Bouncing universes

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Plan of the talk

1. Bouncing scenarios
2. Generation of scale invariant magnetic fields in bouncing universes
3. Duality and scale invariant magnetic fields
4. The tensor bi-spectrum in a matter bounce
5. Tensor-to-scalar ratio in bouncing universes
6. Summary

This talk is based on. . .

- L. Sriramkumar, K. Atmjeet and R. K. Jain, *Generation of scale invariant magnetic fields in bouncing universes*, JCAP **1509**, 010 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- D. Chowdhury, V. Sreenath and L. Sriramkumar, *The tensor bi-spectrum in a matter bounce*, JCAP **1511**, 002 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- D. Chowdhury, L. Sriramkumar and R. K. Jain, *Duality and scale invariant magnetic fields from bouncing universes*, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].
- R. N. Raveendran, D. Chowdhury and L. Sriramkumar, *On the tensor-to-scalar ratio in bouncing universes*, Work in progress.

A few words on the conventions and notations

- We shall work in units such that $c = \hbar = 1$, and define the Planck mass to be $M_{\mathrm{Pl}} = (8\,\pi\,G)^{-1/2}$.

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$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\boldsymbol{x}^2 = a^2(\eta)\,\left(-\mathrm{d}\eta^2 + \mathrm{d}\boldsymbol{x}^2\right),$$

  where t is the cosmic time, $a(t)$ is the scale factor and $\eta = \int \mathrm{d}t/a(t)$ denotes the conformal time coordinate.

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- We shall denote differentiation with respect to the cosmic and the conformal times t and η by an overdot and an overprime, respectively.
- Further, as usual, $H = \dot{a}/a$ shall denote the Hubble parameter associated with the FLRW universe.

Bouncing scenarios: An alternative to inflation[1]

- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

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- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.

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- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- However, matter fields may have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity is expected to fail and quantum gravitational effects are supposed to take over.

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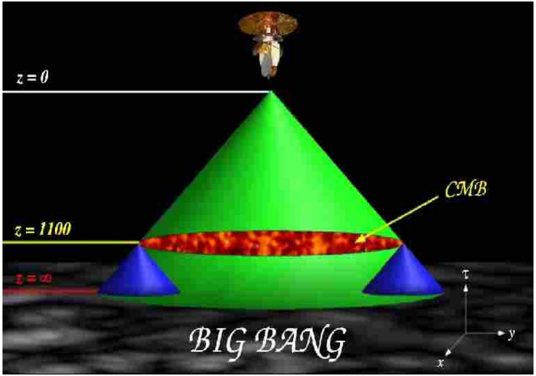
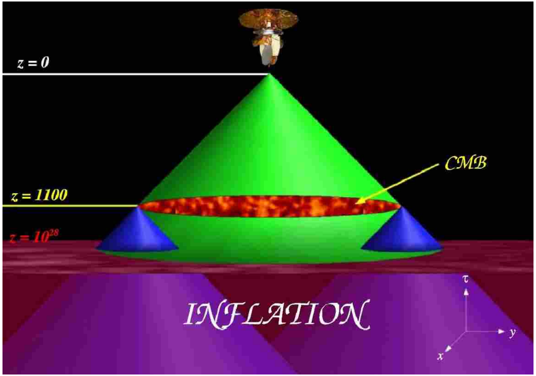
The resolution of the horizon problem in inflation





Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

[2]Images from W. Kinney, astro-ph/0301448.

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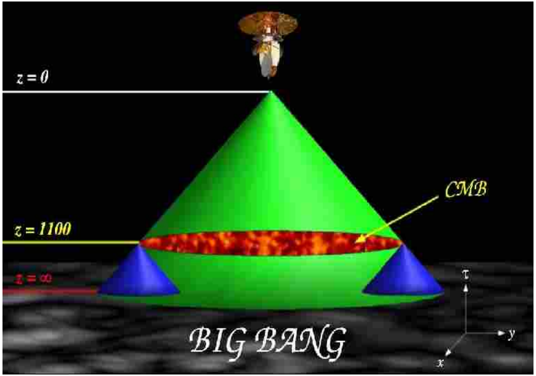

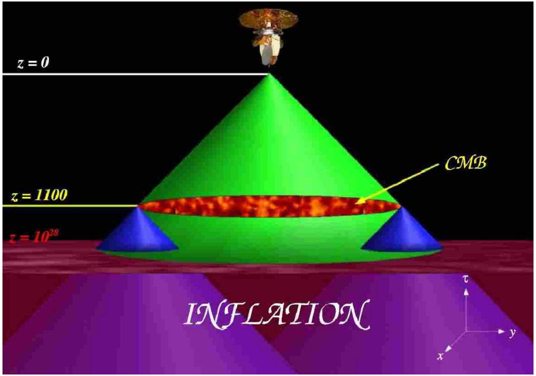

Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[2].

