# Post-Newtonian Theory and Gravitational Wave Physics


K G Arun

Chennai Mathematical Institute


N. R. Sen Lecture

IAGRG meeting 2017

IIT Guwahati

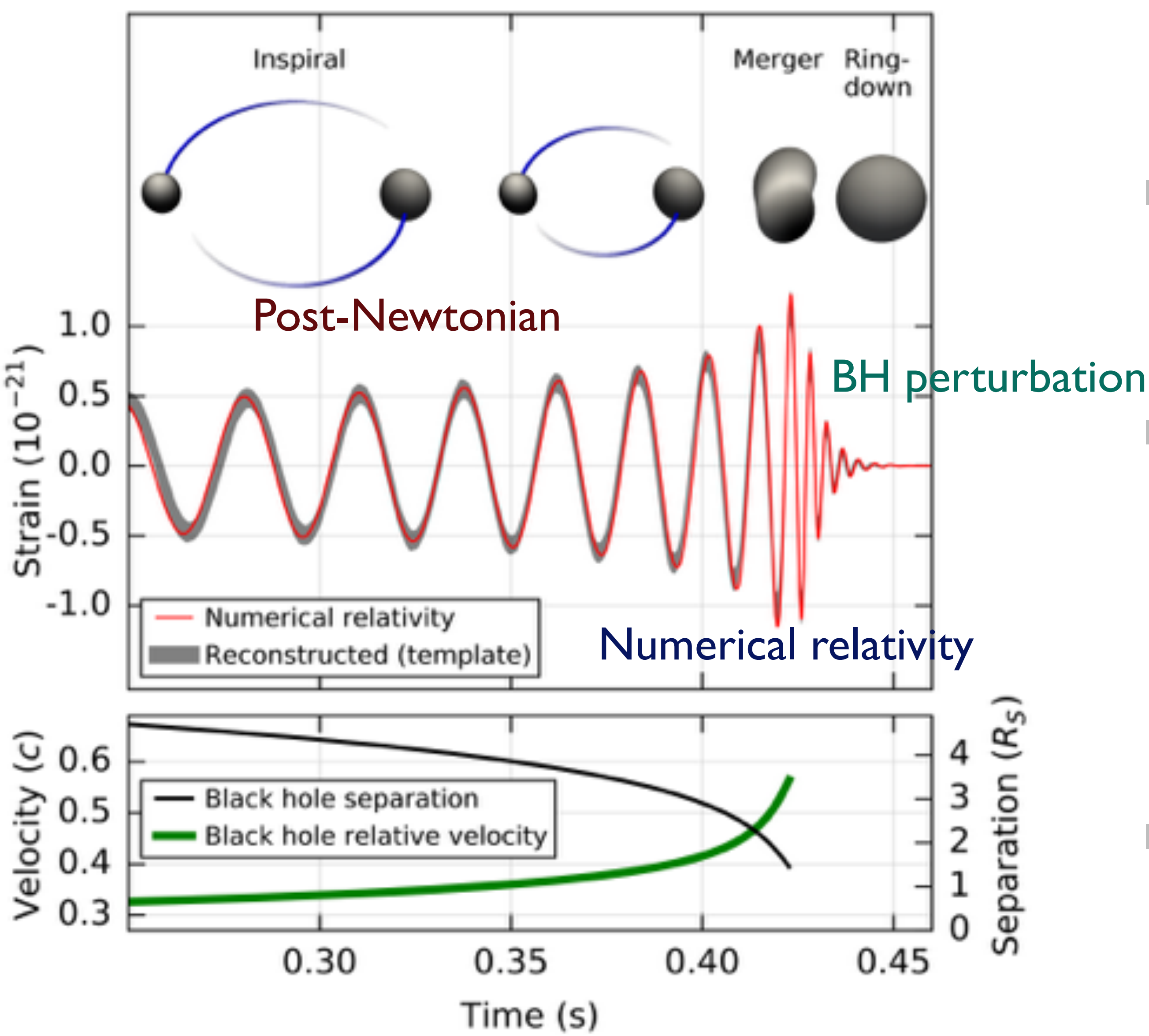
CHENNAI MATHEMATICAL INSTITUTE

indigo

# Outline

- What this talk is about:
  - How elegantly PN theory encodes the highly non-linear physics associated with compact binary mergers.
  - How these can use used to learn physics and astrophysics associated with compact binary mergers.
  - How these can be used to carry out strong-field tests of gravity.

# Post-Newtonian model of compact binary inspiral




LSC + Virgo, Phys. Rev. Lett. 116, 061102 (2016)

## Post-Newtonian Theory

- Solving the two body problem in GR using perturbation theory.
- Uses various approximations to model the dynamics and deduce the gravitational waveforms from the compact binaries.
- Goal: Construct very accurate representation of the phase and amplitude of the gravitational wave signal.

# Energy balance and GW phase

$$\boxed{\frac{d}{dt}E_{\rm orb} = \mathcal{F}}$$

- **Multipole expansion of Flux:**

$$\boxed{\begin{array}{r}U_L\ :\ \text{Mass type Radiative multipoles}\\ V_L\ :\ \text{Current type Radiative multipoles}\end{array}}$$

$$\mathcal{F} = \frac{dE_B}{dT_R} = -\frac{G}{c^5}\left\{\frac{1}{5}U^{(1)}_{ij}U^{(1)}_{ij} + \frac{1}{c^2}\left[\frac{1}{189}U^{(1)}_{ijk}U^{(1)}_{ijk} + \frac{16}{45}V^{(1)}_{ij}V^{(1)}_{ij}\right]\right.$$
$$\left. + \frac{1}{c^4}\left[\frac{1}{9072}U^{(1)}_{ijkm}U^{(1)}_{ijkm} + \frac{1}{84}V^{(1)}_{ijk}V^{(1)}_{ijk}\right] + O(\varepsilon^6)\right\}$$

- **Orbital Energy:**

$$\mathcal{E} = -\frac{c^2}{2}\nu m x\left\{1 - \frac{1}{12}(9+\nu)x - \frac{1}{8}\left(27 - 19\nu + \frac{\nu^2}{3}\right)x^2\right\}$$

Blanchet, Damour, Iyer, 1995

# Frequency Domain GW Phasing

## Using Stationary Phase Approximation

$$\psi_f(t_f) = 2\pi f t_{\rm ref} - \phi_{\rm ref} + 2\int_{v_f}^{v_{\rm ref}} (v_f^3 - v^3)\frac{E'(v)}{\mathcal{F}(v)}dv.$$

## Structure of the Phasing formula

$$\Psi(f) = 2\pi f\, t_c - \phi_c + \frac{3}{128\eta\, v^5}\left[\sum_{k=0}^{N} \Psi_k\, v^{k-5}\right]$$

tidal deformability parameters

$$\Psi(f) = \Psi(f; \overbrace{m_1, m_2}^{\text{masses}}, \overbrace{\chi_1, \chi_2}^{\text{spins}}, \underbrace{e}_{\text{eccentricity}}, \lambda_1, \lambda_2, \cdots)$$

