QCD @ LHC

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- Quantum Chromodynamics
- Strong coupling constant
- Parton Distribution Functions
- Infra-red Safe observables
- Conclusions

Physics at the LHC

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- Excellent discovery reach:
  - Higgs
  - Supersymmery
  - Extra-Dimensional models
  - Anything else

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- Theories:
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- Issues to be tackled:
  - Kinematics
  - Normalisation
  - Renormalisation and factorisation scale uncertainities
  - Parton Distribution Functions
  - Phase Space boundary effects and resummation of large logs

Strong Interaction

- Modern view point: it is the force that binds quarks and gluons inside the hadrons and is the strongest among the four fundamental forces in nature
- Really strong!: About 100 times electromagnetic force, 10^{14} times stronger than Weak interaction and a factor of 10^{40} stronger than the Gravitational force
- Large scale physics is dominated by gravitational and the electro magnetic forces and microspic world is governed by the strong and weak forces as they are short range forces.
- Strong force exists at subatomic distances: a consequence of two features: confinement and asymptotic freedom
- Confinement is the statement that isolated quarks do not showup. But symmetry arguments and scattering experiments in the 1960's established quarks with -1/3 and +2/3 electric charge units and additional colour charge

History

- Gell-Mann and Zweig (1964): the spectroscopy of hadrons can be explained by a fewer number of fundamental particles called quarks. Baryons are made up of 3 quarks and meson are made up of one quark and antiquark

- Proposal: quarks are spin 1/2 particles and carry 1/3 or 2/3 electric charge units.

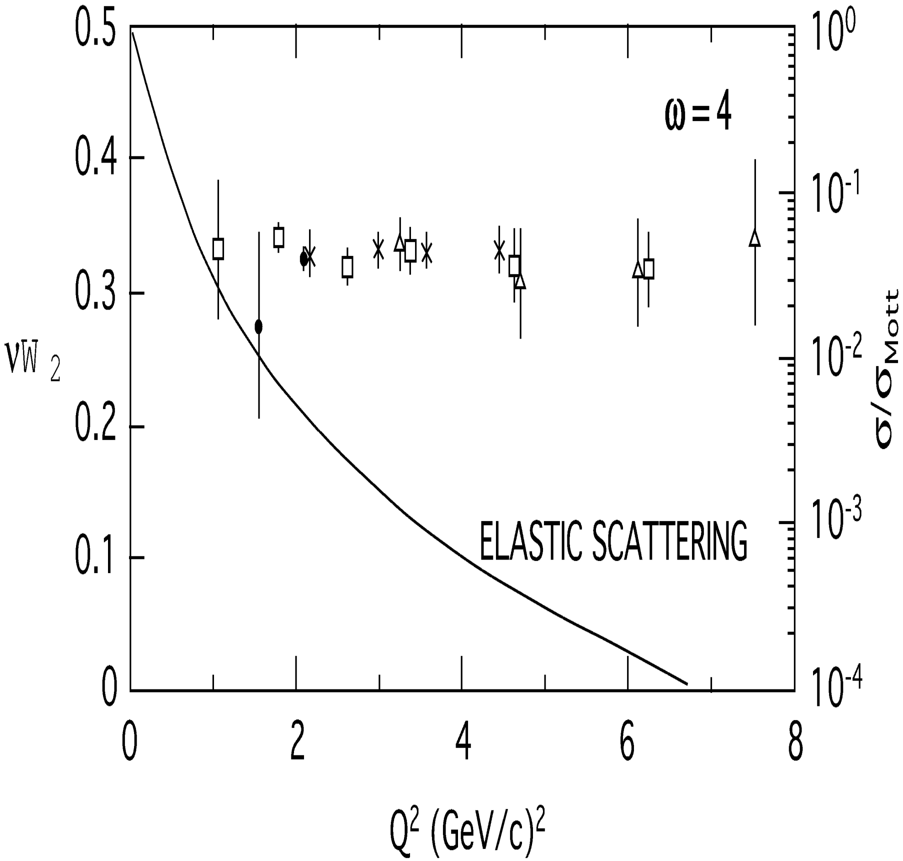
- By end of 1960s, static picture of quarks emerged: quarks being the constituents of hadrons was confirmed through dynamics observed at high $e\ P$ scattering experiments at SLAC

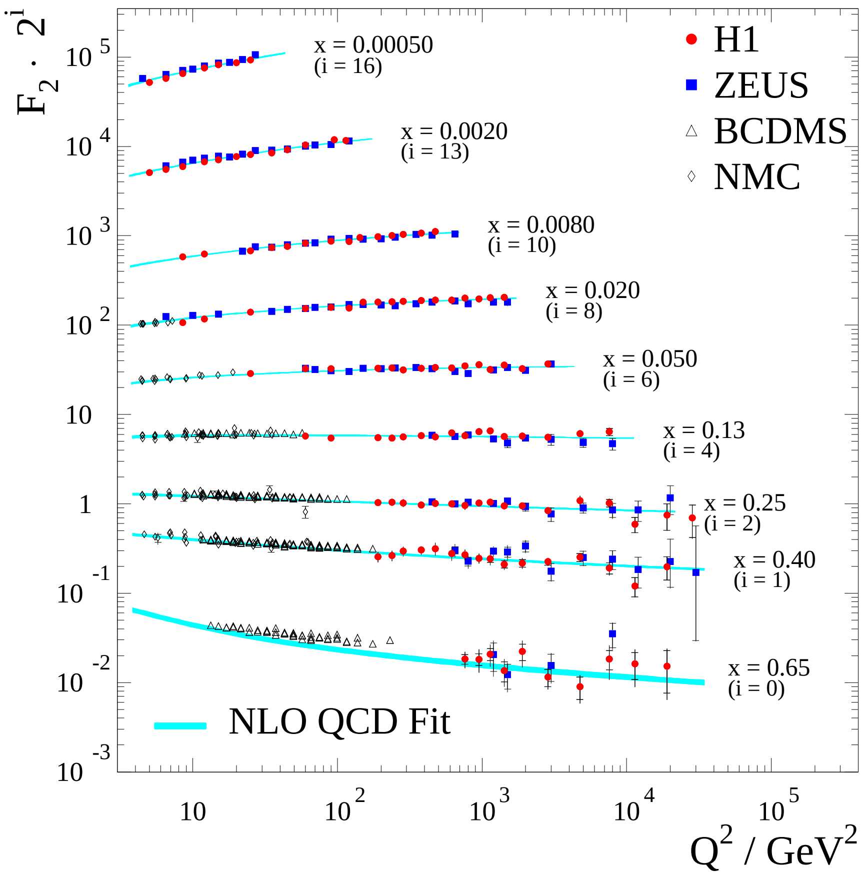
- Instead of decreasing with increasing momentum transfer as expected for elastic scattering of electrons at protons as a whole, the cross section showed a scaling behaviour as it should occur if the electrons scatter on quasi-free, point like and nearly massless constituents inside the proton

- quark model was successful in describing these properties but it had other short comings
  - Violation of the Pauli-principle
  - Prediction of neutral pion lifetime was off by a factor nine
  - No particle of elementary electric charge 1/3 or 2/3 observed in colliders

Deep Inelastic Scattering

- SLAC (1969): sub structure of nucleon— Nobel 1990: Limited range of x and Q^2 in fixed-target lepton-nucleon scattering experiments, prevented unambiguous test of QCD scaling violations and running of α_s





- HERA (2005) at DESY: extended the range of Q^2 by more than 2 orders of magnitude and the range in x by more then 3 orders of magnitude— precise test of scaling violations and running coupling were achieved

Chromodynamics

- Introduction of 3 different colour quantum states for each quark solved the spin-statistics problem and hence saved Pauli-principle and explained the missing factor of nine $(= 3^2)$ for the pion lifetime

  M Y Han and Y Nambu (1965)

- Notion that hadrons consists either of 3 quarks (baryons) or a quark and an anti quark (meson) with the vanishing net colour charge for each hadron— could account for the fact that the strong force is short-ranged

- In early 1970's a quantum field theory of the strong force, namely Quantum Chromodynamics (QCD), was developed using gauge principle. New coloured spin-1 particles called gluons were introduced which couple to colour charges of quarks and also to themselves

  H Fritzsch and M Gell-Mann (1972)

  H Fritzsch, M Gell-Mann and H Leutwyler (1973)

- Chromo-Statics turned into Chromo-Dynamics

Asymptotically Freedom

- Symanzik (1970) showed that in quantum field theories, the couplings may change their effective sizes depending upon the scale at which they are measured through Symanzik's β-function.
- SLAC data on "approximate scaling" and the notion of "free quarks" and gluons inside the proton required a -ive β-function. All field theories probed during that time had a +ive β-function.
- Crucial question in the early 1970s was therefore whether QFT was compatible with ultraviolet stability (asymptotic freedom)?
- Majority view was expressed by Zee (1973); conjecturing that "there are no asymptotically free quantum field theories in 4-dim"
- Coleman and Gross set out to prove that conjecture, their graduate students Politzer and Wilczek (with Gross) tried to close a loophole: β-function for nonabelian gauge theories— still unpublished and probably unknown to everybody except t'Hooft
- Politzer and Gross & Wilczek finally demonstrated in 1973 that Chromo-Dynamics, with coloured quarks and gluons, obeying $SU_c(3)$ symmetry, generated a -ive β-function— the quarks and gluons are asymptotically free

Nobel prize 2004

Asymptotically Freedom

- QCD could explain the approximate scaling in the SLAC data at high energies, and at the same time an increase of coupling strength at low energies lead to confinement

- An important consequence of asymptotic freedom is that the strong coupling α_s is small enough, at sufficiently high energies to allow application of perturbation theory in order to provide quantitative predictions of physical processes

- Quantum Chromodynamics now started its triumphal procession as being the field theory of the strong interaction. Many refined calculations theoretical predictions and experimental verifications were ventured

- Asymptotic freedom and or equivalently the existence of colour charged gluons had to be tested, quantified and proven experimentally. The strong coupling parameter, $\alpha_s(Q^2)$ had to be determined and its energy dependence verified to be compatible with asymptotic freedom

Quantum Chromodynamics (QCD)

- QCD is the gauge field theory of the strong interaction and describes the interaction of quarks through the exchange of massless vector gauge bosons

$$
\begin{aligned}
\mathcal{L}_{QCD} &= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{f=1}^{n_f} \overline{q}_f (i \not{D} - m_f) q_f \\
G^{\mu\nu}_a &= \partial^\mu G^\nu_a - \partial^\nu G^\mu_a - g_s f_{abc} G^\mu_b G^\nu_c \\
(D^\mu)_{ij} &= \delta_{ij} \partial^\mu - i\, g_s \sum_{a=1}^{8} G^\mu_a T^a_{ij}
\end{aligned}
$$

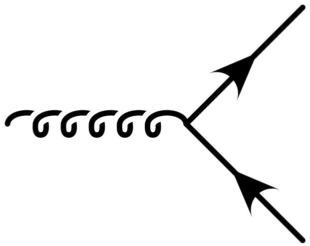
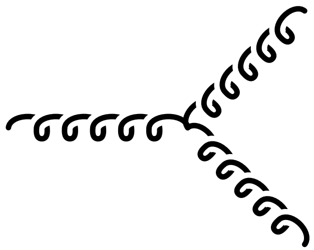
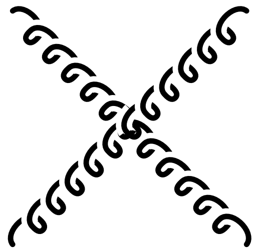
- QCD Lagrangian is invariant under the local $SU_c(3)$ transformation

$$
\begin{aligned}
q_i \longrightarrow q'_i &= U_{ij}(\varepsilon_a) q_j & U_{ij} &= \exp\{-i T^a_{ij} \varepsilon^a\} \\
G_\mu \longrightarrow G'_\mu &= U(\varepsilon) G_\mu U^\dagger(\varepsilon) + \frac{i}{g_s} \left(\partial_\mu U(\varepsilon)\right) U^\dagger(\varepsilon) & G^\mu_{ij} &= G^\mu_a (T_a)_{ij}
\end{aligned}
$$

- QCD does not predict the actual value of $\alpha_s = g_s^2/4\pi$, however it definitely predicts the functional form of its energy dependence

QCD Feynman rules

- Coupling: g_s $\quad$ g_s $\quad$ g_s^2

Colour: $(T_a^F)_{ij}$ $\quad$ $(T_a^A)_{bc} = -if_{abc}$ $\quad$ $f_{abc}f_{abc}$

Lorentz: γ^μ $\quad$ $V^{\mu\nu\rho}$ $\quad$ $W^{\mu\nu\rho\sigma}$

- A theory formulated in terms of quarks and gluons at the Lagrangian level but observed in nature as hadrons
- Hadrons can carry definite flavour quantum number and hence the hadronic wave functions are non-singlets under the falvour symmetry $SU(n_f)$ while hadrons do not carry any colour quantum number and hence transform as singlet under $SU_c(3)$ transformation
  - Baryons $\quad \frac{1}{\sqrt{6}} \sum_{ijk} \epsilon_{ijk} q_i^{f_1} q_j^{f_2} q_k^{f_3}$
  - Mesons $\quad \frac{1}{\sqrt{3}} \sum_{ij} \delta_{ij} q_i^{f_1} \bar{q}_j^{f_2}$

Experimental Group Theory

- Can experimentalists measure all the information contained in the vertices
- Vertices are determined by quark and gluon representation matrices T_a^F and T_a^A (general symmetry group). Combination that appear in measurable quantities are the following traces and sums:

$$tr(T_a^R T_a^R) = T_R \delta_{ab} \qquad \sum_a (T_a^R)_{ij} (T_a^R)_{jk} = C_R \delta_{ij} \qquad (R = F, A)$$

- At LEP, data statistics and precision allowed to actually determine experimentally values of C_A (number of colour charge) and C_F

	SU(3)	LEP
C_F	$\frac{4}{3}$	$1.30 \pm 0.01\ \mathrm{(stat)} \pm 0.09\ \mathrm{(sys)}$
C_A	3	$2.89 \pm 0.03\ \mathrm{(stat)} \pm 0.21\ \mathrm{(sys)}$

∘ Excludes theories exhibiting symmetries other than $SU(3)$

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- The matter fields: Quarks and Anti-quarks with 3 different colours.

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- Properties: Asymptotic freedom, Confinement

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In $\overline{MS}$ renormalisation scheme:

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$$C_A = N, \qquad C_F = \frac{N^2-1}{2N}, \qquad T_f = \frac{1}{2} \qquad \text{for} \qquad SU(N)$$

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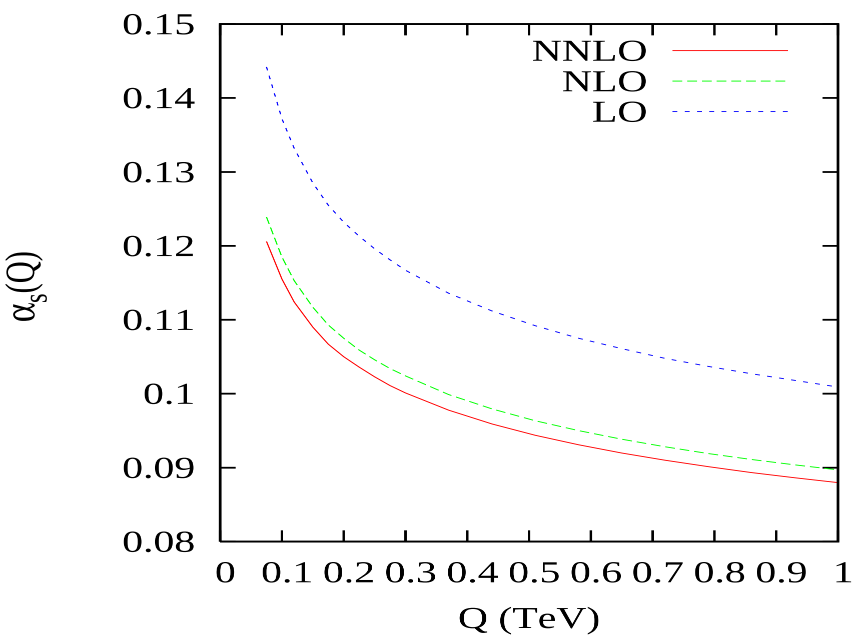

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- We can compute them because they are "infra-red safe" due to their Factorisation properties

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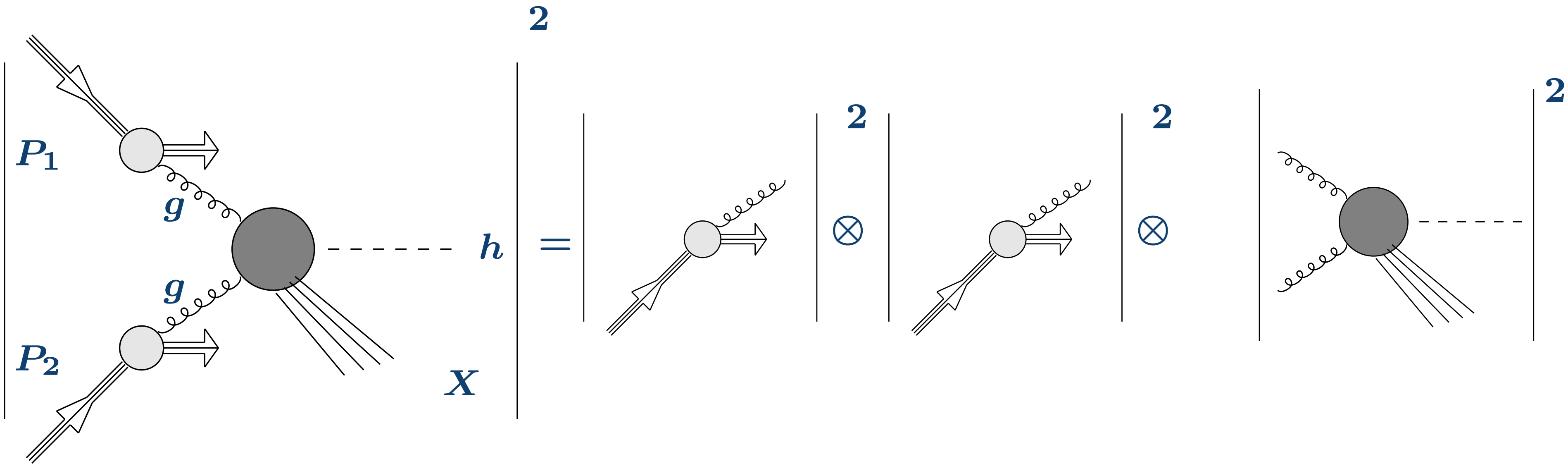

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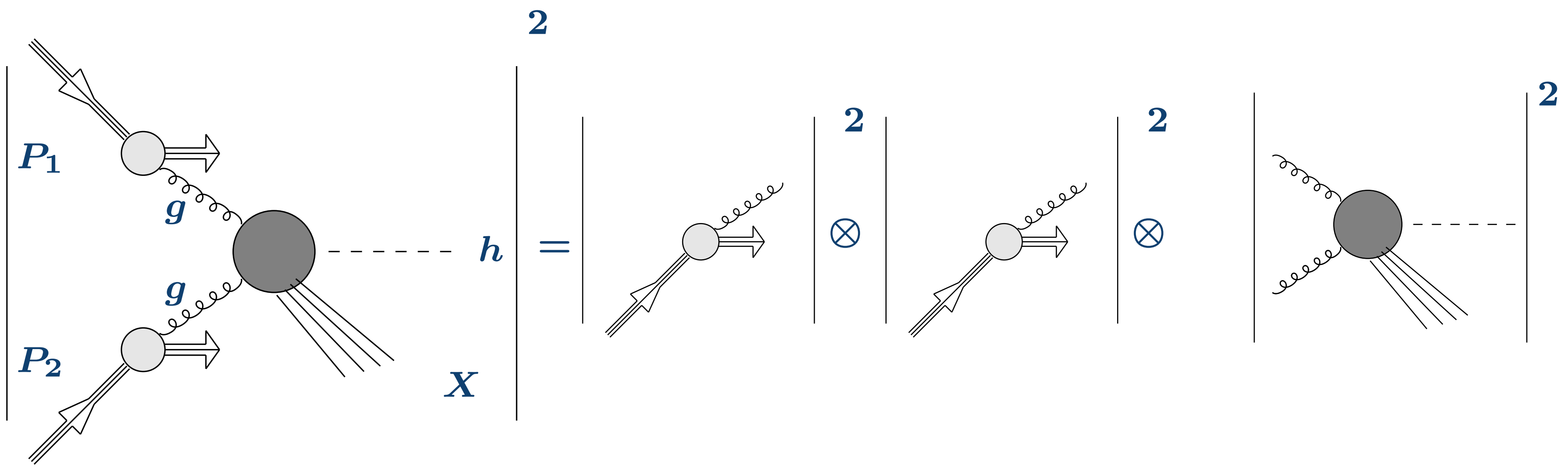