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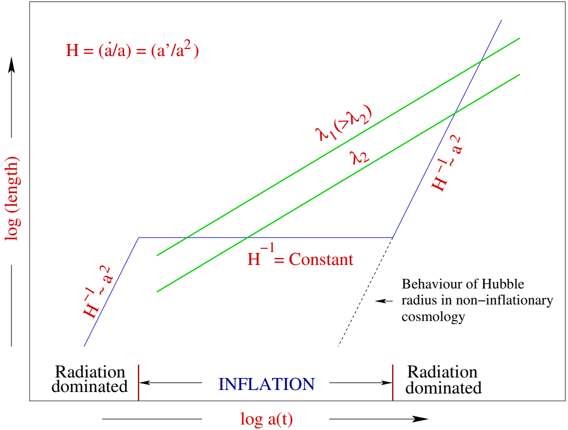
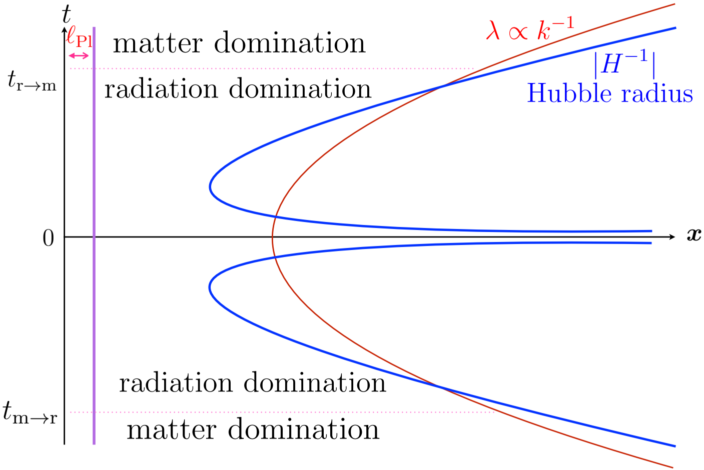
Bringing the modes inside the Hubble radius



A schematic diagram illustrating the behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[3].

[3] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Overcoming the horizon problem in bouncing models



Evolution of the physical wavelength and the Hubble radius in a bouncing scenario[4].

[4] Figure from, D. Battefeld and P. Peter, Phys. Rept. **571**, 1 (2015).

Violation of the null energy condition

Recall that, according to the Friedmann equations

$$\dot{H} = -\,4\,\pi\,G\,(\rho + p)\,.$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

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Evidently, $\dot{H}$ will be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve a bounce.

Classical bounces and sources

Consider for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0 \left(1 + k_0^2\, \eta^2\right)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale $M_{\rm Pl}$.

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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3\,q)$ and $w_2 = (2-q)/(3\,q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2\,q+1)/q}$ and $\rho_2 = M_2/a^{2\,(1+q)/q}$, where $M_1 = 12\, k_0^2\, M_{_{\rm Pl}}^2\, a_0^{1/q}$ and $M_2 = -M_1\, a_0^{1/q}$.

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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0/a^3$ and $\rho_2 = -12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0^2/a^4$.

The non-minimal action and the equation of motion

We shall consider a case wherein the electromagnetic field is coupled non-minimally to a scalar field ϕ and is described by the action

$$S[\phi, A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu},$$

where $F_{\mu\nu}$ denotes the electromagnetic field tensor which is given in terms of the vector potential A^μ as follows:

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu}.$$

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The equation of motion governing the electromagnetic field is given by

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,J^2(\phi)\,F^{\mu\nu}\right] = 0.$$

Quantization of the electromagnetic field

In a spatially flat, FLRW universe, we can choose to work in the Coulomb gauge wherein $A_0 = 0$ and $\partial_i A^i = 0$. In such a gauge, upon quantization, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows[5]:

$$\hat{A}_i(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\lambda=1}^{2} \tilde{\epsilon}_{\lambda\,i}(\boldsymbol{k}) \left[\hat{a}_{\boldsymbol{k}}^{\lambda}\, \bar{A}_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^{\lambda\dagger}\, \bar{A}_k^{*}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where the modes $\bar{A}_k$ satisfy the differential equation

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

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If we define a new variable $\mathcal{A}_k = J\,\bar{A}_k$, then the above equation simplifies to

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0,$$

and one can impose the standard Bunch-Davies initial conditions on the modes $\mathcal{A}_k$ at suitably early times.

