Features in the primordial spectrum and associated non-Gaussianities

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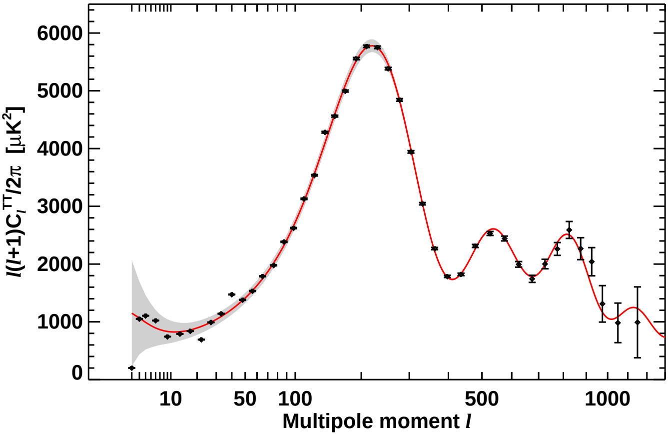
Proliferation of inflationary models[1]

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflation
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models. May be, we should look for models that permit deviations from the standard picture of slow roll inflation.

[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

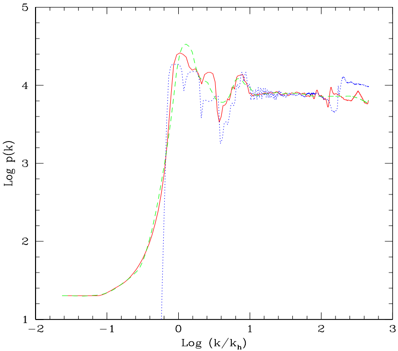
Angular power spectrum from the WMAP 7-year data[2]

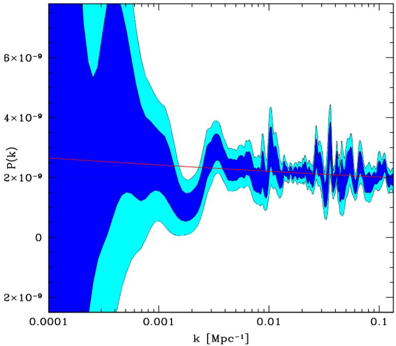


The WMAP 7-year data for the CMB TT angular power spectrum (the black dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid red curve). Note the outliers near the multipoles $\ell = 2$, 22 and 40.

[2] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011).

Reconstructing the primordial spectrum



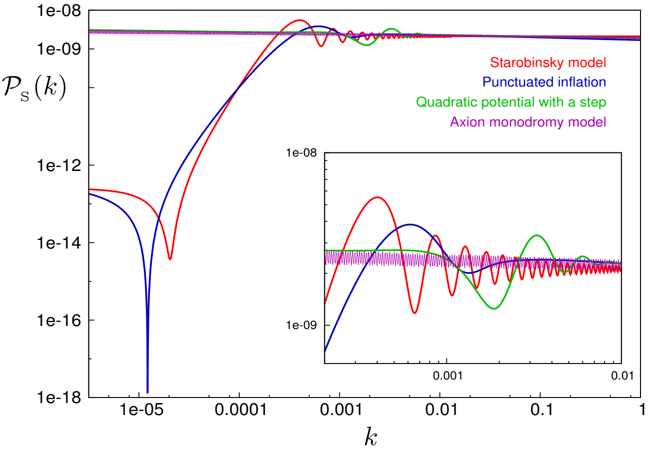


Reconstructed primordial spectra, obtained upon assuming the concordant background ΛCDM model. The recovered spectrum on the left improves the fit to the WMAP 3-year data by $\Delta\chi^2_{\rm eff} \simeq 15$, with respect to the best fit power law spectrum[3]. The spectrum on the right has been recovered from a variety of CMB datasets, including the WMAP 5-year data[4].

[3] A. Shafieloo, T. Souradeep, P. Manimaran, P. K. Panigrahi and R. Rangarajan, Phys. Rev. D **75**, 123502 (2007).

[4] G. Nicholson and C. R. Contaldi, JCAP **0907**, 011 (2009).

Inflationary models leading to features[5]



The scalar power spectra in a few different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum.

[5] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v1 [astro-ph.CO].

