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Simultaneous implementation of Leendertz and Duffy's methods for in-plane displacement measurement

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Abstract

Leendertz two-beam illumination and Duffy's two-aperture arrangements are two well established optical configurations in speckle interferometry for in-plane displacement measurement. However, the measurement sensitivities of both these techniques are quite different. In Leendertz method the interbeam angle between the two illumination beams determines the sensitivity, while in Duffy's arrangement, it is governed by the subtense formed by aperture separation at the object. In the present paper, we implement both these configurations simultaneously for in-plane displacement measurement. The proposed optical arrangement extends the range of in-plane displacement measurement considerably. Detailed theoretical and experimental results are presented.

1. Introduction

Speckle interferometry has been developed for the measurement of a wide range of physical parameters such as displacement, strain, vibration and surface profile of a diffusely reflecting object [1–3]. The speckle correlation interferometer for measuring in-plane displacement components was first suggested by Leendertz [4]. In this method, the object is illuminated by two collimated waves placed symmetrically with respect to the surface normal and the object is imaged by a lens onto the recording plane. The interbeam angle between the two beams determines the sensitivity of the configuration and it can be varied by changing the interbeam angle. A modification of Leendertz method which doubles the sensitivity has recently been proposed by us by increasing the interbeam angle [5]. Another technique in speckle interferometry is Duffy's double aperture speckle interferometer for measuring in-plane displacement [6]. Instead of using dual beam illumination, the object is effectively viewed in two directions by means of two apertures placed symmetrically in front of the imaging lens. the arrangement is sensitive to an in-plane component along the line joining the centers of the two apertures. Even though, the configuration yields almost unit contrast fringes, the measuring sensitivity is extremely poor compared to that of the two-beam illumination method. In the present paper we combine both Leendertz and Duffy's configurations so that in-plane displacements can be measured with corresponding sensitivities from a single setup. Both theory and experimental results are presented.

2. Experimental arrangement and theory

The experimental arrangement is shown in Fig. 1. The object is illuminated by two collimated beams incident at an angle θ on either side of the surface normal.

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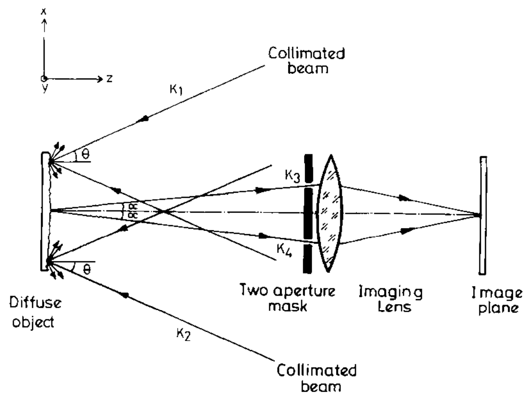

Fig. 1. Schematic of the experimental arrangement.

The imaging lens carries a two-aperture mask. Each illuminating beam generates two scattered beams through the apertures which combine coherently at the image plane. The amplitude distribution at any point on the image plane can be written as

$$A_1 = \sum_{m=1}^{2} \{a_{2m-1} \exp[i(\phi_{2m-1})] + a_{2m} \exp[i(\phi_{2m}+\beta)]\}, \qquad (1)$$

where a's and ϕ's are the random amplitudes and phases of the waves generated from the illuminating beams via two apertures. $\beta = 2\pi\mu x$ is the phase introduced between the waves due to aperture separation, which is responsible for the grating like structure in each speckle.

The irradiance of the speckle pattern at any point in the image plane is given as

$$I_1 = \sum_{m=1}^{4} |a_m|^2 + 2\sum_{m=1}^{3} a_m a_{m+1} \cos(\phi_{m,m+1}+\beta) + 2\sum_{m=1}^{2} a_m a_{m+2} \cos(\phi_{m,m+2}) + 2a_1 a_4 \cos(\phi_{41}+\beta) \qquad (2)$$

where $\phi_{ij} = \phi_i - \phi_j$.

The amplitude of the waves at the same point after the deformation of the object is written as

$$A_2 = \sum_{m=1}^{2} \{a_{2m-1} \exp[i(\phi_{2m-1}+\delta_{2m-1})] + a_{2m} \exp[i(\phi_{2m}+\delta_{2m}+\beta)]\}, \qquad (3)$$

where δ's are the phase changes introduced due to the object deformation. The phase changes can be expressed in terms of the directions of the illumination and the observation beams as

$$\delta_1 = (\boldsymbol{K}_3 - \boldsymbol{K}_1)\cdot \boldsymbol{L}, \quad \delta_2 = (\boldsymbol{K}_4 - \boldsymbol{K}_1)\cdot \boldsymbol{L},$$
$$\delta_3 = (\boldsymbol{K}_3 - \boldsymbol{K}_2)\cdot \boldsymbol{L}, \quad \delta_4 = (\boldsymbol{K}_4 - \boldsymbol{K}_2)\cdot \boldsymbol{L}, \qquad (4)$$

where $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are the propagation vectors of illuminating beams, $\boldsymbol{K}_3$ and $\boldsymbol{K}_4$ are the propagation vectors in the direction of observation and $\boldsymbol{L}(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$ is the deformation vector at any point on the object, u, v, w are the displacement along x, y and z-direction respectively.