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Power spectra of electric and magnetic fields

The energy densities associated with the electric and magnetic fields can be written in terms of the vector potential A_i and its time and spatial derivatives as follows:

$$\begin{aligned}\rho_{\mathrm{E}} &= \frac{J^2}{8\,\pi\,a^2}\, g^{ij}\, A_i'\, A_j',\\ \rho_{\mathrm{B}} &= \frac{J^2}{16\,\pi}\, g^{ij}\, g^{lm}\, \left(\partial_j A_m - \partial_m A_j\right)\, \left(\partial_i A_l - \partial_l A_i\right),\end{aligned}$$

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The expectation values of the corresponding operators, *i.e.* $\hat{\rho}_{\mathrm{E}}$ and $\hat{\rho}_{\mathrm{B}}$, can be evaluated in the vacuum state annihilated by the operator $\hat{a}_{\boldsymbol{k}}^{\lambda}$.

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The spectral energy densities of the magnetic and electric fields are found to be

$$\mathcal{P}_{\mathrm{B}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{\mathrm{B}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\bar{A}_k(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\mathcal{A}_k(\eta)|^2,$$
$$\mathcal{P}_{\mathrm{E}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{\mathrm{E}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,|\bar{A}_k'(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,\left|\mathcal{A}_k'(\eta) - \frac{J'(\eta)}{J(\eta)}\,\mathcal{A}_k(\eta)\right|^2.$$

Power spectra in power law inflation

For power law inflation described by the scale factor $a(\eta) = a_1\,(-\eta/\eta_1)^{\beta+1}$ and for coupling function of the form $J(\eta) = J_0\, a^n(\eta)$, one can show that the power spectrum of the magnetic field is given by[6]

$$\mathcal{P}_{_{\rm B}}(k) = \mathcal{F}(m)\, H^4\, (-k\,\eta)^{4+2\,m},$$

where $m = (\beta+1)\,n = \alpha$ for $\alpha \le 1/2$ and $m = 1-\alpha$ for $\alpha \ge 1/2$, while

$$\mathcal{F}(m) = \left[(2\,\pi)\, 2^{2\,m+1}\,\Gamma^2(m+1/2)\,\cos^2(\pi\, m)\right]^{-1}.$$

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$$\mathcal{P}_{_{\rm E}}(k) = \frac{\mathcal{G}(m)}{2\,\pi^2}\,H^4(-k\,\eta)^{4+2\,m},$$

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It is evident that $m = -2$ leads to a scale invariant spectrum for the magnetic field which corresponds to either $\alpha = 3$ or $\alpha = -2$.

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Modeling the bounce and the non-minimal coupling

We shall model the bounce by assuming that the scale factor $a(\eta)$ behaves as follows:

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We shall assume that the coupling function can be expressed in terms of the scale factor as

$$J(\eta) = J_0\, a^n(\eta).$$

E-$\mathcal{N}$-folds

The conventional e-fold N is defined $N = \log(a/a_0)$ so that $a(N) = a_0 \exp N$. However, the function e^N is a monotonically increasing function of N.

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For want of a better name, we shall refer to the variable $\mathcal{N}$ as e-$\mathcal{N}$-fold since the scale factor grows roughly by the amount $\mathrm{e}^{\mathcal{N}}$ between $\mathcal{N}$ and $(\mathcal{N}+1)$.

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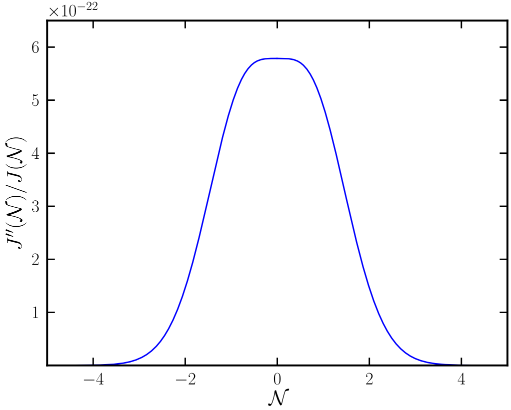
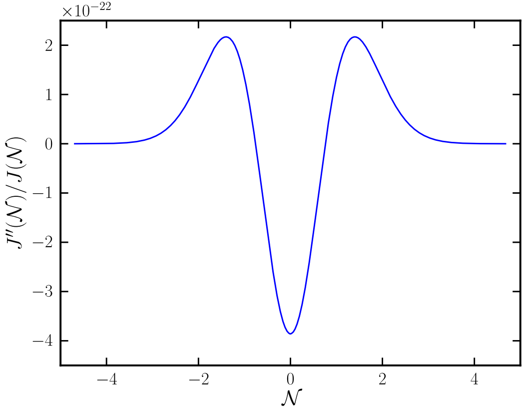
The behavior of J''/J





The behavior of the quantity J''/J has been plotted as a function of $\mathcal{N}$ for $q = 1$ and $n = 3/2$ (on the left) and $n = -1$ (on the right). Note that the maximum value of J''/J is roughly of the order of k_0^2.

