Introduction to PID control

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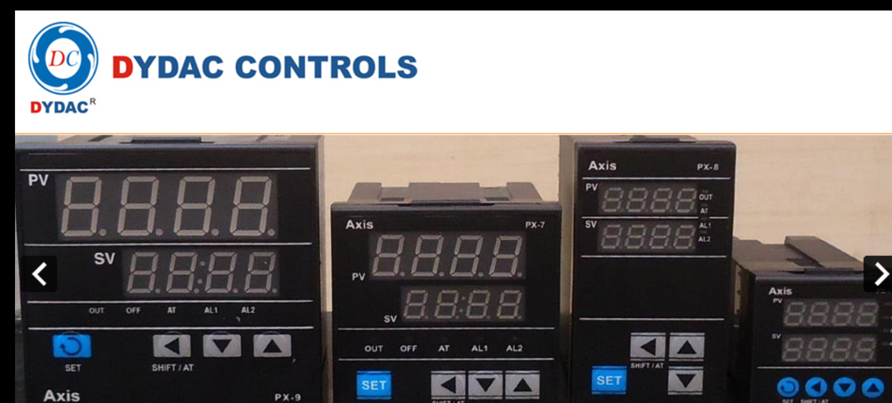
Few commercial PID controllers



Control: The necessity and examples

Common experience

Cycle pedaling

Driving a two/four wheeler

Home

Room temperature and air conditioner

Pressure in a pressure cooker

Industry

Temperature of a furnace

Speed of a motor

Mass flow rate (gas/liquid)

A gadget

Read/write head of CD player or a hard disk

Temperature Control - Illustration

Aim: To control the temperature of a body

The body - A bucket of water

A heater at our disposal to supply heat to the body

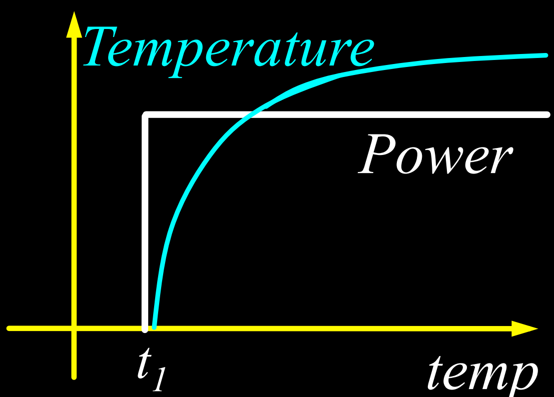
Eg. A simple 220V, 1000W immersion water heater

Understanding Control

No control

Switch on the power to the heater and forget

Let the temperature reach its "own value"



Water temperature does not increase as soon as the heater power is on.

Temperature does not follow the input

Reason?

Left unattended for a while:

Water temperature will reach a value, decided by the heat lost to the surrounding and the heat input from the heater

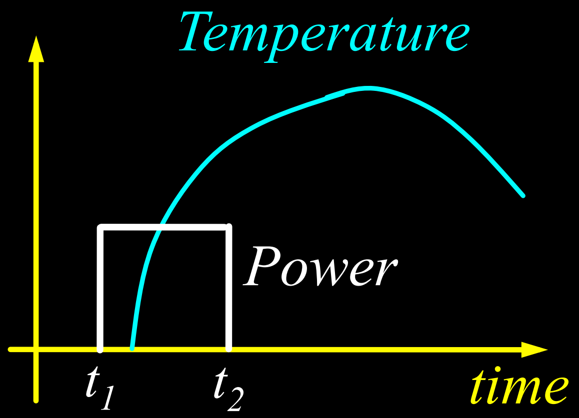
If left unattended indefinitely, the result will be disastrous.

