Nonlocal Effects in Quantum Gravity

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Collaborators

Based on work with

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References

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4. K. Papadodimas and S. Raju, Local Operators in the Eternal Black Hole, Phys. Rev. Lett., 115, 211601 (2015).
5. K. Papadodimas and S. Raju, State-Dependent Bulk-Boundary Maps and Black Hole Complementarity, Phys. Rev. D, 89, 086010 (2014).
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Summary

- Generally accepted that locality breaks down in quantum gravity at the Planck scale

$$[\phi(t, x), \phi(t, x + \vec{\ell_{\text{pl}}})] =?$$

- Will argue that locality also breaks down at macroscopic distances for very high point correlators

$$\langle \phi(x_1), \phi(x_2) \ldots [\phi(x_i), \phi(x_{i+1})] \ldots \phi(x_{n-1})\phi(x_n)\rangle \neq 0$$

even for spacelike x_p for sufficiently large n.

The picture of a smooth spacetime breaks down if it is probed at “too many” points.

Outline

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Nonlocality for high-point correlators can be examined in a simple and precise setting in empty anti-de Sitter space.

[S. Banerjee, J.W. Bryan, K. Papadodimas, S. R. ,JHEP 2016]

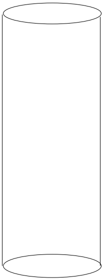
Setup

- Consider anti-de Sitter space,

$$ds^2 = -(\frac{r^2}{\ell_{\text{ads}}^2} + 1)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_{\text{ads}}^2} + 1} + r^2 d\Omega^2$$

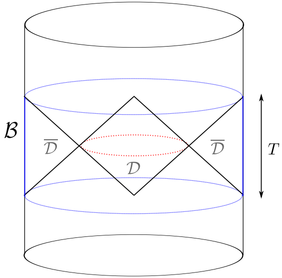
This metric satisfies $G_{\mu\nu} - \frac{3}{\ell_{\text{ads}}^2} g_{\mu\nu} = 0$.

- Define $N \equiv \frac{\ell_{\text{ads}}}{\ell_{\text{pl}}}$.



Setup

- Consider a band, in time, of size $T \leq \pi$ on the boundary.



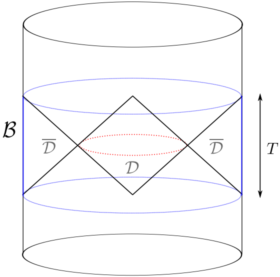
- Equations of motion relate

$$\phi(\bar{D}) \leftrightarrow O(\mathcal{B})$$

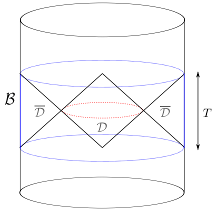
- Within local approximation, operators in $\mathcal{D}$ commute with operators on $\mathcal{B}$.

Nonlocality in empty AdS

Claim Operator at center of $\mathcal{D}$ can be written as complicated polynomial of operators in $\mathcal{B}$ even though they are uniformly spatially separated!



Expansion with IR-UV cutoffs

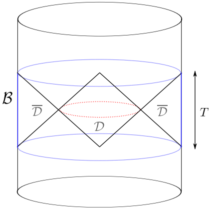


$$\phi(\mathcal{D}) = \sum_{n,m=0}^{N} c_{nm} |n\rangle\langle m|$$

Here we have used that

1. Energies in AdS are quantized.
2. We can throw away matrix elements for energies about M_{pl}.

Reeh-Schlieder theorem



- The Reeh-Schlieder theorem tells us

$$|n\rangle = X_n[\phi(\bar{\mathcal{D}})]|0\rangle$$

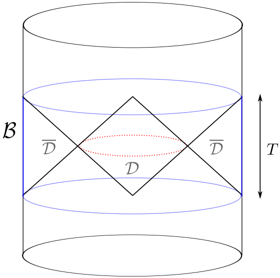
  where X is a simple polynomial.