The irradiance distribution in the image plane after object deformation is given by

$$I_2 = \sum_{m=1}^{4} |a_m|^2 + 2\sum_{m=1}^{3} a_m a_{m+1} \cos(\phi_{m+1,m} + \delta_{m+1,m} + \beta) + 2\sum_{m=1}^{2} a_m a_{m+2} \cos(\phi_{m,m+2} + \delta_{m,m+2}) + 2a_1 a_4 \cos(\phi_{41} + \delta_{41} + \beta), \qquad (5)$$

where $\delta_{ij} = \delta_i - \delta_j$.

The irradiance distributions I_1 and I_2 are sequentially recorded on the same photographic plate. Assuming linear recording, the amplitude transmittance $t(x, y)$ of the specklegram is given as

$$t(x, y) = t_0 - \beta_0 T(I_1 + I_2),$$

where β_0 is a constant and T is the exposure time for each recording.

To obtain the correlation fringes the specklegram is placed in the whole field filtering setup as shown in Fig. 2. At the focal plane of the lens L_1, three halos are

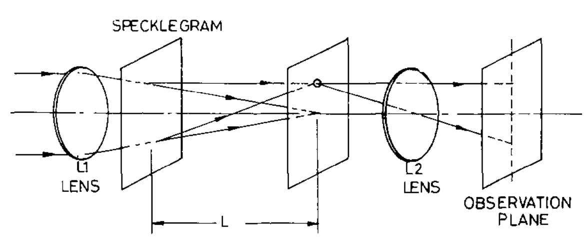

Fig. 2. Schematic of the wholefield filtering arrangement.

formed including the zeroth order. These diffraction halos are formed due to presence of grating frequency μ in the specklegram. Imaging through one of the first order halos results in an interferogram. The irradiance distribution at the image plane when filtered via one of the first order halos is given by

$$\langle I_{\text{first order}}\rangle = C\left[8+2\sum_{n=1}^{3}\sum_{m=n+1}^{4}\cos(\Omega_m-\Omega_n)\right.$$
$$+2\sum_{n=1}^{3}\sum_{m=n+1}^{4}\cos(\Omega_m-\Omega_n+\Delta_m-\Delta_n)$$
$$\left.+2\sum_{n=1}^{4}\sum_{m=1}^{4}\cos(\Omega_m-\Omega_n+\Delta_m)\right], \quad (6)$$

where $\Omega_1=\phi_{21}$; $\Omega_{21}=\phi_{23}$, $\Omega_3=\phi_{43}$; $\Omega_4=\phi_{41}$; $\Delta_1=\delta_{21}$; $\Delta_2=\delta_{23}$; $\Delta_3=\delta_{43}$; $\Delta_4=\delta_{41}$ and C is a constant.

All the terms in the above equation containing ϕ's are random and contribute to speckle noise. The last summation terms, when $n=m$, contributes to fringe formation and they are

$$2\cos\delta_{21}+2\cos\delta_{23}+2\cos\delta_{43}+2\cos\delta_{41}\ .$$

The phase differences for these terms can be derived from Eq. (4) as

$$\delta_{21}=\delta_{43}=(\boldsymbol{K}_4-\boldsymbol{K}_3)\cdot\boldsymbol{L}\ ,$$
$$\delta_{23}=[(\boldsymbol{K}_4-\boldsymbol{K}_3)\cdot\boldsymbol{L}+(\boldsymbol{K}_2-\boldsymbol{K}_1)\cdot\boldsymbol{L}]\ ,$$
$$\delta_{41}=[(\boldsymbol{K}_4-\boldsymbol{K}_3)\cdot\boldsymbol{L}-(\boldsymbol{K}_2-\boldsymbol{K}_1)\cdot\boldsymbol{L}]\ . \quad (7)$$

From the above equation, $(\delta_{23}-\delta_{41})/2=(\boldsymbol{K}_2-\boldsymbol{K}_1)\cdot\boldsymbol{L}=\delta_L$ and $(\delta_{23}+\delta_{41})/2=(\boldsymbol{K}_4-\boldsymbol{K}_3)\cdot\boldsymbol{L}=\delta_D$. Here the phase change δ_L is due to Leendertz two-beam illumination while δ_D is due to Duffy's two-aperture arrangement. Substituting these relations in the cosine terms that are responsible for the fringe formation and after simplification these terms can be written as $8\cos(\delta_D)\cos^2(\delta_L/2)$. The bright fringes are formed when any one or both the terms becomes zero.