Eventually water will evaporate,… heater will start burning the bucket,… and…

Understanding Control

Manual intervention

Switch on the power to the heater and switch off the power after some time



No control over temperature

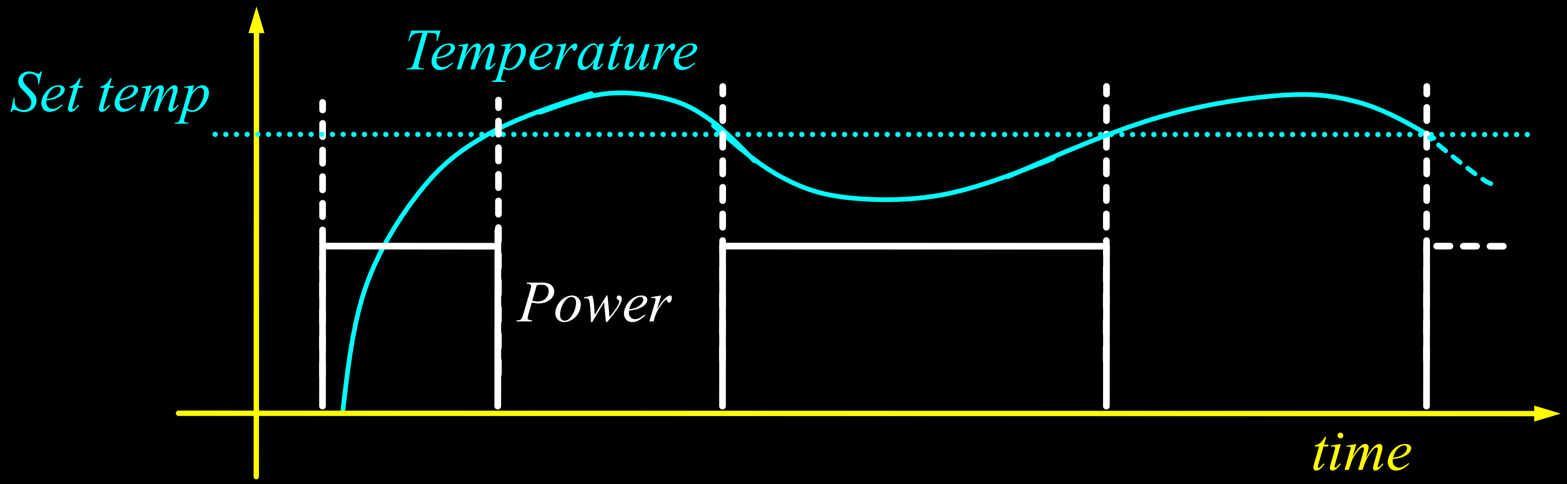
Water temperature will not increase as soon as the heater power is on nor it will fall as soon as the heater power is off. Reason: Thermal inertia.

Simple On-Off control

Switch on power to the heater and monitor the temperature

Switch off the power once the temperature of the body exceeds the set temp. Switch on when the temperature goes below set temp.

Repeat this process



Water temperature will not follow the heater, but it will swing about the set value (goes above and below)

Useful, when precise control is not needed.

Commonly used in:

Iron press (box), refrigerator, air conditioner, kitchen oven, water heater...

Proportional Control

Sense/Measure the temperature of the body continuously
(Temperature sensors: *Thermocouple, Diode thermometer,…*)

Find, $\Delta T(t)$ *the difference between the set and the actual temperature*

Temperatures are measured in terms of voltages: $\Delta T(t) \propto e_R(t)$

$e_R(t)$ *is the error voltage and* $e_R(t)$ *will be zero, when* $\Delta T(t)$ *is zero.*

Proportional control:

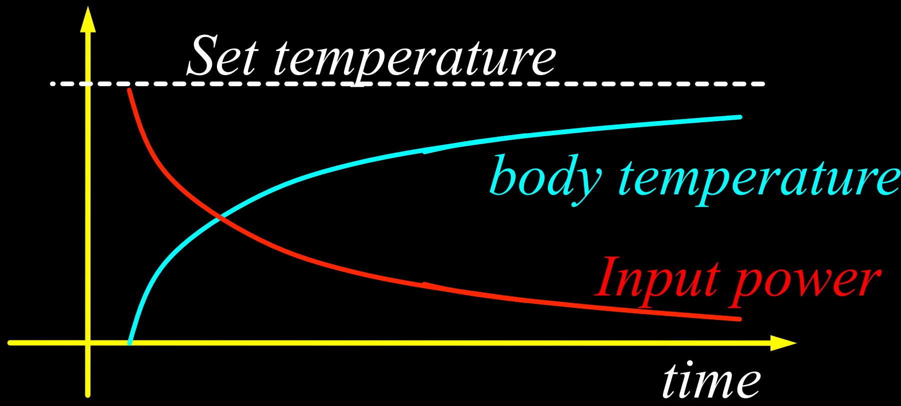
A voltage, $V_P(t)$ *proportional to* $e_R(t)$ *is sent to the heater*

$$V_P(t) = K_P e_R(t)$$

$$e_R(t) = \Delta T\ (= T_{set} - T_{measured})$$

K_p *is the proportional constant (gain)*

Proportional Control



$$V_P(t) = K_P e_R(t)$$

$$e_R(t) \propto \Delta T(t)$$

Temperature rise will not be the same as that in on-off control

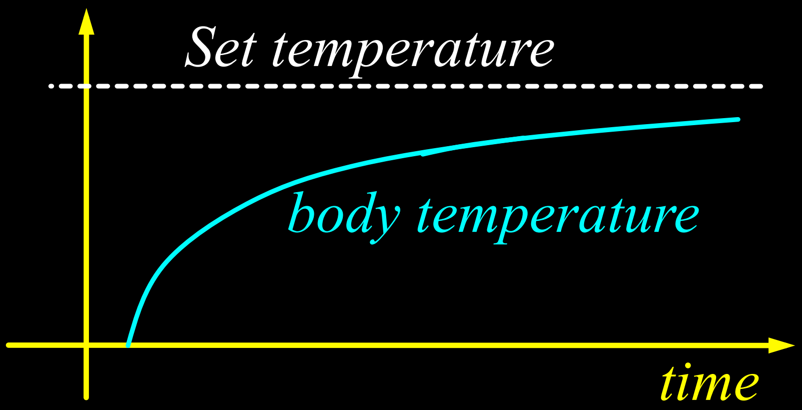
Reason:
Power delivered to the heater is not a step, but $\propto \Delta T(t)$

Temperature will never reach the set value, but will settle at a lower value.