'Large' non-Gaussianities and its possible implications

- The WMAP 7-year data constrains the non-Gaussianity parameter $f_{_{\rm NL}}$ to be $f_{_{\rm NL}} = 32 \pm 21$ in the local limit, at 68% confidence level[6].
- If forthcoming missions such as Planck detect a large level of non-Gaussianity, as suggested by the above mean value of $f_{_{\rm NL}}$, then it can result in a substantial tightening in the constraints on the various inflationary models. For example, canonical scalar field models that lead to a nearly scale invariant primordial spectrum contain only a small amount of non-Gaussianity and, hence, will cease to be viable[7] .
- However, it is known that primordial spectra with features can lead to reasonably large non-Gaussianities[8]. Therefore, if the non-Gaussianity parameter $f_{_{\rm NL}}$ indeed proves to be large, then either one has to reconcile with the fact that the primordial spectrum contains features or we have to turn our attention to non-canonical scalar field models such as, say, D brane inflation models[9].

[6] E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

[7] J. Maldacena, JHEP **05**, 013 (2003).

[8] See, for instance, X. Chen, R. Easther and E. A. Lim, JCAP **0706**, 023 (2007).

[9] See, for example, X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

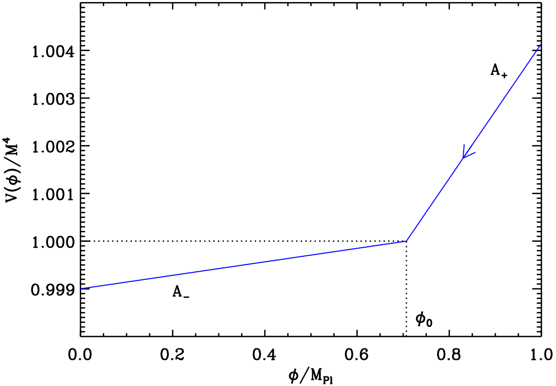
This talk is based on

- J. Martin and L. Sriramkumar, *The scalar bi-spectrum in the Starobinsky model: The equilateral case*, arXiv:1109.5838v1 [astro-ph.CO].
- D. K. Hazra, L. Sriramkumar and J. Martin, *On the discriminating power of* $f_{\rm NL}$, In preparation.

Outline of the talk

1. The scalar power spectrum in the Starobinsky model
2. The definition of the non-Gaussianity parameter $f_{_{\rm NL}}$
3. The method for evaluating $f_{_{\rm NL}}$
4. $f_{_{\rm NL}}$ in the Starobinsky model (in the equilateral limit)
5. Can $f_{_{\rm NL}}$ help in discriminating between inflationary models?
6. Summary

The Starobinsky model[10]



The Starobinsky model involves the canonical scalar field which is described by the potential

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0) & \text{for}\ \ \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0) & \text{for}\ \ \phi < \phi_0. \end{cases}$$

[10] A. A. Starobinsky, Sov. Phys. JETP Lett. **55**, 489 (1992).

Assumptions and properties

- It is assumed that the constant V_0 is the dominant term in the potential for a range of ϕ near ϕ_0. As a result, over the domain of our interest, the expansion is of the de Sitter form corresponding to a Hubble parameter H_0 determined by V_0.
- The scalar field rolls slowly until it reaches the discontinuity in the potential. It then fast rolls for a brief period as it crosses the discontinuity before slow roll is restored again.
- Since V_0 is dominant, the first slow roll parameter ϵ_1 remains small even during the transition. This property allows the background to be evaluated analytically to a good approximation.

Analytic expressions for the slow roll parameters

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One can show that, after the transition, the evolution of the first slow roll parameter ϵ_1 can be expressed in terms of the number of e-folds N as follows:

$$\epsilon_{1-} \simeq \frac{A_-^2}{18\, M_{_{\rm Pl}}^2\, H_0^4}\, \left[1 - \frac{\Delta A}{A_-}\, {\rm e}^{-3\,(N-N_0)}\right]^2,$$

where $\Delta A = (A_- - A_+)$, while N_0 is the e-fold at which the field crosses the discontinuity.