Assuming that the illuminating beams with an inter beam angle of 2θ are confined to the $x-z$ plane and also that the two apertures in the mask are aligned in the x-direction, the bright fringes are formed when

$$\delta_D=\frac{2\pi}{\lambda}2u\sin\alpha=2n_1\pi\ ,\quad n_1=\pm1,2,3,\ldots\ . \quad (8)$$

The fringe spacing corresponds to an incremental displacement

$$\Delta u_D=\frac{\lambda}{2\sin\alpha}\ .$$

Similarly the bright fringes are formed whenever

$$\delta_L=\frac{2\pi}{\lambda}2u\sin\theta=2n_2\pi\ ,\quad n_2=1,2,3,\ldots\ , \quad (9)$$

and the incremental displacement

$$\Delta u_L=\frac{\lambda}{2\sin\theta}\ .$$

It can be seen from Eqs. (8) and (9) that the phase terms δ_D and δ_L will follow fixed relation with each other as $\delta_D=(\sin\alpha/\sin\theta)\,\delta_L$. Taking all these parameters into account, the normalized irradiance distribution plot is shown in Fig. 3. While plotting the irradiance distribution, we have not considered into account the associated random phase terms ϕ in the Eq. (6). These terms add background speckle noise in the fringe pattern. The fringes generated due to the phase term δ_L will be inside the coarse fringes obtained from the phase term δ_D. High contrast fringes can be observed when ever the fine fringes fall in the regions of minima of coarse fringes, while the visibility of these fringes is poor in the areas of maximum intensity. The poor vis-

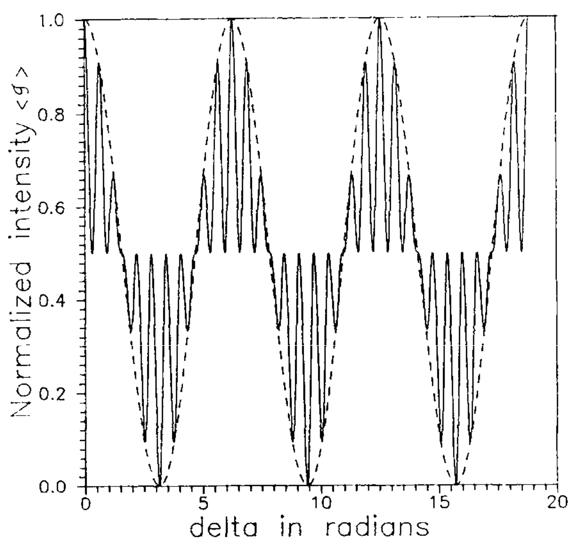

Fig. 3. Irradiance distribution profile generated from one of the first order halos.

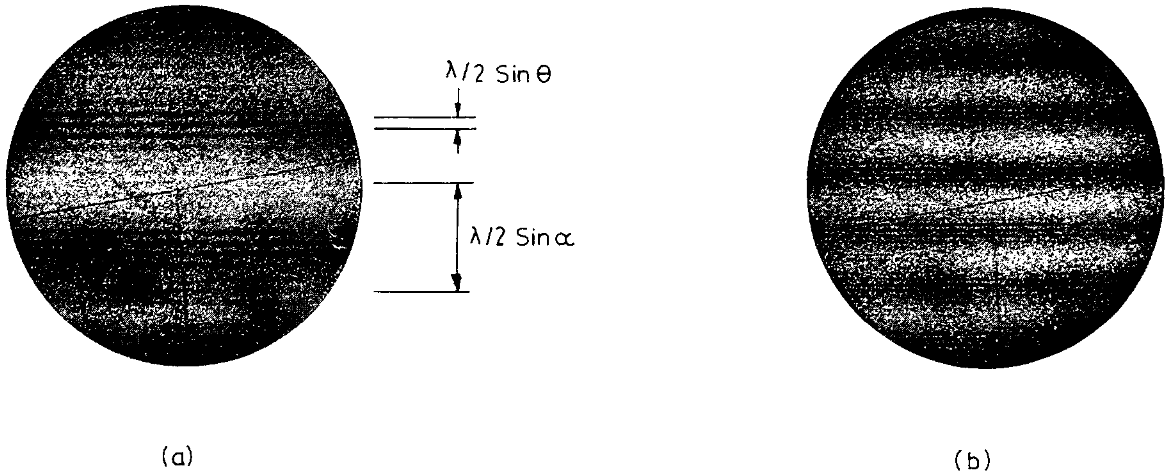

Fig. 4. u-family of rotation fringes, for an in-plane rotation of about (a) 0.45 mrad (b) 0.9 mrad: angles α and θ are 1°56″, and 20° respectively.