Reason:
When $\Delta T = 0$, $e_R(t) = 0$. $V_P = 0$, *no power is given to the heater*

Proportional + Integral control

With proportional control, the body temperature will never reach the set value



Send an additional voltage, $V_I(t)$ to meet the difference

How to determine $V_I(t)$ needed to compensate the difference?

Integrate the error voltage $e_R(t)$ from the start:

$$V_I(t) = K_I \int_{t_0}^{t} e_R(\tau)d\tau$$

$$V_{out}(t) = V_P(t) + V_I(t) = K_P e_R(t) + K_I \int_{t_0}^{t} e_R(\tau)d\tau$$

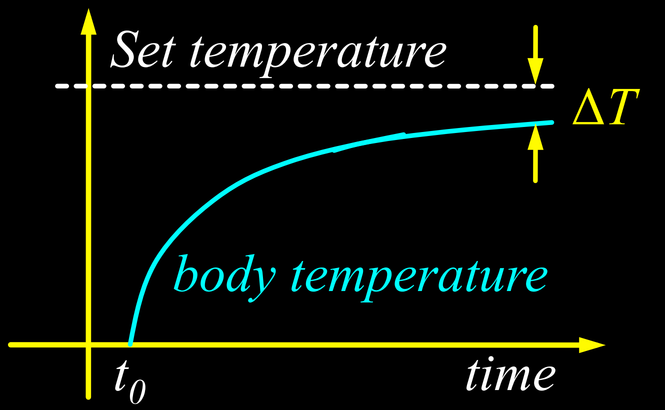
K_I is the integral gain. Choice of K_I is critical ($K_I < K_P$)

After a long time, which term will dominate and why?

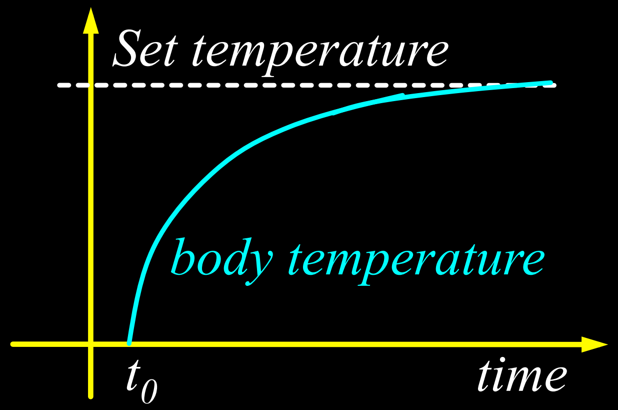
Proportional + Integral control

Inclusion of a signal proportional to the integral of $e_R(t)$ results in controlling the temperature of the body at any desired value.

$$V_{out}(t) = V_P(t) + V_I(t) = K_P e_R(t) + K_I \int_{t_0}^{t} e_R(\tau)\, d\tau$$



Proportional alone



Proportional + integral

Supposing there is a sudden deviation from the set value, Which terms will act and why?

Proportional + Integral + Differential control

Proportional + Integral parts will achieve control

A sudden change in system conditions may lead to sudden change in its temperature