Analytical solutions for the modes at early times

At very early times (*i.e.* for $\eta \ll -\eta_0$), the scale factor simplifies to the power law form $a(\eta) \propto \eta^{2\,q}$. During such times, the non-minimal coupling function J also behaves as $J(\eta) \propto \eta^{\gamma}$, where we have set $\gamma = 2\,n\,q$.

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In such a case, we have $J''/J \simeq \gamma\,(\gamma-1)/\eta^2$ and it is easy to show that the solutions to the modes of the electromagnetic vector potential $\mathcal{A}_k$ can be expressed as

$$\mathcal{A}_k(\eta) = \sqrt{-k\,\eta}\,\left[C_1(k)\,J_{\gamma-1/2}(-k\,\eta) + C_2(k)\,J_{-\gamma+1/2}(-k\,\eta)\right].$$

One finds that, for the Bunch-Davies initial conditions, $C_1(k)$ and $C_2(k)$ are given by

$$C_1(k) = \sqrt{\frac{\pi}{4\,k}}\,\frac{\mathrm{e}^{-i\,\pi\,\gamma/2}}{\cos\,(\pi\,\gamma)} \quad \text{and} \quad C_2(k) = \sqrt{\frac{\pi}{4\,k}}\,\frac{\mathrm{e}^{i\,\pi\,(\gamma+1)/2}}{\cos\,(\pi\,\gamma)}.$$

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It can also be shown that

$$\mathcal{A}_k'(\eta) - \frac{J'}{J}\,\mathcal{A}_k(\eta) = k\,\sqrt{-k\,\eta}\,\left[C_1(k)\,J_{\gamma+1/2}(-k\,\eta) - C_2(k)\,J_{-\gamma-1/2}(-k\,\eta)\right].$$

Analytical solutions near the bounce

Note that, when $n > 0$, J''/J has a maximum at the bounce. In such a case, for $k \ll k_0$, $k^2 \ll J''/J$ around the bounce. Hence, upon ignoring the k^2 in the equation governing $\bar{A}_k$, we can integrate the equation to yield

$$\bar{A}_k'(\eta) \simeq \bar{A}_k'(\eta_*)\, \frac{J^2(\eta_*)}{J^2(\eta)},$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce. The above equation can be integrated to arrive at

$$\bar{A}_k(\eta) \simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \int\limits_{\eta_*}^{\eta} \mathrm{d}\eta\, \frac{J^2(\eta_*)}{J^2(\eta)} = \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, a^{2\,n}(\eta_*) \int\limits_{\eta_*}^{\eta} \frac{\mathrm{d}\eta}{a^{2\,n}(\eta)},$$

where we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

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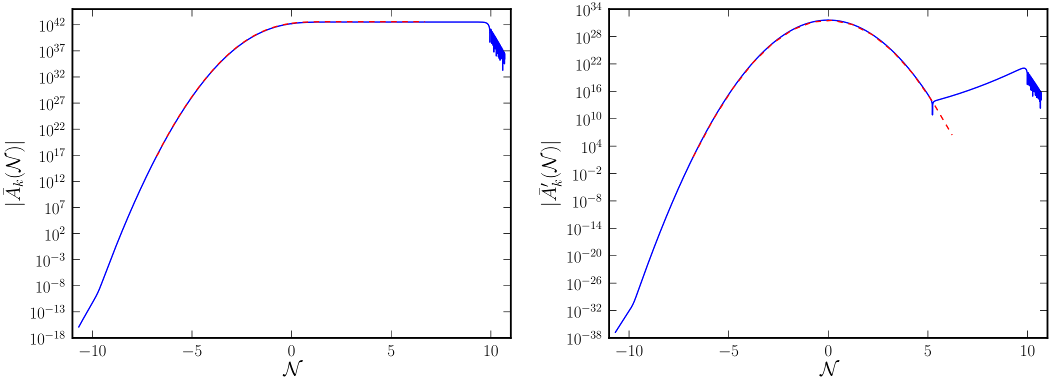
where we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

