DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory   Assignment 1   19.01.2015   (due: 26.01.2015)

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1. Using Dirac delta functions in the appropriate coordinates, express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{x})$.
   (a) In spherical polar coordinates, a charge Q uniformly distributed over a spherical shell of radius R.
   (b) In cylindrical polar coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b.
   (c) In spherical polar coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.
   (d) In cylindrical polar coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.

2. Each of three charged spheres of radius a, one conducting, one having a uniform charge density within its volume, and one having a spherically symmetric charge density that varies radially as $r^n (n > -3)$, has a total charge Q. Use Gauss's theorem to obtain the electric fields both inside and outside each sphere. Sketch the behavior of the fields as a function of radius for the first two spheres, and for the third with $n = -2, +2$.

3. The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right)$$

   where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

4. A spherically symmetric charge distribution of radius R has a charge density given by

$$\rho(r) = \begin{cases} \rho_0 \left(1 - \frac{r}{R}\right) & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

   Find the electric field as a function of r.

5. Verify Gauss law by explicitly evaluating the expression for the electric field $\mathbf{E}$, due to a point charge Q at the origin, over the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

6. Derive the two dimensional form of Green's boundary value theorem: if $\phi(x, y)$ is the two dimensional potential, show that

$$\phi(\mathbf{r}) = \frac{1}{2\pi\epsilon_0} \int_S G(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}')d^2r' + \frac{1}{2\pi} \int_C \Big(G(\mathbf{r}, \mathbf{r}')\nabla'\phi - \phi\nabla' G(\mathbf{r}, \mathbf{r}') \Big) \cdot \hat{n}\ dl\ ,$$

where S is the area bounded by the contour C and $\hat{n}$ is the outward normal unit vector.