K_P *and* K_I *both will act and the system temperature may overshoot*

Term proportional to the rate of change in temperature is required

Term proportional to derivative of $e_R(t)$ *will counter sudden changes*

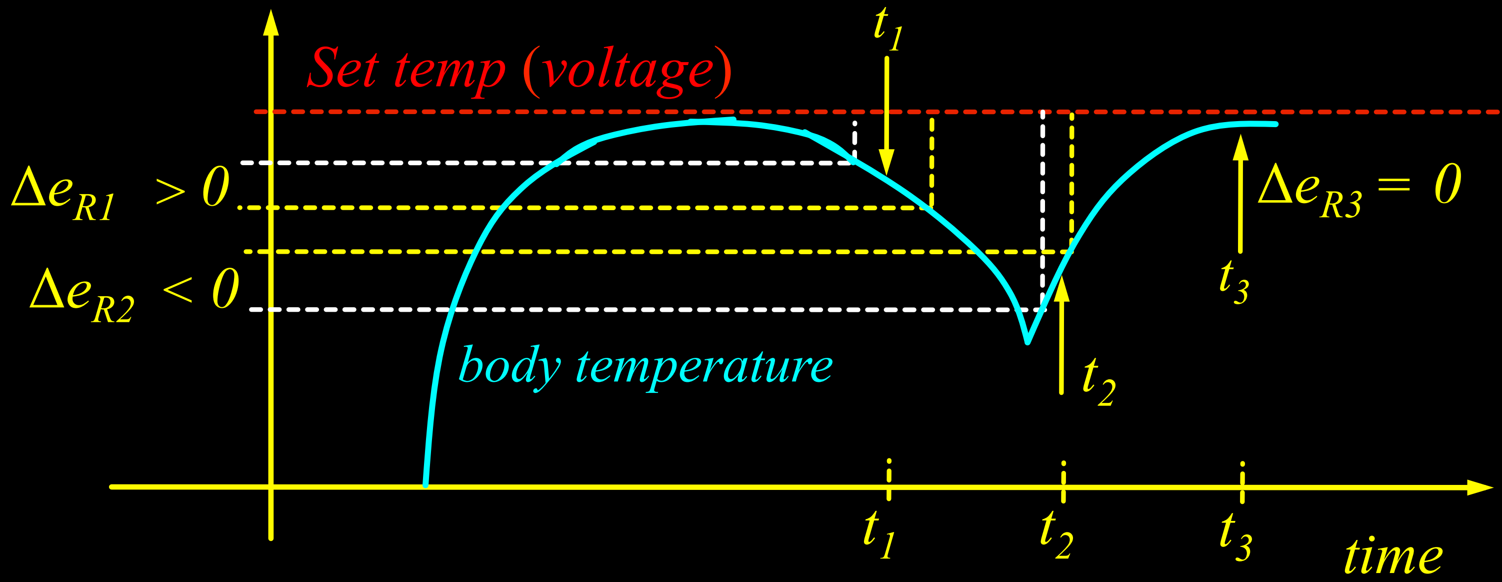
$$V_D(t) = K_D \frac{de_R}{dt}$$

K_D *is the differential gain*

The PID control output:

$$V_{out}(t) = V_P(t) + V_I(t) + V_D(t) = K_P e_R(t) + K_I \int_{t_0}^{t} e_R(\tau)\, d\tau + K_D \frac{de_R}{dt}$$

The role of Differential term



$\Delta T = T_{set} - T_{measured}$ and $e_R(t) \propto \Delta T(t)$

$e_R(t)$ is the difference between the set voltage (temperature) and the measured voltage (temperature) at any instant of time.

de_R/dt is derivative of the difference voltage (error voltage, $e_R(t)$).

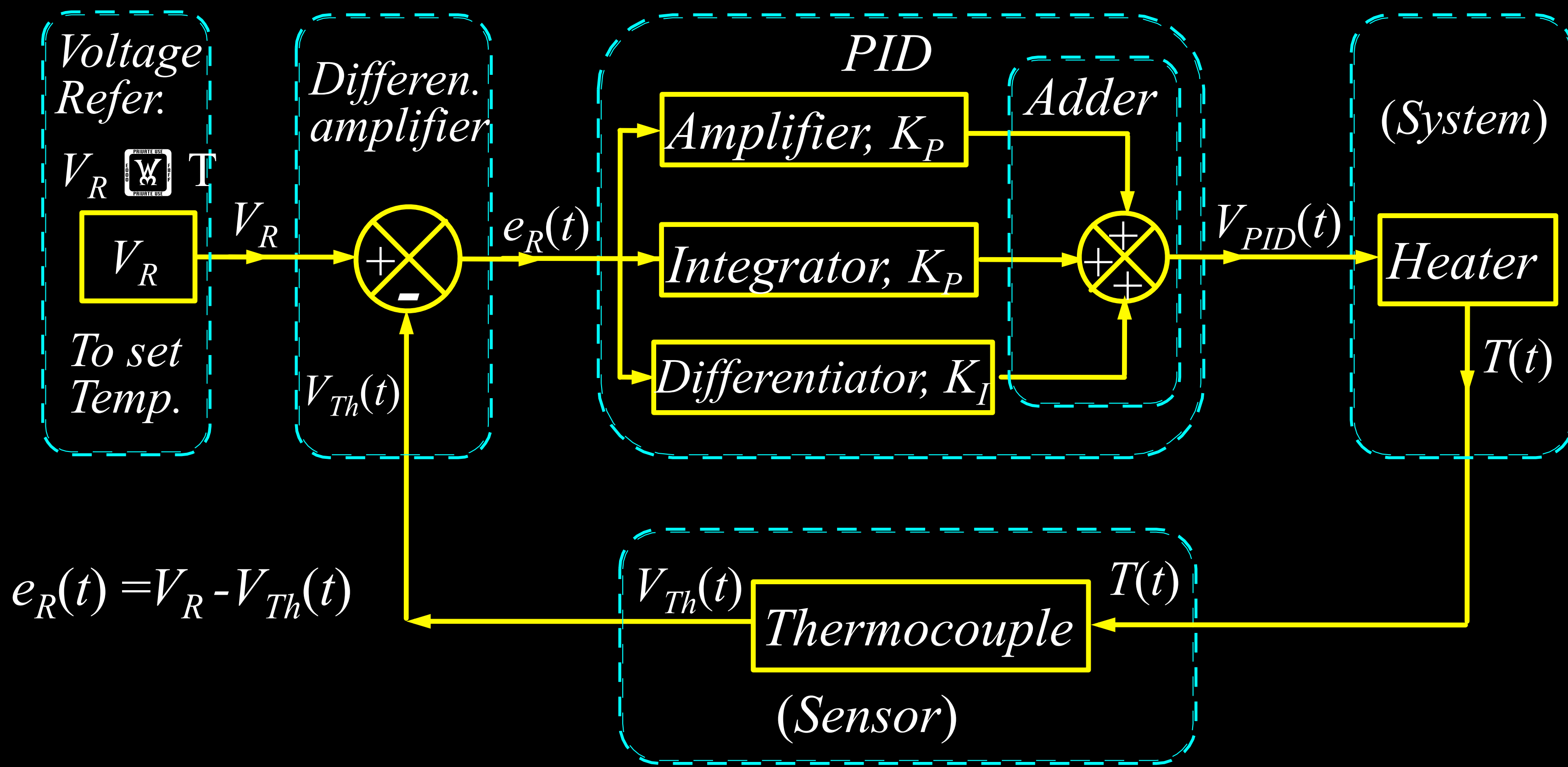
$\Delta e_R/\Delta t = [e_R(t+\Delta t) - e_R(t)]/\Delta t$