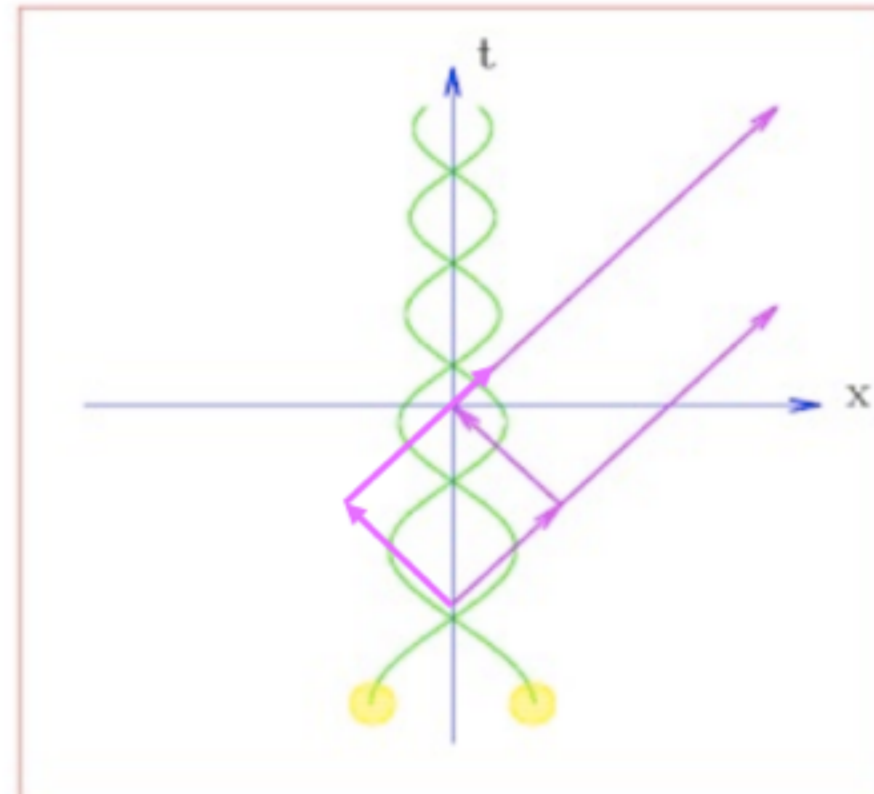
# Table of Physical effects: Quick glance

| PN Order | Effect |
| --- | --- |
| 0PN | Chirp Mass |
| 1PN | Possibility to measure component masses, Periastron Advance, |
| 1.5PN | Tails of GWs, Spin-orbit interaction |
| 2PN | Spin-spin interaction, spin-induced quadrupole |
| 2.5PN | Black hole Horizon Flux (spinning) |
| 3PN | Tails of Tails, Tail^2 |
| 3.5PN | Spin-induced octupole |
| 4PN | Black hole Horizon Flux (nonspinning) |
| 5PN | Tidal interactions |

# Hereditary effects

- Contributions which depend on the dynamics if the system in the past (Vs instantaneous which is function of the retarded time).
- Tails: Due to back-scattering of GWs by the background space time.
- Tails of Tails: Tails being scattered by background curvature.
- Memory: Re-radiation of stress-energy tensor of GWs.

Gravitational wave tails




Blanchet and Schaefer (1994)

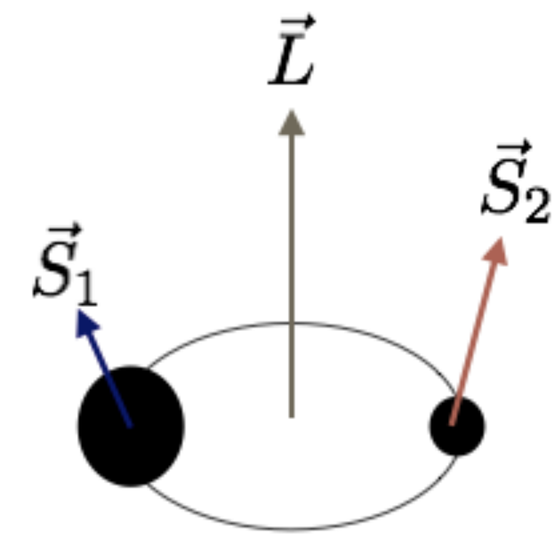
Computation of Hereditary terms:

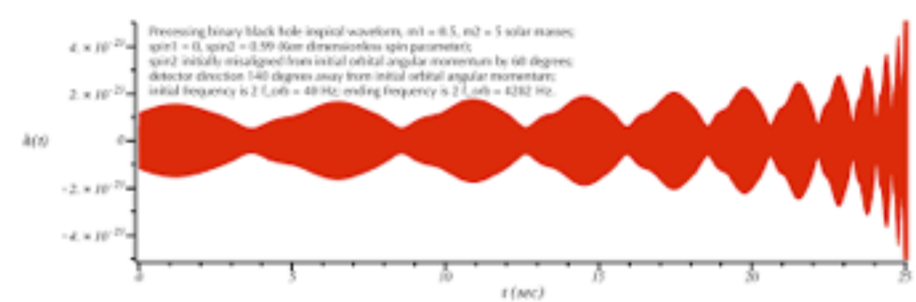
Blanchet, Damour, Iyer, Schafer, Will, Wiseman, KGA, Qusailah, Favata, Sinha, Mishra, …

# Spin effects

*Clues to formation channels of BBHs*

- Spin-orbit interaction (can lead to spin-orbit resonances).
- Spin-spin interactions (carry information about spin-induced multipoles of the compact binaries)
- When spins are arbitrarily aligned w.r.t orbital angular momentum, precession can lead to complicated modulations in the waveform.
- Many of these carry unique imprints about the formation scenario of the binary black holes.



soundsofspacetime.org

# Orbital Eccentricity

*Another way to track the binary formation*

- GW emission is expected to circularise the binary.
- Residual eccentricity may carry important information about binary formation channel.

- GW Flux and polarisation are computed up to 3PN. [KGA, Blanchet, Damour, Gopakumar, Iyer, Mishra, Quasailah, Sinha…]
- Frequency domain representations are also available [KGA, Yunes, Berti, Will, Gopakumar, Haney, Kapadia, Huerta, Favata, Moore,…]

# Tidal interaction

## Inferring Equation of State of compact objects

- Tidal interaction is a very unique aspect of the late time dynamics of compact binaries and carry signatures of the Equation of State of the compact object.

- Can be handy to test whether the observed system is a binary Black hole or not.

- In PN theory this is a higher order effect, though this may be magnified in the late time dynamics.