When $\gamma = 3$, we can evaluate the above integral to obtain that

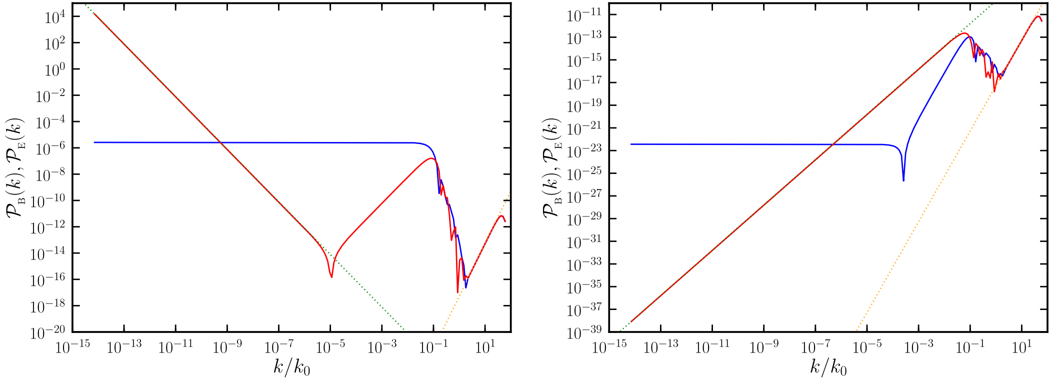
$$\begin{aligned}\bar{A}_k(\eta) \quad &\simeq \quad \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, \frac{a^{2\,n}(\eta_*)}{a_0^{2\,n}}\, \frac{\eta_0}{8} \left\{\frac{\eta}{\eta_0}\, \frac{5 + 3\,(\eta/\eta_0)^2}{[1 + (\eta/\eta_0)^2]^2} + 3\tan^{-1}\left(\frac{\eta}{\eta_0}\right)\right. \\ &\left. - \frac{\eta_*}{\eta_0}\, \frac{5 + 3\,(\eta_*/\eta_0)^2}{[1 + (\eta_*/\eta_0)^2]^2} - 3\tan^{-1}\left(\frac{\eta_*}{\eta_0}\right)\right\}.\end{aligned}$$

Comparison of the numerical and analytical results



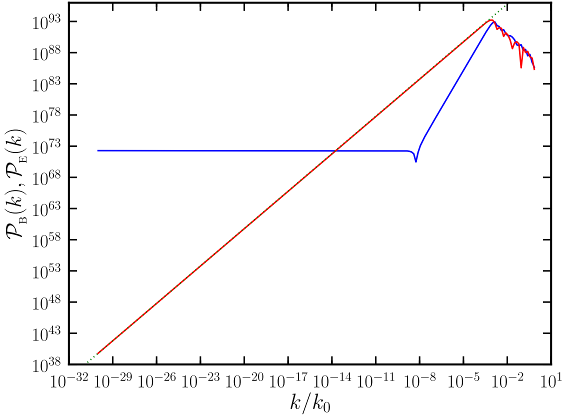
The behavior of the absolute values of $\bar{A}_k$ (on the left) and its derivative $\bar{A}'_k$ (on the right) has been plotted for the mode $k = 10^{-10}\,k_0$ with $k_0/M_{\rm Pl} = {\rm e}^{-25} = 1.389 \times 10^{-11}$ for the case wherein $n = 3/2$, $q = 1$, $a_0 = 10^{-10}$ and $J_0 = 10^4$. The dashed red curves represent the analytical approximation around the bounce that can be arrived at for modes such that $k \ll k_0$.

Power spectra of magnetic and electric fields



The power spectra of the magnetic (in blue) and the electric (in red) fields for the cases wherein $(q,n)=(1,3/2)$ (corresponding to $\gamma=3$, on the left) and $(q,n)=(1,-1)$ (corresponding to $\gamma=-2$, on the right). We have worked with the same values of k_0, a_0 and J_0 as in the previous figure. The power spectra of the electric field are along expected lines, behaving as $k^{4-2\gamma}=k^{-2}$ when $\gamma=3$ and $k^{6+2\gamma}=k^{2}$ when $\gamma=-2$ (indicated by the dotted green lines).

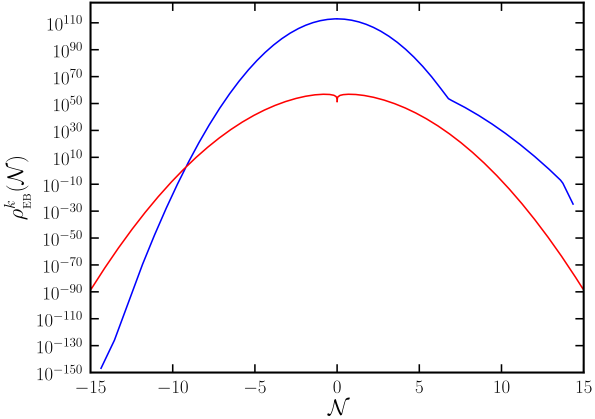
Spectrum of observable strengths



The power spectra with $q = 1$ and $n = -1$, corresponding to $\gamma = -2$ has been plotted for a wide range of wavenumbers. We have set $k_0/M_{\mathrm{Pl}} = 1$, $a_0 = 4 \times 10^{-29}$ and $J_0 = 10^4$, which lead to magnetic fields in the early universe that correspond to observable strengths today[8].