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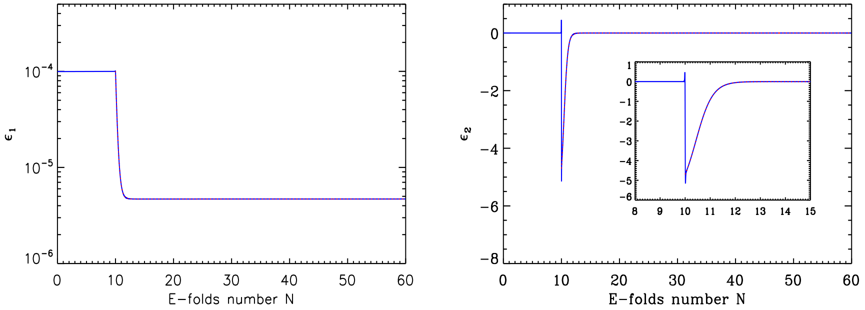
where $\Delta A = (A_- - A_+)$, while N_0 is the e-fold at which the field crosses the discontinuity.

It is found that, *immediately after the transition*, the second slow roll parameter ϵ_2 is given by

$$\epsilon_{2-} \simeq \frac{6\,\Delta A}{A_-}\, \frac{{\rm e}^{-3\,(N-N_0)}}{1 - (\Delta A/A_-)\, {\rm e}^{-3\,(N-N_0)}}.$$

Evolution of the slow roll parameters



The evolution of the first slow roll parameter ϵ_1 on the left, and the second slow roll parameter ϵ_2 on the right in the Starobinsky model. While the blue curves describe the numerical results, the dotted red curves represent the analytical expressions mentioned in the previous slide.

The modes before and after the transition

It can be shown that, under the assumptions that one is working with, the quantity $z = a\, M_{_{\rm Pl}}\, \sqrt{2\, \epsilon_1}$, which determines the evolution of the modes, say, f_k, associated with the curvature perturbation $\mathcal{R}_k$, simplifies to

$$z''/z \simeq 2\, \mathcal{H}^2$$

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As a result, while the solution to the Mukhanov-Sasaki variable $v_k = (z\, f_k)$ corresponding to the Bunch-Davies vacuum before the transition is given by

$$v_k^{+}(\eta) = \frac{1}{\sqrt{2\,k}}\, \left(1 - \frac{i}{k\,\eta}\right)\, {\rm e}^{-i\,k\,\eta},$$

after the transition, it can be expressed as a linear combination of the positive and the negative frequency modes as follows:

$$v_k^{-}(\eta) = \frac{\alpha_k}{\sqrt{2\,k}}\, \left(1 - \frac{i}{k\,\eta}\right) {\rm e}^{-i\,k\,\eta} + \frac{\beta_k}{\sqrt{2\,k}}\, \left(1 + \frac{i}{k\,\eta}\right) {\rm e}^{i\,k\,\eta},$$

where α_k and β_k are the usual Bogoliubov coefficients.

The scalar power spectrum in the Starobinsky model

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The scalar power spectrum, given by

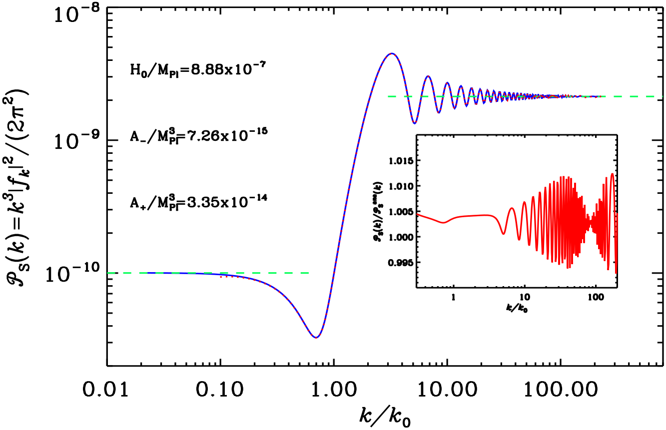
$$\mathcal{P}_{\rm S}(k) = (k^3/2\,\pi^2)\,\vert f_k\vert^2 = (k^3/2\,\pi^2)\,(\vert v_k\vert/z)^2,$$

