DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory   Assignment 4   04.03.2015   (due: 10.03.2015)

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1. Find the potential of a dielectric sphere in a uniform electric field $\mathbf{E}_0$. Show that, outside the sphere, the potential due to the polarized sphere is that of a dipole with dipole moment

$$\mathbf{p} = \frac{\epsilon - 1}{\epsilon + 2}\mathbf{E}_0 R^3$$

Here R is the radius of the dielectric sphere and ϵ is its permittivity.

2. Consider a point charge q located at $\mathbf{r}_0$ outside a dielectric sphere of radius R. Find the potential both inside and outside the sphere. Show that, at large distances the leading term of the potential is that of a dipole with dipole moment

$$\mathbf{p} = \left(\frac{1-\epsilon}{2+\epsilon}\right)\frac{qR^3}{r_0^3}\,\mathbf{r}_0$$

3. A localized cylindrically symmetric current distribution is such that the current flows only in the azimuthal direction; the current density is a function only of r and θ (or ρ and z): $\mathbf{J} = \hat{e}_\varphi J(r,\theta)$. The distribution is "hollow" in the sense that there is a current-free region near the origin, as well as outside.

   (a) Show that the magnetic field can be derived from the azimuthal component of the vector potential, with a multipole expansion

$$A_\varphi(r,\theta) = -\frac{\mu_0}{4\pi}\sum_L m_L r^L P_L^1(\cos\theta)$$

   in the interior and

$$A_\varphi(r,\theta) = -\frac{\mu_0}{4\pi}\sum_L \mu_L r^{-L-1} P_L^1(\cos\theta)$$

   outside the current distribution.

(b) Show that the internal and external multipole moments are

$$m_L = -\frac{1}{L(L+1)} \int d^3x \ r^{-L-1} \ P_L^1(\cos\theta) J(r,\theta)$$

and

$$\mu_L = -\frac{1}{L(L+1)} \int d^3x \ r^{L} \ P_L^1(\cos\theta) J(r,\theta)$$

4. Consider a system of two circular coils of radius a and separation b. The current density for this system can be described in cylindrical coordinates by

$$\mathbf{J} = \hat{e}_\varphi I \delta(\rho - a)[\delta(z - b/2) \ + \ \delta(z + b/2)]$$

(a) Using the formalism of the previous problem, calculate the internal and external multipole moments for $L = 1, ..., 5$.

(b) Using the internal multipole expansion of the previous problem, write down explicitly an expression for B_z on the z axis.
