DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory   Assignment 2   28.01.2015 (due: 04.02.2015)

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1. Using the Green's function technique, obtain the solution of the problem
$$\frac{d^2\phi(\theta)}{d\theta^2} = -\frac{\rho(\theta)}{\epsilon_0} \ , \ 0 < \theta < \pi$$
subjected to the boundary condition $\phi(0) = A$ and $\phi(\pi) = B$, where A and B are constants.

2. Consider the electrostatic problem in a two dimensional region $0 \leq x \leq a, y \geq 0$ without any source. Show that the solution to the Laplace equation in the above region with boundary condition
$$\Phi = \begin{cases} 0 & \text{at } x = 0, x = a \ (y > 0) \\ V_0 & \text{at } y = 0 \ (V_0 = \text{const.}) \\ 0 & \text{for } y \to \infty \end{cases}$$
is given by
$$\Phi(x, y) = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} e^{-\frac{(2n+1)\pi}{a} y} \sin\left(\frac{(2n+1)\pi}{a} x\right).$$

3. Show that the above series can be summed to give
$$\Phi(x, y) = \frac{2V_0}{\pi} \tan^{-1}\left(\frac{\sin(\pi x/a)}{\sinh(\pi y/a)}\right)$$

4. Consider a two dimensional rectangular region $0 \leq x \leq a, \ 0 \leq y \leq b$. Show that the Green function, which vanishes on the boundary, can be expressed as
$$G(\mathbf{r}, \mathbf{r}') = \frac{2}{a} \sum_{n=1}^{\infty} g_n(y, y') \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi x'}{a}\right) \ ,$$
where the function $g_n(y, y')$ satisfies the differential equation
$$\left(\frac{\partial^2}{\partial y^2} - \frac{n^2\pi^2}{a^2}\right) g_n(y, y') = -4\pi\delta(y - y') \ ,$$
with the boundary condition $g_n(0, y') = g_n(b, y') = 0$. Solve the above differential equation to determine $g_n(y, y')$. [1]

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[1]You may find the following Fourier series expansion, for the Dirac δ-function in an interval $0 \leq x \leq a$, useful:
$$\delta(x - x') = \frac{2}{a} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi x'}{a}\right) \ .$$