can be evaluated at late times to be

$$\begin{aligned}\mathcal{P}_{\rm S}(k) = \left(\frac{9\,H_0^6}{4\,\pi^2\,A_-^2}\right)\,\Biggl\{1 - \frac{3\,\Delta A}{A_+}\,\frac{k_0}{k}\left[\left(1-\frac{k_0^2}{k^2}\right)\,\sin\left(\frac{2\,k}{k_0}\right) + \frac{2\,k_0}{k}\,\cos\left(\frac{2\,k}{k_0}\right)\right]\\ +\frac{9\,\Delta A^2}{2\,A_+^2}\,\frac{k_0^2}{k^2}\,\left(1+\frac{k_0^2}{k^2}\right)\left[\left(1+\frac{k_0^2}{k^2}\right) - \frac{2\,k_0}{k}\,\sin\left(\frac{2\,k}{k_0}\right)\right.\\ \left.+\left(1-\frac{k_0^2}{k^2}\right)\,\cos\left(\frac{2\,k}{k_0}\right)\right]\Biggr\},\end{aligned}$$

where the quantity k_0 is the wavenumber of the mode that leaves the Hubble radius when the field crosses the discontinuity. Note that the power spectrum depends on the wavenumber only through the ratio (k/k_0).

Comparison with the numerical result



The scalar power spectrum in the Starobinsky model. While the blue solid curve denotes the analytic result, the red dots represent the corresponding numerical scalar power spectrum that has been obtained through an exact integration of the background as well as the perturbations.

The scalar bi-spectrum

The scalar bi-spectrum $\mathcal{B}_{\rm S}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time $\eta_{\rm e}$, as follows[11]:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_{\rm e}) \rangle = (2\,\pi)^3\; \mathcal{B}_{\rm S}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\; \delta^{(3)}\,(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\,.$$

For convenience, we shall set

$$\mathcal{B}_{\rm S}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\,\pi)^{-9/2}\; G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

[11] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

The introduction of $f_{\rm NL}$

The observationally relevant non-Gaussianity parameter $f_{\rm NL}$ is introduced through the equation[12]

$$\mathcal{R}(\eta, \mathbf{x}) = \mathcal{R}^{\rm G}(\eta, \mathbf{x}) - \frac{3\, f_{\rm NL}}{5}\, \left[\mathcal{R}^{\rm G}(\eta, \mathbf{x})\right]^2,$$

where $\mathcal{R}^{\rm G}$ denotes the Gaussian quantity, and the factor of $(3/5)$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

[12] J. Maldacena, JHEP **0305**, 013 (2003);
S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

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It ought to be emphasized here that the non-Gaussianity parameter $f_{\rm NL}$ has been introduced through the above equation with the local limit in mind.

In Fourier space, the above equation can be written as

$$\mathcal{R}_{\mathbf{k}} = \mathcal{R}_{\mathbf{k}}^{\rm G} - \frac{3\, f_{\rm NL}}{5} \int \frac{{\rm d}^3\mathbf{p}}{(2\,\pi)^{3/2}}\, \mathcal{R}_{\mathbf{p}}^{\rm G}\, \mathcal{R}_{\mathbf{k}-\mathbf{p}}^{\rm G}.$$

[12] J. Maldacena, JHEP **0305**, 013 (2003);
S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

The introduction of $f_{\rm NL}$ . . . continued

Using the above relation and Wick's theorem, one can arrive at the three point correlation of the curvature perturbation in Fourier space in terms of the parameter $f_{\rm NL}$. It is found to be

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = -\left(\frac{3\, f_{\rm NL}}{10}\right) (2\,\pi)^4\, (2\,\pi)^{-3/2}\, \left(\frac{1}{k_1^3\, k_2^3\, k_3^3}\right) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times \left[k_1^3\, \mathcal{P}_{\rm S}(k_2)\, \mathcal{P}_{\rm S}(k_3) + \text{two permutations}\right].$$

The relation between $f_{\rm NL}$ and the bi-spectrum

Upon using the above expression for the three point function of the curvature perturbation and the definition of the bi-spectrum, we can, in turn, arrive at the following relation:

$$\begin{aligned} f_{\rm NL} &= -\left(\frac{10}{3}\right)\,(2\,\pi)^{-4}\,(2\,\pi)^{9/2}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,\mathcal{B}_{\rm S}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\ &\qquad\times\left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3)+\text{two permutations}\right]^{-1} \\ &= -\left(\frac{10}{3}\right)\,(2\,\pi)^{-4}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\ &\qquad\times\left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3)+\text{two permutations}\right]^{-1}. \end{aligned}$$

