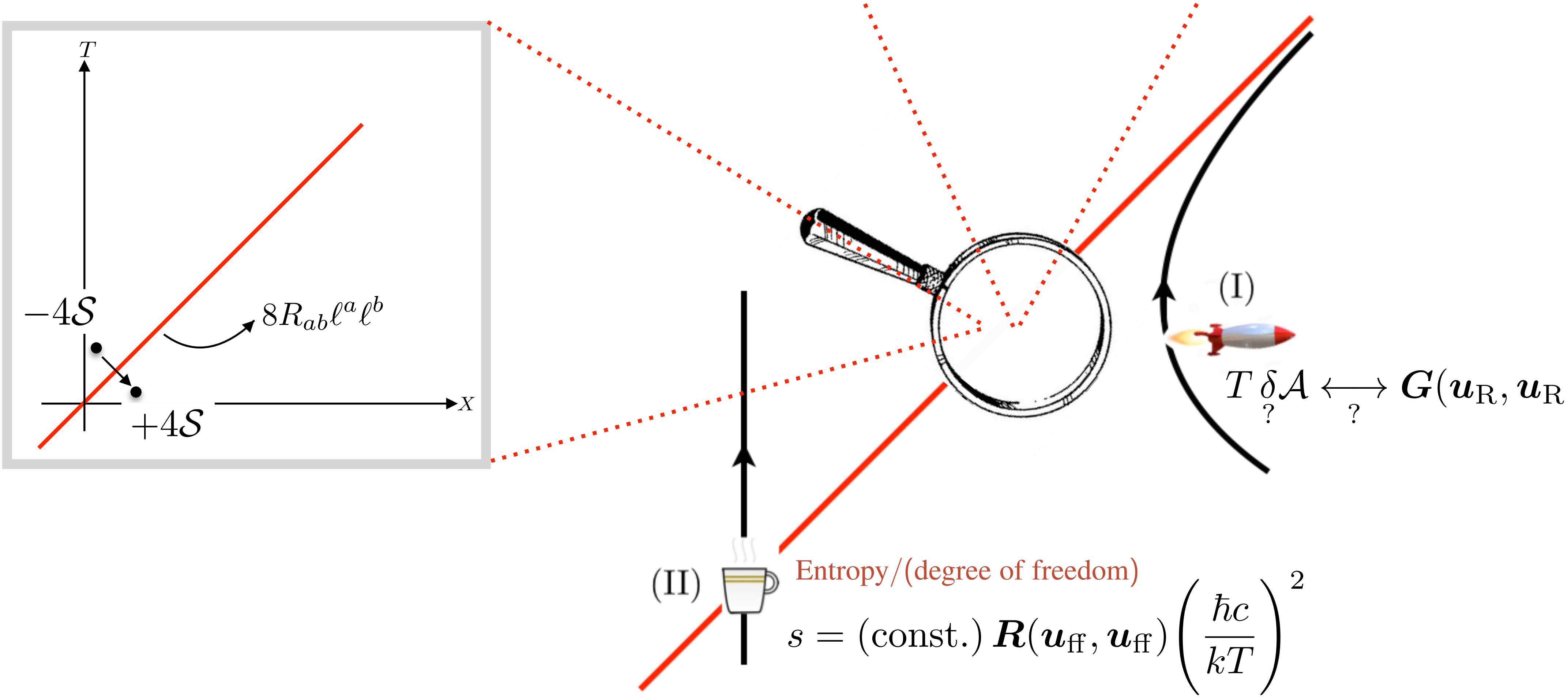
The Cosmological constant

as a relic of the small scale structure of spacetime

The atoms of spacetime and the cosmological constant

T. Padmanabhan [arXiv:1702.06136]





Future Outlook

- implications for the cosmological constant problem

  our approach seems to connect the notion of the cosmological constant being a non-local relic of quantum gravity and it's role in the emergent gravity paradigm

- implications of the non-locality for QFT in curved spacetime

- implications for spacetime singularities

- implications for the emergent gravity paradigm

  for a first step in this direction, see:

  T. Padmanabhan, Distribution function of the Atoms of Spacetime and the Nature of Gravity (arXiv:1508.06286)

Thank You