Are First and Second Order Formulations of Gravity Equivalent?

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The Institute of Mathematical Sciences, Chennai

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Talk based on the work done with Sandipan Sengupta

RKK and S. Sengupta:

"Degenerate spacetimes in first order gravity",
Phys. Rev. D93 (2016) no.8, 084026, arXiv:1602.04559 [gr-qc];

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Einstein-Hilbert Action

One way to describe theory of gravity is through Einstein-Hilbert action functional (with Euclidean signature):

$$\mathcal{S} \ = \ \frac{1}{2\kappa^2} \int d^4x \ \sqrt{g} \ g^{\mu\alpha} g^{\nu\beta} \ R_{\mu\nu\alpha\beta}(\Gamma)$$

where Riemann curvature tensor is:

$$R_{\mu\nu\lambda}{}^{\rho}(\Gamma) = \partial_\mu \Gamma_{\nu\lambda}{}^{\rho} - \partial_\nu \Gamma_{\mu\lambda}{}^{\rho} + \Gamma_{\mu\eta}{}^{\rho} \Gamma_{\nu\lambda}{}^{\eta} - \Gamma_{\nu\eta}{}^{\rho} \Gamma_{\mu\lambda}{}^{\eta} \ ;$$

$$R_{\mu\nu\alpha\beta} = g_{\rho\beta} R_{\mu\nu\alpha}{}^{\rho}; \qquad g \equiv det \ g_{\mu\nu}$$

and Christoffel symbols are given in terms of the metric as:

$$\Gamma_{\mu\nu}{}^{\lambda} \equiv \frac{1}{2} g^{\lambda\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$$

We could add cosmological constant and also matter fields to this action, but for the purpose of this talk, we do not need to do so.

This action functional for pure gravity on its own is of sufficient interest.

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Here the metric $g_{\mu\nu}$ is the independent field in the action (Γ is not an independent field).

Variation of the action functional with respect to the metric $g_{\mu\nu}$ leads to the Euler-Lagrange equations of motion which are identical with the Einstein field equations for pure gravity (vacuum equations):

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

where the Ricci tensor, $R_{\mu\alpha} \equiv g^{\nu\beta} R_{\mu\nu\alpha\beta}$ and the Rici scalar is: $R \equiv g^{\mu\nu} R_{\mu\nu}$

These equations have, besides others, the remarkable property that they admit Schwarzschild black hole as a solution.

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Solutions of Einstein equations for pure gravity (that is, in absence of any matter like fermions), by construction, do not possess any torsion.

The important point to register here is that, by construction, the metric here is non-degenerate; it has no zero eigen values so that its inverse and determinant are defined.

This formulation of gravity theory is also called second order formulation in contrast to another framework which is known as the first order formulation.

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An alternate formulation of theory of gravity is the first order formulation. It is described by the Hilbert-Palatini action functional given in terms of tetrads e^I_μ and connection fields $\omega_\mu{}^{IJ}$ as :

$$\mathcal{S} \;=\; \frac{1}{8\kappa^2}\int d^4x\ \epsilon^{\mu\nu\alpha\beta}\ \epsilon_{IJKL}\ e^I_\mu e^J_\nu\ R_{\alpha\beta}{}^{KL}(\omega)$$

where now the curvature $R_{\mu\nu}{}^{IJ}(\omega)$ is the field strength of the gauge connections $\omega_\mu{}^{IJ}$ of the local Lorentz group $SO(4)$ in the Euclidean version of the theory:

$$R_{\mu\nu}{}^{IJ}(\omega) \;\equiv\; \partial_{[\mu}\omega_{\nu]}{}^{IJ} \;+\; \omega_{[\mu}{}^{IK}\omega_{\nu]}{}^{KJ}$$

The completely antisymmetric epsilon symbols have constant values 0 and ±1 with $\epsilon^{xyz\tau} = +1$ and $\epsilon_{1234} = +1$.

The tetrad can be thought of to be square root of the metric as:

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Both the tetrad e^I_μ and connection fields $\omega_\mu{}^{IJ}$ in the action functional are to be taken as independent.

Euler-Lagrange equations of motion are obtained by varying with respect to both these fields independently.