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In the equilateral limit (i.e. when $\mathbf{k}_1=\mathbf{k}_2=\mathbf{k}_3$), this expression for $f_{\rm NL}$ simplifies to

$$f_{\rm NL}^{\rm eq} = -\left(\frac{10}{9}\right)\,(2\,\pi)^{-4}\,\left(\frac{k^6\,G(k)}{\mathcal{P}_{\rm S}^2(k)}\right).$$

The action at the cubic order[13]

It can be shown that the third order term in the action describing the curvature perturbations is given by

$$\mathcal{S}_3[\mathcal{R}] = M_{_{\rm Pl}}^2 \int {\rm d}\eta \int {\rm d}^3\mathbf{x} \biggl[a^2\, \epsilon_1^2\, \mathcal{R}\, \mathcal{R}'^2 + a^2\, \epsilon_1^2\, \mathcal{R}\, (\partial\mathcal{R})^2 - 2\, a\, \epsilon_1\, \mathcal{R}'\, (\partial^i\mathcal{R})\, (\partial_i\chi) + \frac{a^2}{2}\, \epsilon_1\, \epsilon_2'\, \mathcal{R}^2\, \mathcal{R}' + \frac{\epsilon_1}{2}\, (\partial^i\mathcal{R})\, (\partial_i\chi)\, (\partial^2\chi) + \frac{\epsilon_1}{4}\, (\partial^2\mathcal{R})\, (\partial\chi)^2 + \mathcal{F}\left(\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\right)\biggr],$$

where $\mathcal{F}(\delta\mathcal{L}_2/\delta\mathcal{R})$ denotes terms involving the variation of the second order action with respect to $\mathcal{R}$, while χ is related to the curvature perturbation $\mathcal{R}$ through the relations

$$\Lambda = a\, \epsilon_1\, \mathcal{R}' \quad \text{and} \quad \partial^2\chi = \Lambda.$$

[13] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

Evaluating the bi-spectrum

At the leading order in the perturbations, one then finds that the three point correlation in Fourier space is described by the integral

$$\langle\hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e)\,\hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_{\rm e})\rangle = -i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a(\eta)\; \left\langle\left[\hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_{\rm e}), \hat{H}_{\rm I}(\eta)\right]\right\rangle,$$

where $\hat{H}_{\rm I}$ is the operator corresponding to the above third order action, while $\eta_{\rm i}$ is the time at which the initial conditions are imposed on the modes when they are well inside the Hubble radius, and $\eta_{\rm e}$ denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are evaluated in the initial vacuum state (viz. the Bunch-Davies vacuum in the situation of our interest).

Evaluating the bi-spectrum . . . continued

In the equilateral limit, the quantity $G(k)$, evaluated towards the end of inflation at the conformal time $\eta = \eta_{\rm e}$, can be written as

$$G(k) \equiv \sum_{C=1}^{7} G_{_C}(k) = M_{_{\rm Pl}}^2 \sum_{C=1}^{6} \left[f_k^3\,(\eta_{\rm e})\; \mathcal{G}_{_C}(k) + f_k^{*3}\,(\eta_{\rm e})\; \mathcal{G}_{_C}^{*}(k)\right] + G_7(k),$$

where the quantities $\mathcal{G}_{_C}(k)$ are integrals that correspond to six terms that arise in the action at the third order in the perturbations, while, recall that, f_k are the modes associated with the curvature perturbation $\mathcal{R}_k$.

The additional, seventh term G_7 arises due to a field redefinition, and its contribution is given by[14]

$$G_7(k) = \frac{3\,\epsilon_2(\eta_{\rm e})}{2}\,\vert f_k(\eta_{\rm e})\vert^4.$$

[14] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
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Evaluating $f_{\rm NL}$ in the Starobinsky model

When there exist deviations from slow roll, it is often found that the fourth term $\mathcal{G}_4$ provides the dominant contribution to $f_{\rm NL}$.

It is described by the following integral

$$\mathcal{G}_4(k) = 3\, i \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\, \epsilon_1\, \epsilon_2'\, f_k^{*2}\, f_k'^{*}.$$

In the case of the Starobinsky model, as ϵ_2 is a constant before the transition, ϵ_2' vanishes, and hence the above integral $\mathcal{G}_4$ is non-zero only post-transition.

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We find that the integral involved can be computed analytically.