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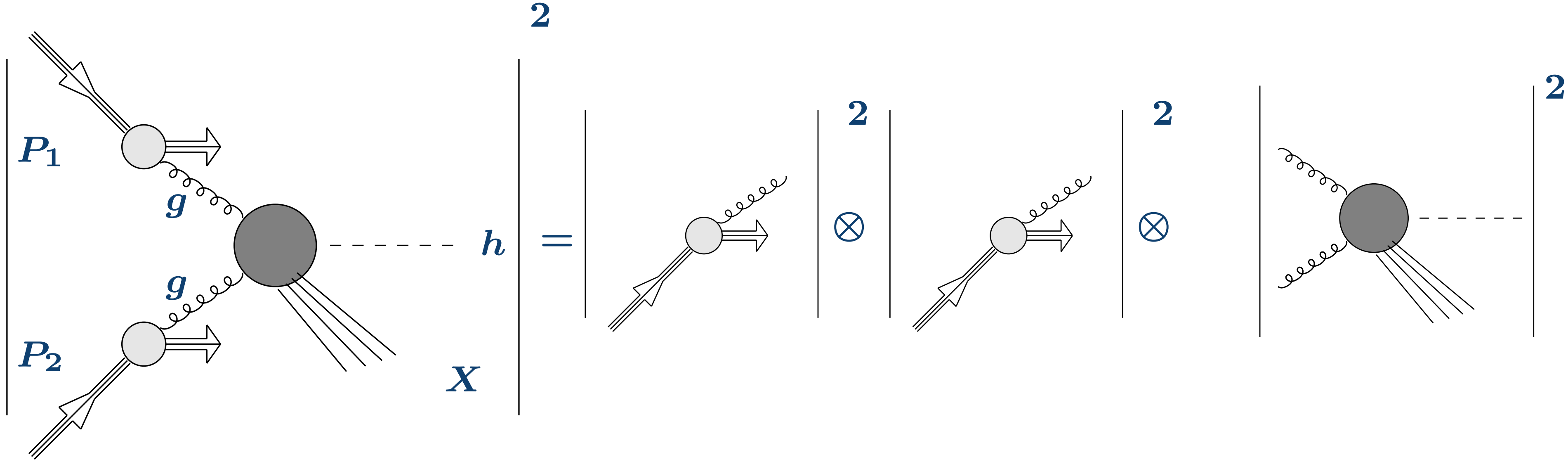

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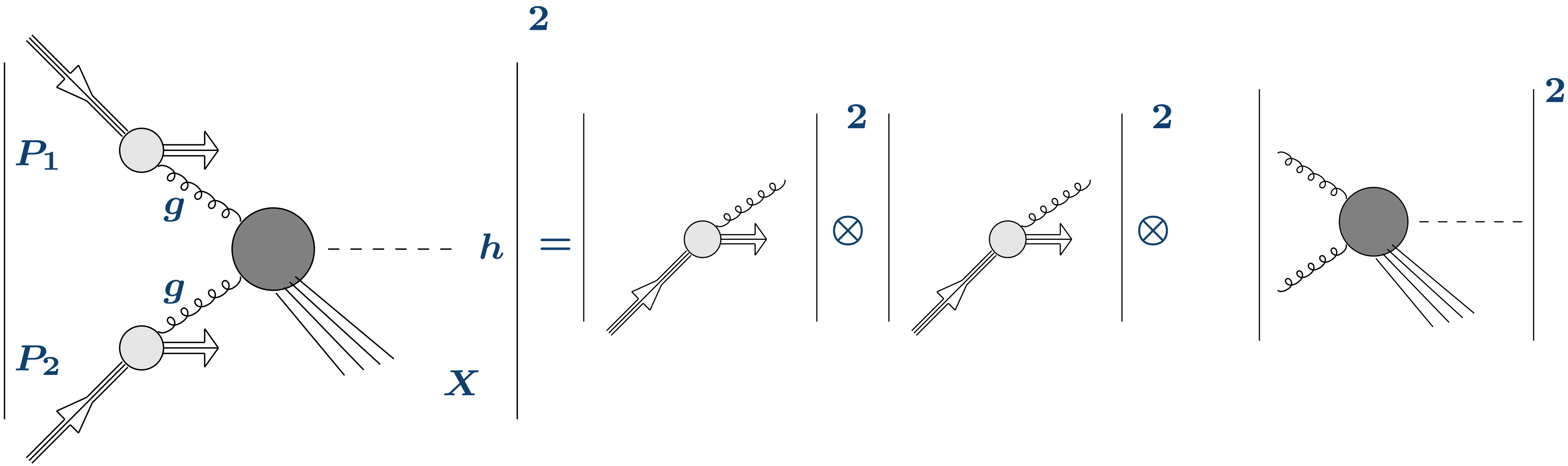

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$$f_1(x) \otimes \cdots \otimes f_n(x) = \int_0^1 dx_1 \cdots \int_0^1 dx_n f_1(x_1) \cdots f_n(x_n) \delta(x - x_1 x_2 \cdots x_n)$$

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

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$$\frac{d}{d\mu}\sigma^{P_1 P_2}(\tau, m_h^2) = 0, \qquad \mu = \mu_F, \mu_R$$

Factorisation

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$$d\hat{\sigma}^{B}_{ab}(z, \frac{1}{\varepsilon_{\rm IR}}) = \sum_{c,d} \Gamma_{ca}\left(z, \mu_F, \frac{1}{\varepsilon_{\rm IR}}\right) \otimes \Gamma_{db}\left(z, \mu_F, \frac{1}{\varepsilon_{\rm IR}}\right) \otimes d\hat{\sigma}_{cd}(z, \mu_F)$$

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DGLAP evolution (Renormalisation Group) Equations

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Perturbatively Calculable:

$$\begin{aligned} P_{ab}(z,\mu_F) \quad &= \quad \left(\frac{\alpha_s(\mu_F)}{4\pi}\right) P^{(0)}(z) && \text{one loop} \quad (LO) \\ &+ \left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^2 P^{(1)}(z) && \text{two loop} \quad (NLO) \\ &+ \left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^3 P^{(2)}(z) && \text{three loop} \quad (NNLO) \end{aligned}$$

NNLO is computed recently (summer 2004)

Scale Variation of Flux at LHC

$$\Phi^{I}_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} \; f^I_a(z, \mu_F) \; f^I_b\left(\frac{x}{z}, \mu_F\right) \qquad I = LO, NLO, NNLO$$

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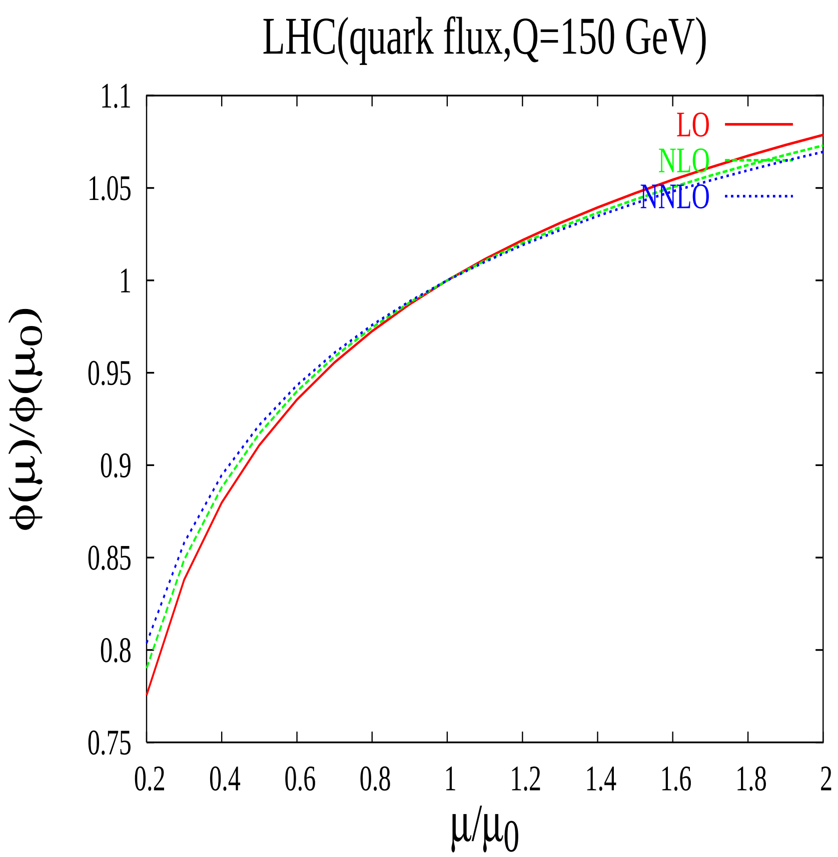

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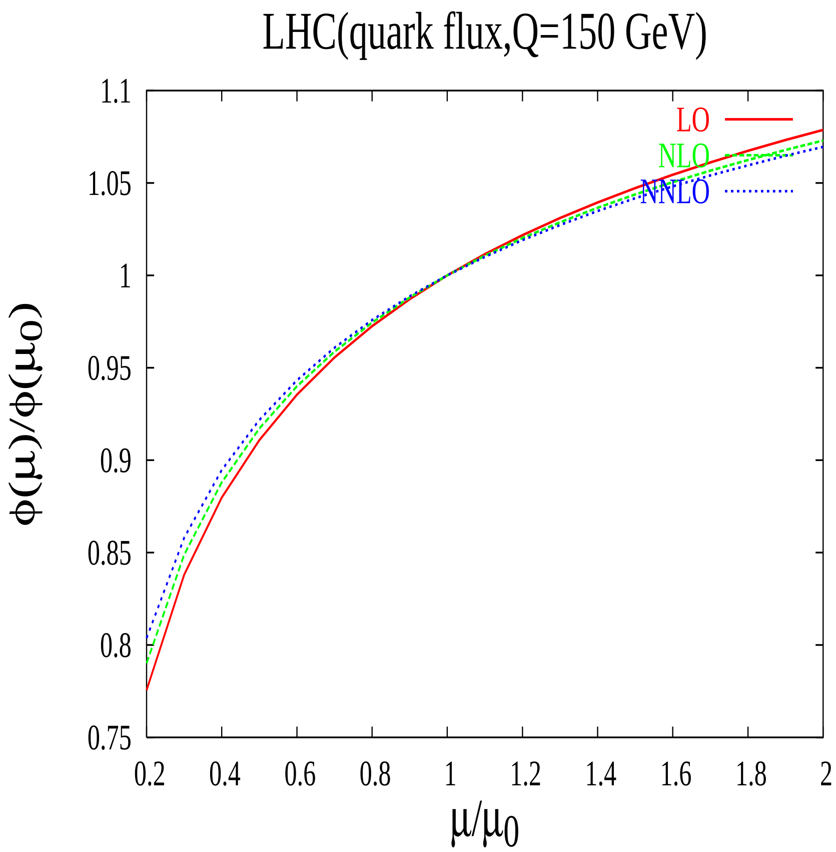

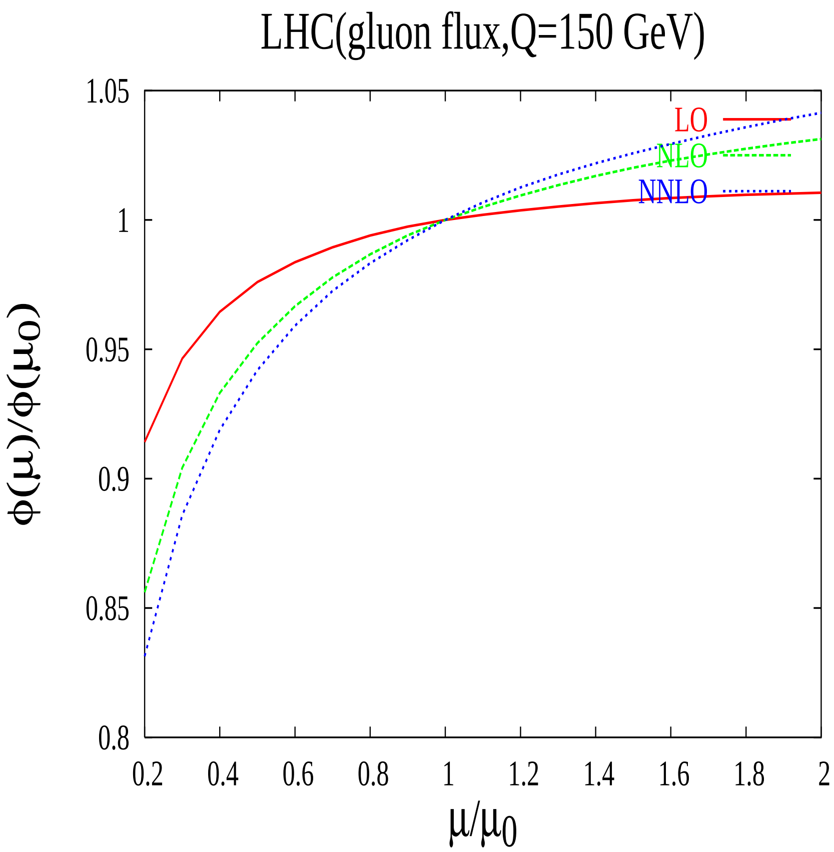

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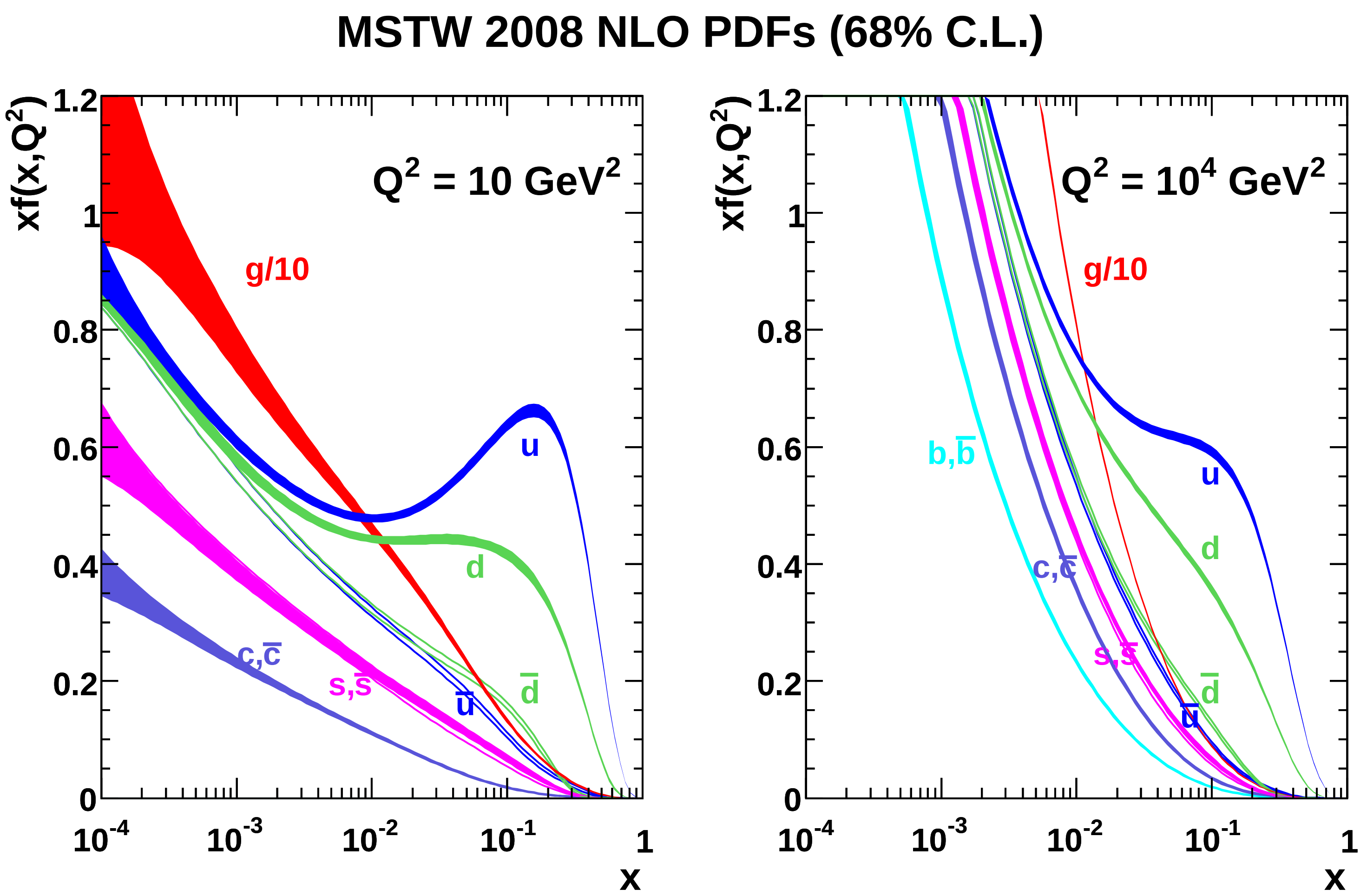
MSTW NLO Parton Distribution Functions



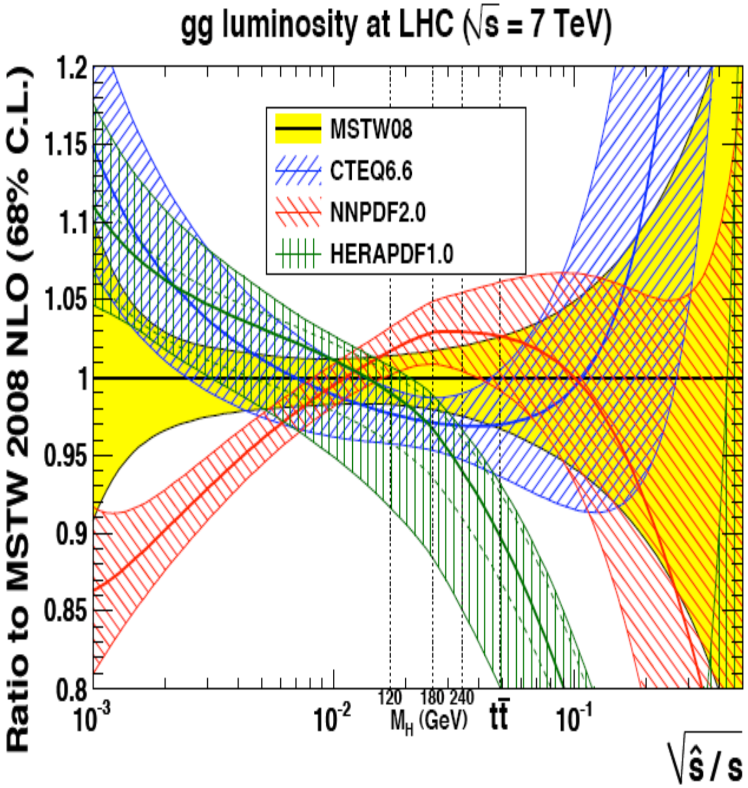
MSTW 2008 NLO PDFs at $Q^2 = 10$ GeV^2 and $Q^2 = 10^4$ GeV^2.