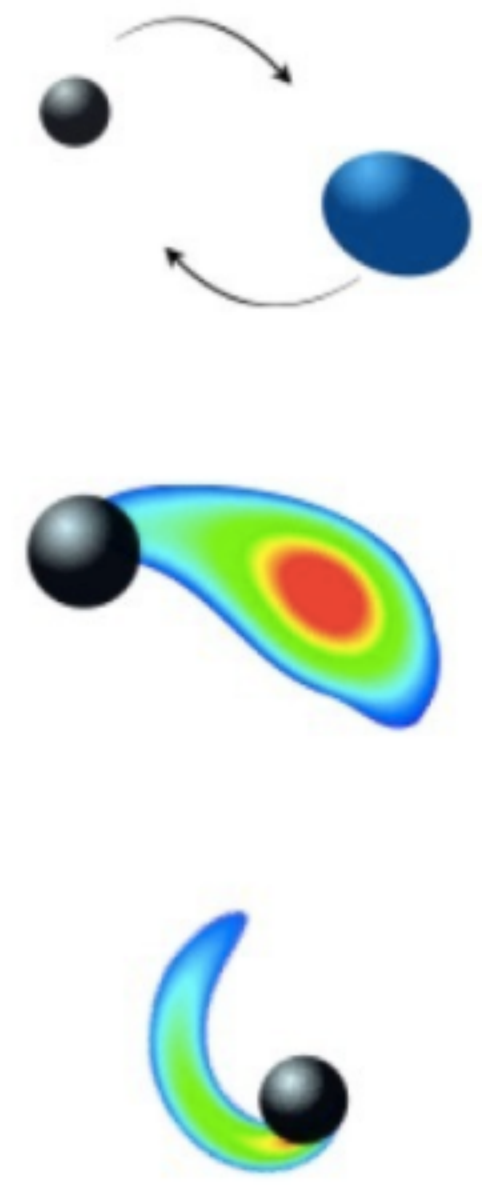


Fig: AEI, Potsdam

# Impact on the Discovery

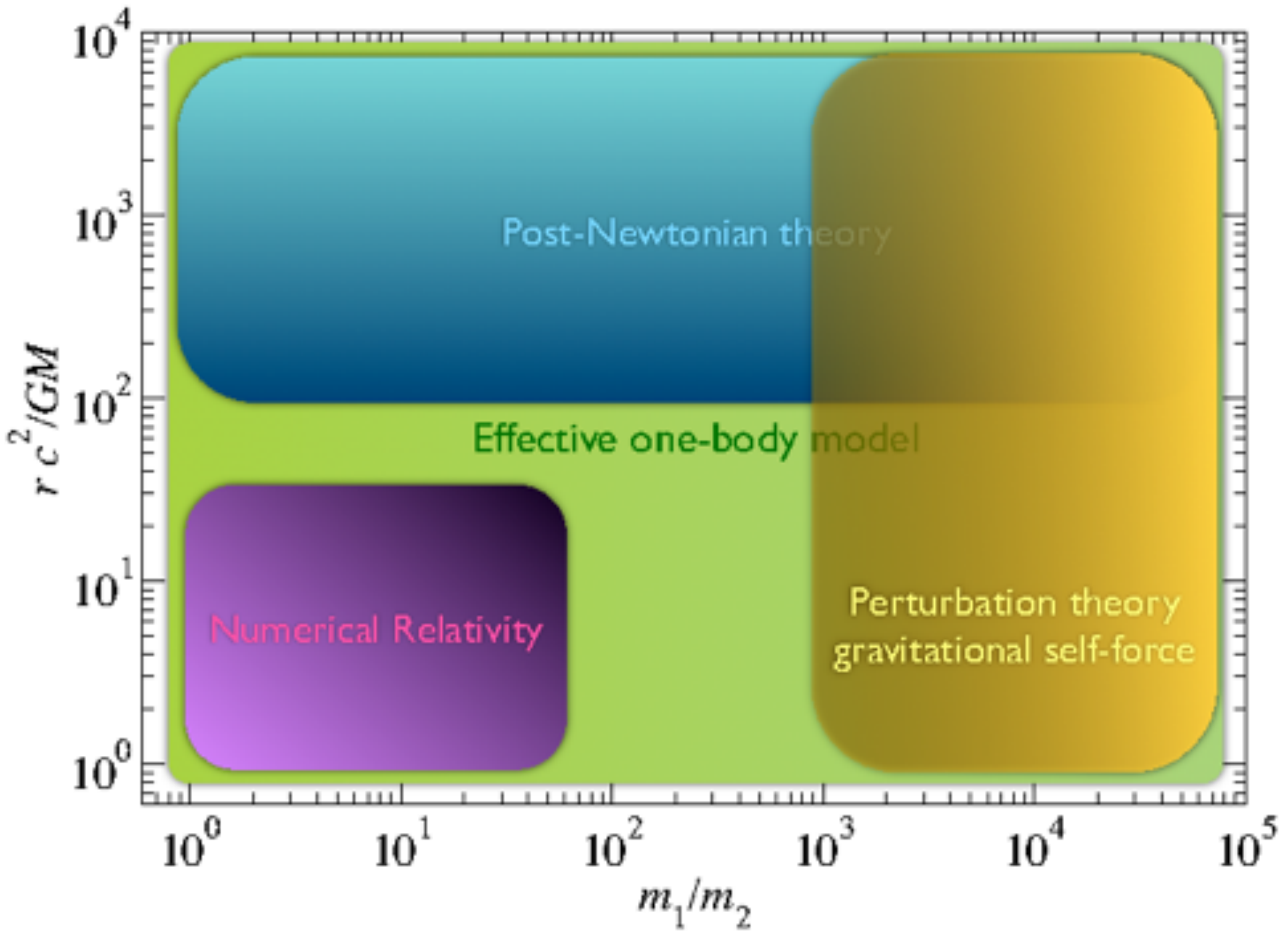
# PN inspired waveforms

## *Effective One-body approach*

[Buonanno, Damour, Nagar, Iyer, ..]

- A semi-analytical model of compact binary merger which crucially exploits the best of the perturbation theory, and calibrated to Numerical Relativity simulations.

- Ongoing efforts to improve the model to include spin interactions, tides, and even extend it to alternative theories of gravity!

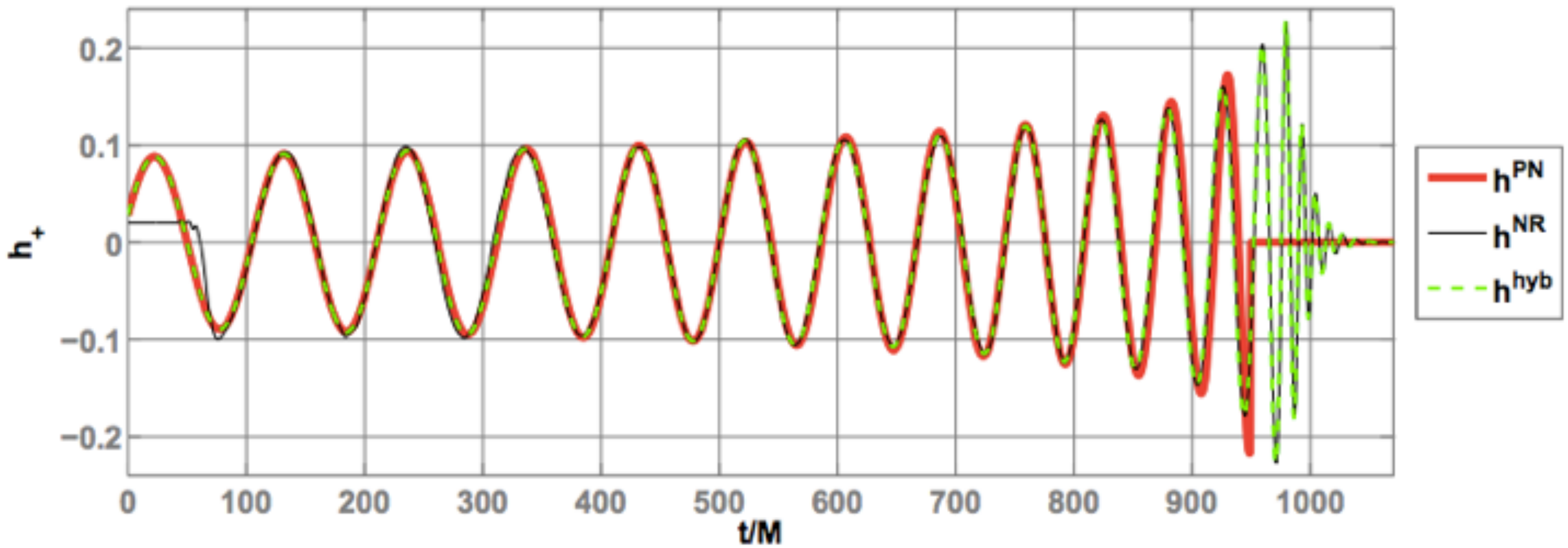



Buonanno, Sathyaprakash 2014

# PN inspired waveforms

## *Phenomenological waveform*

[Ajith, Krishnan, Chen, Babak, Husa, Hannam, Schmidt, Mishra, Mc-Daniel…]




Ajith+, 2007.

- Constructs a NR+PN hybrid.
- Effective analytical parametrisation in frequency domain.
- Captures physical effects such as spins, higher modes etc.

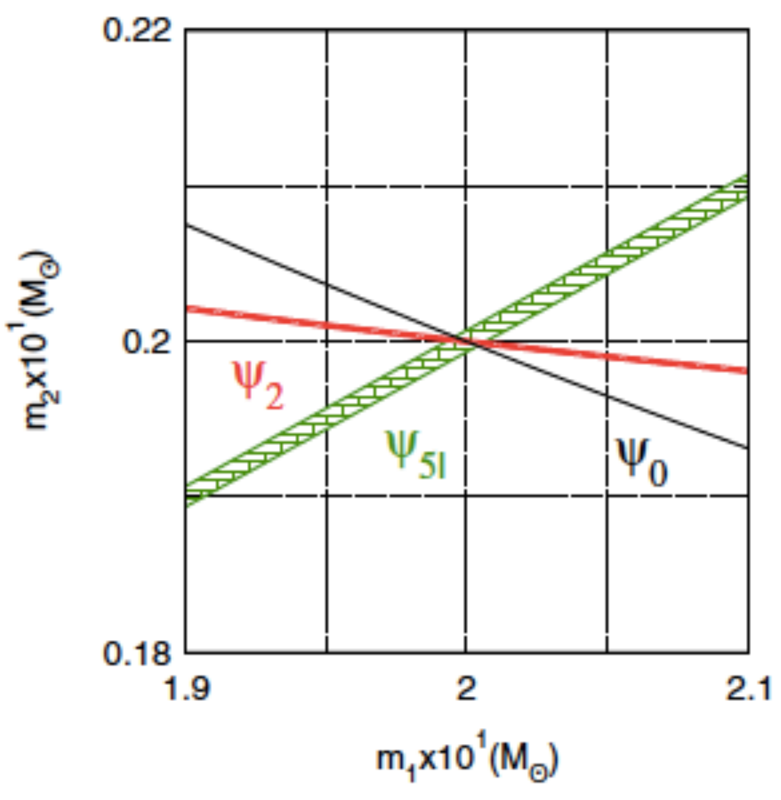
Both Effective One Body and Phenomenological waveforms crucially uses results from Post-Newtonian theory.