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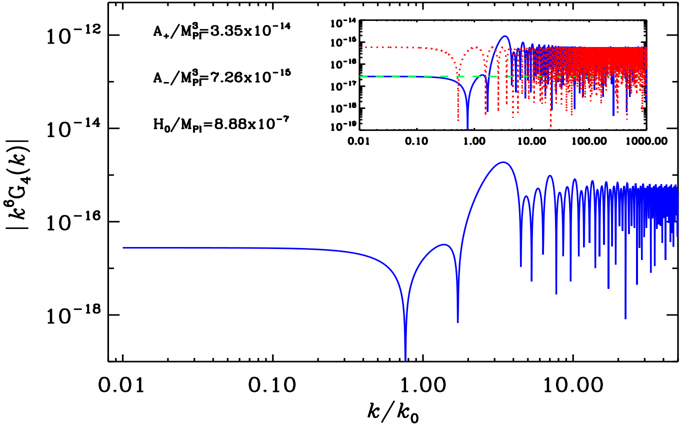
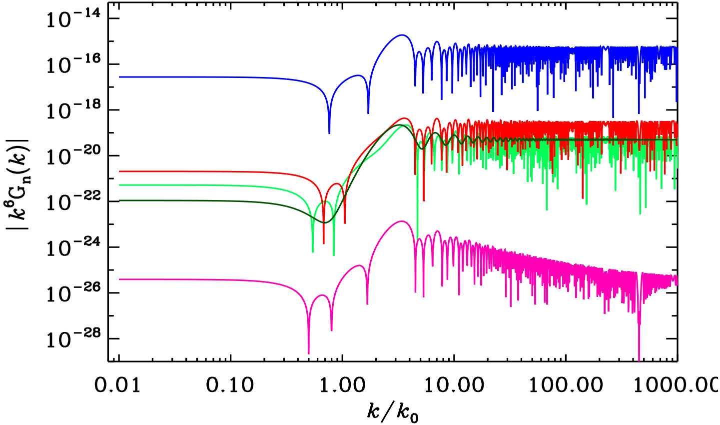
In fact, with some effort, analytic expressions can be arrived at for all the $\mathcal{G}_n$.

The dominant contribution in the Starobinsky model



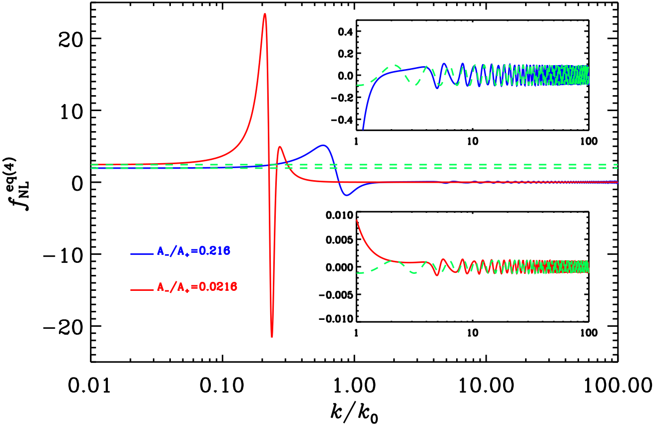
The absolute value of the quantity $[k^6\, G_4]$ has been plotted as a function of (k/k_0) (the blue curve). We have worked with the same of values of A_+, A_- and V_0 as in the earlier figure wherein we had plotted the power spectrum. The green and the red curves in the inset represent the limiting values for $k \ll k_0$ and $k \gg k_0$, respectively.

The different contributions



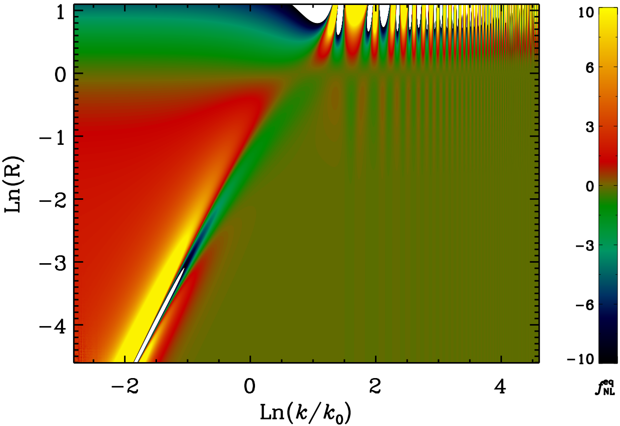
The quantities k^6 times the absolute values of $G_1 + G_3$ (in light green), G_2 (in red), G_4 (in blue), $G_5 + G_6$ (in purple) and G_7 (in dark green) have been plotted as a function of (k/k_0) for the Starobinsky model. We have worked with the same values of the parameters as in the previous three figures.