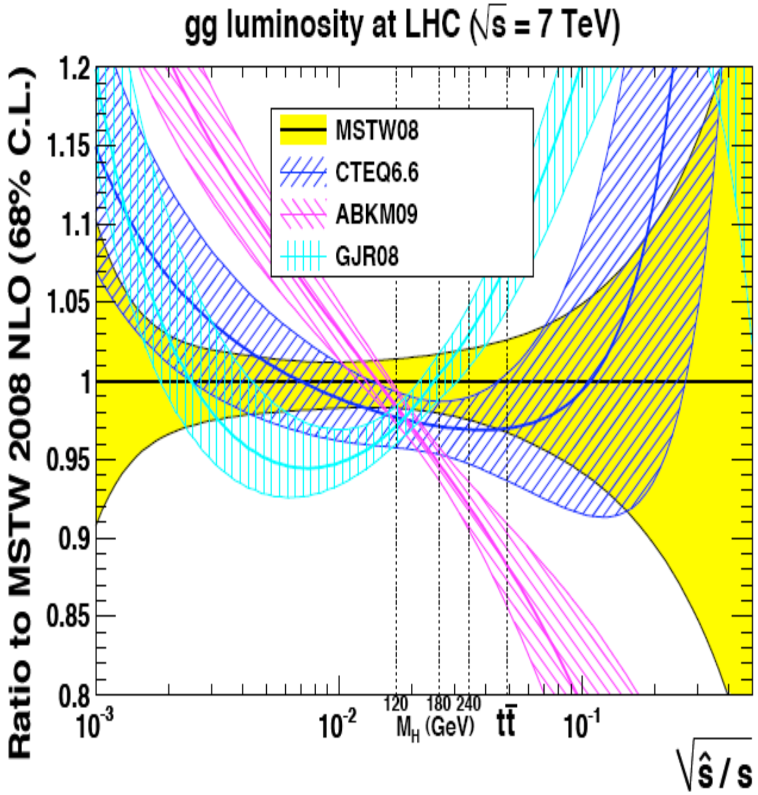
Uncertainty due to PDFs

[$PDF4LHC$]

Uncertainty due to PDFs

[*PDF4LHC*]





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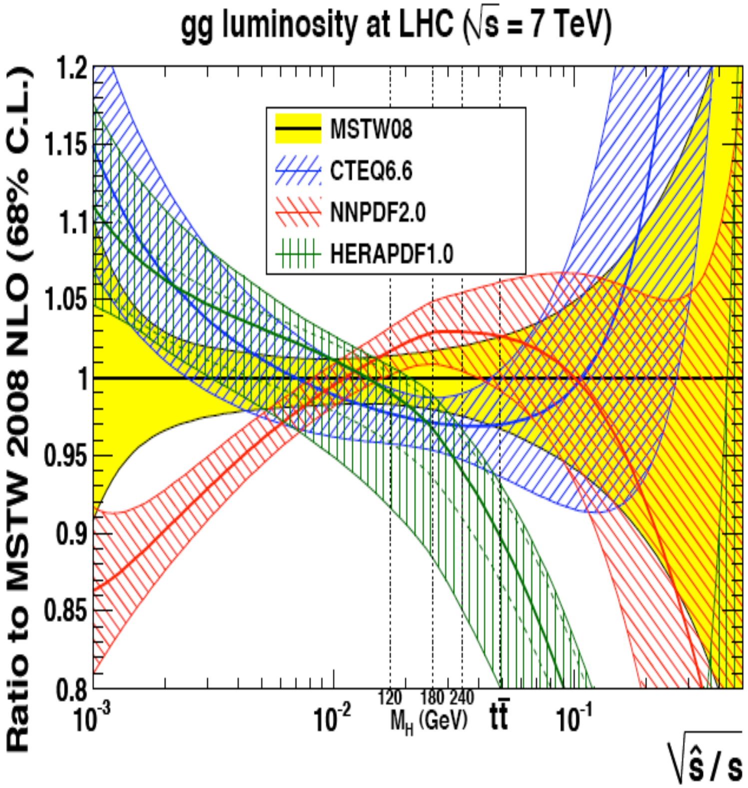
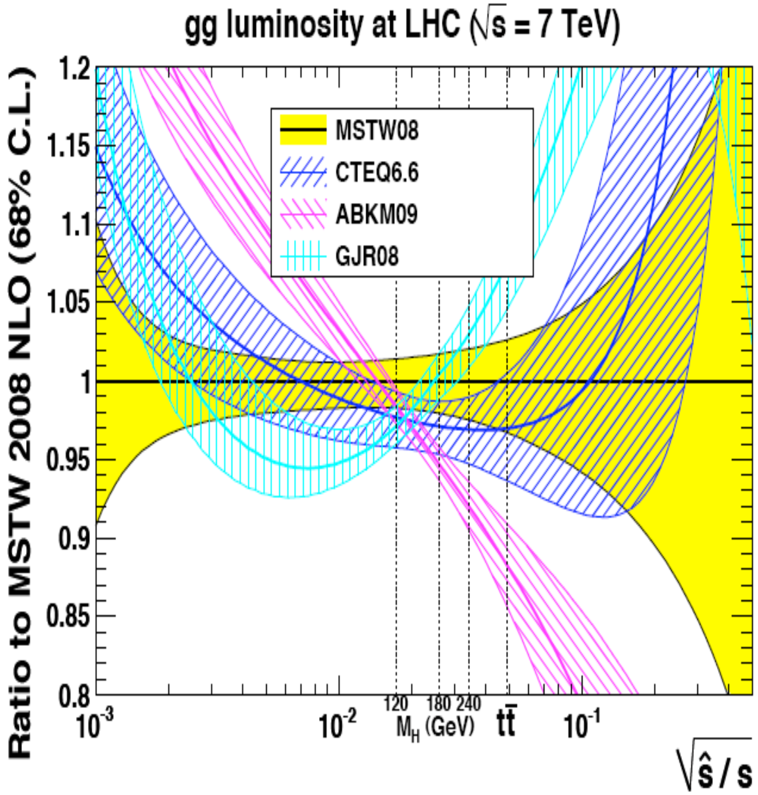
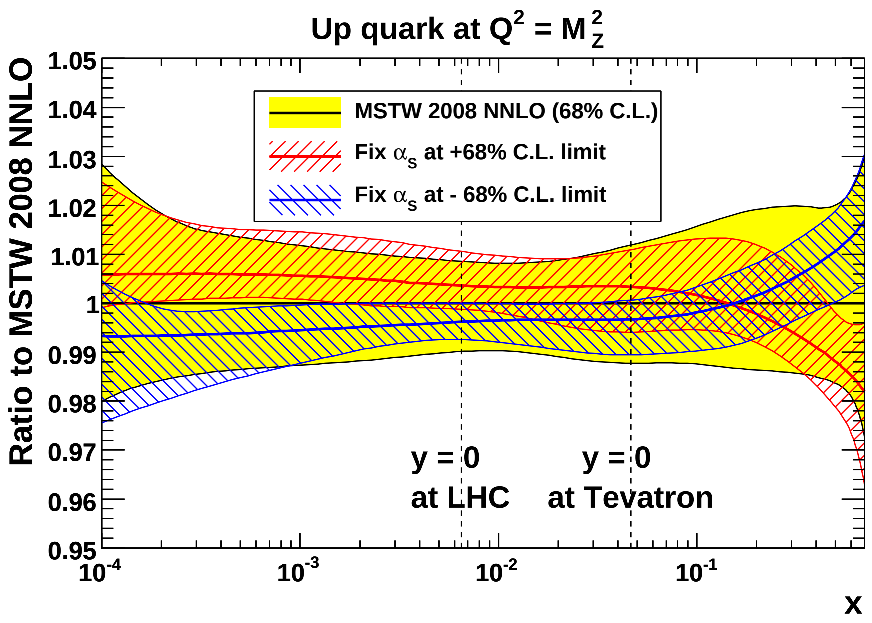




- Data sets: Electroproduction, hadron production (fixed target and collider)
- Fits procedure: Hessian and Monte Carlo
- Treatment: α_s, m_b and m_c

Uncertainty in Quark and Gluon distribution function



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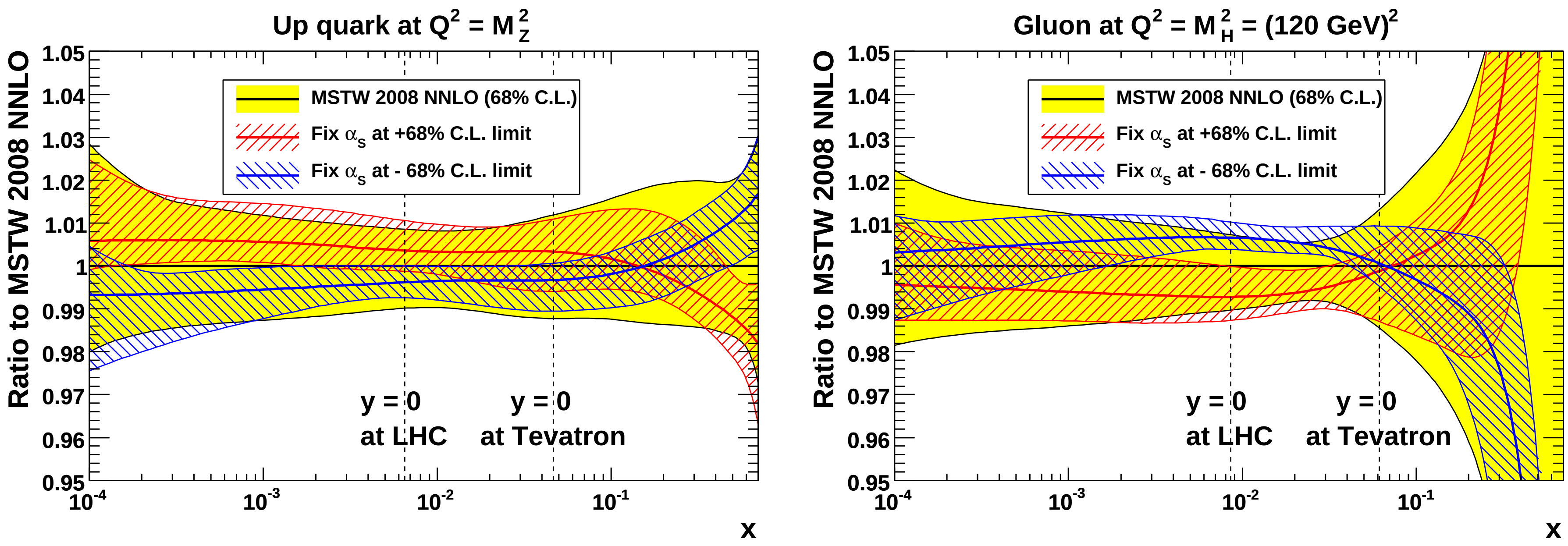
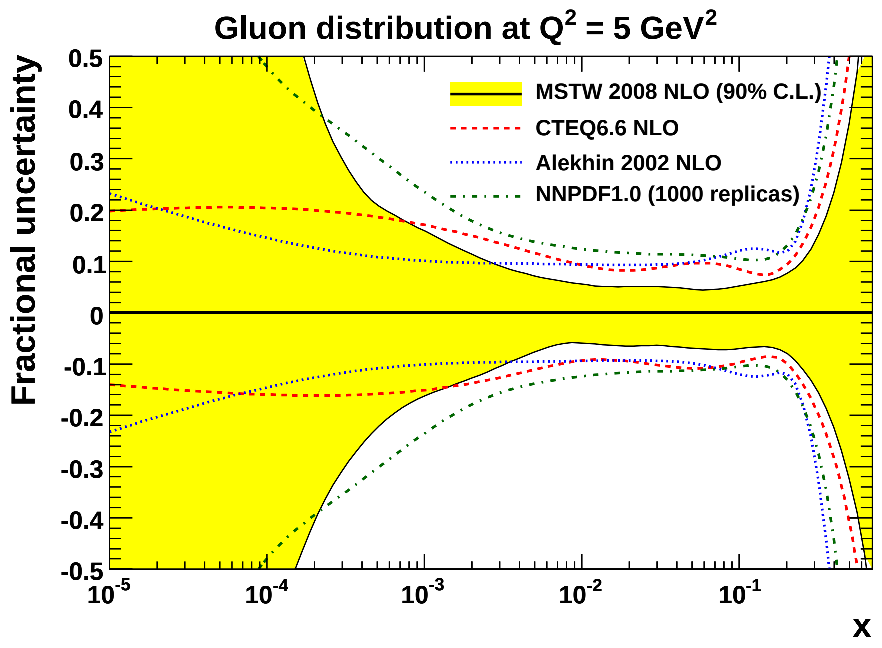


A comparison of the fractional uncertainty for the present MSTW, CTEQ6.6 , Alekhin and NNPDF1.0 NLO gluon distributions at $Q^2 = 5$ GeV^2. All uncertainty bands represent a 90% C.L. limit.

Uncertainty in gluon distribution



PDF and α_s

- For consistent prediction, PDFs along with appropriate α_s have to be used.
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NNLO	$\alpha_S(M_Z^2)$ (expt. unc. only)
MSTW	$0.1171\ ^{+0.0014}_{-0.0014}$
AMP	$0.1128\ \pm 0.0015$
BBG	$0.1134\ ^{+0.0019}_{-0.0021}$
ABKM	$0.1129\ \pm 0.0014$
JR	$0.1158\ \pm 0.0035$

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Redefinition using $Z(x, \mu_F^2)$:

$$\begin{aligned} \Phi_{ab}(x, \mu_F^2) &= Z(\mu_F^2) \otimes \overline{\Phi_{ab}}(\mu_F^2) \\ d\hat{\sigma}_{ab}(x, \mu_F^2) &= Z^{-1}(\mu_F^2) \otimes d\overline{\hat{\sigma}_{ab}}(\mu_F^2) \end{aligned}$$

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Observables do not depend on μ_F^2

$$\mu_F^2 \frac{d}{d\mu_F^2} d\sigma^{P_1 P_2}(\tau, Q^2) = 0$$

Scale variation

$$\mu_F \to \lambda \mu_F \qquad \text{where} \qquad \lambda^2 > 0$$

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Redefinition using $Z(x, \mu_F^2)$:

$$\begin{aligned}
\Phi_{ab}(x, \mu_F^2) &= Z(\mu_F^2) \otimes \overline{\Phi_{ab}}(\mu_F^2) \\
d\hat{\sigma}_{ab}(x, \mu_F^2) &= Z^{-1}(\mu_F^2) \otimes d\overline{\hat{\sigma}_{ab}}(\mu_F^2)
\end{aligned}$$

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- Collinear finite partonic cross sections are calculable in perturbative QCD in powers of α_s^B, bare strong coupling constant.

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- $Soft$ divergences disappear thanks to KLN theorem

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$$+ \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \left(\mu_R^2 \frac{d}{d\mu_R^2} d\sigma^{(i)}(\mu_R^2) \right)$$

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Using RG equation for $a_s(\mu_R^2)$,

$$-\sum_{i=1}^{\infty} i\, a_s^{i+1}(\mu_R^2) \beta_0\, d\sigma^{(i)}(\mu_R^2) + \cdots + \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \mu_R^2 \frac{d}{d\mu_R^2} d\sigma^{(i)}(\mu_R^2) = 0$$

The cancellation of μ_R dependence is not order by order in a_s.

Higgs production at Leading Order(LO)

Hinchcliff, many others

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$$2S\, d\sigma^{PP}(x, m_H) = \int_x^1 \frac{dz}{z} \Phi_{gg}^{(0)}(z, \mu_F)\, 2\hat{s}\, d\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) + \cdots$$

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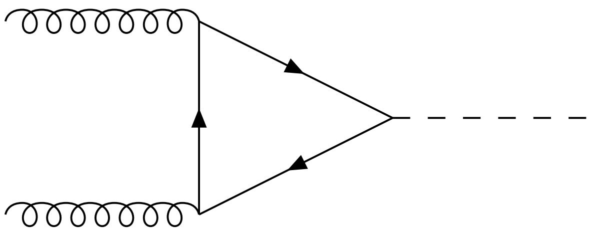

- μ_R-renormalisation scale
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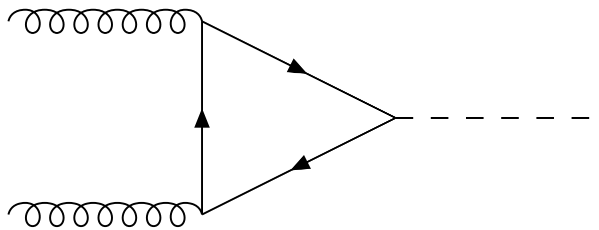

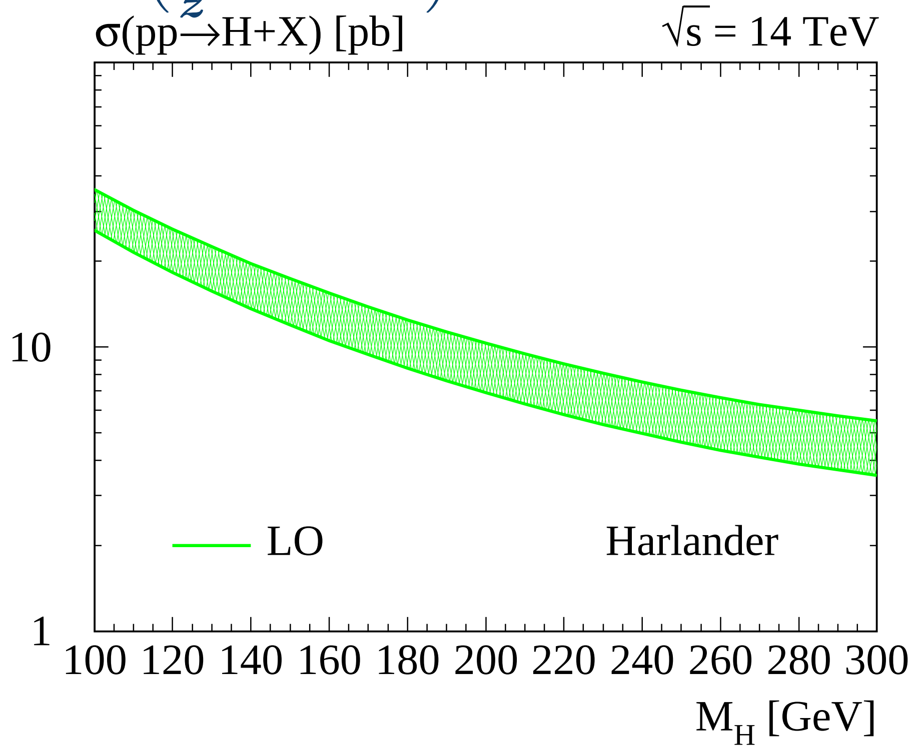

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$$2\hat{s}\, \hat{\sigma}_{gg}^{(0)}(\hat{s}, \mu_R) \sim \alpha_s^2(\mu_R)\, G_F \left[\frac{4m_t^2}{m_H^2} F\left(\frac{4m_t^2}{m_H^2}\right)\right], \qquad \frac{m_H}{2} < \mu_R = \mu_F < 2m_H$$

LO prediction is Unreliable due $100 - 200\%$ scale uncertainity

NNLO QCD corrected Higgs Cross section at $\sqrt{S} = 14$ TeV

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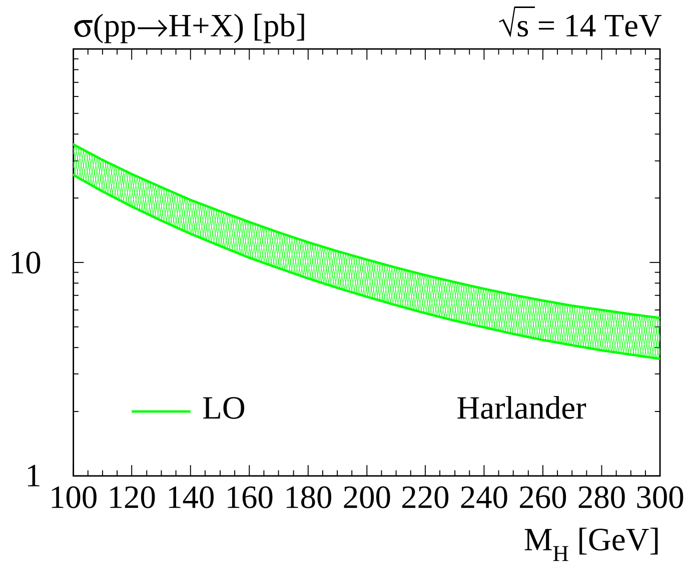

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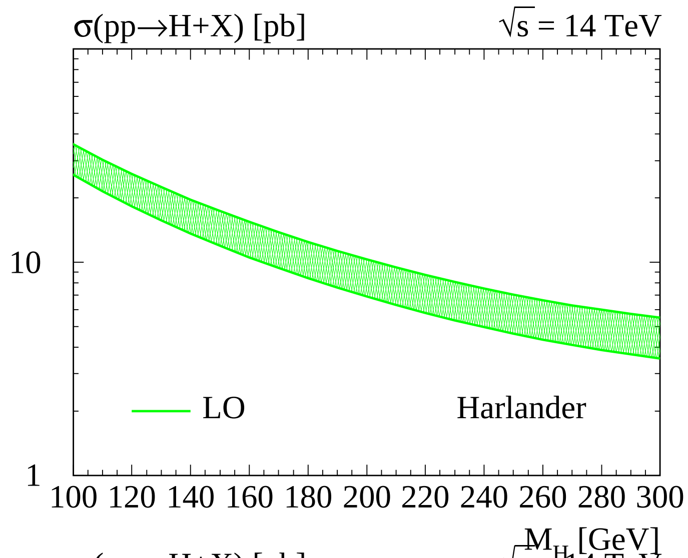

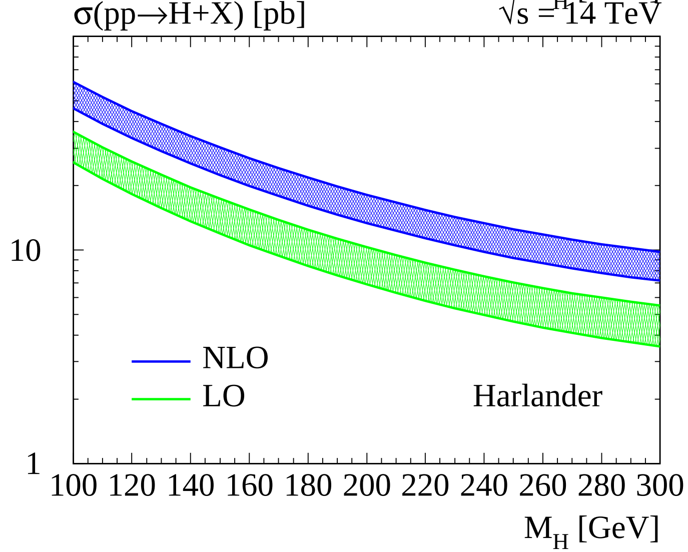

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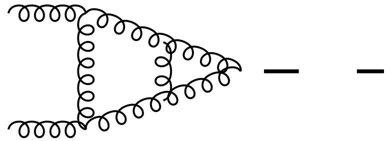