ibility in this region is due to high dc term which is evident from the irradiance profile. In the absence of second beam illumination, the configuration reduces to Duffy's method and the measuring sensitivity will follow Eq. (8) alone. The sensitivity of the method can be adjusted by varying the interbeam angle 2θ of the symmetric illumination in a way that is similar to the Leendertz method [4]. Fringes corresponding to the v-component of the in-plane displacement are obtained when the illuminating beams and the two aperture mask lie in the $y-z$ plane.

3. Experimental results

The experiments are conducted on a circular plate of 60 mm diameter. The plane is coated with aluminium paint and is illuminated with two collimated beams at an angle of 20° on either side of the object normal. A Linhoff camera lens ($f=150$ mm, $f/5.6$) is used for imaging the specimen onto the recording plane. The aperture mask in front of the lens contains two 5 mm diameter holes with an interspacing of 20 mm. The orientation of the apertures is aligned such that they are confined along the x-direction. Two exposures, one before and other after the specimen is subjected to rigid body in-plane rotation, are made on 10E75 holographic plate. The filtering of double exposure specklegram is done on a Fourier filtering setup. In the present analysis, white light illumination with a narrow band red filter is used to extract the information. Figs. 4a and 4b show the u-component of in-plane displacement fringes obtained for two different in-plane rotations. It can be seen from these photographs that the fine fringes lie within the coarse fringes as expected. The visibility of these fringes in the brighter regions of the coarse pattern is poor as explained in the theory. From the fringe patterns the measured incremental displacement between the adjacent fringe are $\Delta u_L = 0.9$ μm and $\Delta u_D = 9.3$ μm. In the absence of second beam illumination, the measuring sensitivity of the configuration is dependent only on aperture separation and follows Duffy's arrangement. The usefulness of the method is further demonstrated by conducting an experiment on a cantilever beam which is displaced in its own plane and the configuration is so chosen that v-family of three fringes are formed due to deformation. The free end of

Fig. 5. v-family of in-plane fringes of a cantilever beam for a displacement of about 40 μm.

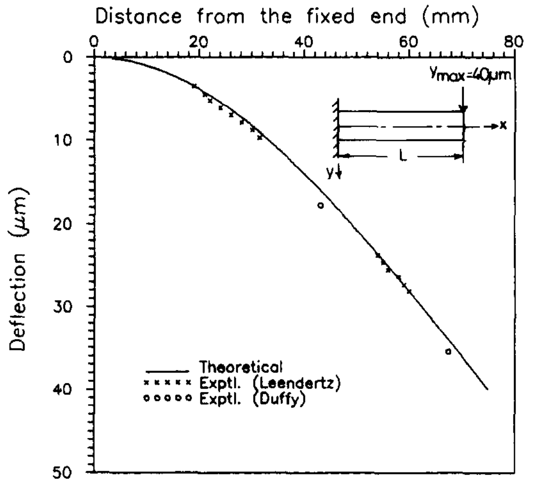

Fig. 6. Comparison between theory and experiment of a Cantilever experiment.

cantilever is given a displacement of 40 μm which generates only three bright Duffy's fringes, while fine Leendertz's fringes are seen in the dark Duffy's fringes. Fig. 5 shows the photograph of the v-family of in-plane displacement fringes as obtained by filtering via one of the diffraction halos. In Fig. 6 we have plotted the experimental values of in-plane displacement, v, at the midplane of the cantilever using both Duffy's and Leendertz's fringes and compared with the theoretical data [8]. It may be seen that near the clamped end where the in-plane component is very small, Leendertz's fringes can be used to obtain the values of deformation, while towards the end both Duffy's and Leendertz's fringes could be used. In fact one can fashion the sensitivity depending on the requirement as interbeam angle can be easily varied. This analysis clearly shows that the proposed optical configuration extends the range of in-plane displacement measurements considerably.

4. Conclusion

An optical configuration is presented by illuminating the object with two beams and recording via two aperture arrangement for in-plane displacement measurement. It is also demonstrated that the configuration offers considerable increase in range of in-plane displacement measurement. Since multiaperture configuration provides facility to multiplex various information [7], the proposed method can be extended to extract in-plane and out-of-plane displacement components with comparable sensitivities from a single setup. We are carrying out experiments in this direction; the results will be communicated separately.

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