$\Rightarrow \cancel{\{\langle\psi|A|\phi\rangle\} = 0}$ $A = 0$!

IIly, in order to prove (i) $\Rightarrow$ (ii)

consider $|\omega\rangle = |\phi\rangle - |\psi\rangle$

then $(\langle\phi| - \langle\psi|)\, A\, (|\phi\rangle - |\psi\rangle) = \langle\omega|A|\omega\rangle = 0$

$\Rightarrow \underbrace{\langle\phi|A|\phi\rangle}_{0} - \langle\phi|A|\psi\rangle - \langle\psi|A|\phi\rangle + \underbrace{\langle\psi|A|\psi\rangle}_{0} = 0$

$\Rightarrow$ $\cancel{\langle\phi|A|\psi\rangle = 0}$ $A = 0$!

Also,

$\langle u|A|u\rangle - \langle\omega|A|\omega\rangle = 2\langle\phi|A|\psi\rangle + 2\langle\psi|A|\phi\rangle$.

consider,

$|v\rangle = |\phi\rangle + i|\psi\rangle$ then $\langle v|A|v\rangle = 0$.

$(\langle\phi| - i\langle\psi|)\, A\, (|\phi\rangle + i|\psi\rangle) = \underbrace{\langle\phi|A|\phi\rangle}_{=0} + \underbrace{\langle\psi|A|\psi\rangle}_{=0}$
$+ i\langle\phi|A|\psi\rangle - i\langle\psi|A|\phi\rangle$

$= +i(\langle\phi|A|\psi\rangle - \langle\psi|A|\phi\rangle)$

then $i\langle v|A|v\rangle = -(\langle\phi|A|\psi\rangle - \langle\psi|A|\phi\rangle)$.

finally take $|x\rangle = |\phi\rangle - i|\psi\rangle$, then

$\langle x|A|x\rangle = 0$

$\Rightarrow (\langle\phi| + i\langle\psi|)\, A\, (|\phi\rangle - i|\psi\rangle) = \underbrace{\langle\phi|A|\phi\rangle}_{0} + \underbrace{\langle\psi|A|\psi\rangle}_{0}$
$+ i\langle\psi|A|\phi\rangle - i\langle\phi|A|\psi\rangle$

$\Rightarrow i\langle x|A|x\rangle = \langle\phi|A|\psi\rangle - \langle\psi|A|\phi\rangle$

$\Rightarrow i\langle v|A|v\rangle - i\langle x|A|x\rangle = -2\langle\phi|A|\psi\rangle + 2\langle\psi|A|\phi\rangle$

So,

$\langle u|A|u\rangle - \langle\omega|A|\omega\rangle + i\langle v|A|v\rangle - i\langle x|A|x\rangle$
$= 4\langle\psi|A|\phi\rangle = 0$

$\Rightarrow \langle\psi|A|\phi\rangle = 0$!