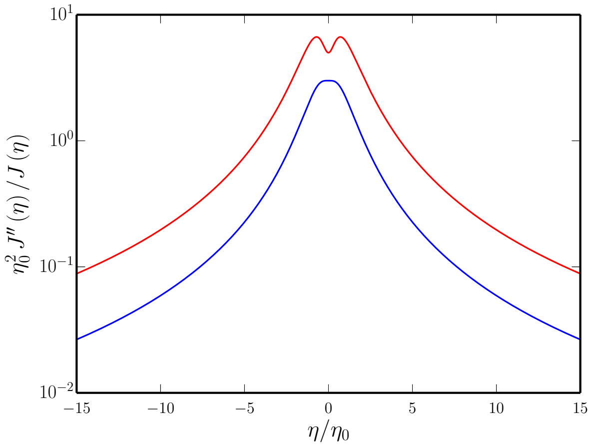
[8] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

The issue of backreaction



The behavior of the energy density in the electric and magnetic fields for the mode $k = 10^{-20}\, k_0$ has been plotted (in blue) along with the energy density of the background (in red). We have worked with the same values of the various parameters as in the last figure.

The behavior of J''/J



The behavior of $\eta_0^2\, J''/J$, which depends only on η/η_0, has been plotted for $\gamma = 3$ (in blue) and $\gamma = 5$ (in red). The figure has been plotted over a very narrow range of η/η_0 in order to illustrate the presence of a single maximum for $\gamma = 3$ and two maxima and one minimum for $\gamma = 5$.

Analytical solutions near the bounce for arbitrary γ

Recall that, near the bounce, when $n > 0$, for scales of cosmological interest such that $k \ll k_0$, we had obtained that

$$\bar{A}_k(\eta) \simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \int_{\eta_*}^{\eta} \mathrm{d}\tilde{\eta}\, \frac{J^2(\eta_*)}{J^2(\tilde{\eta})} = \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, a^{2\,n}(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^{2\,n}(\tilde{\eta})},$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce and we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

[9] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016).

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where η_* is a time when $k^2 \ll J''/J$ before the bounce and we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

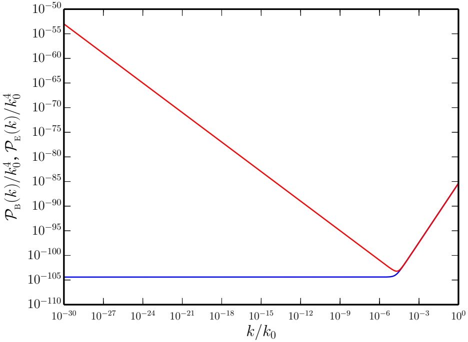
The above integral can, in fact, be carried out for an arbitrary γ to arrive at

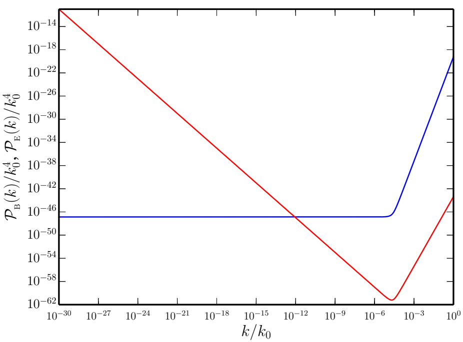
$$\begin{aligned} \bar{A}_k(\eta) \;\; &\simeq \;\; \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, \frac{a^{2\,n}(\eta_*)}{a_0^{2\,n}} \\ &\times \left[\eta\; {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta^2}{\eta_0^2}\right) - \eta_*\; {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta_*^2}{\eta_0^2}\right)\right], \end{aligned}$$

where ${}_2F_1(a, b, c, z)$ denotes the hypergeometric function[9].

[9] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016).

Power spectra before and after the bounce





Left: The dimensionless power spectra of the magnetic (in blue) and electric (in red) fields, evaluated before the bounce at $\eta = -\alpha\,\eta_0$ have been plotted as a function of k/k_0 for $\gamma = 3$, $q = 1$, $a_0 = 8.73 \times 10^{10}$ and $\alpha = 10^5$.