Near t_1, de_R/dt is +ve, differential term will reinforce the other terms

Near t_2, de_R/dt is -ve differential term will oppose the other terms

Near t_3, de_R/dt is zero, differential term will have no contribution

System with PID control



A closed loop system, with feedback provided by the sensor

The heater should attain the set temperature as fast as possible and should continue to remain at the set temperature.

System analysis: Time domain Vs s-domain

Aim:

Find the response (temp), when the input, $V_R(t)$ is set at a given value

Analysis in time domain

Find the impulse response function, $h(t)$ of individual blocks

Find the impulse response function of the whole system

Convolute the impulse response function with input to get the output

Analysis in s-domain (complex domain)

Find the Laplace transform of individual blocks – s-domain

Find the transfer function, $H(s)$ (ratio of output to input in s-domain) of the individual blocks

Find the transfer function of the whole system

Multiply the transfer function of the whole system by the input (s-domain) to get the output in d-domain

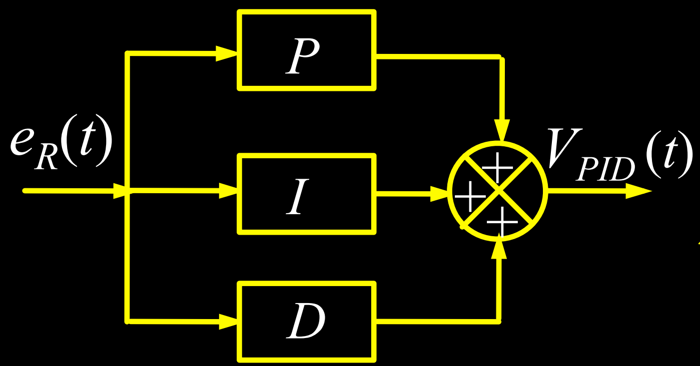
Take inverse Laplace transform to get the output in time domain

Which procedure is simpler and why?

Integral/differential equations become algebraic equations in "s"

Laplace transform and s-Domain representation

1. The PID block



$$V_{PID}(t) = K_P(t)e_R(t) + K_I\int_{t_0}^{t} e_R(\tau)d\tau + K_D\frac{de_R}{dt}$$

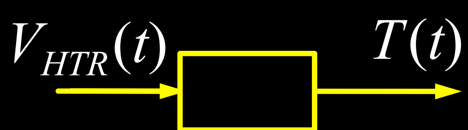
$$L\{V_{PID}(t)\} = \mathcal{V}_{PID}(s)$$

$$\mathcal{V}_{PID}(s) = K_P\,\mathcal{E}_R(s) + \frac{K_I}{s}\,\mathcal{E}_R(s) + sK_D\,\mathcal{E}_R(s)$$

(assuming "0" initial values)

The Transfer Function: $H_{PID}(s) = \mathcal{V}_{PID}(s)/E_R(s) = K_P + K_I/s + sK_D$