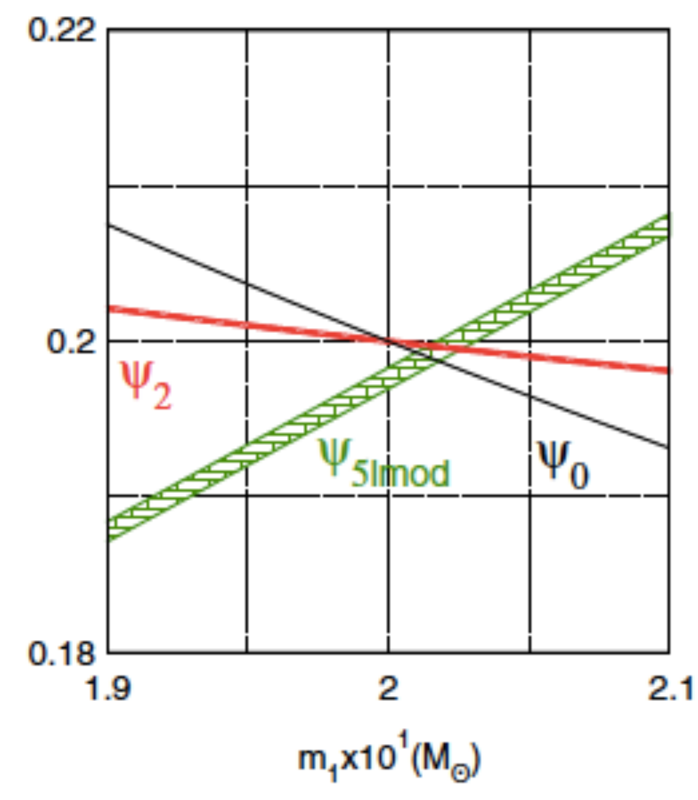
# Tests of strong-field gravity

# Testing PN gravity using GW observations

- Since PN waveforms encode so much physics related to BBH mergers, measuring the PN coefficient accurately will help us put GR to test!

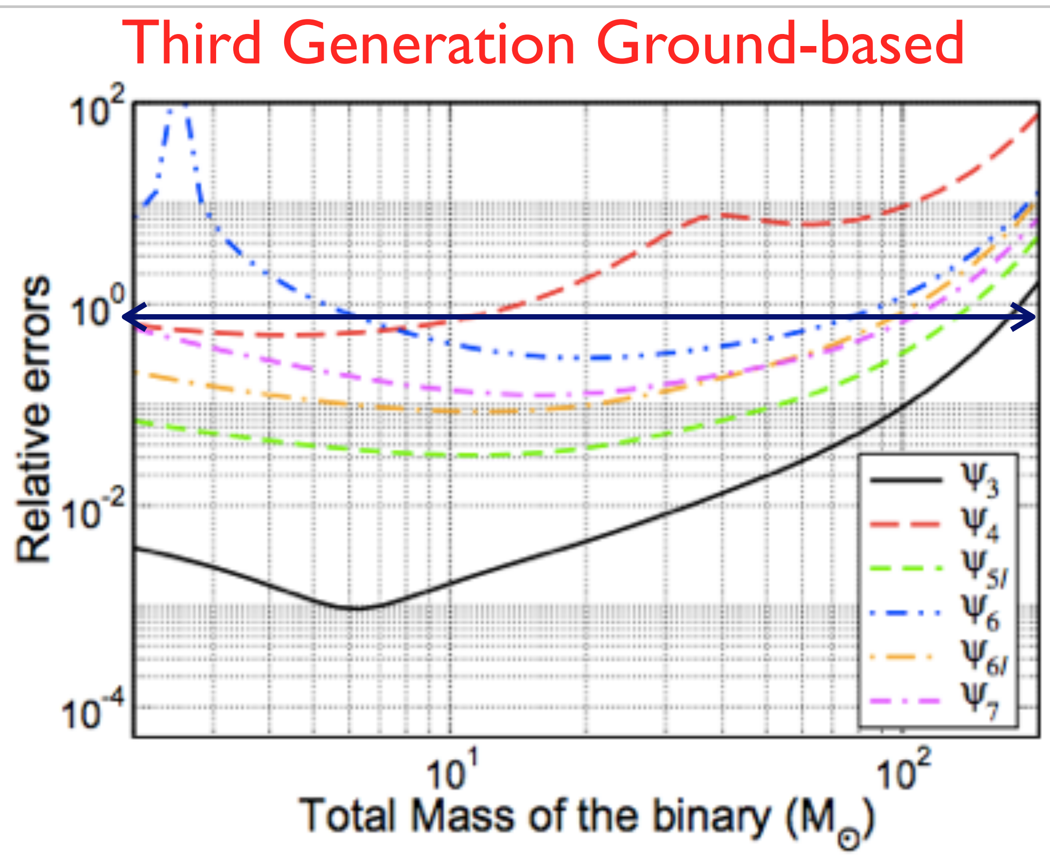
- The test may be casted as a consistency test in the mass plane of the binary.

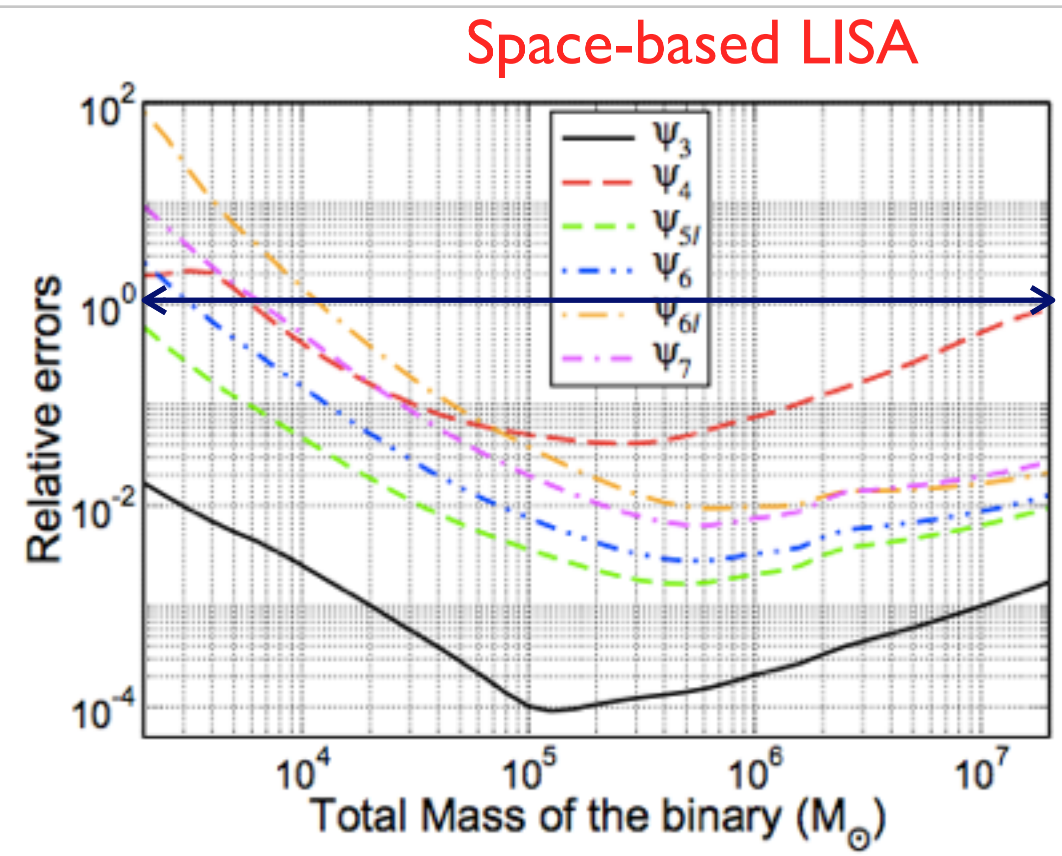



KGA, Iyer, Quasailah,Sathyaprakash, 2006a,2006b, Mishra, KGA, Iyer, Sathyaprakash, 2010, Pai, KGA, 2013