- This is true in any quantum field theory. Not itself a sign of nonlocality.

Consequence of Reeh-Schlieder

Reeh-Schlieder implies

$$\phi(\mathcal{D}) = \sum_{n,m=0}^{N} c_{nm} X_n[\phi(\bar{\mathcal{D}})] \; |0\rangle\langle 0| \; X_m^{\dagger}[\phi(\bar{\mathcal{D}})]$$



Projector onto the vacuum

- Use complicated polynomials in $\mathcal{B}$ to approximate $P_0 = |0\rangle\langle 0| \approx \mathcal{P}[\phi(\bar{\mathcal{D}})]$.

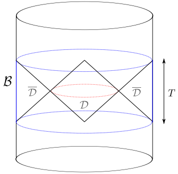
$$\mathcal{P} = \lim_{\alpha \to \infty} e^{-\alpha H} \approx \sum_{n=0}^{n_c} \frac{(-\alpha H)^n}{n!}$$

- But H is an operator in $\bar{\mathcal{D}}$. So with

$$\alpha = l_{\text{ads}} \log\left(\frac{l_{\text{ads}}}{l_{\text{pl}}}\right); \quad n_c = \frac{l_{\text{ads}}}{l_{\text{pl}}} \log\left(\frac{l_{\text{ads}}}{l_{\text{pl}}}\right)$$

$\mathcal{P}$ is a very complicated polynomial in $\bar{\mathcal{D}}$.

Empty AdS complementarity



Combining the previous formulas we get

$$\boxed{\phi(\mathcal{D}) = \sum_{nm} c_{nm} X_n[\phi(\bar{\mathcal{D}})] \mathcal{P}[\phi(\bar{\mathcal{D}})] X_m^{\dagger}[\phi(\bar{\mathcal{D}})]}$$

where X_n are explicit simple polynomials, and $\mathcal{P}$ is a complicated polynomial. Explicitly realizes nonlocality in complicated correlators and also consistent with approximate locality in simple correlators.

Outline

Gauss Law nonlocality

- The Hamiltonian in AdS is a boundary term $\Rightarrow$ nonlocal Gauss law commutators.

$$H = \lim_{r \to \infty} N \int T(r) d^2\Omega$$

- Implies natural nonlocal commutators of size $\frac{1}{N}$.

Breakdown of perturbation theory

- To enhance $\frac{1}{N}$ commutators to $O(1)$ requires breakdown of $\frac{1}{N}$ perturbation theory in AdS.
- Breakdown of gravitational perturbation theory is necessary for significant nonlocal effects.

Flat space Gauss law

- Hamiltonian is a boundary term even in flat-space

$$H = \frac{1}{16\pi G} \int_{\infty} d^2 s_k (g_{ik,i} - g_{ii,k})$$

[ADM, de Witt,Regge, Teitelboim . . ., 62–74]

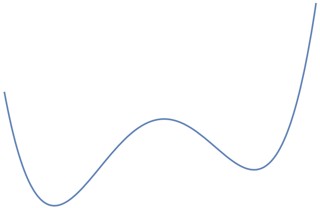
- Suggests nonlocal commutators controlled by $\frac{E}{M_{\text{pl}}}$

- Large-scale nonlocality $\Leftrightarrow$ breakdown of $\frac{E}{M_{\text{pl}}}$ perturbation theory.

A Path Integral Argument

- A semi-classical spacetime is a saddle point of the QG path-integral.

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$



- Breakdown of perturbation theory $\Rightarrow$ change in saddle-point $\Rightarrow$ different semi-classical metric with different notion of locality.

Outline

String perturbation theory in flat space breaks down for a large number of particles, even if they have low energy.