2. The Heater



$(V_{HTR}(t) = \textit{input voltage})$

For a short duration voltage (input) applied at t = 0, the heater temperature rises to a finite value and for t > 0 decreases exponentially. $T(t) \propto e^{-at}$

The Transfer function: $H_{HTR}(s) = \mathcal{V}_{HTR}(s) / \mathcal{T}(s) = \dfrac{1}{1+s}$

(assuming $a = 1$*)*

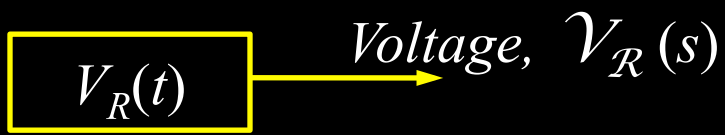
s-Domain representation

3. Temperature sensor (Thermocouple)

Transfer function: $H_{Th}(s) = \mathcal{V}_{Th}(s)/\mathcal{T}(s)$

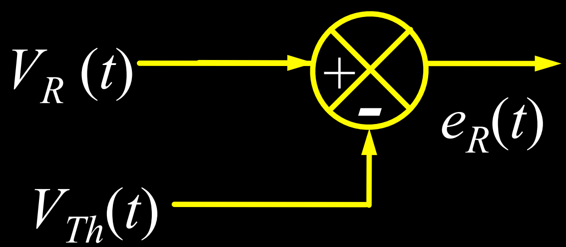
If the sensor gives 1V for 1K(after signal conditioning), $H_{Th}(s) = 1$

4. Reference voltage source



A constant voltage source, calibrated to set temperature

5. Differential amplifier

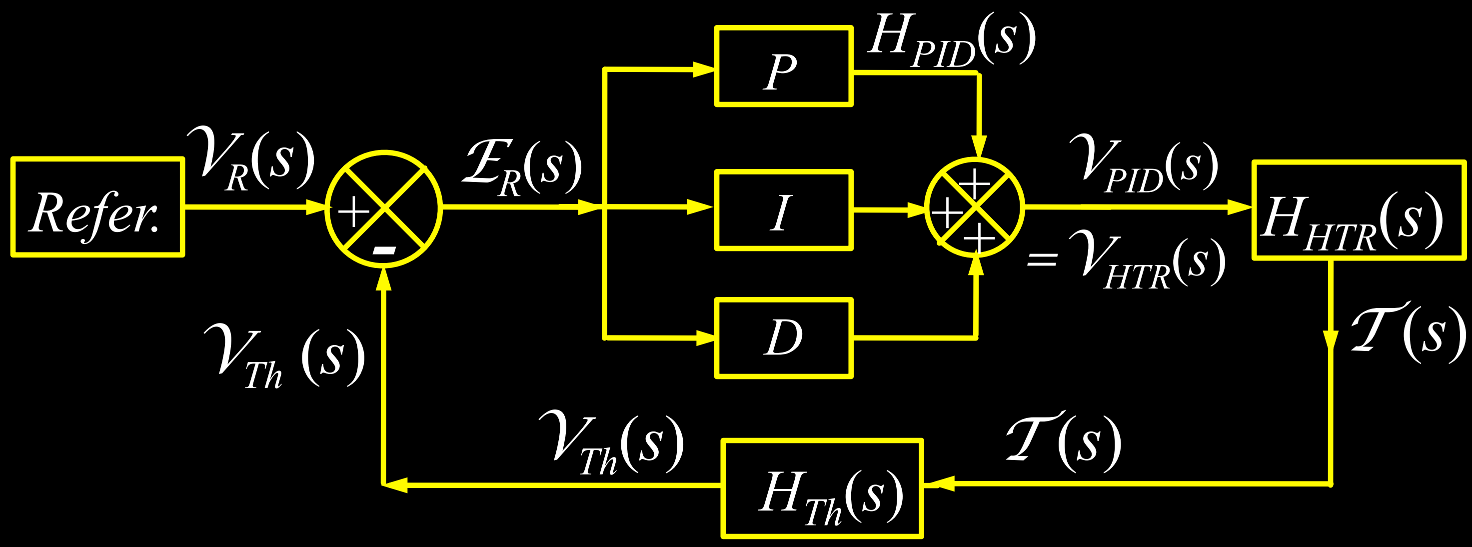


Compares the temperature sensor voltage and the reference voltage and gives a voltage proportional to the difference between them