# Projected Accuracy of measuring PN coefficients




KGA, Iyer, Qusailah, Sathyaprakash, 2006a

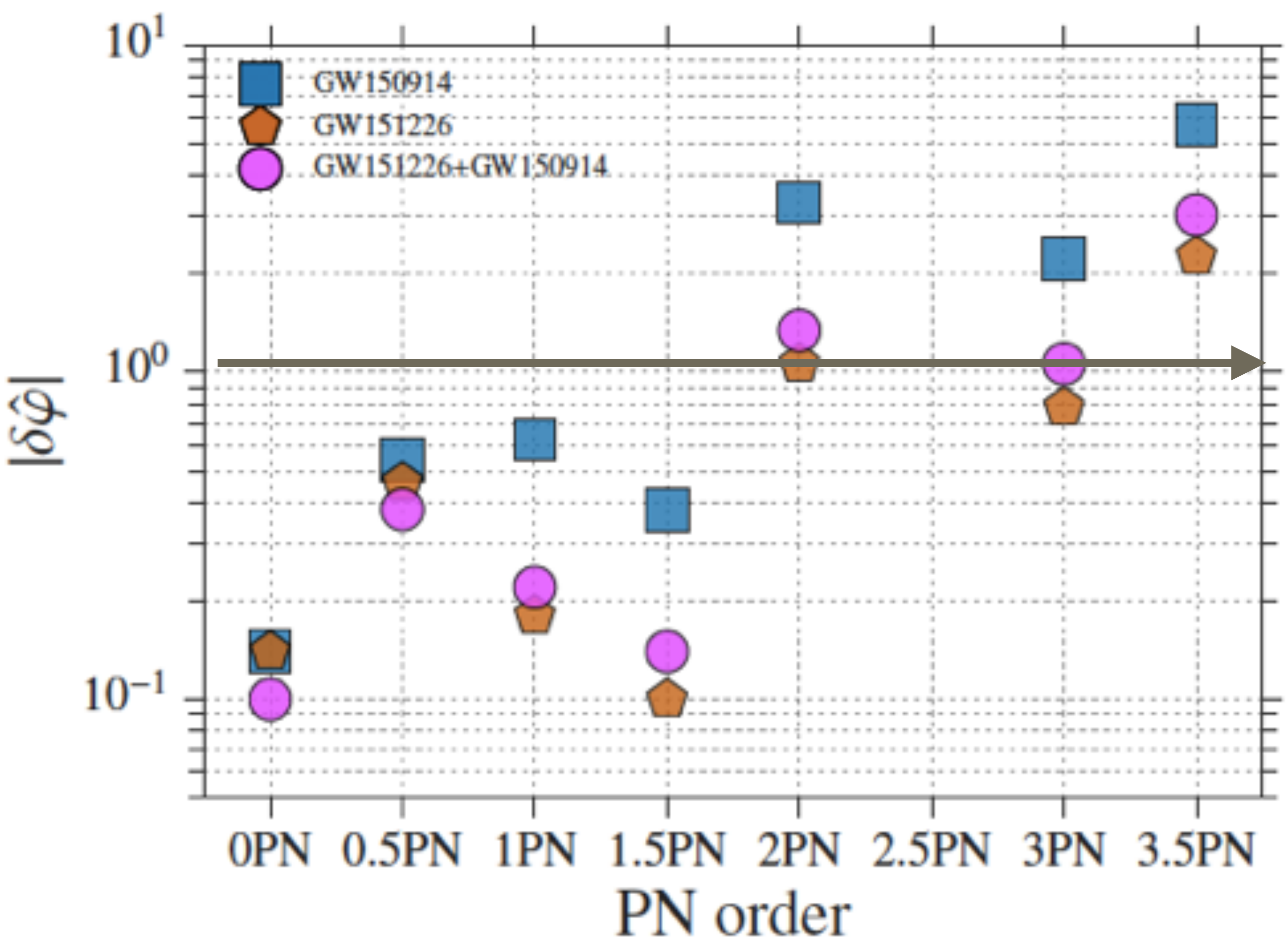
- This proposal was recast in the framework of Bayesian inference which facilitated direct application on detected GW signals.

Li+ 2013, Agathos+ 2014

$$\boxed{\Psi_k \to \Psi_k^{\rm GR}\,(1+\delta\chi_k)}$$

Deviation Parameter

# On the detected GW events




LSC+Virgo, 2016, Phys. Rev. X 6, 041015 (2016)

- This is the current Observational Limit on the deviations allowed on the PN coefficients.
- Combines the two events during First observation run.
- With more events, the limits will get narrower (or we will see a violation?!!)

# Multipolar interpretation of the Test

- Can we map the Test to be constraints on the multipoles of the compact binary?

$$I_L \to \mu_l\, I_L$$
$$J_L \to \epsilon_l\, J_L$$

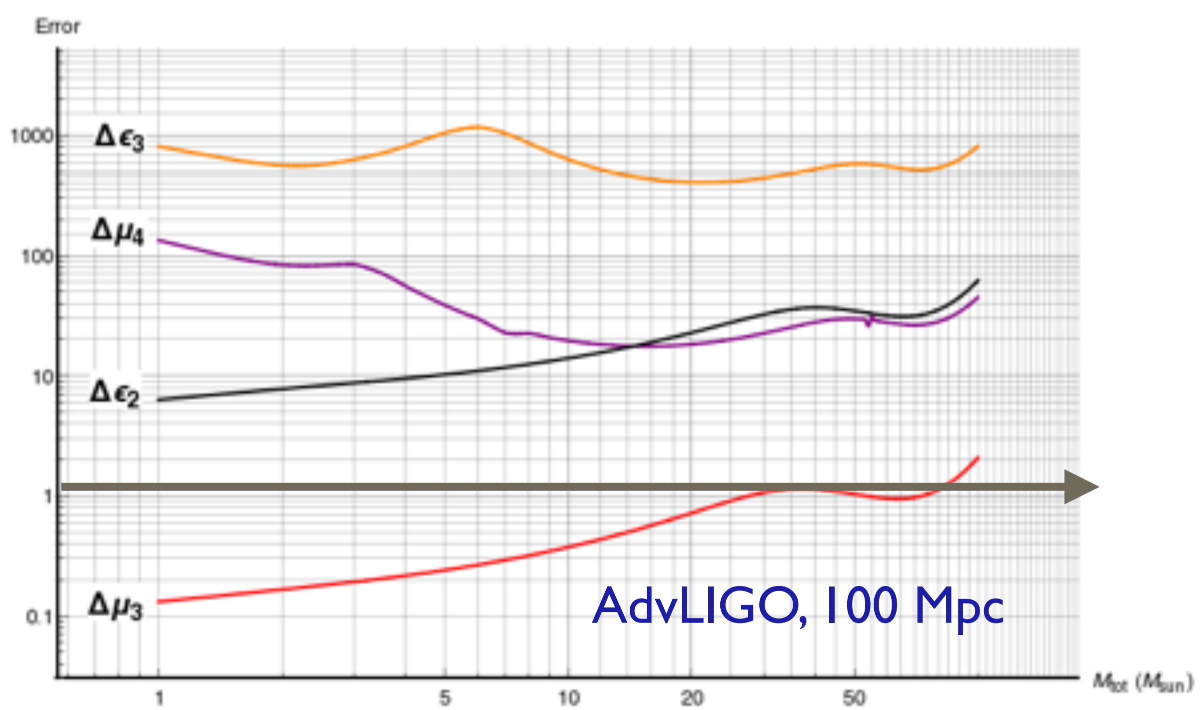
$$\Psi(f) \to \Psi(f; \mu_l, \epsilon_l)$$