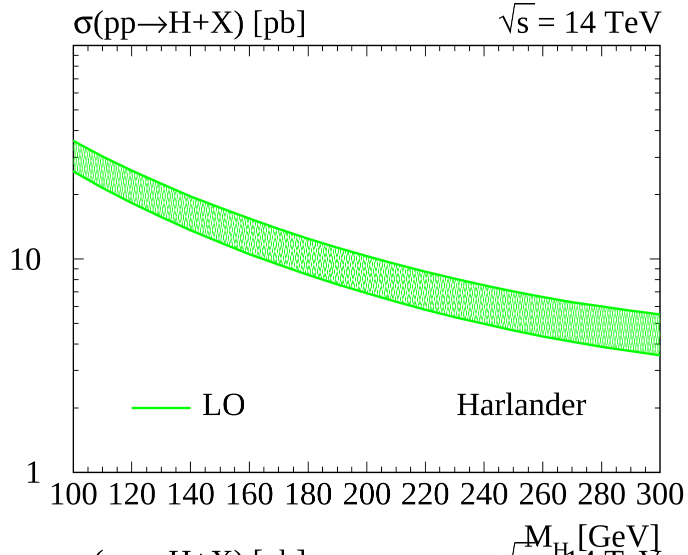

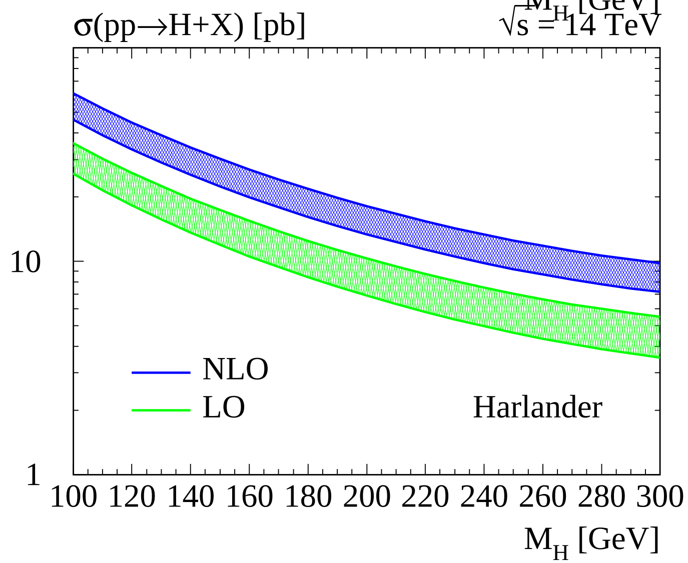

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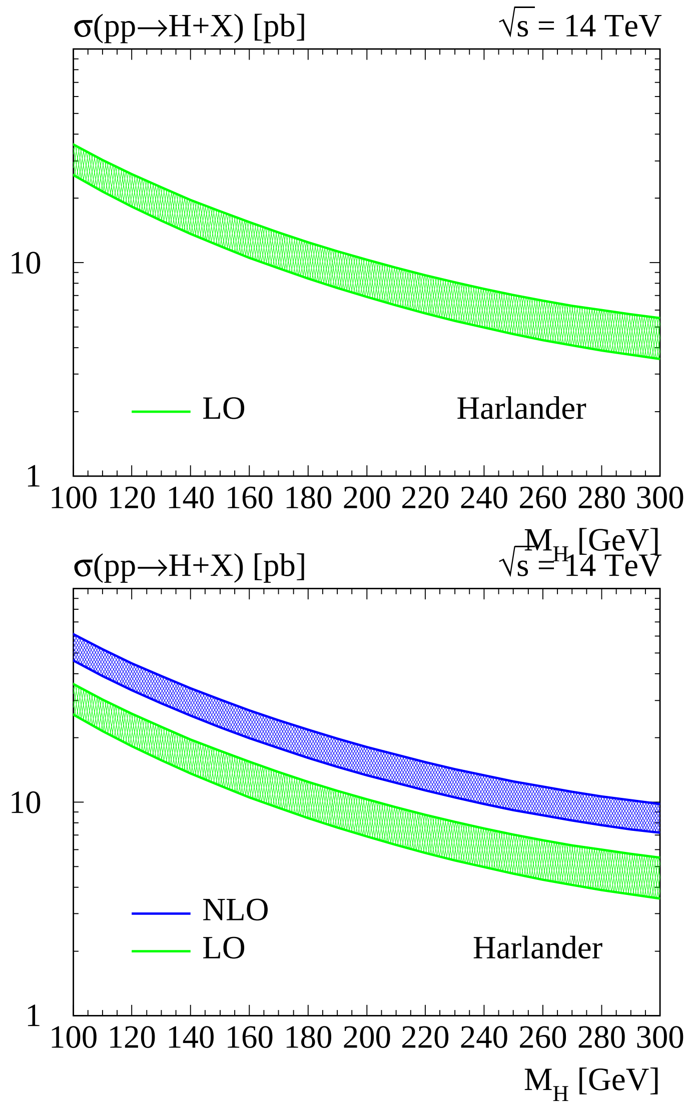
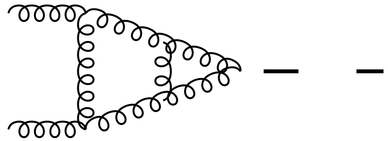
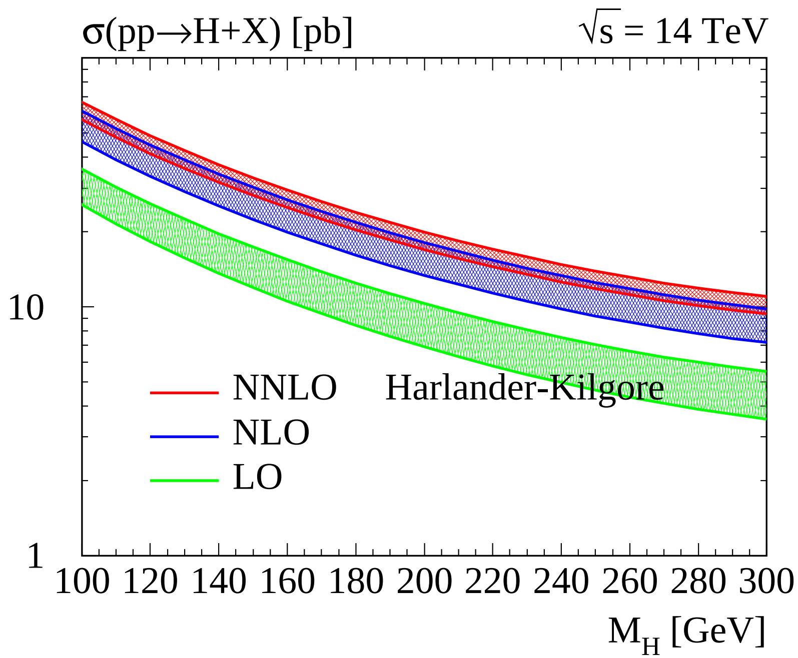
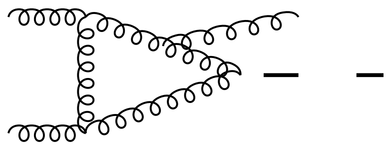


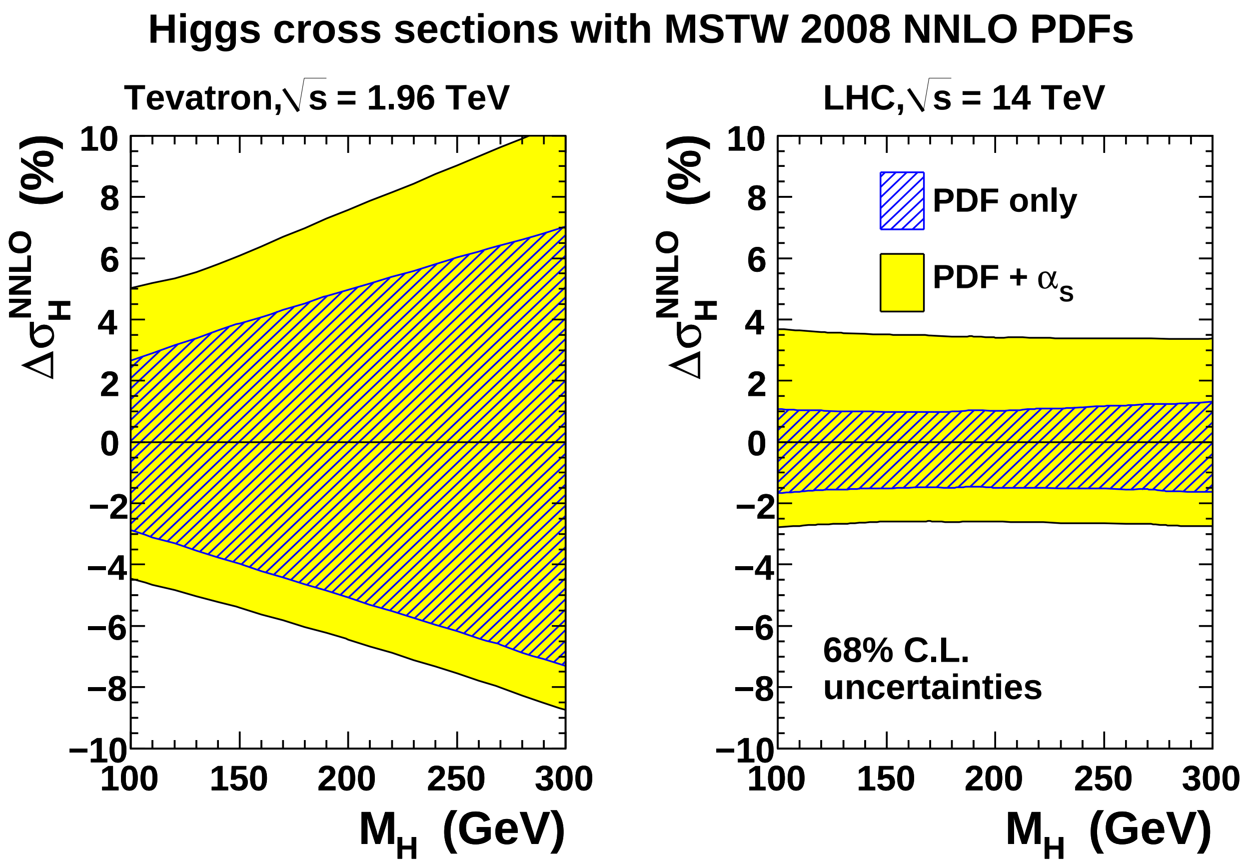






"NNLO": Harlander, Kilgore; Anastasiou Melnikov; Smith, Ravindran, van Neerven

Uncertainty in Higgs cross section



NNLO gluon distribution at $Q^2 = M_H^2 = (120\,\mathrm{GeV})^2$. The values of $x = M_H/\sqrt{s}$ relevant for central production (assuming $p_T^H = 0$) at the Tevatron and LHC are indicated.

Scale variation at N^3LO_{pSV} for Higgs production

Vogt, Moch, V. Ravindran

$$R = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$

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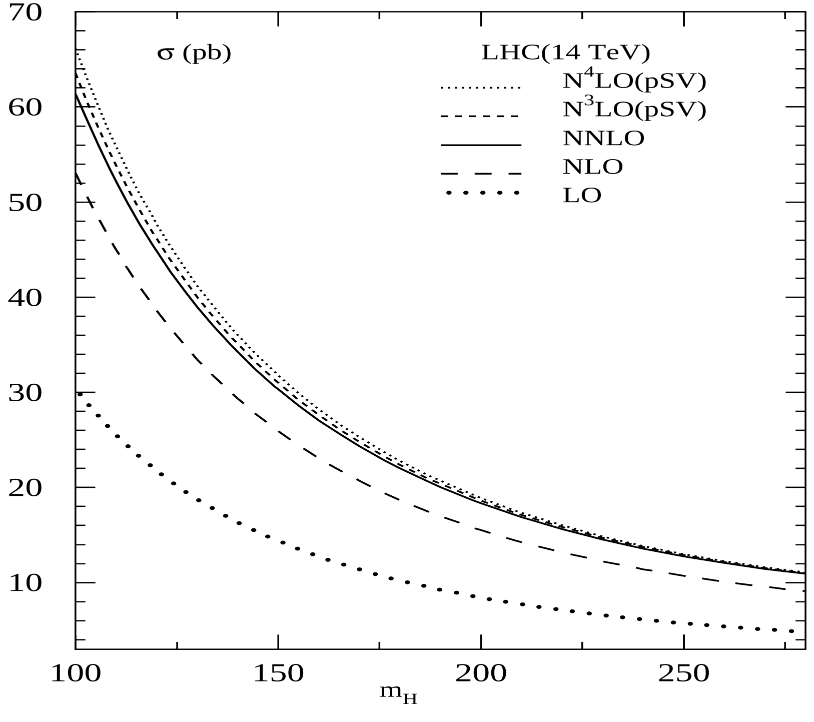

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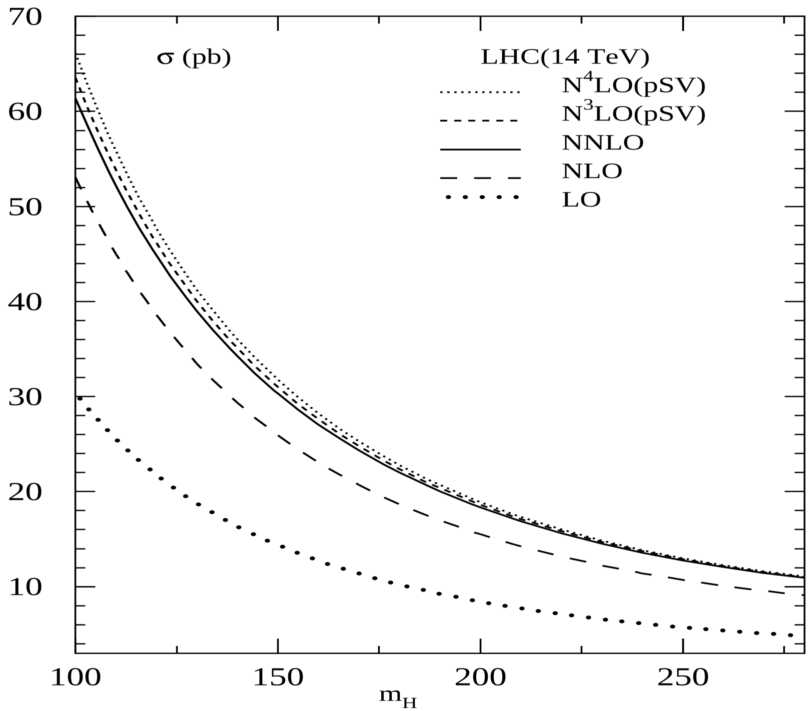

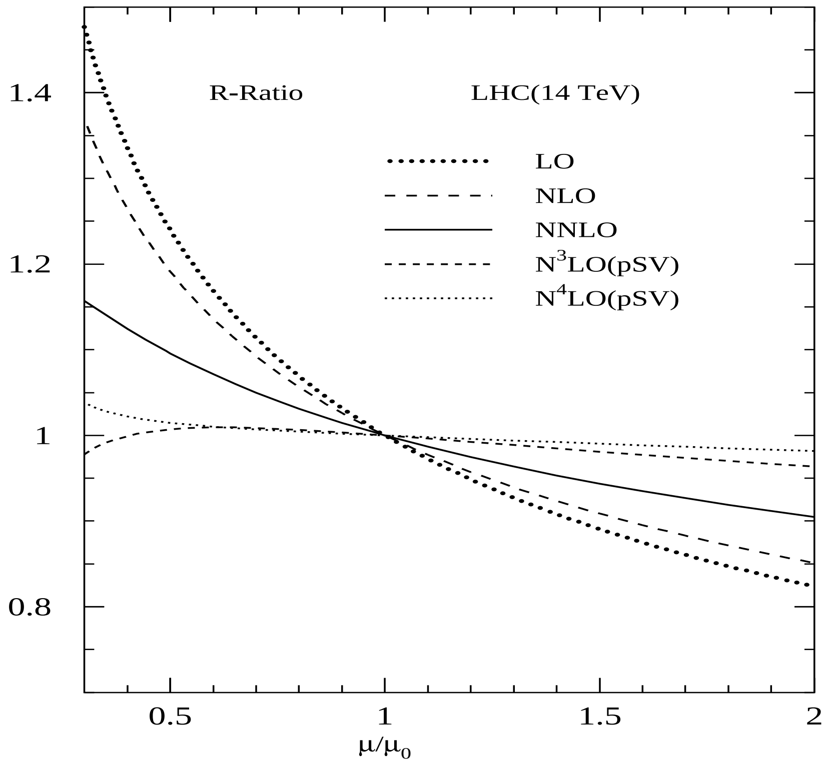

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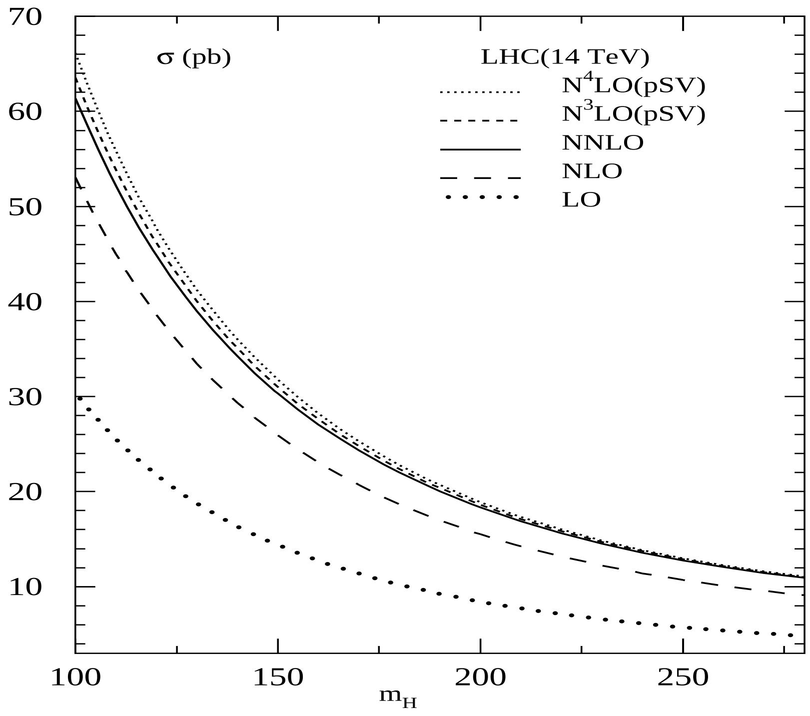

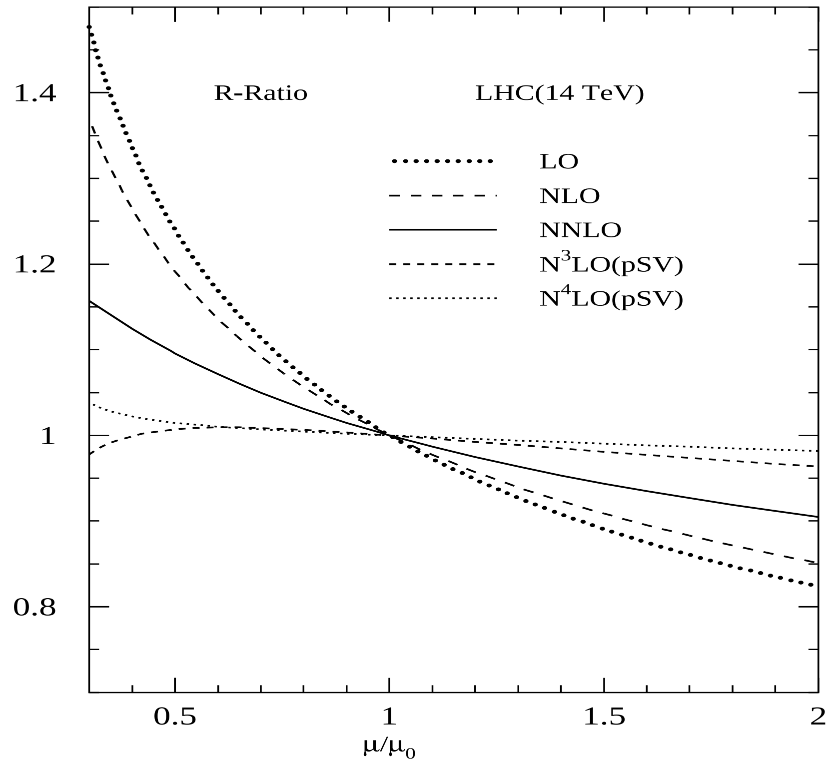

- Scale uncertainity improves a lot
- Additional $7-9\%$ increase in cross section due to N^3LO soft gluons.