$V_R(t) - V_{Th}(t) = e_R(t)$, the error signal

$\mathcal{V}_R(s) - \mathcal{V}_{Th}(s) = \mathcal{E}_R(s)$

PID control – Analysis in s-domain



Aim: To find the transfer function of the whole system that is to find the ratio of output ($\mathcal{T}(s)$) to input ($\mathcal{V}_R(s)$)

$$\mathcal{T}(s) = \mathcal{V}_{HTR}(s)\, H_{HTR}(s) = \mathcal{V}_{PID}(s)\, H_{HTR}(s) \qquad [\, \mathcal{V}_{HTR}(s) = \mathcal{V}_{PID}(s)]$$

$$\mathcal{T}(s) = \mathcal{E}_R(s)\, H_{PID}(s) H_{HTR}(s) \qquad [\, \mathcal{V}_{PID}(s) = H_{PID}(s)\, \mathcal{E}_R(s)]$$

$$\mathcal{E}_R(s) = \mathcal{V}_R(s) - \mathcal{V}_{Th}(s) = \mathcal{V}_R(s) - H_{Th}(s)\, \mathcal{T}(s) \qquad [\, \mathcal{V}_{Th}(s) = H_{Th}(s)\, \mathcal{T}(s)]$$

$$\mathcal{T}(s) = [\, \mathcal{V}_R(s) - H_{Th}(s)\, \mathcal{T}(s)]\, H_{PID}(s)\, H_{HTR}(s)$$

$$\mathcal{T}(s)\, [1 + H_{Th}(s)\, H_{PID}(s)\, H_{HTR}(s)\,] = \mathcal{V}_R(s)\, H_{PID}(s)\, H_{HTR}(s)$$

PID control – Analysis in s-domain

The transfer function for the whole system is:

$$H(s) = \frac{\mathcal{T}(s)}{\mathcal{V}_R(s)} = \frac{H_{PID}(s)\, H_{HTR}(s)}{[1 + H_{Th}(s)\, H_{PID}(s)\, H_{HTR}(s)\,]}$$

Where the individual transfer functions are: $H_{Th}(s) = 1$

$$H_{PID}(s) = K_P + K_I/s + sK_D = \frac{(s^2K_D + sK_P + K_I)}{s} \qquad H_{HTR}(s) = \frac{1}{1+s}$$

$$H(s) = \frac{s^2K_D + sK_P + K_I}{[s^2(1+K_D) + s(1+K_P) + K_I\,]}$$

Output can be found for any input, as H(s) is known

If the input $V_R(t) = \delta(t)$, *then output* $\mathcal{T}(s) = H(s)$; $L\{\delta(t)\} = 1$

If the input $V_R(t) = u(t)$, *then output* $\mathcal{T}(s) = H(s)/s$; $L\{u(t)\} = 1/s$

Transfer Function analysis of PID control

Let $V_R(t) = u(t)$; $u(t)$ *is unit step,* $K_P = 1,\ K_I = 5\ \&\ K_D = 0$

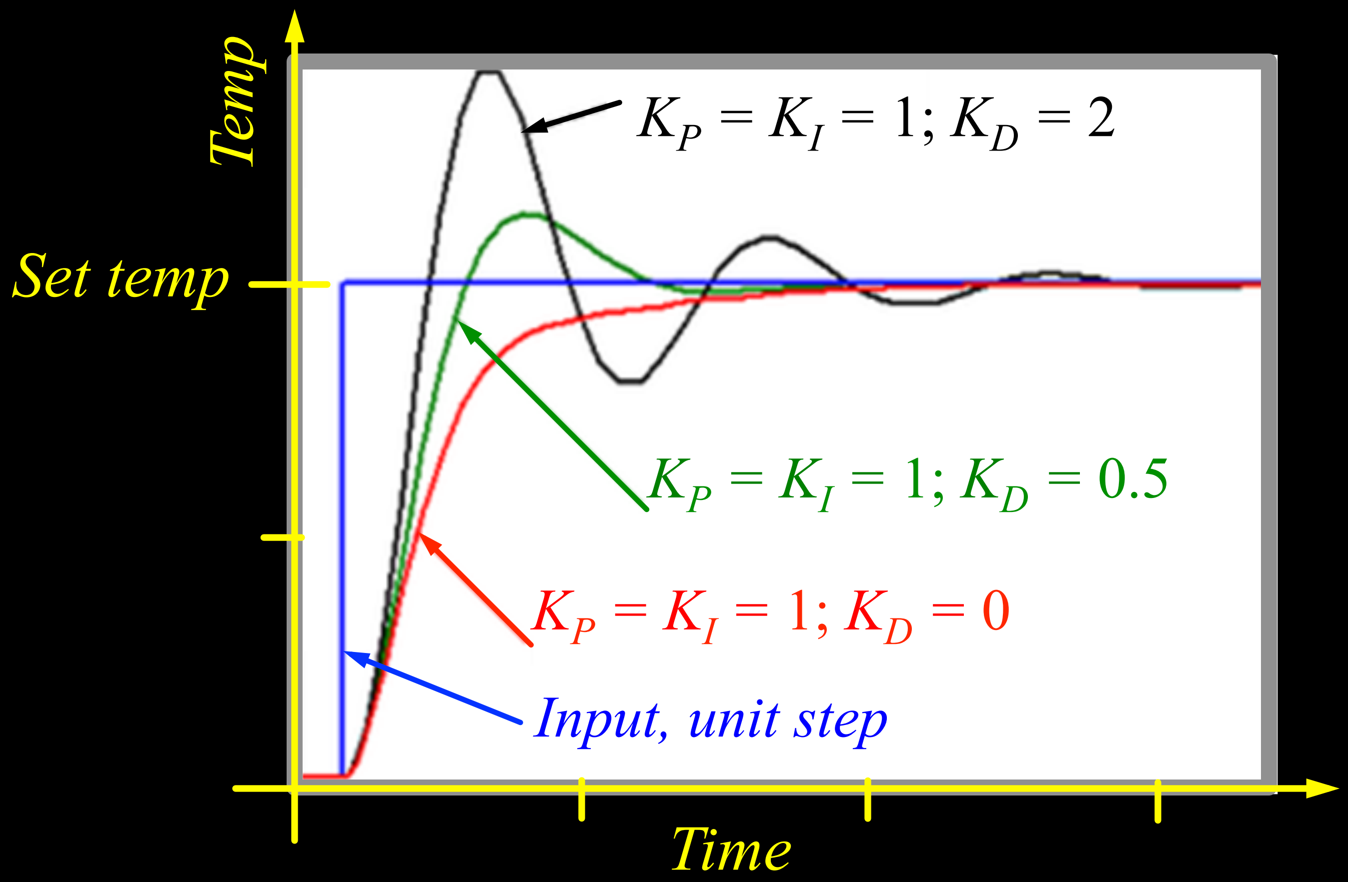
$$H(s) = \frac{s^2K_D + sK_P + K_I}{[s^2(1+K_D) + s(1+K_P) + K_I]} = \frac{s+5}{s^2+2s+5}$$