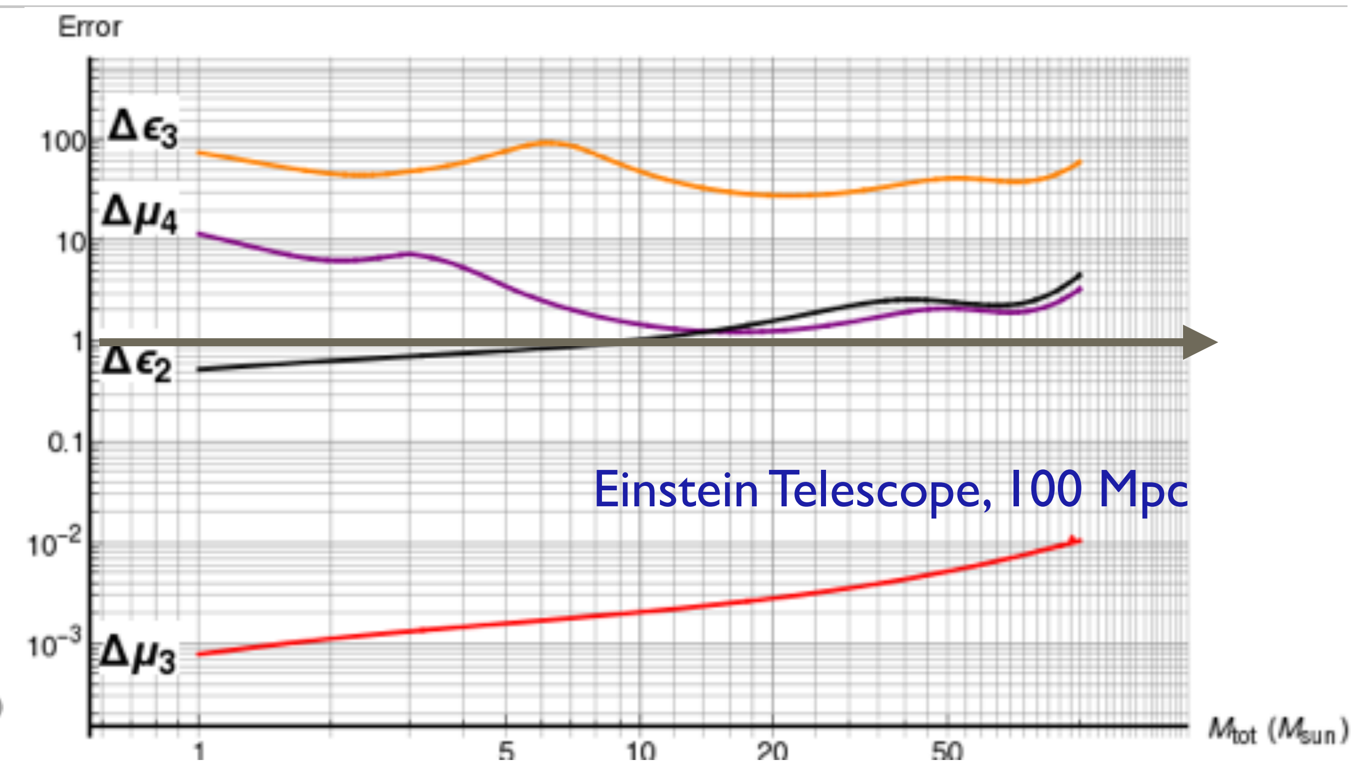
$$\mu_l^{\rm GR} \equiv 1$$
$$\epsilon_l^{\rm GR} \equiv 1$$

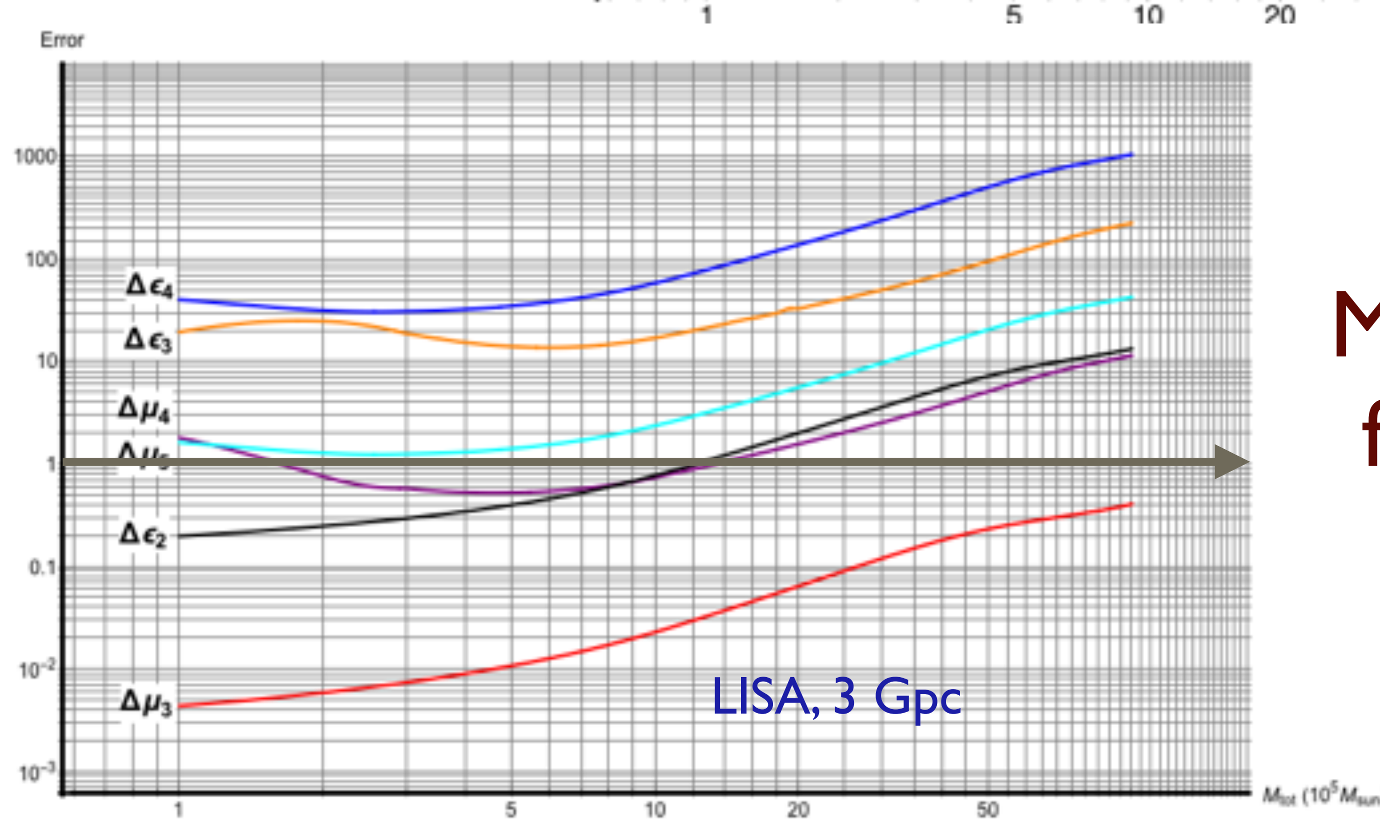
How well can we estimate the multipole coefficients from GW observations?

[S Kastha, A Gupta, KGA, B Sathyaprakash (Ongoing)]

# Preliminary results




Mass ratio=2 for all plots



S. Kastha, A. Gupta, KGA, B Sathyaprakash (In progress)

# Tests of Binary Black hole nature

How can we test we have indeed seen Black holes and not some exotic stars?

[Talk by Johnson-McDaniel on 20th].