Resummed results for total cross section

Catani and Grazzini; Vogt and Moch

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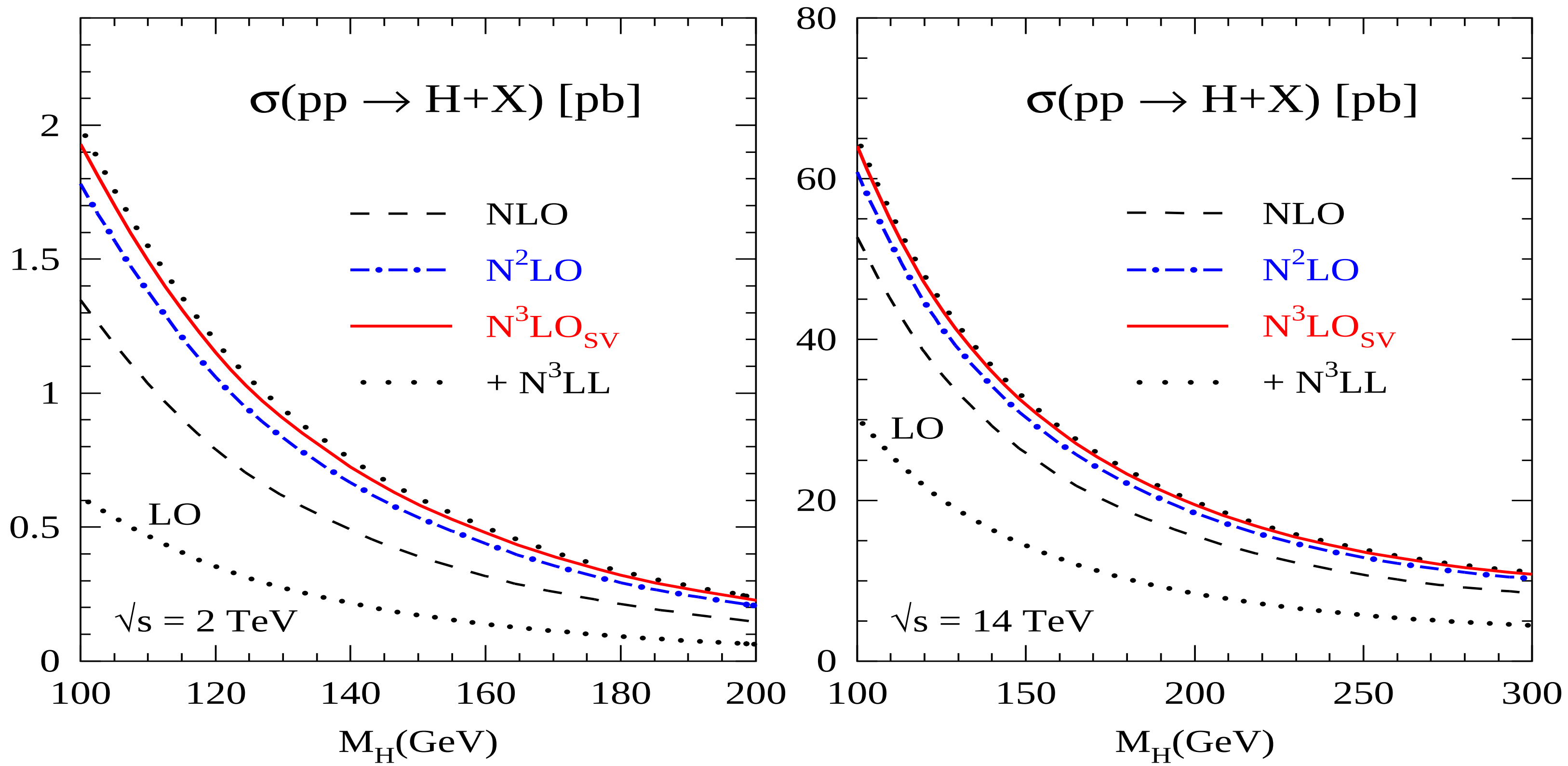

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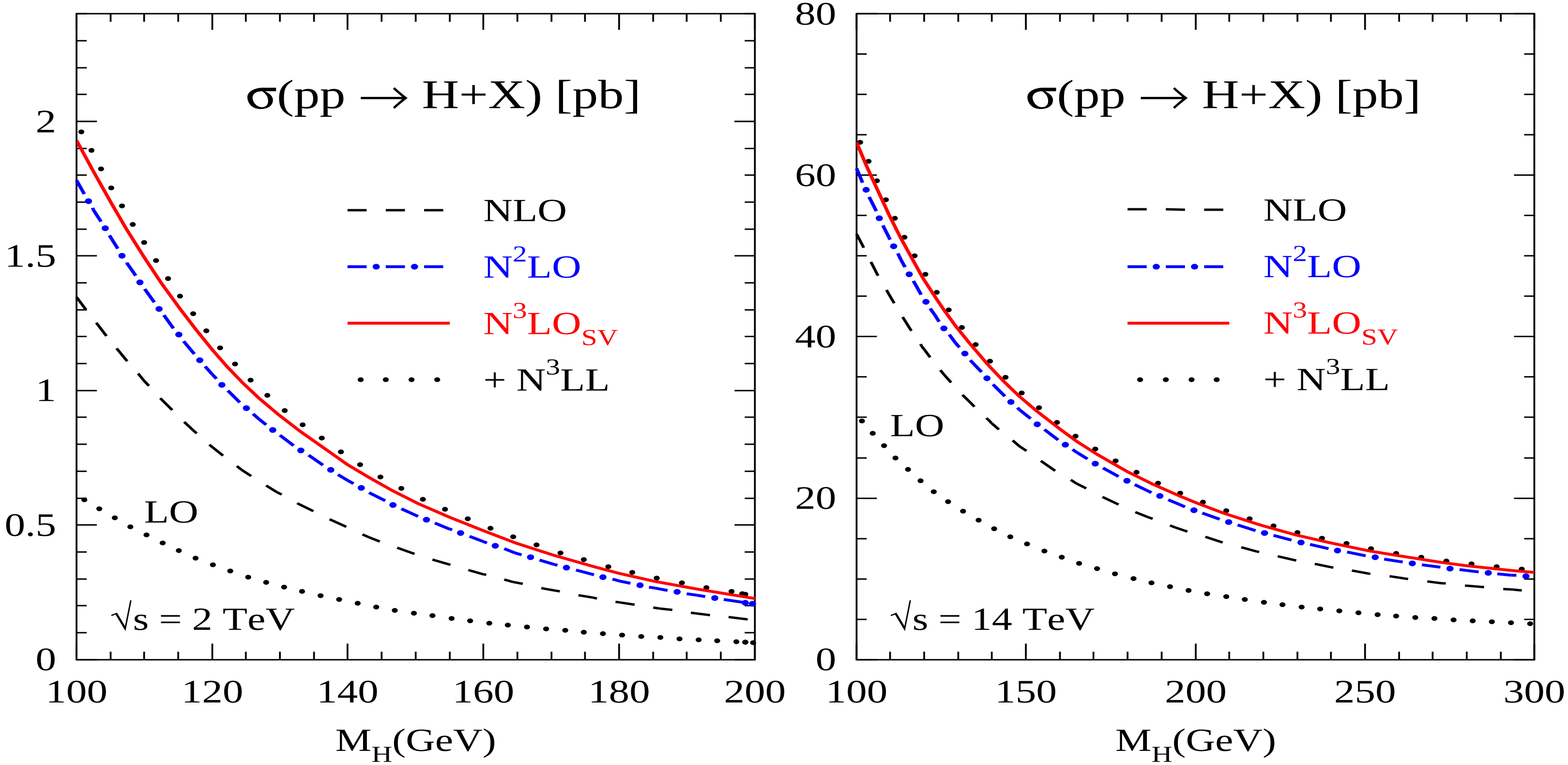

- N^3LL resummation exponents are available now.

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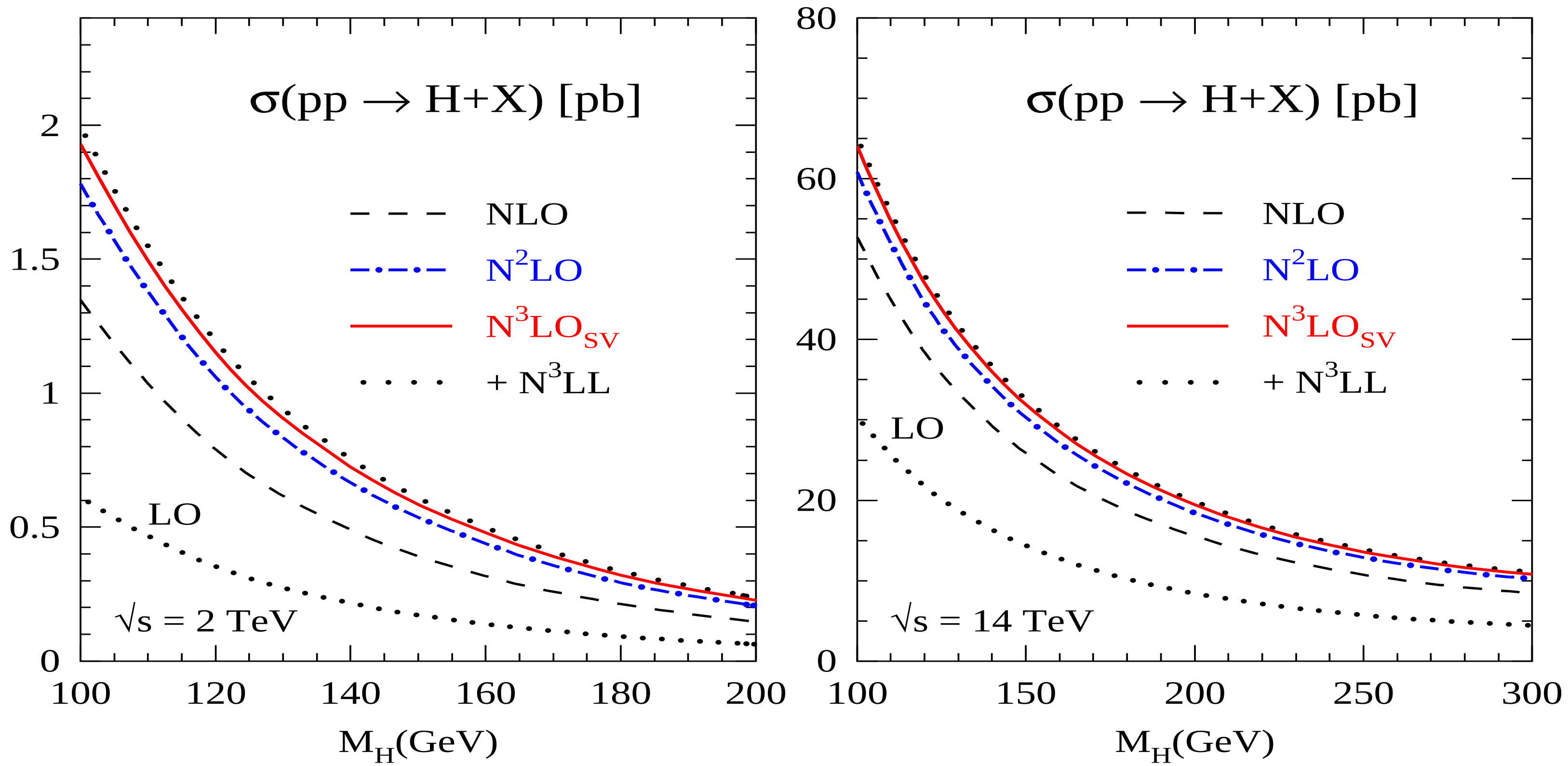

- N^3LL resummation exponents are available now.
- N^3LL resummation does not change the picture much. Fixed order N^3LO_{pSV} is very close to the N^3LL resummed result.

Flux at LHC and Tevatron

$$\Phi_{ab}(x,\mu_F) \quad = \quad \int_x^1 \frac{dz}{z} f_a\left(z,\mu_F\right) f_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad x = \frac{Q^2}{S}$$

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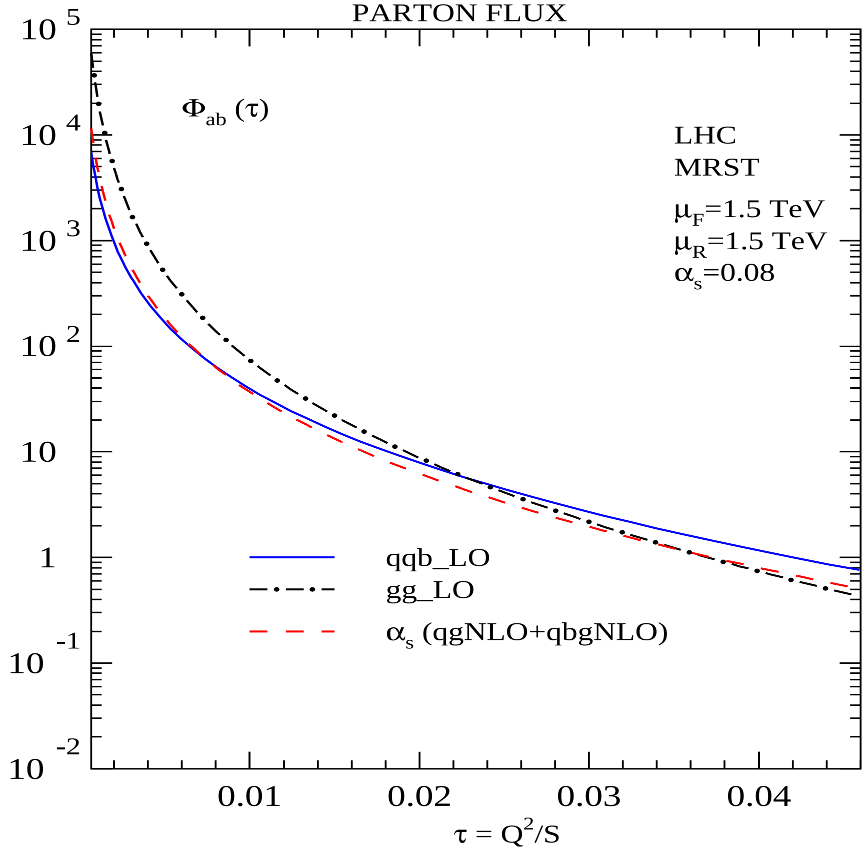

Gluon flux is largest at LHC

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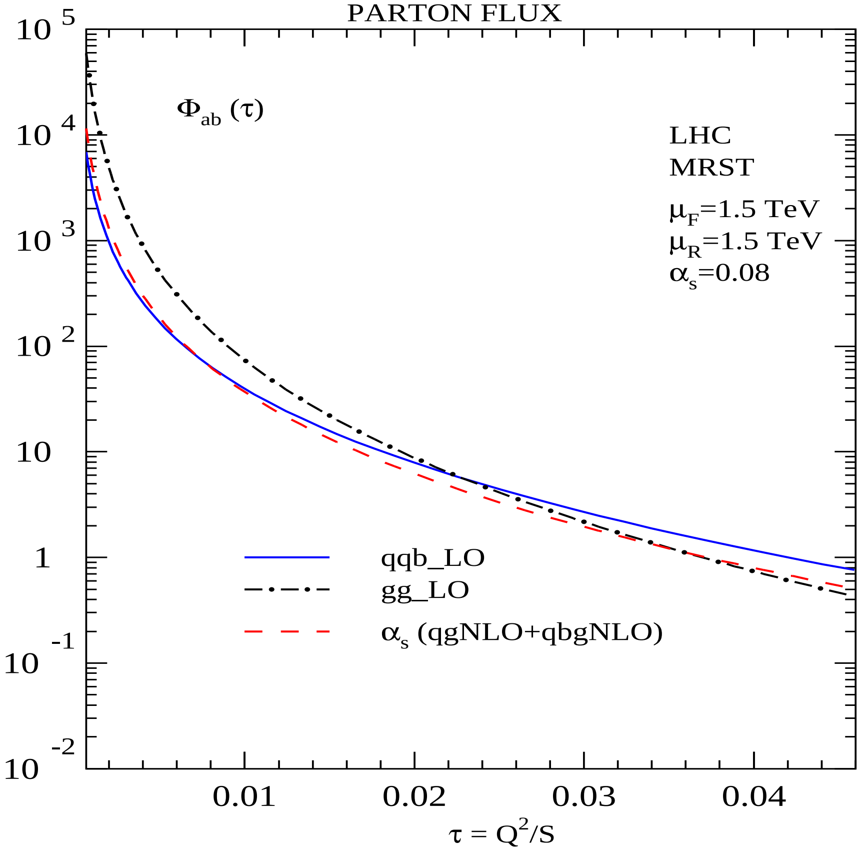

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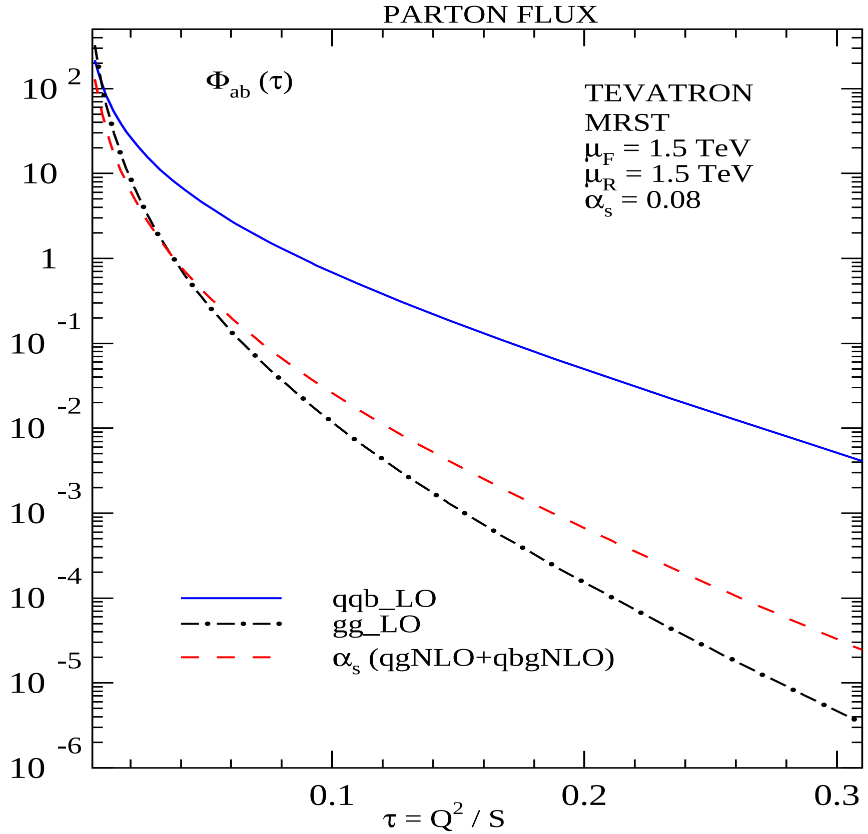
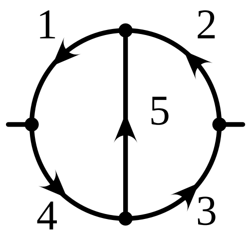


Quark-anti quark flux is largest at Tevatron

Hurdles at NNLO

Hurdles at NNLO

- Virtual processes:



$$I^{\mu_1\mu_2\cdots\nu_1\nu_2}(n_1,\ldots,n_5) = \int \frac{\mathrm{d}^n p}{(2\pi)^n} \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{p^{\mu_1} p^{\mu_2} \cdots k^{\nu_1} k^{\nu_2} \cdots}{(p^2+m_1^2)^{n_1} \cdots (k^2+m_5^2)^{n_5}} \,,$$

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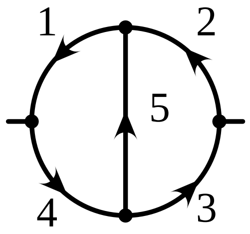

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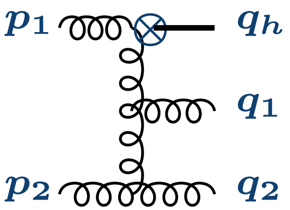
- Real emission processes:

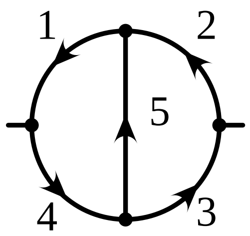


$$\left| \quad \right|^2 \sim \int d^n q_1 \int d^n q_2 \; \frac{\delta(q_1^2)\;\delta(q_2^2)\;\delta(q_h^2 - m_h^2)}{[(q_h-p_1)^2]^2\,[(q_2-p_2)^2]^2} \quad [\ldots]$$

Integration by Parts

Chetyrkin and Tkachov

Integration by Parts



Chetyrkin and Tkachov

$$I(n_1,\ldots,n_5) = \int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{(p^2+m_1^2)^{n_1} \cdots (k^2+m_5^2)^{n_5}}\,,$$

Integration by Parts

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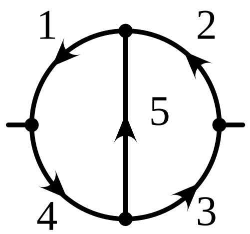

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Integration by parts:

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Integration by Parts

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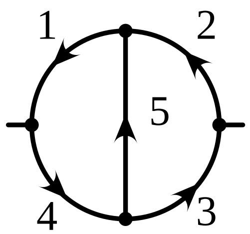

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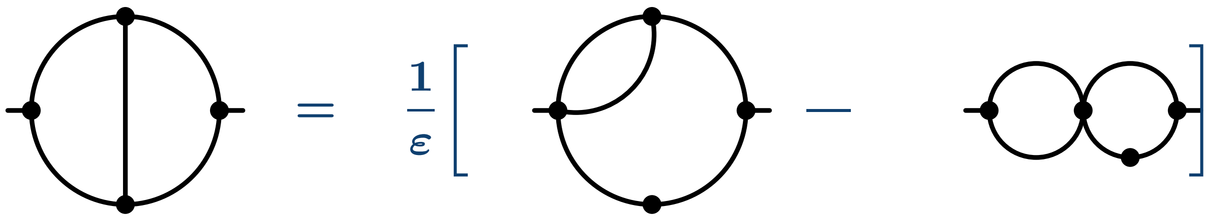
Recursion Relation($m_i = 0$):

$$I(n_1,\ldots,n_5) = \frac{1}{n-2n_5-n_2-n_3}\left[n_2\mathbf{2}^+\left(\mathbf{5}^- - \mathbf{1}^-\right)+n_3\mathbf{3}^+\left(\mathbf{5}^- - \mathbf{4}^-\right)\right] I(n_1,\ldots,n_5)\,.$$



Tensorial Reduction

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$$\frac{1}{k^2 - m^2 + i\epsilon} = \frac{1}{i} \int_0^\infty \mathrm{d}\alpha e^{i\alpha(k^2 - m^2 + i\epsilon)} ,$$

We arrive at

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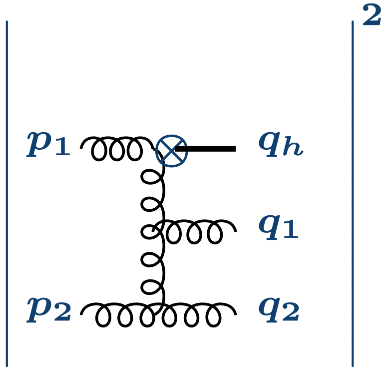
$$T_{\mu\nu} \left\langle \frac{1}{D_1 D_2 D_3} \right\rangle_n = (4\pi)^2 \left[2\, p_{1\mu} p_{1\nu} \left\langle \frac{1}{D_1 D_2^3 D_3} \right\rangle_{n+4} + \cdots \right]$$

Most convenient for Two loop integrals with higher rank tensors.

