DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH5020 Electromagnetic Theory   Assignment 5   17.03.2015   (due: 24.03.2015)

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1. Consider a Lorentz transformation (boost) along x-axis. What is the matrix representation of $\Lambda^\alpha{}_\beta$ for this Lorentz transformation? Show that this matrix satisfies the relation $\Lambda^T \eta \Lambda = \eta$.

2. Consider the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Find the matrix representations of $F_{\mu\nu}$ as well as $F^{\mu\nu}$ and express them in terms of the components of the electric field $\mathbf{E}$ and the magnetic induction $\mathbf{B}$.

3. Consider the dual field strength $\tilde{F}_{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ of the gauge field A_μ. Here $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor with $\epsilon^{0123} = 1$. $\tilde{F}_{\mu\nu}$ can be represented in terms of a 4×4 matrix. Express the components of $\tilde{F}_{\mu\nu}$ in terms of the electric and magnetic fields and find its matrix representation.

4. Use the Lorentz transformation properties of $F_{\mu\nu}$ to derive the expressions for the transformed electric and magnetic fields in terms of the original ones for a boost along x-axis.

5. Consider the Maxwell's Lagrangian
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + c_0 J^\mu A_\mu \ .$$
Find the Euler-Lagrange equation for the gauge field A_μ. For what value of c_0 this equation coincides the Maxwell's equation as discussed in the class? Is this term $J^\mu A_\mu$ gauge invariant? Justify your answer.

6. If you add a term
$$\mathcal{L}_{\text{top}} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$
to the Maxwell's Lagrangian
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + c_0 J^\mu A_\mu \ ,$$
how will it effect the equations of motion (i.e. the Euler-Lagrangian equations)?

7. Show that $\mathcal{L}_{\text{top}}$ can be expressed as a total derivative, i.e., $\mathcal{L}_{\text{top}} = \partial_\mu B^\mu$ for certain B^μ.

8. Express both $\mathcal{L}$ as well as $\mathcal{L}_{\text{top}}$ in terms of the non-covariant quantities $\mathbf{E}, \mathbf{B}, \mathbf{A}, \phi, \rho$ and $\mathbf{J}$.