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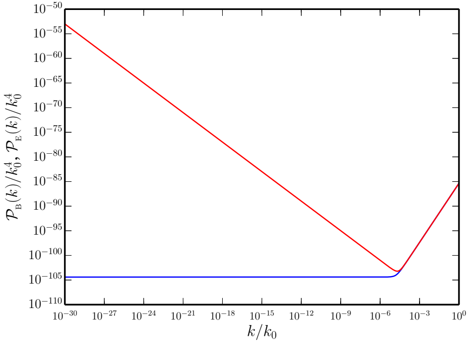

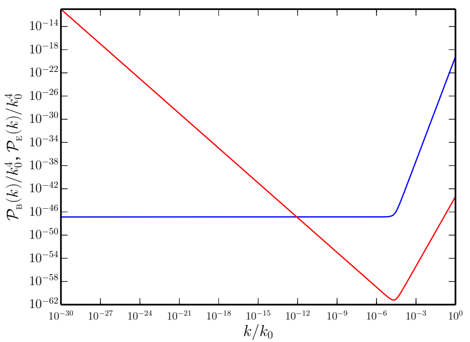

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Right: The corresponding power spectra evaluated after the bounce at $\eta = \beta\,\eta_0$, with $\beta = 10^2$. We should mention that the values of the parameters we have worked with lead to magnetic fields of observed strengths today corresponding to a few femto gauss[10].

[10] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016).

Duality transformations

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation[11]. For instance, it can be shown that the dual solution to the de Sitter case corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

[11] D. Wands, Phys. Rev. D **60**, 023507 (1999).

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In the case of electromagnetic fields of our interest here, given a coupling function J, its dual function, say, $\tilde{J}$, which leads to the same $\tilde{J}''/\tilde{J}$ is found to be

$$J(\eta) \to \tilde{J}(\eta) = C\, J(\eta) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{J^2(\bar{\eta})},$$

where C and η_* are constants. These constants can be suitably chosen to arrive at a physically reasonable form for $\tilde{J}$.

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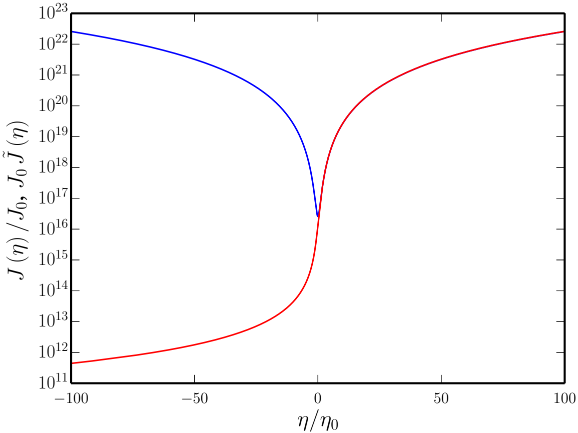
It can be shown that the cases corresponding to $\gamma = 3$ and $\gamma = -2$ in the bouncing models which had led to scale invariant spectra are dual to each other.

[11] D. Wands, Phys. Rev. D **60**, 023507 (1999).

A symmetric coupling function and its asymmetric dual



The coupling function J (in blue) and its dual $\tilde{J}$ (in red) have been plotted as a function of η/η_0 for $\gamma = 3$ and $\eta_* \to -\infty$. Also, we have chosen the constant C to be $C/k_0 = 5.7 \times 10^{32}$ so that the dual function $\tilde{J}$ matches the original coupling function J after the bounce[12].

[12] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **60**, 023507 (1999).

Equation governing the tensor perturbations

Upon quantization, the tensor perturbations can be written in terms of the corresponding modes, say, h_k, as follows:

$$\begin{aligned}\hat{\gamma}_{ij}(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\gamma}_{ij}^{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \\ &= \sum_s \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \left(\hat{b}_{\boldsymbol{k}}^s\, \varepsilon_{ij}^s(\boldsymbol{k})\, h_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\mathbf{k}}^{s\dagger}\, \varepsilon_{ij}^{s*}(\boldsymbol{k})\, h_k^{*}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right),\end{aligned}$$

where $\hat{b}_{\boldsymbol{k}}^s$ and $\hat{b}_{\boldsymbol{k}}^{s\,\dagger}$ are the usual creation and annihilation operators that satisfy the standard commutation relations, while $\varepsilon_{ij}^s(\boldsymbol{k})$ represents the transverse and traceless polarization tensor describing gravitational waves.