Reduction of Phase space

Melnikov, Anastasiou



$$\sim \int d^d q_1 \int d^d q_2 \, \frac{\delta(q_1^2)\,\delta(q_2^2)\,\delta(q_h^2 - m_h^2)}{[(q_h - p_1)^2]^2\,[(q_2 - p_2)^2]^2}\,[\ldots]$$

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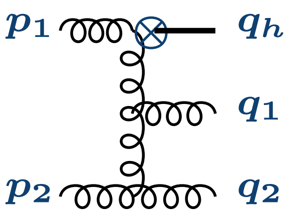

$$\left| \; \cdots \; \right|^2 \sim \int d^d q_1 \int d^d q_2 \frac{\delta(q_1^2)\,\delta(q_2^2)\,\delta(q_h^2 - m_h^2)}{\left[(q_h - p_1)^2\right]^2 \left[(q_2 - p_2)^2\right]^2} [\ldots]$$

Use the trick:

$$2i\pi\delta\left(p^2 - m^2\right) \to \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}.$$

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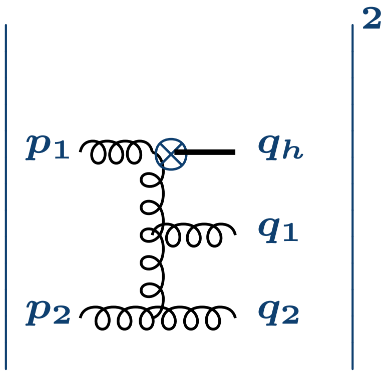

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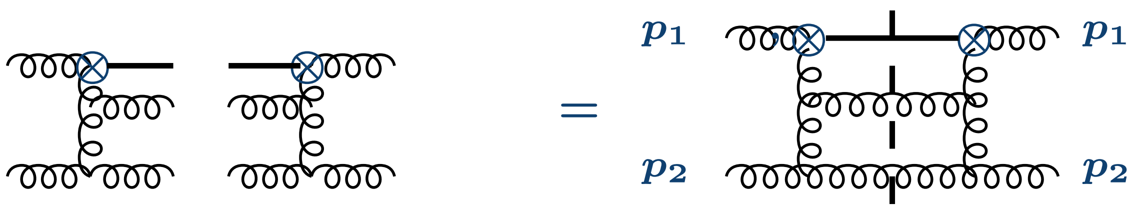

- They look like Two loop virtual diagrams and hence loop techniques can be applied.
- Integration by parts
- Recursion Relations

Additional trics

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- Two loop virtual processes can be computed using Cutkosky rules.

  Two loop diagrams reduce to many cut diagrams with one loop and 3-body processes having simpler kinematics.

  Use dispersion relation to obtain the two loop results.

$$A^{2-loop}(m_h) = \int \frac{ds}{s - m_h^2} \sum_{cuts} \mathcal{A}^{2-loop}$$

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- Real emission processes can be computed by clever choice of frames.

  1) CM frame of incoming partons

  2) CM frame of 3rd and 4th partons

  3) CM frame of 4th and 5th partons

Phenomenology with Extra-Dimension

In the SM, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\frac{d}{dQ^2}\hat{\sigma}_{ab}^{SM}\left(\hat{s}, Q^2\right) \sim \frac{1}{\hat{s}Q^2}$$

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- The processes where the virtual/real KK gravitons contribute significantly:

  (1) Di-lepton or Drell-Yan production at large invariant mass Q

  (2) Di-photon or Di-boson production at large Q, P_T

  (3)Observables with missing energy (...) $\cdots$

Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \to [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$

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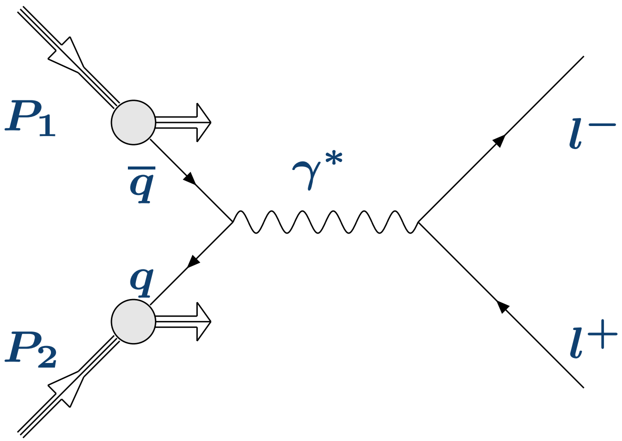

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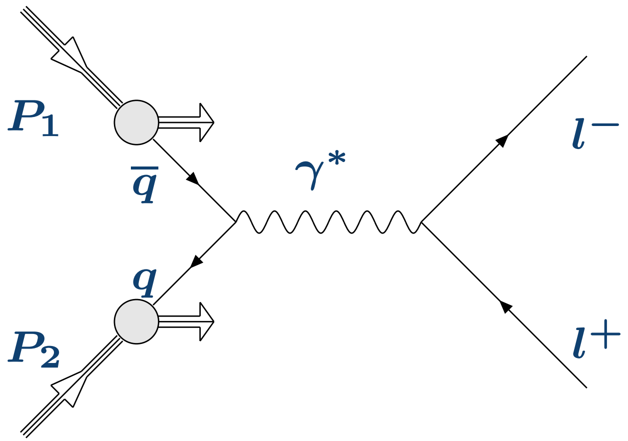

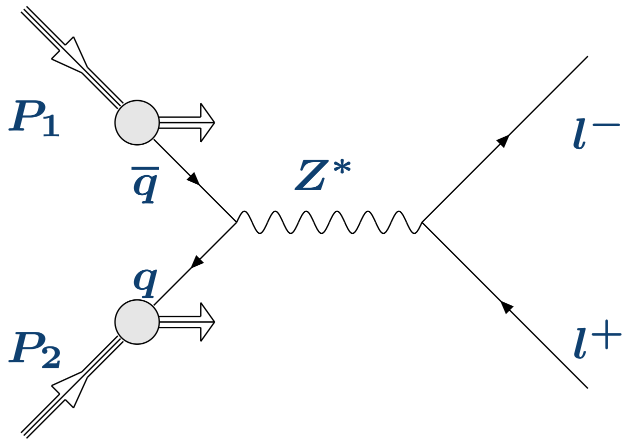

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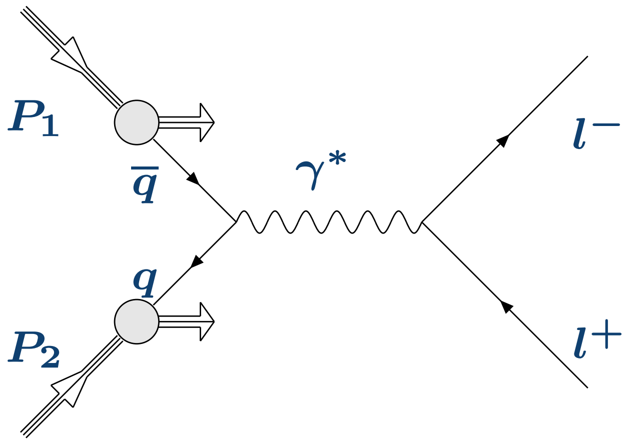

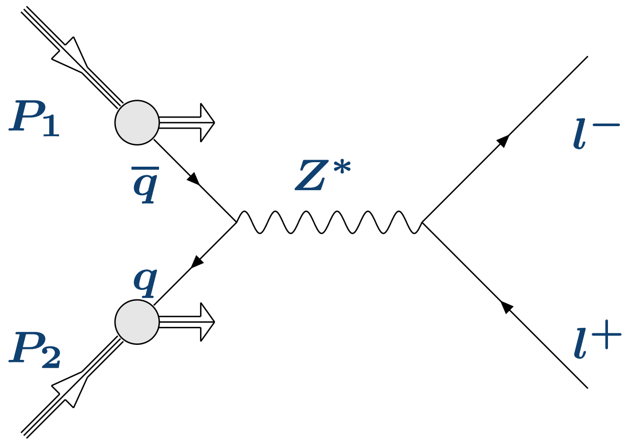

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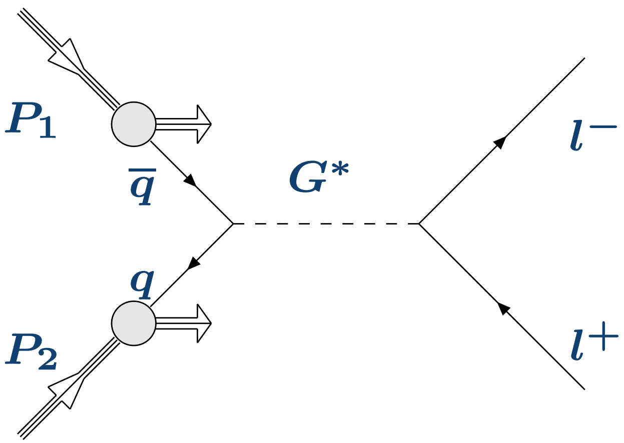

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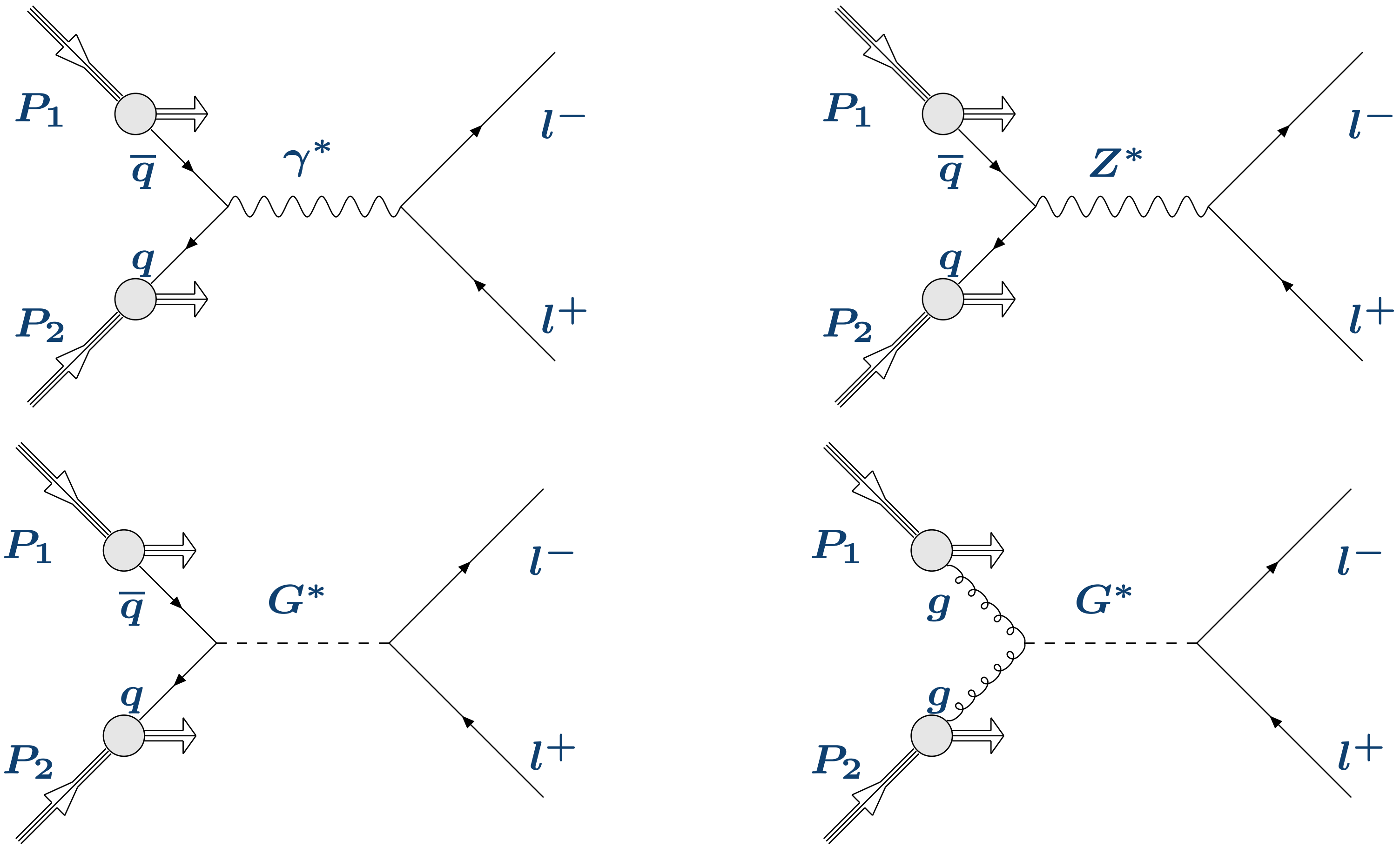


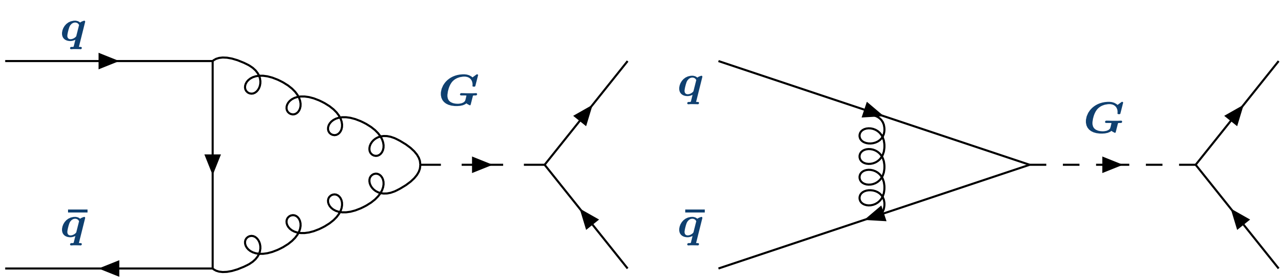
SM

Gravity

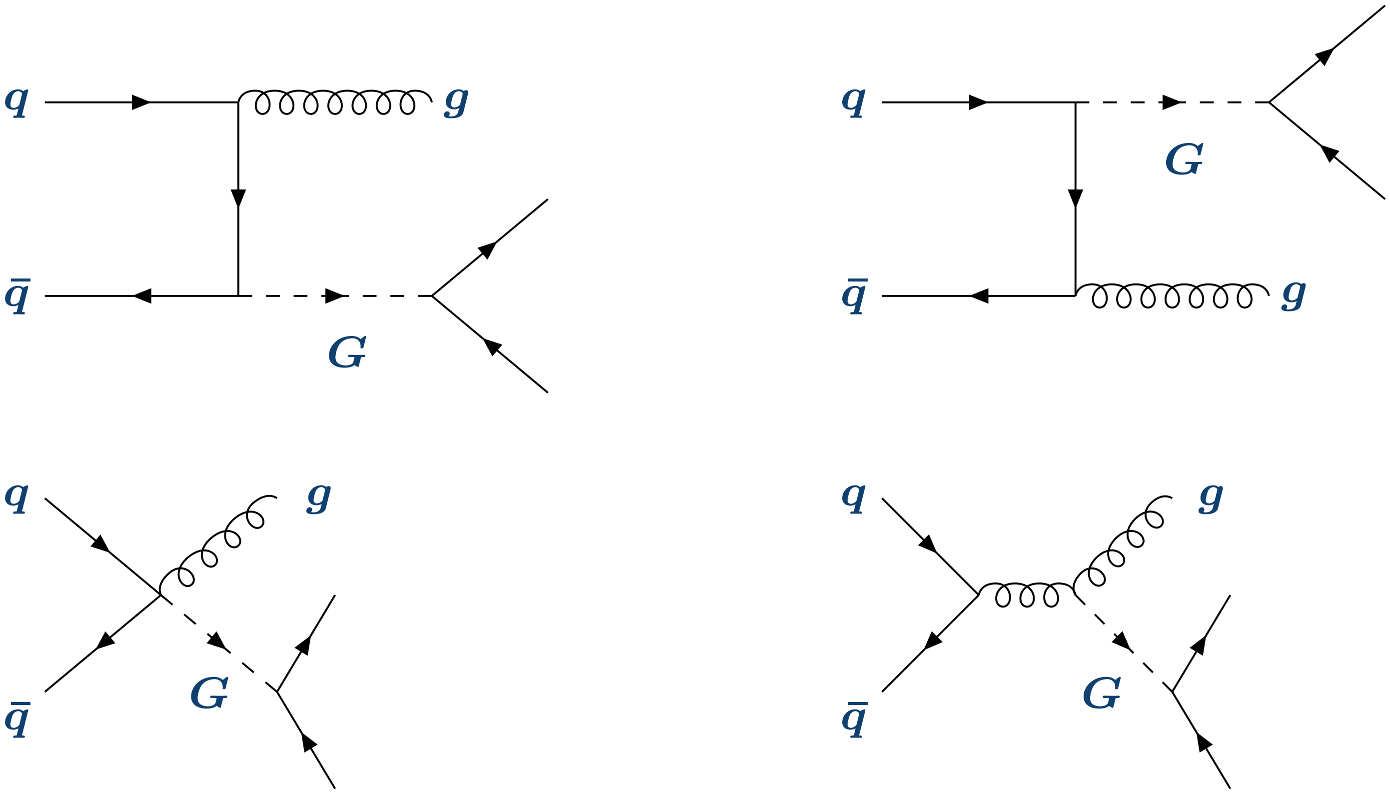
Virtual Corrections, $q\,\bar{q} \to G$

$$\bar{\Delta}^{G}_{q\bar{q}} = \Delta^{(0)G}_{q\bar{q}} + a_s \frac{2}{\varepsilon} \Gamma^{(1)}_{qq} \otimes \Delta^{(0)G}_{q\bar{q}} + a_s \Delta^{(1)G}_{q\bar{q}}$$

$q + \bar{q} \to G$ (1 loop):