Equations of motion:

$$\frac{\delta S}{\delta e^I_\mu} : \qquad \epsilon^{\mu\nu\alpha\beta}\, \epsilon_{IJKL}\; e^J_\nu\; R_{\alpha\beta}{}^{KL}(\omega) \;=\; 0 \qquad\qquad (16\ eqns.)$$

$$\frac{\delta S}{\delta \omega_\mu{}^{IJ}} : \qquad \epsilon^{\mu\nu\alpha\beta}\, \epsilon_{IJKL}\;\; e^K_\mu\;\; D_\nu(\omega)\; e^L_\alpha \;=\; 0 \qquad\qquad (24\ eqns.)$$

where $\quad D_\mu(\omega)\; e^I_\nu \;\equiv\; \partial_\mu e^I_\nu \;+\; \omega_\mu{}^{IJ} e^J_\nu .$

We need to solve these equations for the tetrads and connection fields.

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Both the tetrad e^I_μ and connection fields ${\omega_\mu}^{IJ}$ in the action functional are to be taken as independent.

Euler-Lagrange equations of motion are obtained by varying with respect to both these fields independently.

Equations of motion:

$$\frac{\delta \mathcal{S}}{\delta e^I_\mu} : \qquad \epsilon^{\mu\nu\alpha\beta}\, \epsilon_{IJKL}\, e^J_\nu\, {R_{\alpha\beta}}^{KL}(\omega) \;=\; 0 \qquad (16 \; eqns.)$$

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Consider the second set of equations of motion first.

$$e^{[I}_{[\mu} D_\nu(\omega)\ e^{J]}_{\alpha]} \ = \ 0 \qquad\qquad (24\ eqns.)$$

Assume tetrads are *invertible*, that is, $det\ e^I_\mu \ \neq \ 0$.
Inverse of the tetrad, e^μ_I, is defined by $e^I_\mu e^\nu_I = \delta^\nu_\mu$ and $e^I_\mu e^\mu_J = \delta^I_J$.

By multiplying by e^μ_I sucessively, this set of 24 equations can be easily solved to yield equivalent 24 equations:

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These equations can further be solved for 24 connection fields :

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Hilbert-Palatini Action: invertible Tetrads

Other set of Euler-Lagrange equations of motion, obtained by varying the action with respect to tetrad fields, can be rewritten as:

$$e^{[I}_{[\mu}\; R_{\nu\alpha]}{}^{JK]}(\omega) \;= 0 \qquad\qquad (16\; eqns.)$$

Multiplying by the inverse tetrads $e^\mu_I e^\nu_J$, these can be seen to yield the following 16 equations:

$$R_\alpha{}^K \;-\; \tfrac{1}{2}\, e^K_\alpha\; R \;= 0$$

where

$$R_\alpha{}^K \;\equiv\; e^\mu_I\; R_{\mu\alpha}{}^{IK}(\omega) \quad and \quad R \;\equiv\; e^\alpha_K\; R_\alpha{}^K$$

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Further notice that the above equations are really Einstein equations, which becomes transparent, by realizing that, for invertible tetrads, the local Lorentz field strength and Reimann curvature are related:

$$R_{\mu\nu}{}^{IJ}(\omega)\ e^I_\lambda e^\rho_J \ = \ R_{\mu\nu\lambda}{}^\rho(\Gamma)$$

Thus, for invertible tetrads, the first order and second order formalisms are exactly equivalent.

In both these formulations, the configurations that satisfy vacuum equations of motion, *(for first order formalism, with invertible tetrads)*, do not possess any torsion. This is both necessary and also sufficient.

Torsion is introduced in both these frameworks by matter couplings, say by coupling fermions to the theory.

Before we are tempted to assert the equivalence of two frameworks too quickly, let us examine the action functional for the first order formulation a bit more closely.

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Notice that this action is defined both for the **invertible** as well as **non-invertible** tetrads.

There are no $det\ e^I_\mu$ nor inverse of the tetrad here.

It could admit configurations which involve degenerate *(non-invertible)* tetrads.

Such configurations would be particularly important in the quatum theory where we need to integrate over all possible configurations including those with degenerate tetrads, in the fuctional integral, each with a weight $\exp(-\mathcal{S})$ as dictated by Feynman path integral quantum theory.

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Such configurations would be particularly important in the quatum theory where we need to integrate over all possible configurations including those with degenerate tetrads, in the fuctional integral, each with a weight $\exp(-\mathcal{S})$ as dictated by Feynman path integral quantum theory.

Also, as we shall see in the following, such configurations could as well possess torsion even without presence of any torsion inducing matter.

