① Computing with qubits :-

(a) Deutsch algorithm (see Lecture - 1)

(b) Quantum copying circuit ?

No-cloning principle : (Wootters and Zurek, Nature 299, 802 1982)

Is it possible to make a copy of an unknown quantum state ? No!

$$\underset{\text{data}}{|\psi\rangle} \otimes \underset{\text{"target" slot}}{|s\rangle} \xrightarrow{U} |\psi\rangle \otimes |\psi\rangle$$

Suppose it works for $|\psi\rangle, |\phi\rangle$.

$$|\psi\rangle|s\rangle \xrightarrow{U} |\psi\rangle|\psi\rangle$$

$$|\phi\rangle|s\rangle \xrightarrow{U} |\phi\rangle|\phi\rangle$$

$$\Rightarrow \langle\phi|\psi\rangle = (\langle\phi|\psi\rangle)^2 \Rightarrow \langle\phi|\psi\rangle = 0 \text{ or } \langle\phi|\psi\rangle = 1$$

* Works only for orthogonal states!

i.e. No universal quantum copying machine is possible!

* Resolution : Use "entanglement" and implement "Quantum teleportation"!

## ② Composite Systems & Entanglement

$$|\psi_A\rangle \in \mathcal{H}_A \ , \ |\psi_B\rangle \in \mathcal{H}_B .$$

$\mathcal{H}_A$, $\mathcal{H}_B$ are called subsystems of $\mathcal{H}_{AB}$

$$|\psi_{AB}\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B .$$

$\otimes$ ↓ Kronecker product

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ \vdots & & & \\ a_{n1}B & a_{n2}B & \cdots & a_{nn}B \end{pmatrix}$$

Eg. Basis for $\mathbb{C}^2 \otimes \mathbb{C}^2$ :-

$$\{|0\rangle, |1\rangle\}_A \otimes \{|0\rangle, |1\rangle\}_B .$$

$$\equiv \{|0\rangle\otimes|0\rangle, |0\rangle\otimes|1\rangle, |1\rangle\otimes|0\rangle, |1\rangle\otimes|1\rangle\}$$

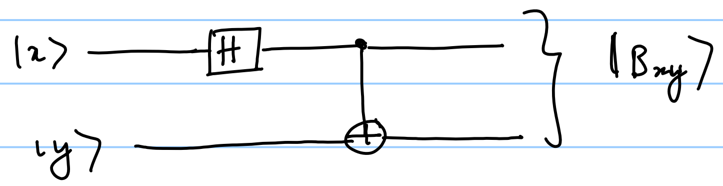
* Rules:

* $\langle v_A \otimes w_B, e_A \otimes f_B\rangle = \langle v_A, e_A\rangle\langle w_B, f_B\rangle$

* $(A \otimes B)(|v\rangle \otimes |w\rangle) = A|v\rangle \otimes B|w\rangle$

* Bell states:-

Consider this circuit:-




$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \ , \ |\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \ , \ |\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

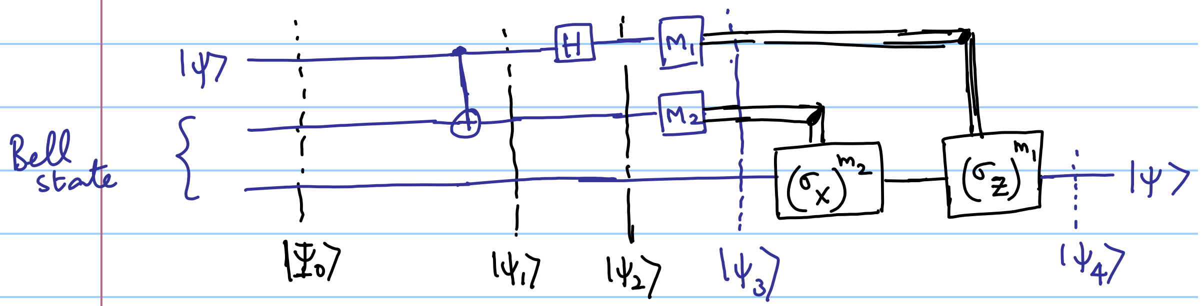
$\{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$ forms an ON basis for $\mathbb{C}^2 \otimes \mathbb{C}^2$.

* There do not exist states $|\alpha\rangle, |\beta\rangle \in \mathbb{C}^2$ such that

$$|\beta_{xy}\rangle = |\alpha\rangle \otimes |\beta\rangle$$

$\Rightarrow$ These are not <u>product</u> states; <u>entangled</u>!

(2a) <u>Teleportation</u>:- Entangled states are an important resource for quantum information and computing.




$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ ("unknown" quantum state)

- Single copy of $|\psi\rangle$ given to Anita.
- Her task is to communicate this state to Bharat without destroying the state! (i.e. without measuring the state!)
- <u>Resource</u>: Anita & Bharat share a <u>Bell state</u>.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle \in \mathcal{H}_{A_1}$$

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \in \mathcal{H}_{A_2} \otimes \mathcal{H}_B$$

- $|\psi_0\rangle_{A_1A_2B} = (\alpha|0\rangle + \beta|1\rangle)_{A_1} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2B}$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$

- $|\psi_1\rangle_{A_1A_2B} = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$

- $|\psi_2\rangle_{A_1A_2B} = \frac{1}{2}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$

$$= \frac{1}{2}\Big[|00\rangle_A(\alpha|0\rangle + \beta|1\rangle)_B + |11\rangle_A(\alpha|1\rangle - \beta|0\rangle)_B + |01\rangle_A(\alpha|1\rangle + \beta|0\rangle)_B + |10\rangle_A(\alpha|0\rangle - \beta|1\rangle)_B\Big]$$

- Anita measures her qubits in $\{|00\rangle, |11\rangle, |01\rangle, |10\rangle\}$ basis.

| $(m_1, m_2)$ | B's state |
|---|---|
| $(0,0)$ | $\alpha|0\rangle + \beta|1\rangle = |\psi\rangle$ ! |
| $(0,1)$ | $\alpha|1\rangle + \beta|0\rangle = \sigma_x|\psi\rangle = (\sigma_x)^{m_2}(\sigma_z)^{m_1}|\psi\rangle$ |
| $(1,0)$ | $\alpha|0\rangle - \beta|1\rangle = \sigma_z|\psi\rangle = (\sigma_x)^{m_2}(\sigma_z)^{m_1}|\psi\rangle$ |
| $(1,1)$ | $\alpha|1\rangle - \beta|0\rangle = \sigma_x\sigma_z|\psi\rangle = (\sigma_x)^{m_2}(\sigma_z)^{m_1}|\psi\rangle$ |

Note: No faster than light communication!
* B gets $|\psi\rangle$ only after A has communicated her measurement outcomes.
* Without the classical channel, teleportation conveys No information.

History:

Quantum teleportation discovered by Bennett, Brassard, Crepeau, Jozsa, Peres and Wootters, PRL 70, 1895 (1993).

- Experimentally:- Bouwmeester et al (1997) (photon polarization)
  Furasawa et al (1998) ("squeezed" states of light)
  Nielsen et al (1998) (NMR)