$f_{\rm NL}^{\rm eq}$ due to the dominant contribution



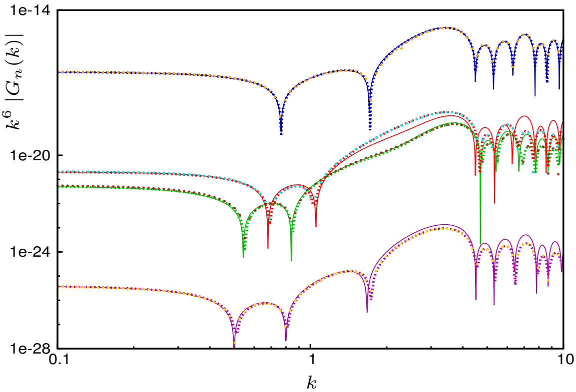
The non-Gaussianity parameter $f_{\rm NL}^{\rm eq}$ due to the dominant term in the Starobinsky model, plotted as a function of (k/k_0) for $(A_-/A_+) = 0.216$ and $(A_-/A_+) = 0.0216$. Larger the difference between A_- and A_+, larger is the corresponding $f_{\rm NL}^{\rm eq}$.

$f_{\rm NL}^{\rm eq}$ for a range of values of the parameters



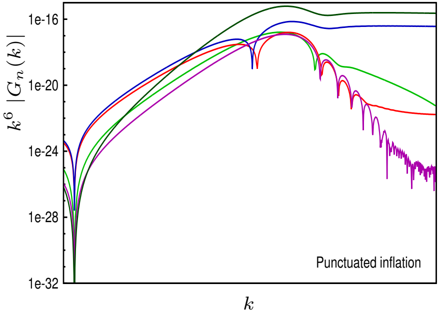
The non-Gaussianity parameter $f_{\rm NL}^{\rm eq}$ due to the dominant contribution in the Starobinsky model, plotted as a function of (k/k_0) and the ratio $R = (A_-/A_+)$. The white contours indicate regions wherein $f_{\rm NL}^{\rm eq}$ can be as large as 50. Note that, provided R is reasonably small, $f_{\rm NL}^{\rm eq}$ can be of the order of 20 or so, as is indicated by the currently observed mean value.

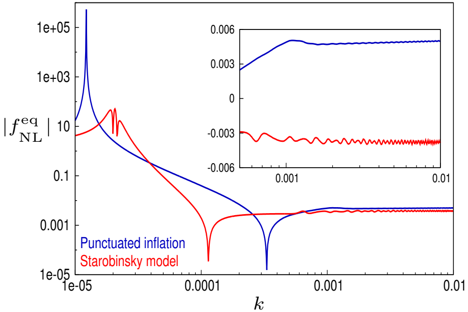
Calibrating using the Starobinsky model



A comparison of the analytical expressions in the Starobinsky model with the corresponding numerical results. The solid curves represent the analytical expressions that we discussed earlier, while the dashed curves denote the numerical results computed using a Fortran 90 code. The dots of an alternate color denote the corresponding numerical values that have been arrived at independently using a Mathematica code.

$f_{\rm NL}^{\rm eq}$ in punctuated inflation[15]