Hilbert-Palatini Action: Degenerate Tetrads

Thus, even at the classical level, we could ask the following questions:

(i) Are there any degenerate tetrad solutions of the Euler-Lagrange equations of motion in the first order formulation?

(ii) Do such solutions possess torsion even without any matter?

(iii) Do these configurations have finite action?

If we do find such solutions of the equation of motion, then two formulations of the gravity theory, based on Einstein-Hilbert and Hilbert-Palatini action functionals, would not be equivalent even classically.

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Let us start with degenerate tetrads with one zero eigen-value.

$$Ansatz: \qquad e^I_\mu = \begin{pmatrix} e^1_x & e^2_x & e^3_x & 0 \\ e^1_y & e^2_y & e^3_y & 0 \\ e^1_z & e^2_z & e^3_z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} e^i_a & 0 \\ 0 & 0 \end{pmatrix};$$

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Four dimensional length element is: $ds^2_{(4)} = 0 + g_{ab} dx^a dx^b$

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Recall the 24 connection eqns of motion: $e^{[I}_{[\mu} \; D_\nu(\omega) \; e^{J]}_{\alpha]} \;=\; 0$

Unlike the case of invertible tetrads, these equations now can not be solved for all the 24 connection fields ω_μ^{IJ}.

It turns out that 12 of the connection fields are fixed other 12 connection fields stay undetermined.

After some bit of analysis, it can be shown that the most general solutions of these equations are provided by configurations which satisfy the following constraints:

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Hilbert-Palatini Action: Degenerate Tetrads

For degenerate tetrads with one zero eigenvalue, this set of equations are exactly equivalent to original 24 equations of motion we had obtained by varying the action functional with respect to the connection fields ω_μ^{IJ} .

An important property to note here is: these equations imply that such configurations would generically have non-zero contorsion even for the case of pure gravity without any matter fields such as fermions.

Next we analyse the rest of equations of motion; those obtained by varying the action with respect to the tetrad fields e_μ^I :

$$e_{[\mu}^{[I} R_{\nu\alpha]}{}^{JK]}(\omega) = 0 \qquad\qquad (16\ eqns).$$

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After some bit of analysis, it can be shown that the most general solutions of these equations are provided by configurations which satisfy the following 16 constraints:

$$\hat{e}_i^a \; R_{\tau a}{}^{ij}(\omega) \;=\; 0,$$

$$R_{a\tau}{}^{4k}(\omega) \;=\; -D_\tau(\omega)\; M_a^k \;=\; 0\;, \qquad\qquad (\; \omega_a^{4k} \equiv M_a^k \;=\; M^{kl} e_a^l \;)$$

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This allows us to write the last condition on the curvatures as:

$$\xi \equiv N^{ij}N^{ji} - N^{ii}N^{jj} = 6\lambda^2 - \hat{e}^a_i \, \hat{e}^b_j \, \bar{R}_{ab}{}^{ij}(\bar{\omega})$$

All we need to do now is find a set of triads e^i_a which satisfy all the earlier conditions. Evaluate the corresponding spatial (three-) curvature scalar $\hat{e}^a_i \, \hat{e}^b_j \, \bar{R}_{ab}{}^{ij}(\bar{\omega})$ to fix the above combination of the contorsion matrices as represented by ξ.

This is what we shall do next. There are many possible solutions.

There are a set of solutions for homgeneous three-geometries described by the triads e^i_a, which can be put in eight classes given by Thurston's model three-geometries.

These eight geometries are: E_3, S_3, H_3, $S_2 \times R$, $H_2 \times R$, Sol, Nil, $\widetilde{SL(2,R)}$.

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Degenerate Tetrads: Explicit Solutions

Let us now present a few examples of such explicit degenerate tetrad solutions based on some of these model three-geometries.

Example 1: (H_3 geometry). The metric is given by:

$$ds^2_{(4)} = \frac{\ell^2}{z^2}\left(dx^2 + dy^2 + dz^2\right), \qquad z > 0.$$

The triads and associated Levi-Civita connection fields and field strength fields are:

$$e^1_x = e^2_y = e^3_z = \frac{\ell}{z}, \qquad \textit{and all others zero}$$

$$\bar{\omega}^{31}_x(e) = \frac{1}{z} = -\bar{\omega}^{23}_y(e) \qquad \textit{and all others zero}$$

$$\bar{R}_{xy}{}^{12}(\bar{\omega}) = \bar{R}_{yz}{}^{23}(\bar{\omega}) = \bar{R}_{zx}{}^{31}(\bar{\omega}) = -\frac{1}{z^2} \qquad \textit{and all others zero}$$