$$\mathcal{V}_R(s) = 1/s$$

$$\mathcal{T}(s) = H(s)\,\mathcal{V}_R(s) = \frac{s+5}{s^2+2s+5}\,\frac{1}{s} = \frac{1}{s} - \frac{s+1}{(s+1)^2+4)}$$

$$T(t) = L^{-1}\{\mathcal{T}(s)\} = 1 - e^{-t}\cos 2t$$

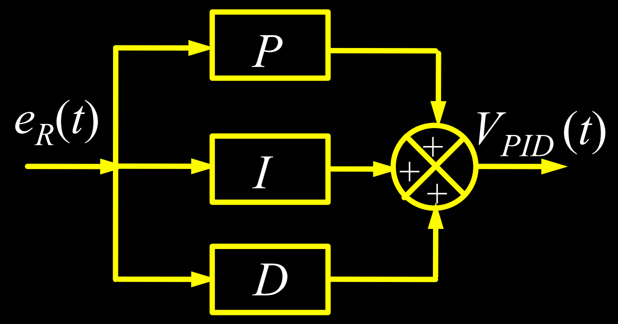
PID Control – MATLAB simulation



A quick analogy

The PID controller output is:

$$K_P e_R(t) + K_I \int_{t_0}^{t} e_R(\tau)\,d\tau + K_D \frac{deR}{dt}$$



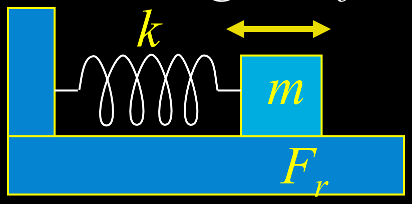
Rewrite the equation as

$$K_D \frac{d^2 e_R}{dt^2} + K_P \frac{de_R}{dt} + K_I e_R = f(t)$$

Let $e_R = x$; $K_D = m$; $K_P = b$ and $K_I = k$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = g(t)$$

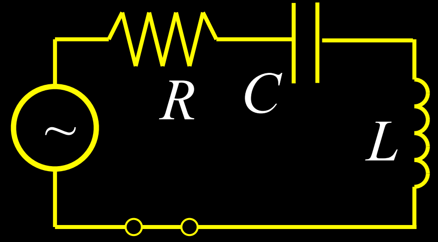
Damped SHM: spring-mass system on rough surface



Let $e_R = I$; $K_D = L$; $K_P = R$ and $K_I = 1/C$

$$L\frac{d^2I}{dt^2} + b\frac{dI}{dt} + (1/C)I = h(t)$$

Current in an LCR circuit



Bound electron in alkali halide under EM radiation

$$m_e \ddot{\vec{r}} + b\dot{\vec{r}} + k\vec{r} = -e\vec{E}_L e^{i\omega t}$$

Quick summary

Necessity for control

The meaning of PID control

Example of PID control of a Heater

Modeling using Transfer function

s-domain analysis of Heater with PID control

Simulation and effect of PID constants