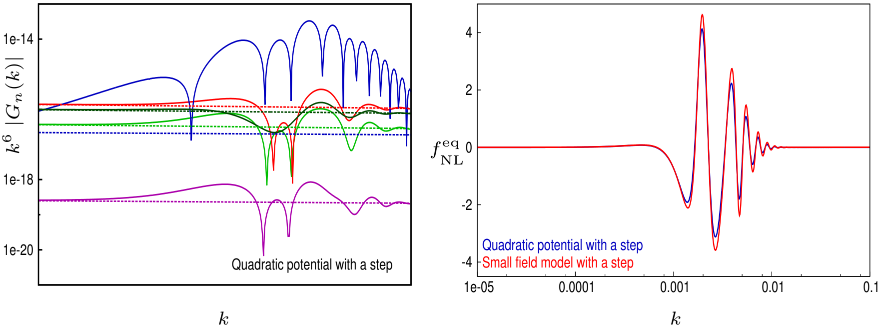




The contributions due to the various terms (on the left), and the absolute value of $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right), in the punctuated inflationary scenario. The absolute value of $f_{\rm NL}^{\rm eq}$ in a Starobinsky model that closely resembles the power spectrum in punctuated inflation has also been displayed. Note that $f_{\rm NL}^{\rm eq}$ can become rather large in punctuated inflation. The large difference in $f_{\rm NL}^{\rm eq}$ between punctuated inflation and the Starobinsky model can be attributed to the considerable difference in the background dynamics.

[15] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010).

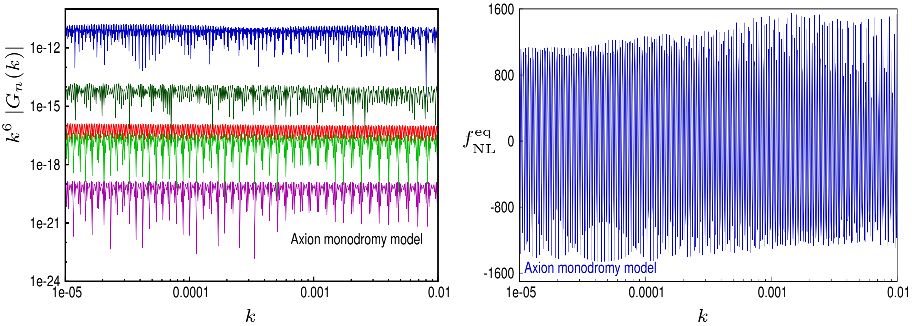
Models with a step in the inflaton potential[16]



The contributions due to the various terms (on the left) and $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right) when a step has been introduced in the popular chaotic inflationary model involving the quadratic potential. The $f_{\rm NL}^{\rm eq}$ that arises in a small field model with a step has also been illustrated. The background dynamics in these two models are very similar, which lead to almost the same $f_{\rm NL}^{\rm eq}$.

[16] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010).

Oscillating inflation potentials[17]



The contributions due to the various terms (on the left) and $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right) in the axion monodromy model. The model is described by a linear potential with superposed modulations. These modulations give rise to a certain resonant behavior, leading to a large $f_{\rm NL}^{\rm eq}$.

In contrast, oscillations introduced in the quadratic potential in a certain fashion do not result in such a large level of non-Gaussianity.

[17] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006**, 009 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v1 [astro-ph.CO].

Summary

- Remarkably, we find that, for a certain range of values of the parameters involved, the non-Gaussianity parameter $f_{\rm NL}^{\rm eq}$ can be evaluated analytically, to a good accuracy (as is confirmed by comparison with numerical computations) in the Starobinsky model.

Summary

- Remarkably, we find that, for a certain range of values of the parameters involved, the non-Gaussianity parameter $f_{\rm NL}^{\rm eq}$ can be evaluated analytically, to a good accuracy (as is confirmed by comparison with numerical computations) in the Starobinsky model.
- Interestingly, for suitably small values of $R = (A_-/A_+)$, $f_{\rm NL}^{\rm eq}$ in the Starobinsky model can be as large as indicated by the currently observed mean values.

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- Interestingly, for suitably small values of $R = (A_-/A_+)$, $f_{\rm NL}^{\rm eq}$ in the Starobinsky model can be as large as indicated by the currently observed mean values.
- The analytic results obtained in the case of the Starobinsky model can be utilized to calibrate numerical codes, thereby permitting the computation of bi-spectrum in a wide variety of inflationary models that contain departures from slow roll.

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- Interestingly, for suitably small values of $R = (A_-/A_+)$, $f_{\rm NL}^{\rm eq}$ in the Starobinsky model can be as large as indicated by the currently observed mean values.
- The analytic results obtained in the case of the Starobinsky model can be utilized to calibrate numerical codes, thereby permitting the computation of bi-spectrum in a wide variety of inflationary models that contain departures from slow roll.
- We find that certain differences in the background dynamics—reflected in the behavior of the slow roll parameters—can lead to a reasonably large difference in the $f_{\rm NL}^{\rm eq}$ generated by the competing models.

Thank you for your attention