# Multipole moments of a Kerr BH

***No-hair conjecture***

Mass-type multipoles

Mass

$$Q_l + iS_l = M^{l+1}(i\chi)^l$$

Dimensionless spin $\chi = |\mathbf{S}|/M^2$

Current type multipoles

BH mimickers

## Mass quadrupole of a compact object

$$Q_2 = -\kappa\, M^3\chi^2, \text{with } \kappa_{\rm BH} \equiv 1$$

$$\kappa_{\rm NS} \simeq 2 - 14$$
$$\kappa_{\rm BS} \simeq 10 - 150$$
$$\kappa_{\rm GS} \simeq +/-$$

## Spin octupole of a compact object

$$S_3 = -\lambda\, M^4\,\chi^3 \text{ with } \lambda_{\rm BH} \equiv 1$$

$$\lambda_{\rm NS} \sim 4 - 30$$
$$\lambda_{\rm BS} \sim 10 - 200$$

# Post-Newtonian waveforms

PN phasing

PN coefficients (encode various non-linear interactions in the binary dynamics)

$$\Psi(f) = \frac{3}{128\eta\, v^5} \sum_{k=0}^{N} \psi_k\, v^k$$

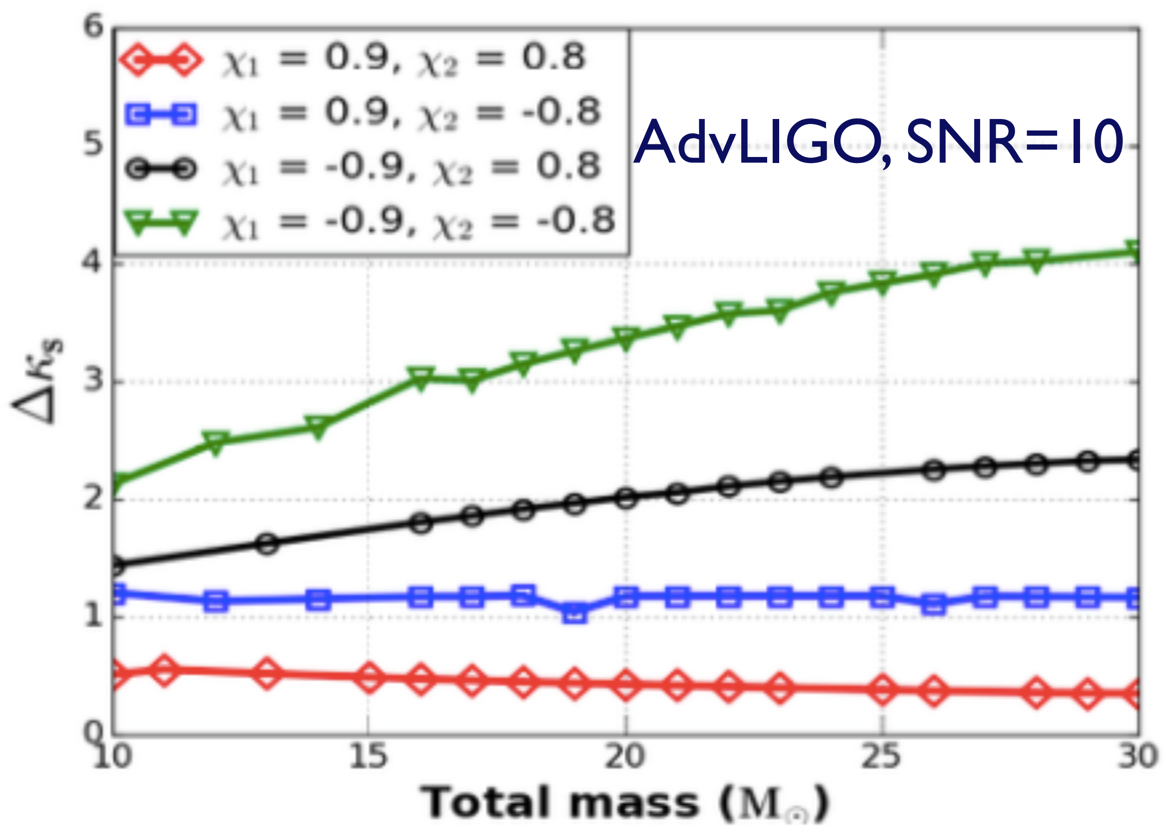
[Spin-orbit, spin-spin, Tails, Tails of Tails, Tail^2, Memory, …]

Spin terms at 2PN [v^4], 3PN [v^6] and 3.5PN [v^7] contain quadratic in spin interactions (kappa) and at 3.5PN the cubic interactions (lambda).

KGA, Blanchet, Bohe, Buonanno, Faye, Kidder, Marsat, Poisson, Porter,…(over the past several years)

Can we estimate $\kappa$ from GW observations and probe the nature of the compact object?

# No-hair Test of Binary Black holes




Krishnendu, KGA, Mishra, 2017

- Very accurate estimation of spin-quadrupole is possible with AdvLIGO.
- A New way to probe the nature of the compact binary system.

Details: Poster by Krishnendu

# Tidal deformability of the compact object

Induced quadrupole moment

External tidal field

$$Q_{ij} = -\lambda\,\epsilon_{ij}$$

Tidal love number

Dimensionless tidal love number

$$k_2 = \frac{3}{2} G\lambda\, R^{-5}$$

$k_2 \equiv 0$ for BHs

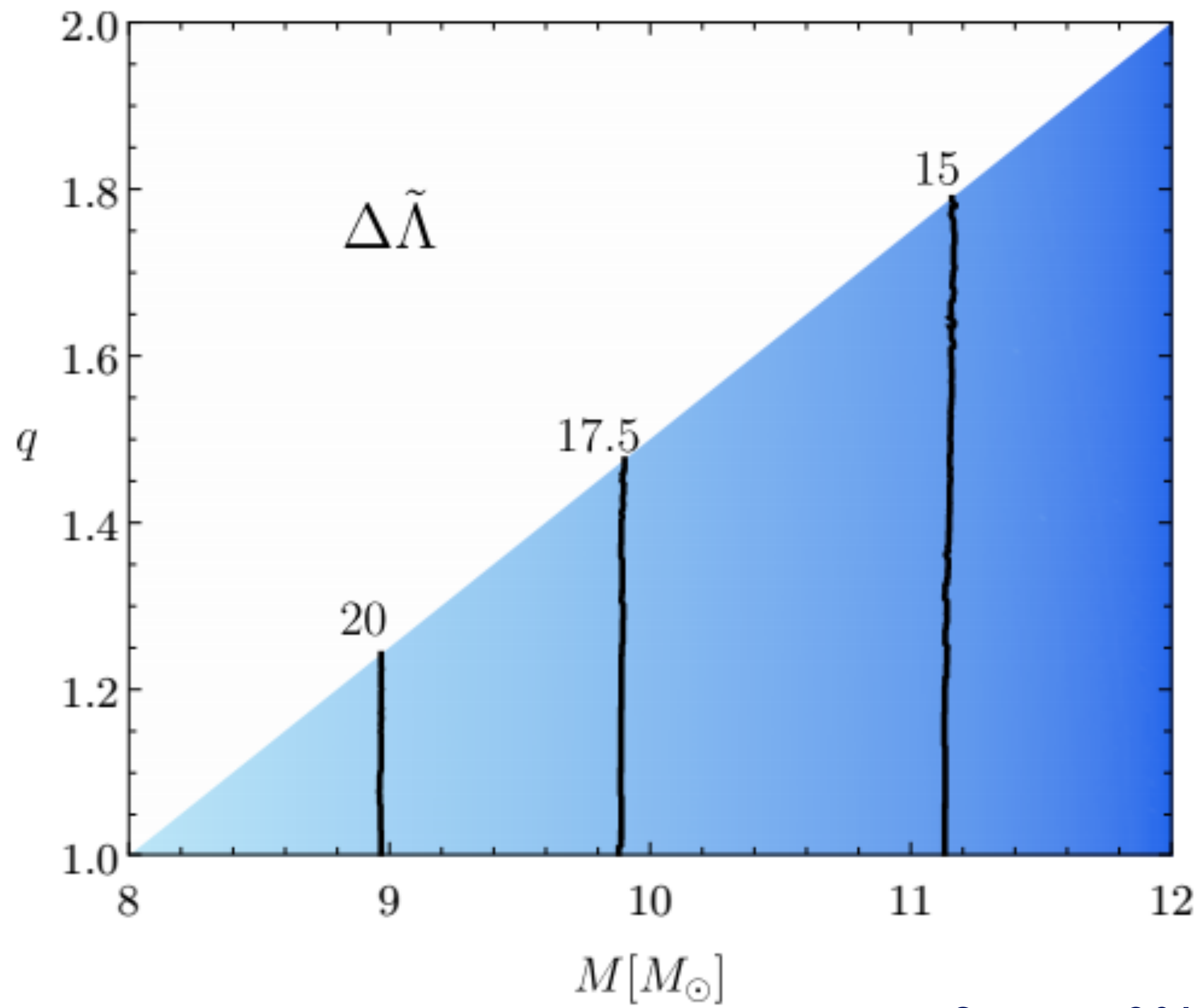
Damour & Nagar, 2009
Bennington & Poisson, 2009

| | | $k_2^E$ |
|---|---|---|
| NSs | | 210 |
| ECOs | Boson star | 41.4 |
| | Wormhole | $\frac{4}{5(8+3\log\xi)}$ |
| | Perfect mirror | $\frac{8}{5(7+3\log\xi)}$ |
| | Gravastar | $\frac{16}{5(23-6\log 2+9\log\xi)}$ |