Real emission, $q\ \bar{q} \to g\ G$



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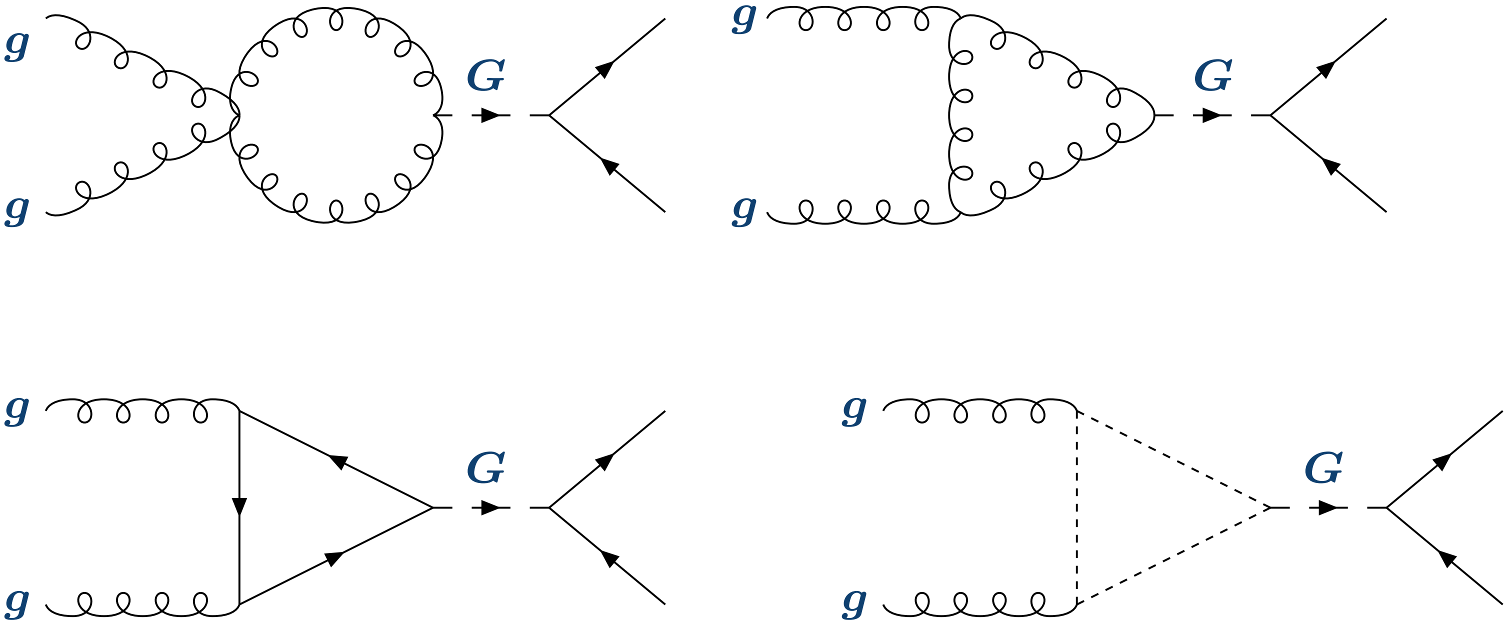

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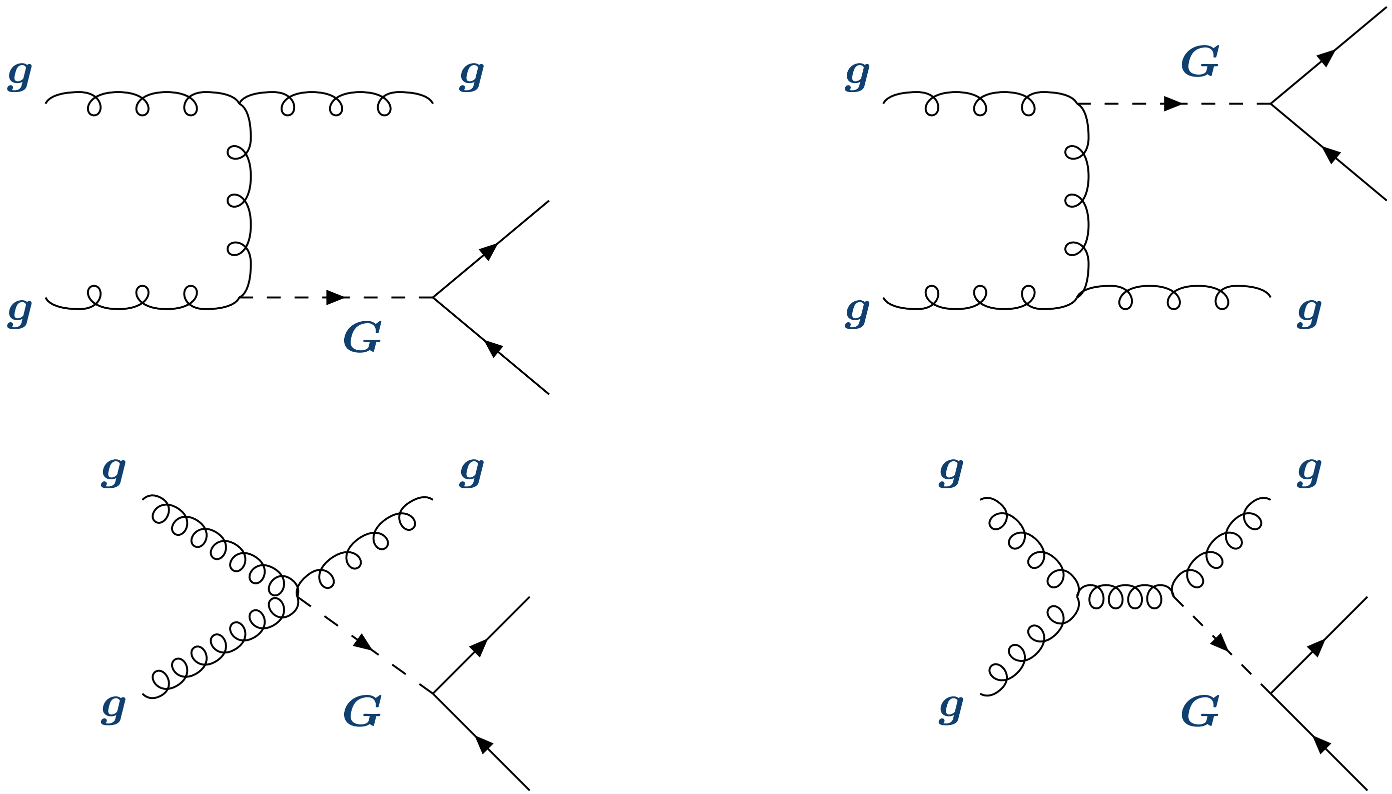

Real emissions, $q\,\bar{g} \to q\,G$

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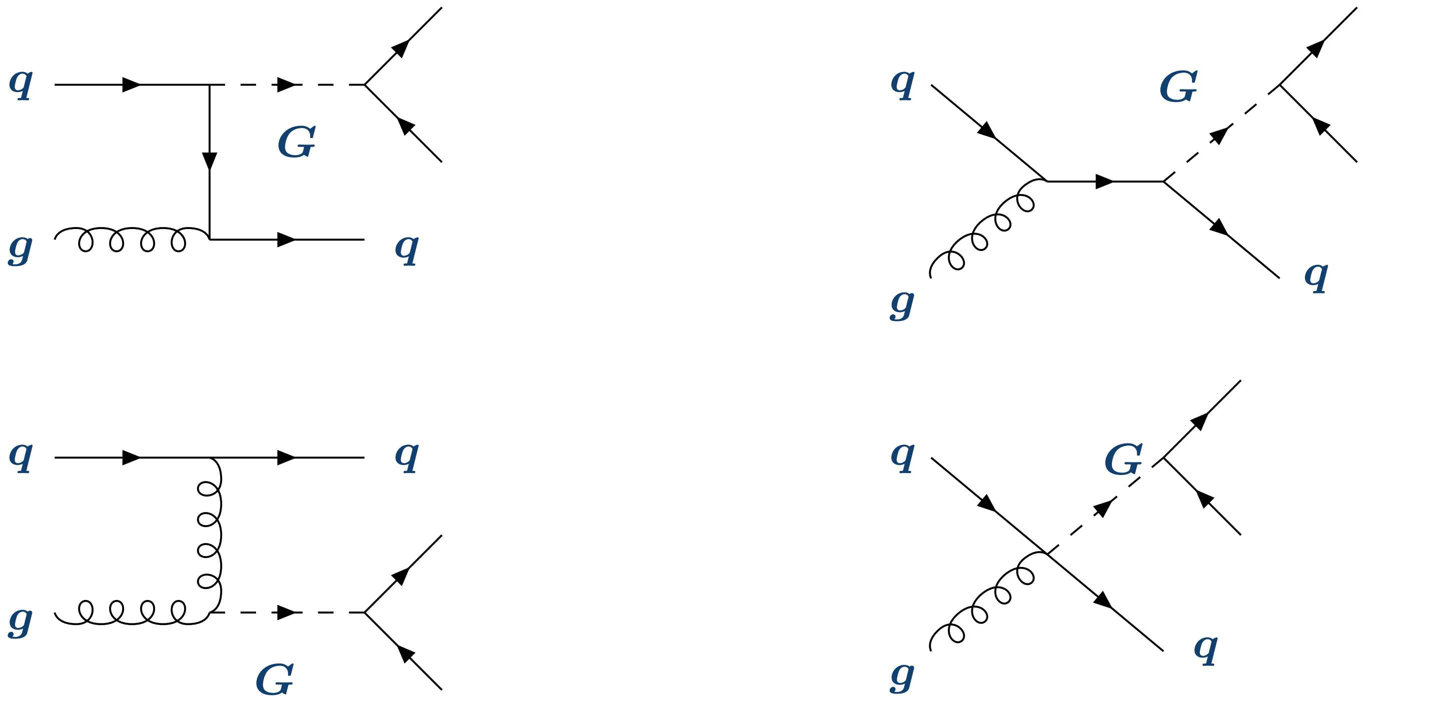

Invariant mass distribution of lepton pair

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Coefficient Functions Independent of ADD or RS Model

RS Results

	LHC	TEVATRON
$\sqrt{S}$	14 TeV	1.96 TeV

PDF Choice of scale	MRST 2001 LO & NLO $\mu_F = \mu_R$ & $\mu_F = Q$

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Partonic Cross Section versus its Flux

- Parton level cross section increases monotonically with invariant lepton pair mass Q upto m_o.

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- At large Q the parton "cross section" dominates over "Flux" leaving "observable effect".

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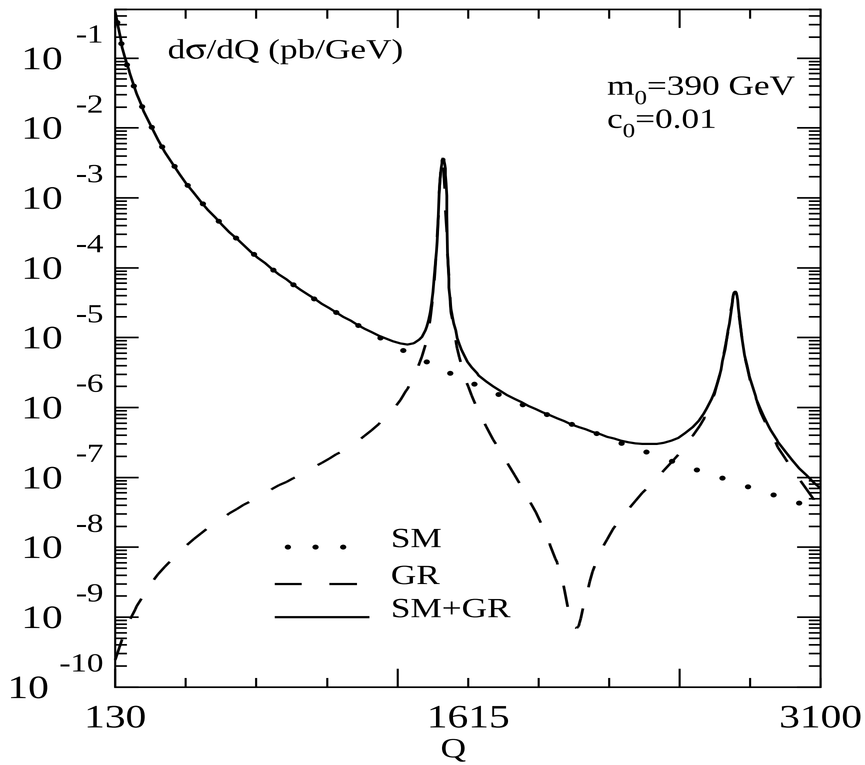
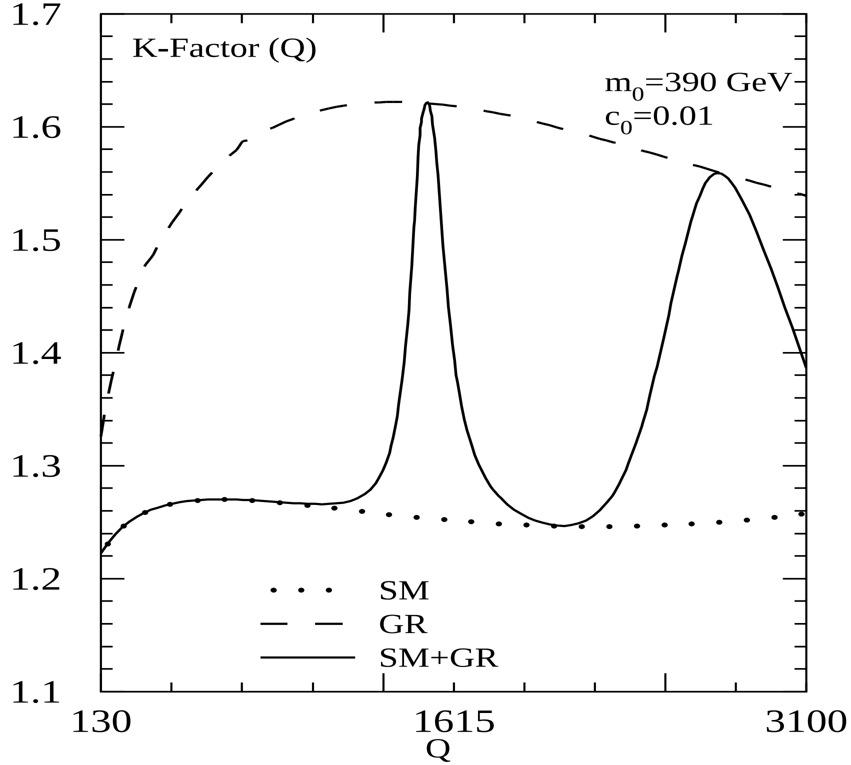
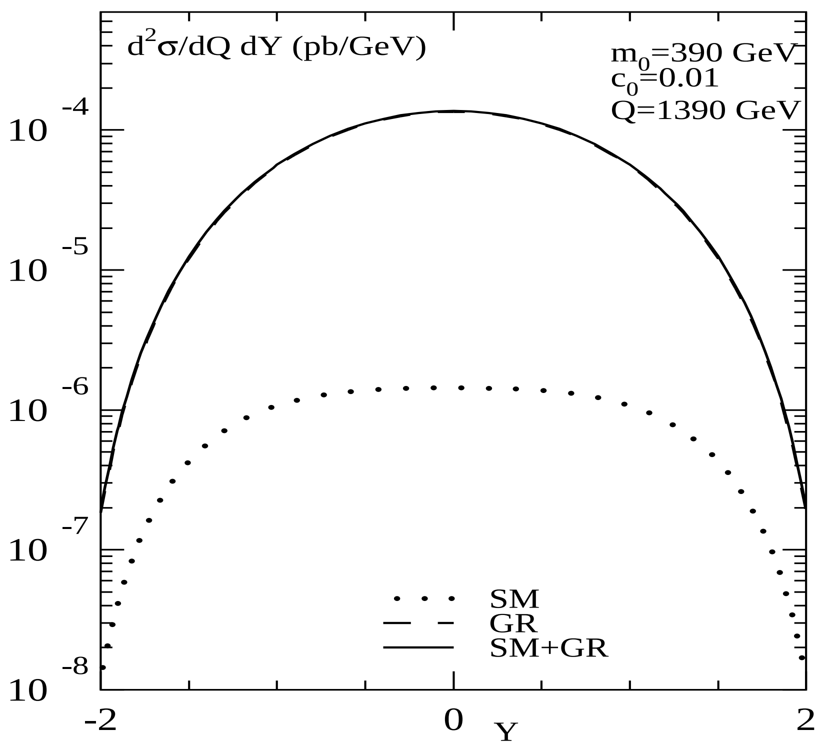
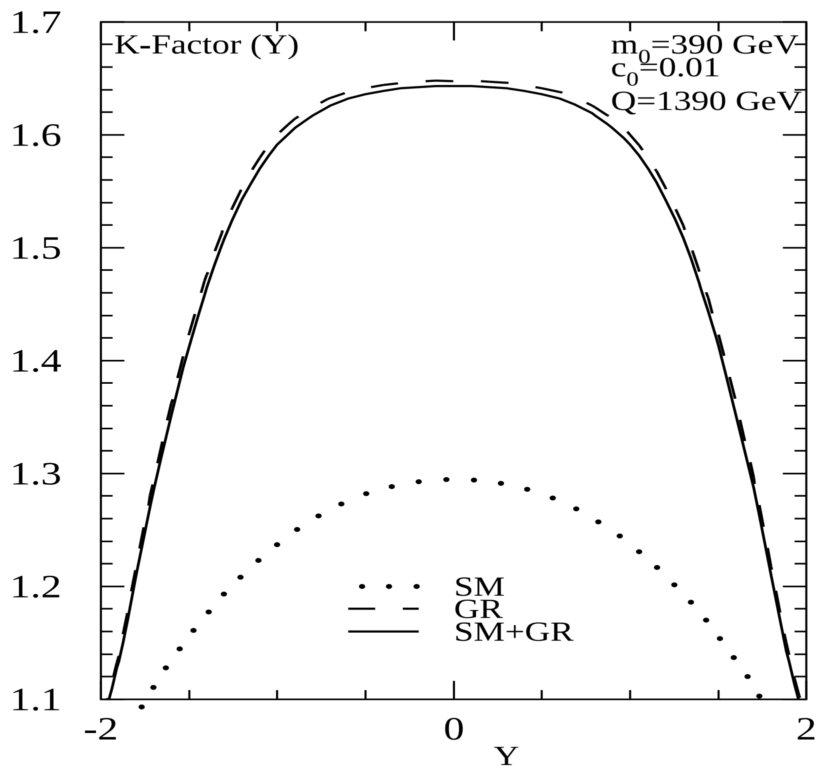




- Away from the resonance region gravity contribution is negligible
- K-Factor behavior can be understood from the $K^{(0)}$ behavior for the RS model.

Rapidity Y distribution:



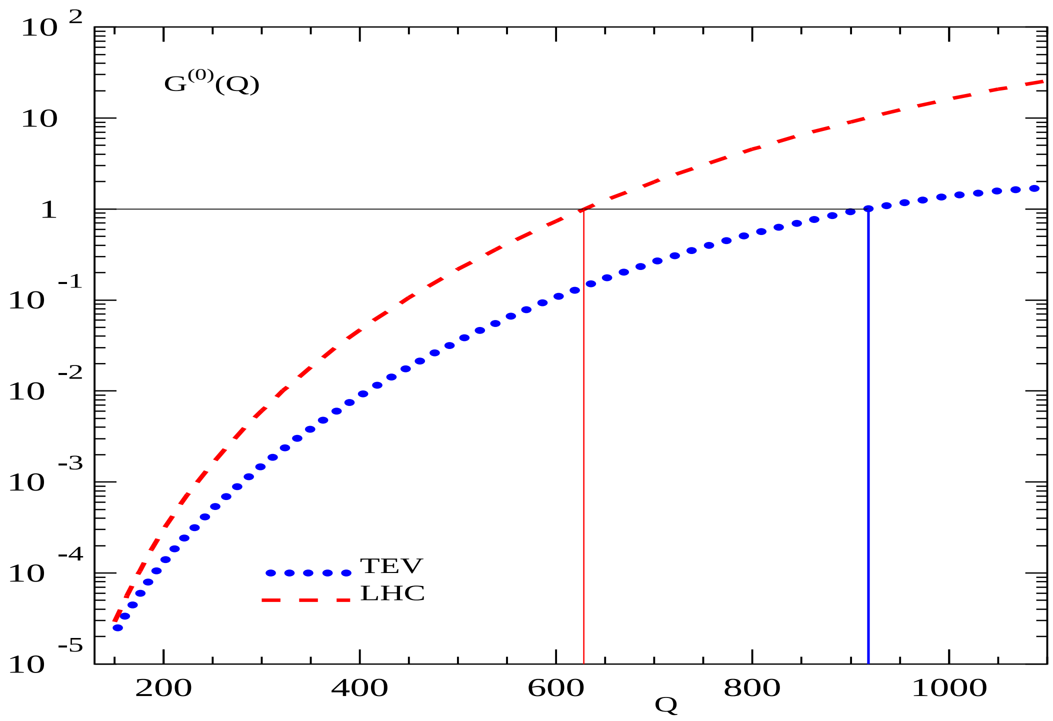


- K-Factor for rapidity distribution, close to to the first KK resonance $m_1 \sim 1.5$ TeV
- Gravity dominates the resonance region as can be understood from the $K^{(0)}$ behavior of RS model.