[Sudip Ghosh, S. R. ,PRL 2017]

String Perturbation theory limits

- Consider a limit where string-scale and Planck scale are widely separated, number of particles is large, and energy-per-particle is small.

$$g_s^2 = \frac{2\pi l_{pl}^{d-2}}{(2\pi\sqrt{\alpha'})^{d-2}} \to 0$$

$$n \to \infty$$

$$\frac{\log(E\sqrt{\alpha'})}{\log(n)} \to -\gamma \leq 0$$

- String perturbation theory breaks down n that satisfies

$$\frac{\log(g_s)}{\log(n)} = \frac{1}{2}((d-2)\gamma - 1)$$

$$\Rightarrow n \propto g_s^{\frac{2}{(d-2)\gamma-1}}$$

Requires $(d-2)\gamma \leq 1$.

Growth of String Amplitudes

To show this, we show that massless string amplitudes in the bosonic string and superstring grow at least as fast as

$$\frac{\log\left(M_{\rm pl}^{\frac{(d-2)n-2d}{2}} M(k_1 \ldots k_{\frac{n}{2}} \rightarrow p_1 \ldots p_{\frac{n}{2}})\right)}{n\log(n)} = 1$$

$$\Rightarrow M(k_1 \ldots k_{\frac{n}{2}} \rightarrow p_1 \ldots p_{\frac{n}{2}}) \sim \frac{n!}{M_{\rm pl}^{\frac{(d-2)n}{2}-d}}$$

for large n. This is a lower bound.

Growth from volume of moduli space

- Amplitude can be written as

$$M \sim g_s^n \int_{\mathcal{M}_n} d(W.P.)(\det P_1^\dagger P_1)^{\frac{1}{2}} (\det \Delta)^{\frac{-d}{2}} \langle\langle W_1(\zeta_1) \ldots W_n(\zeta_n) \rangle\rangle$$

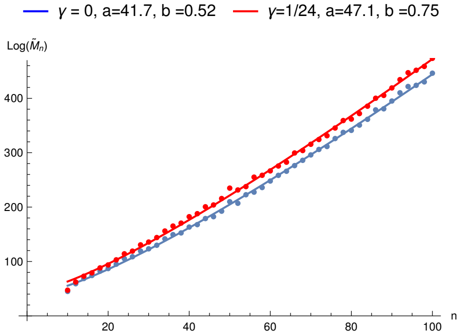
[D'Hoker, Giddings, 87]

- $$V_{g,n} = \int_{\mathcal{M}_{n,g}} d(W.P) \underset{g+n \to \infty}{\longrightarrow} (4\pi^2)^{2g+n-3}(2g+n-3)!$$

[Zograf, Mirzakhani, 2008–2013]

Factorial growth from numerics

Possible to numerically estimate the string-scattering amplitude.



$$\log(\tilde{M}_n) = \log(\langle M_n \rangle) - (n-2)\log(4\pi g_s) + n\log(d-2)$$
$$= a + bn + \log((n-3)!)$$

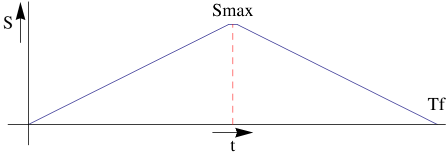
Outline

Cloning Paradox

This loss of locality is precisely sufficient to resolve various versions of the information paradox.

Page Curve

The entropy of the emitted black-hole radiation varies with time as follows



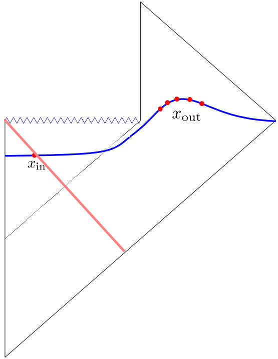
[Page, 1993]

Nice slices

- Nice slices can capture the incoming matter and a large fraction of the outgoing radiation.

- Quantization on nice slices seems to lead to a "cloning" paradox.