Cardoso et al, 2017

# Projected constraints on Tidal Parameter

Distance=400 Mpc



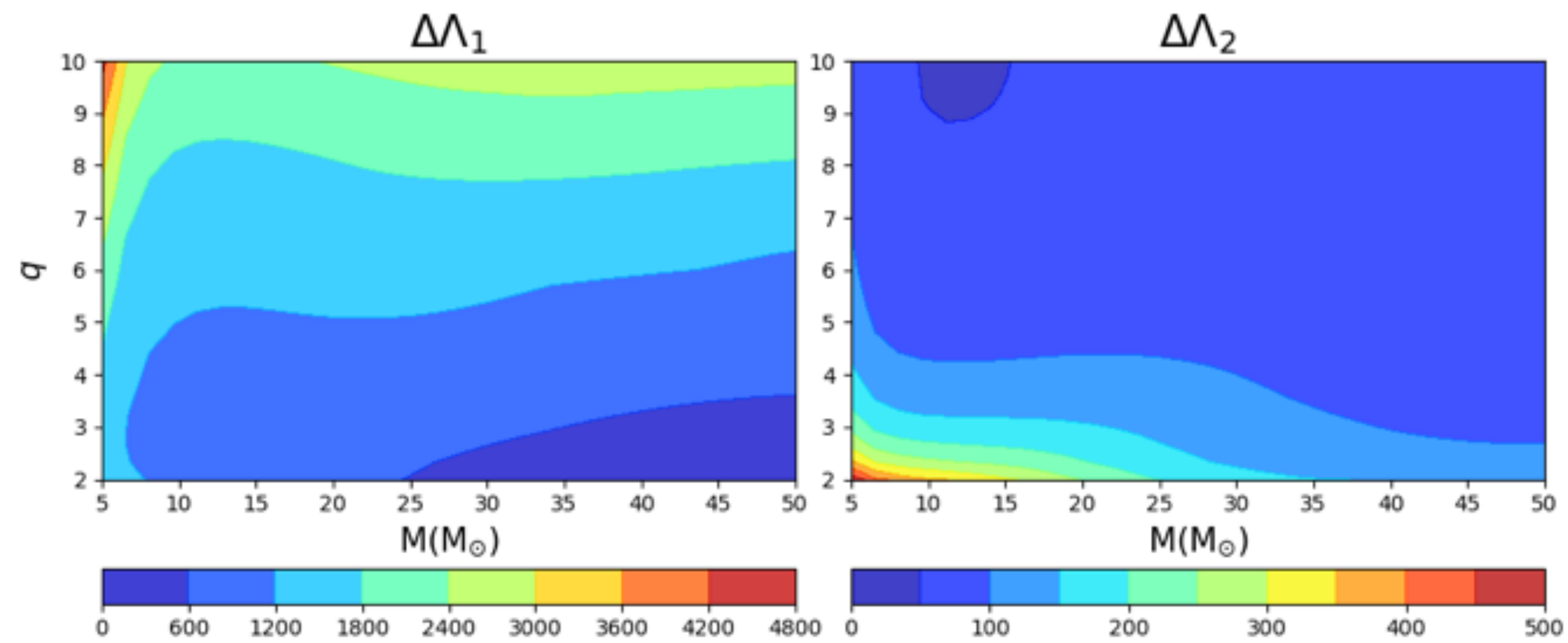

Sennet+ 2017

Errors on a linear combination of Tidal deformability Parameter of BHs

Cardoso+, 2017, Sennet+ 2017

# Using higher modes of GWs

100 Mpc




A. Gupta, N.V. Krishnendu, KGA (In Progress)

$$\Delta\Lambda_{1,2}^{\text{projected}} \sim 100s\,(\text{AdvLIGO})$$

$$\Delta\Lambda_{1,2}^{\text{projected}} \sim 10s\;(\text{ET})$$

$$\Lambda_{\text{MiniBS}} \geq 900$$

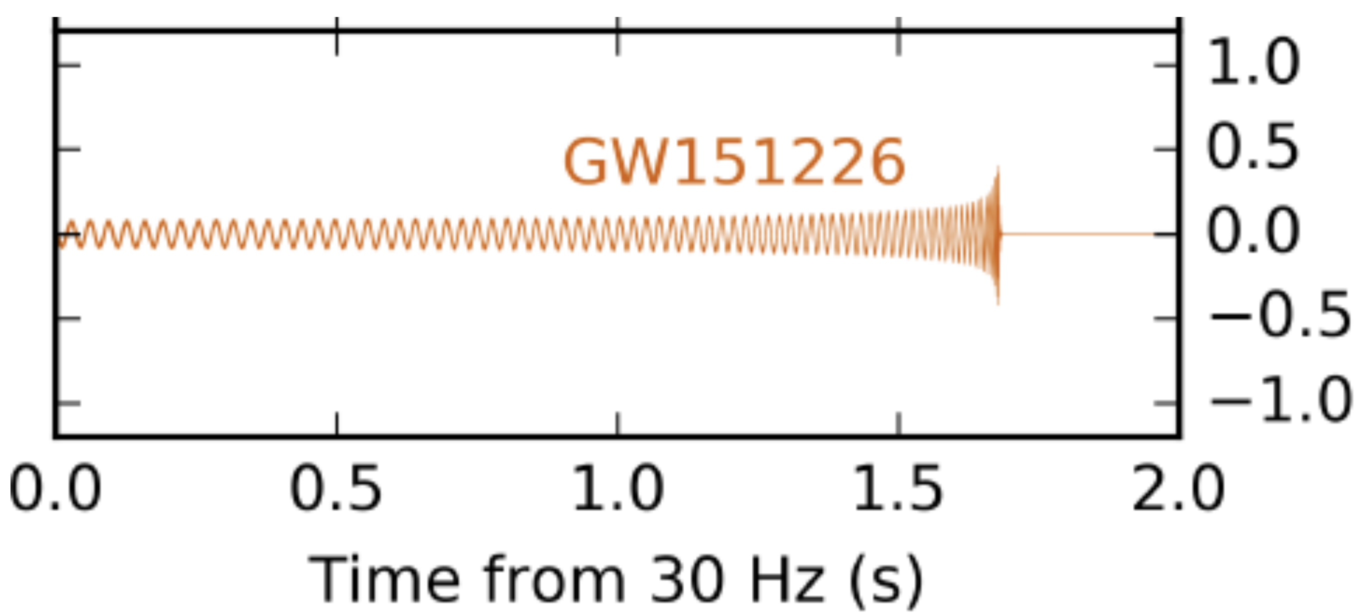
$$\Lambda_{\text{MassiveBS}} \geq 280$$

Sennet+, 2017

# Summary

- Post-Newtonian theory has been very successful in modelling the compact binary dynamics and elegantly captures the highly nonlinear evolution of the binary.
- It has also been crucial for developing the waveform families such as Effective One Body and Phenomenological waveforms.
- These results have even widely used in carrying out strong-field tests of gravity.
- Some of the predictions of PN theory is already put to test by the GW  detections in future it is expected that many more of them will be put to test.

# Back up slides




- Required the use theoretical waveform model to detection.
- Very long inspiral (55 orbital cycles).
- Stringent constraints on post-Newtonian coefficients.