K-Factor

$$K^{(SM+GR)}(Q) = \frac{K^{SM} + K^{GR}\, G^{(0)}}{1 + G^{(0)}}$$

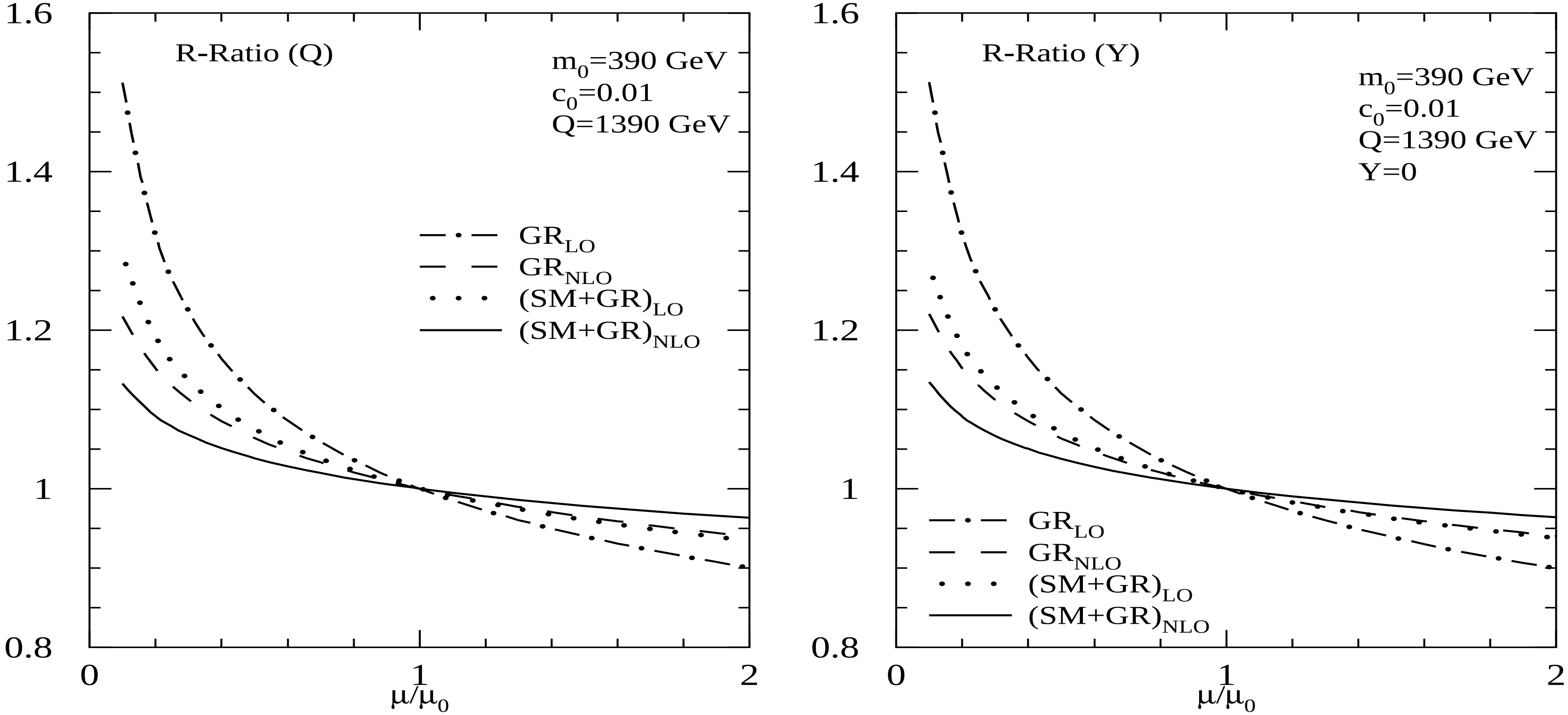
$$G^{(0)}(Q) = \left[\frac{d\sigma^{SM}_{LO}(Q)}{dQ}\right]^{-1} \left[\frac{d\sigma^{GR}_{LO}(Q)}{dQ}\right]$$



- $G^{(0)}(Q)$ behavior is governed by a competing 'couplings' and PDF flux at LHC and TEV
- At high Q when Gravity contribution becomes comparable to SM, the PDF flux dictates the proceedings

R-Factor:

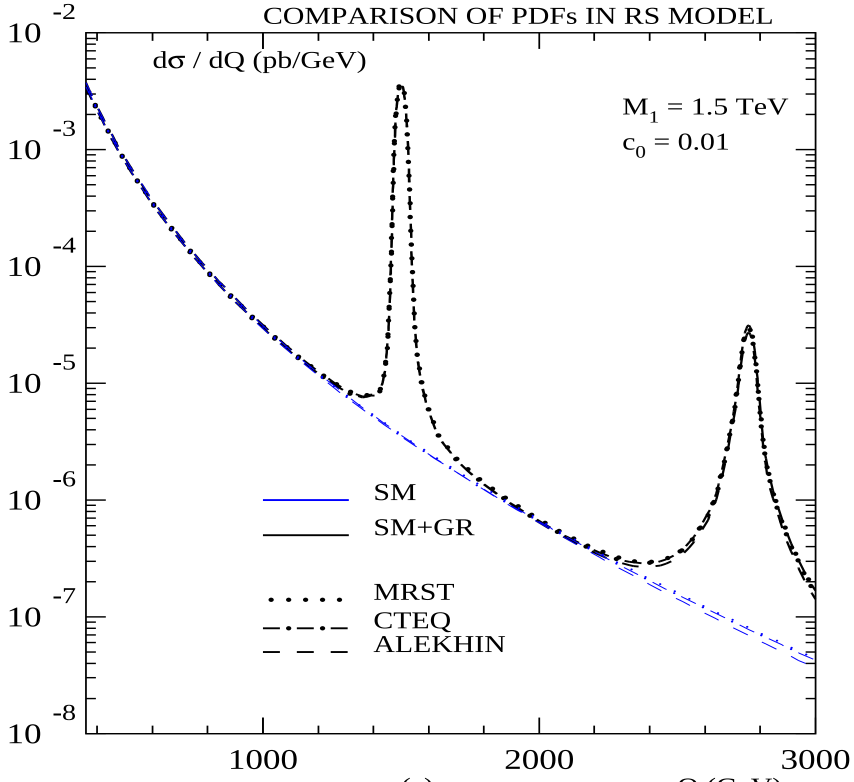
$$R^{I}_{LO,NLO} = \left[\frac{d\sigma^{I}_{LO,NLO}(Q,\mu=\mu_0)}{dQ}\right]^{-1}\left[\frac{d\sigma^{I}_{LO,NLO}(Q,\mu)}{dQ}\right]\Bigg|_{Q=Q_0}$$



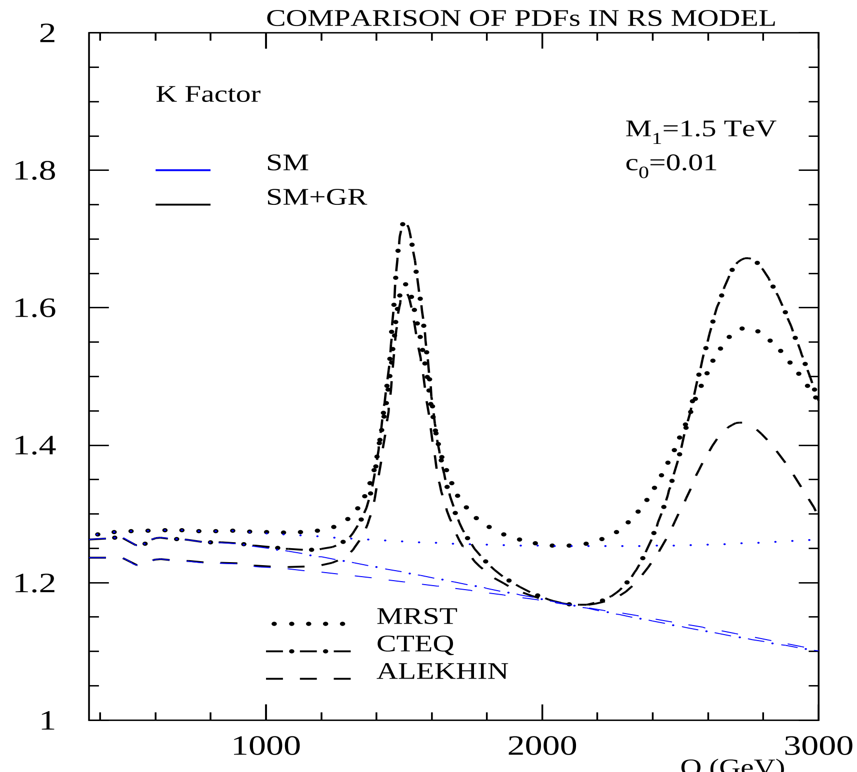
- Scale variation reduced considerably in going from LO $\longrightarrow$ NLO
- Inclusion of SM to GR also reduces scale variation

RS Scenario Results

$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) \quad = \quad \frac{c_0^2}{m_0^2}\sum_{n=1}^{\infty}\frac{1}{s - M_n^2 + iM_n\Gamma_n} \quad \equiv \frac{c_0^2}{m_0^4}\lambda\left(\frac{Q}{m_0}\right)$$



(a)

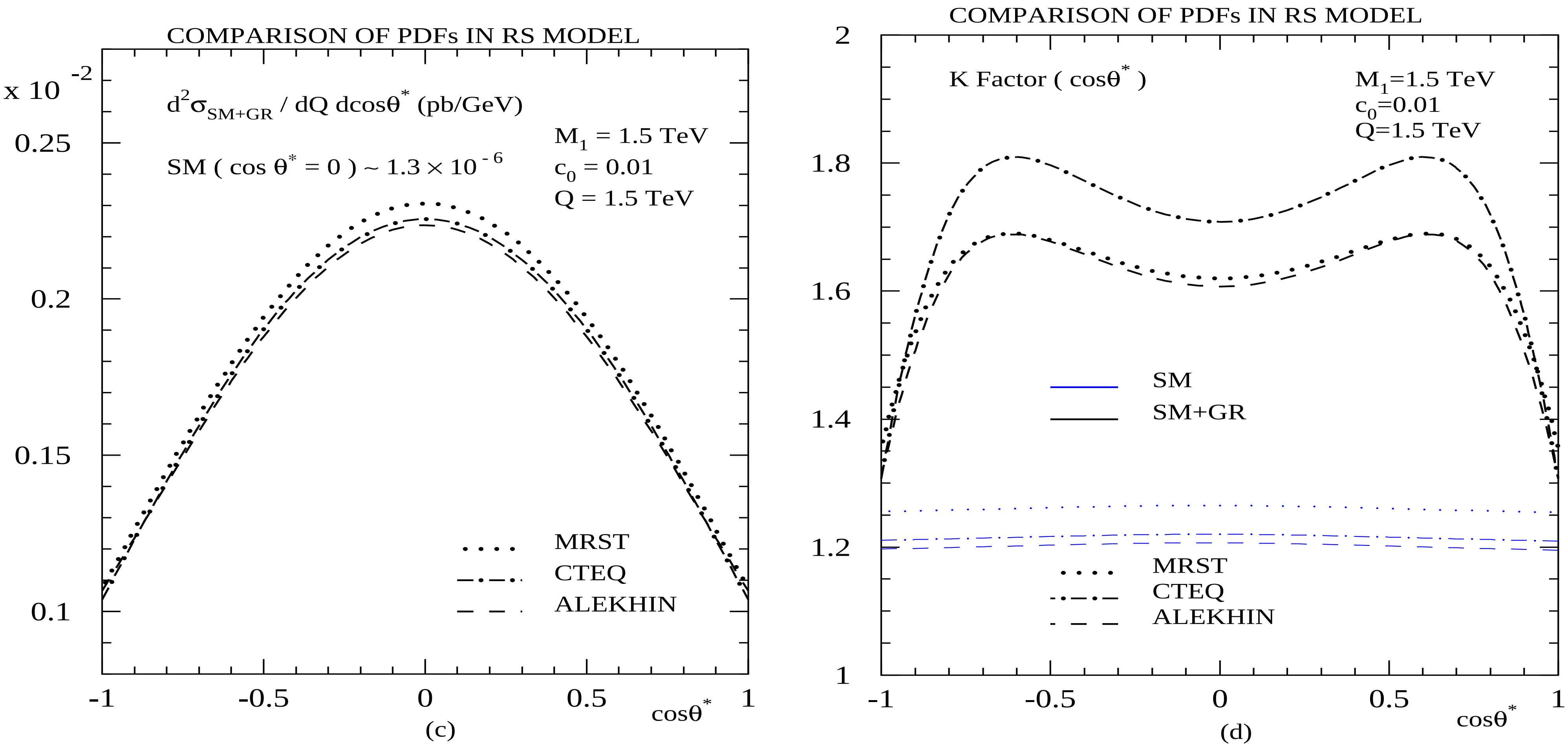


(b)

- Away from the resonance region gravity contribution is negligible
- K-Factor behavior can be understood from the $K^{(0)}$ behavior for the RS model

Angular distribution:

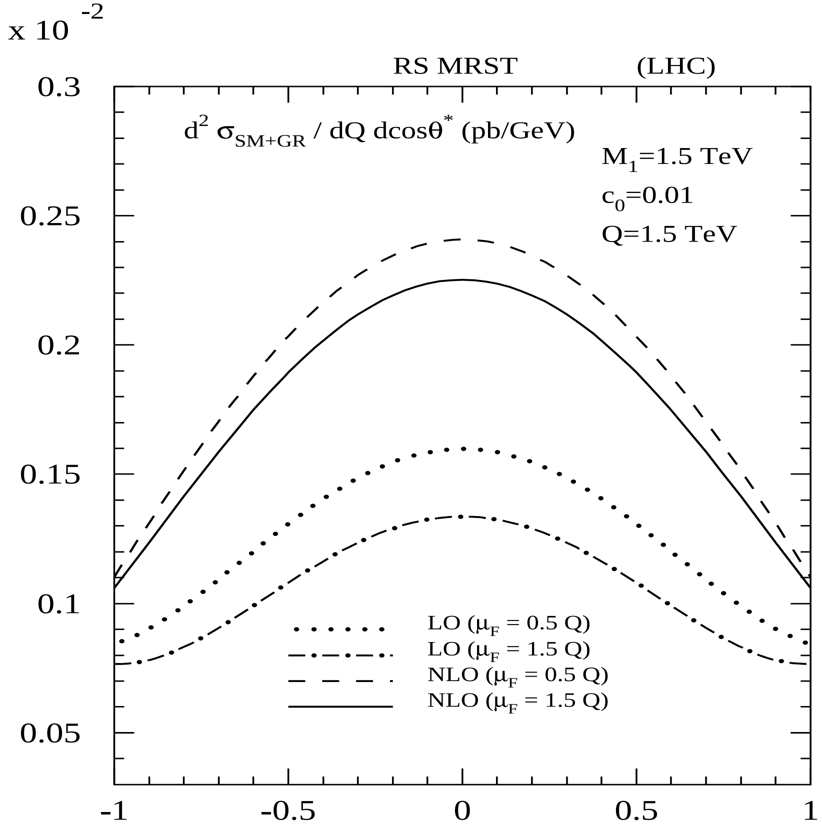
$$\left.\frac{d\sigma(Q,\cos\theta)}{dQ\, d\cos\theta}\right|_{Q_0}$$



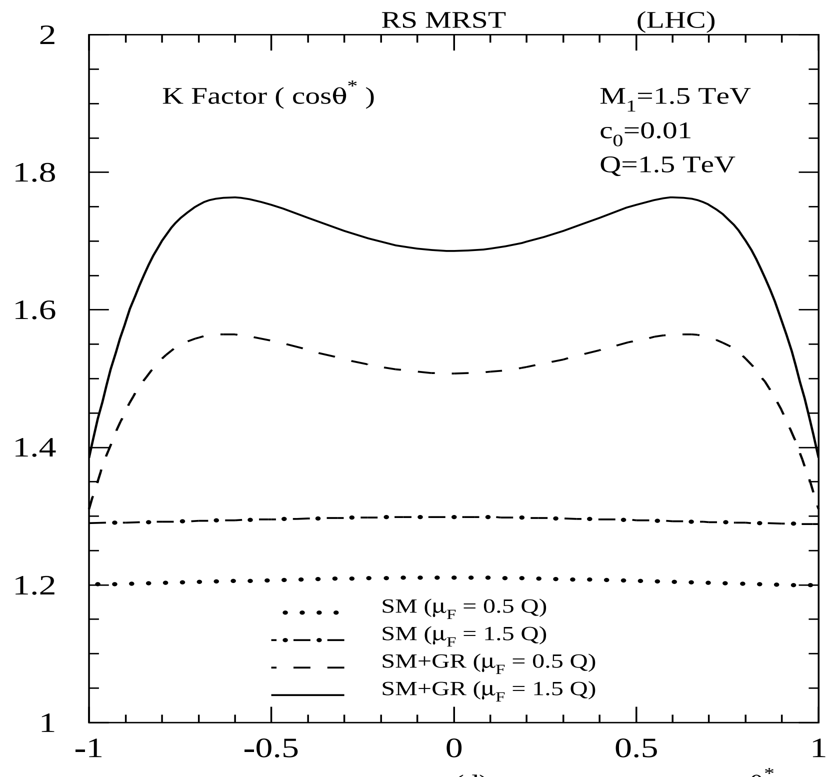
Factorisation scale dependence of angular distribution:

$$\frac{d\sigma(Q,\cos\theta)}{dQ\,d\cos\theta}\Big|_{Q_0}$$

Distributions	Tevatron		LHC	
	LO	NLO	LO	NLO
$d^2\sigma/dQdY$	23.2	7.7	18.7	6.9
$d^2\sigma/dQdcos\theta$	24.2	8.0	18.4	6.8

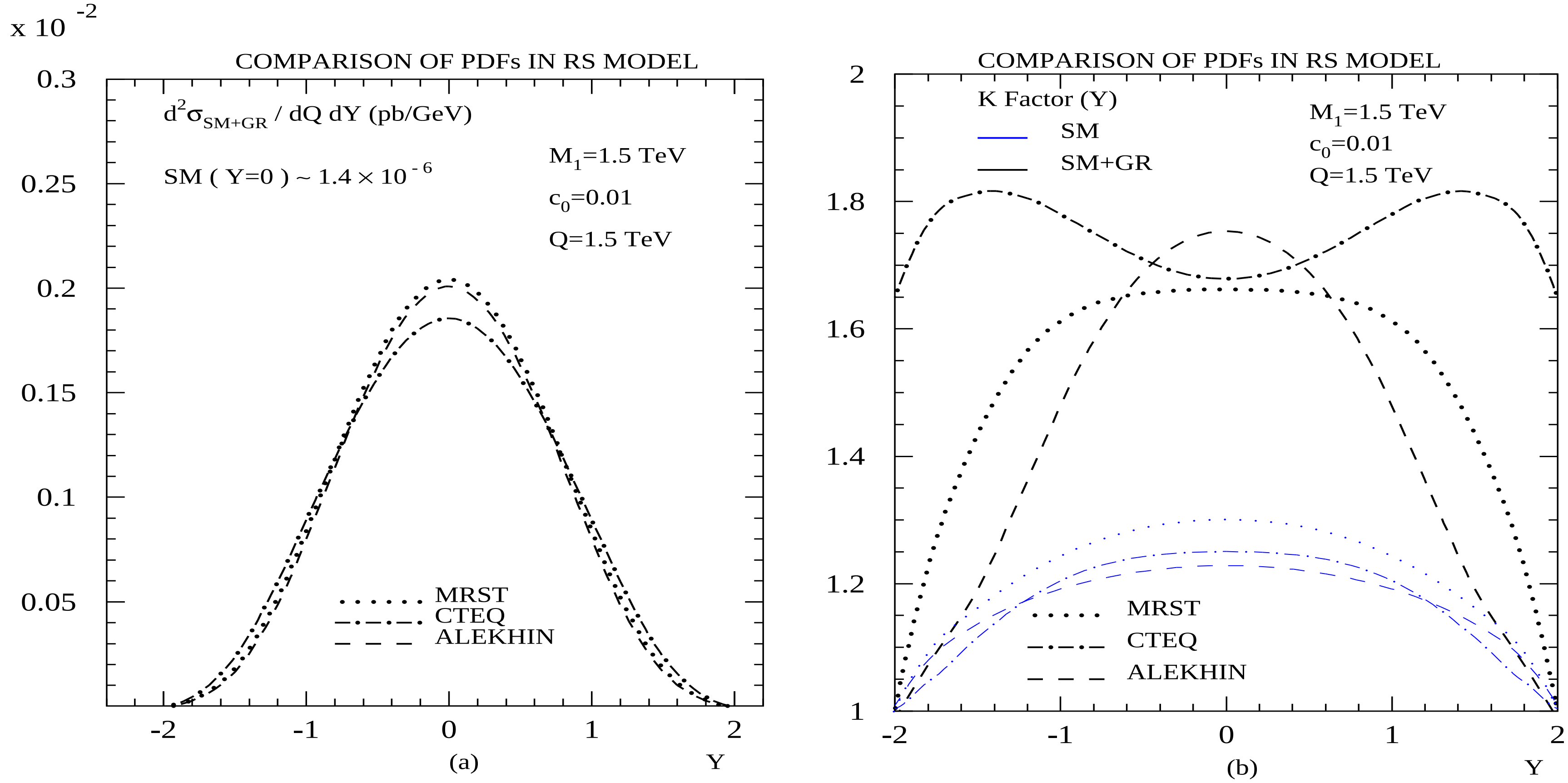


(c)



(d)

RS Rapidity



x 10 -2
COMPARISON OF PDFs IN RS MODEL
d²σSM+GR / dQ dY (pb/GeV)
SM (Y=0) ~ 1.4 × 10 - 6
M1=1.5 TeV
c0=0.01
Q=1.5 TeV
0.3
0.25
0.2
0.15
0.1
0.05
MRST
CTEQ
ALEKHIN
-2
-1
0
1
2
(a)
Y
COMPARISON OF PDFs IN RS MODEL
K Factor (Y)
SM
SM+GR
M1=1.5 TeV
c0=0.01
Q=1.5 TeV
2
1.8
1.6
1.4
1.2
1
MRST
CTEQ
ALEKHIN
-2
-1
0
1
2
(b)
Y