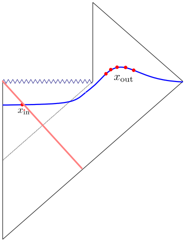
  [Susskind, Thorlacius, Uglum, 93]



$$|\Psi\rangle \to |\Psi\rangle \otimes |\Psi\rangle ?$$

Complementarity and Locality

- But, if
$$\phi(x_{in}) \cong P(\phi(x_2), \phi(x_3), \ldots, \phi(x_n)),$$
then no cloning paradox. Called black hole complementarity



- Complementarity ensures no violation of QM; subtle loss of locality instead.

Are the effects of nonlocality important for the observables required to frame this paradox?

Black Hole Evaporation in d-dimensions

- Recall that in d-dimensions

$$T \propto \left(\frac{M_{\text{pl}}^{d-2}}{M}\right)^{\frac{1}{d-3}} \propto \frac{1}{R_h}$$

$$S \propto \left(M_{\text{pl}} R_h\right)^{d-2}$$

- To observe the cloning contradiction, we need to measure at least some connected S-point correlators in the emitted Hawking radiation.

Breakdown of perturbation theory

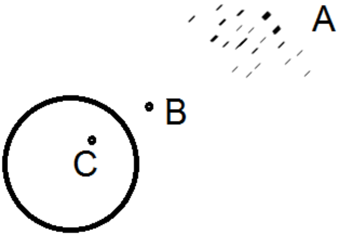
- Corresponds to S-matrix elements with S-insertions and typical momentum of order T
- But perturbation theory breaks down for

$$\frac{(2-d)\log\frac{T}{M_{\rm pl}}}{\log(n)} = 1 + O\left(\frac{1}{\log(n)}\right)$$

Reached precisely at n = S

- So S-point correlators may receive non-perturbative corrections. Suggests a version of complementarity.

Three Subsystems



A related paradox is the strong-subadditivity paradox. Think of three subsystems

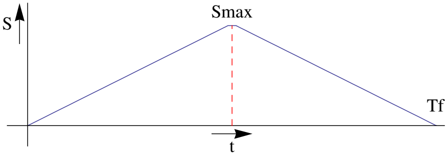
1. The radiation emitted long ago – A
2. The Hawking quanta just being emitted – B
3. Its partner falling into the BH – C

Entropy of A

- Say the black hole is formed by the collapse of a pure state.
- Consider the entropy of system A

$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A$$

- This follows the Page curve.



Strong Subadditivity contradiction?

- Now, consider an old black hole, beyond its “Page time” where S_A is decreasing. We must have

$$S_{AB} < S_A$$

since B is purifying A.

- Second, the pair B, C is related to the Bogoliubov transform of the vacuum of the infalling observer, and almost maximally entangled.

$$S_{BC} < S_C$$

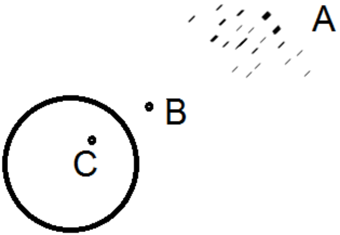
- However, strong subadditivity tells us

$$S_A + S_C \leq S_{AB} + S_{BC}$$

[Mathur, AMPS, 2009–12]

- We seem to have a violation at $O(1)$.

Complementarity and Strong Subadditivity



In the presence of

$$\phi(x_C) \cong P(\phi(x_{A_1}), \phi(x_{A_2}), \ldots)$$

A, B, C are not independent subsystems. So, strong subadditivity inapplicable.

[Kyriakos Papadodimas, S.R., 2013–15]

Summary

- Robust principles of gravity suggest non-local effects

$$\phi(x_C) \cong P(\phi(x_{A_1}), \phi(x_{A_2}), \ldots, \phi(x_{A_n}))$$

- These effects must preserve approximate locality in low-point correlation functions.
- Such a relation can be explicitly realized in empty AdS.
- The breakdown of string-perturbation theory for a large number of particles indicates the existence of a similar effect in flat space.
- This effect is precisely what is required to resolve some versions of the information paradox.