The scalar (spatial) three-curvature is: $\hat{e}^a_i\ \hat{e}^b_i\ \bar{R}_{ab}{}^{ij}(\bar{\omega}) = -\frac{6}{\ell^2}$

And the final constraint is: $\xi \equiv N^{ij}N^{ji} - N^{ii}N^{jj} = 6\lambda^2 + \frac{6}{\ell^2}$

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DEGENERATE TETRADS: EXPLICIT SOLUTIONS

Let us now present a few examples of such explicit degenerate tetrad solutions based on some of these model three-geometries.

Example 1: (H_3 geometry). The metric is given by:

$$ds^2_{(4)} = \frac{\ell^2}{z^2}\left(dx^2 + dy^2 + dz^2\right) , \qquad z > 0.$$

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New solutions and Sen Gupta spacetimes

We have constructed a variety of solutions for the classical eqns of motion in the first order formulation with degenerate tetrads.

These all generically contain torsion.

The torsion is parametrized through the symmetric 3×3 matrix of fields N^{kl} (where contorsion $\kappa_a^{ij} = \epsilon^{ijk} N^{kl} e_a^l$) .

Components of N^{kl} depend on all the four coordinates, x, y, z, τ.

Six components $N^{kl}(x, y, z, \tau)$ are all independent except for one constraint so that the combination $\xi \equiv \left(N^{ij} N^{ji} - N^{ii} N^{jj}\right)$ has fixed values as dictated by the various solutions.

Unlike the usual framework where torsion enters through matter couplings such as fermions, we have here solutions which exhibit torsion in the pure gravity without any matter fields.

The first and second order formulations of gravity theory are not equivalent even classically.

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We have constructed a variety of solutions for the classical eqns of motion in the first order formulation with degenerate tetrads with one zero eigenvalue.

Let us try to understand the nature of geometry these new solutions represent.

These do not represent the usual geometry as seen in Einstein gravity.

To analyse this question, we shall go to the flat space-time limit.

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In the first order formulation, whereas invertible tetrads correspond to the usual Einsteinian curved space-time, the degenerate tetrads *(with one zero eigenvalue)* lead us to another (dual) phase described by the curved space-time generalizations of the Sen Gupta space-time.

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Concluding Remarks

We have constructed a variety of solutions for the classical equations of motion in the first order formulation with degenerate tetrads with one zero eigenvalue.

These all generically contain torsion.

The torsion is parametrized through the symmetric 3×3 matrix of fields N^{kl} (where contorsion $\kappa_a^{ij} = \epsilon^{ijk} N^{kl} e_a^l$) .

Components of N^{kl} depend on all the four coordinates, x, y, z, τ.

Six components $N^{kl}(x, y, z, \tau)$ are all independent except for one constraint so that the combination $\xi \equiv \left(N^{ij} N^{ji} - N^{ii} N^{jj}\right)$ has fixed values as dictated by the various solutions.

Concluding Remarks

We have constructed a variety of solutions for the classical equations of motion in the first order formulation with degenerate tetrads with one zero eigenvalue.

These all generically contain torsion.

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Concluding Remarks

Thus the phase with degenerate tetrads leads to solutions of the equation of motion which generically carry non-zero torsion even in absence of any torsion inducing matter.

In contrast, in the second order formulation, we need matter fields such as fermions to introduce torsion.

A specific class of these degenerate tetrad solutions with one zero eigenvalue are associated with Thurston's homogeneous model eight three-geometries.

There is a profound connection with deep mathematics here.

Gravity theories in the second order formulation based on Einstein-Hilbert action functional and the first order formulation based on Hilbert-Palatini action are not equivalent even classically.

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Concluding Remarks

This analysis can also be extended to obtain additional degenerate solutions of the equations of motion containing tetrads with two zero eigenvalues.

These also exhibit non-zero torsion even without usual torsion inducing matter.

A special class of such solutions (degenerate $4-$geometries) correspond to three fundamental geometries that closed two-surfaces can accommodate, namely, two dimensional Euclidean plane E^2, two-sphere S^2 and two dimensional hyperbolic plane H^2.

For details of solutions containing tetrads with two and more zero eigenvalues see:

RKK and S. Senguta, New solutions in pure gravity with degenerate tetrads, Phys. Rev. D 94, 104047 (2016).

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Thank you!