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The modes h_k are governed by the differential equation

$$h_k'' + 2\,\mathcal{H}\,h_k' + k^2\,h_k = 0$$

where $\mathcal{H} = a'/a$ and, in terms of the variable $u_k = M_{_{\rm Pl}}\,a\,h_k/\sqrt{2}$, the above equation reduces to

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)\,u_k = 0.$$

The tensor power spectrum: Definition

The tensor power spectrum $\mathcal{P}_{_{\mathrm{T}}}(k)$ is defined through the relation

$$\langle\hat{\gamma}^{\boldsymbol{k}}_{m_1n_1}\,\hat{\gamma}^{\boldsymbol{p}}_{m_2n_2}\rangle = \frac{(2\,\pi)^2}{8\,k^3}\,\Pi^{\boldsymbol{k}}_{m_1n_1,m_2n_2}\,\mathcal{P}_{_{\mathrm{T}}}(k)\,\delta^3\left(\boldsymbol{k}+\boldsymbol{p}\right),$$

where

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In terms of the quantities h_k and u_k, the tensor power spectrum $\mathcal{P}_{_{\rm T}}(k)$ in the Bunch-Davies vacuum is given by

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,|h_k|^2 = \frac{8}{M_{_{\rm Pl}}^2}\,\frac{k^3}{2\,\pi^2}\,\left(\frac{|u_k|}{a}\right)^2,$$

with the right hand side being evaluated at suitably late times[13].

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The matter bounce

We shall assume that the scale factor describing the bouncing scenario is given in terms of the conformal time coordinate η by the relation

$$a(\eta) = a_0\,\left(1 + \eta^2/\eta_0^2\right) = a_0\,\left(1 + k_0^2\,\eta^2\right).$$

As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch[14].

[14] See, for example, R. Brandenberger, arXiv:1206.4196.

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The quantity a''/a corresponding to the above scale factor is given by

$$\frac{a''}{a} = \frac{2\, k_0^2}{1 + k_0^2\, \eta^2},$$

which has essentially a Lorentzian profile.

[14] See, for example, R. Brandenberger, arXiv:1206.4196.

The tensor modes in the first domain

Let us divide the period before the bounce into two domains, with the first domain be determined by the condition $-\infty < \eta < -\alpha\,\eta_0$, where α is a relatively large number, which we shall set to be, say, 10^5.

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0\, k_0^2\, \eta^2$, so that $a''/a = 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.

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The modes h_k can be easily obtained in such a case and the positive frequency modes that correspond to the vacuum state at early times are given by

$$h_k(\eta) = \frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{a_0\,k_0^2\,\eta^2}\,\left(1 - \frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta}.$$

The modes in the second domain

Let us now consider the behavior of the modes in the domain $-\alpha\,\eta_0 < \eta < 0$. Since we are interested in scales much smaller than k_0, we shall assume that $\eta_k \ll -\alpha\,\eta_0$, which corresponds to $k \ll k_0/\alpha$.

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In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h_k'(\eta_*)\,a^2(\eta_*)\,\int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^2(\tilde{\eta})},$$

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where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

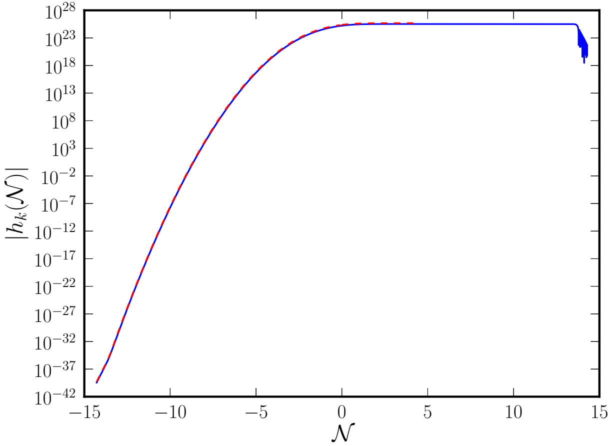
If we choose $\eta_* = -\alpha\,\eta_0$, we can make use of the solution in the first domain to obtain the following solution in the second domain:

$$h_k = A_k + B_k\, f(k_0\,\eta),$$

where

$$f(k_0\,\eta) = \frac{k_0\,\eta}{1 + k_0^2\,\eta^2} + \tan^{-1}(k_0\,\eta)\,.$$

Evolution of the tensor modes across the bounce



A comparison of the numerical results (in blue) with the analytical results (in red) for the amplitude of the tensor mode $|h_k|$ corresponding to the wavenumber $k/k_0 = 10^{-20}$. We have set $a_0 = 10^5$, and we have chosen $\alpha = 10^5$ for plotting the analytical results[15].

[15] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The third domain and the tensor power spectrum

The quantities A_k and B_k are given by

$$\begin{aligned}
A_k &= \frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{a_0\,\alpha^2}\,\left(1+\frac{i\,k_0}{\alpha\,k}\right)\,{\rm e}^{i\,\alpha\,k/k_0} + B_k\,f(\alpha),\\
B_k &= \frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{2\,a_0\,\alpha^2}\,\left(1+\alpha^2\right)^2\,\left(\frac{3\,i\,k_0}{\alpha^2\,k}+\frac{3}{\alpha}-\frac{i\,k}{k_0}\right)\,{\rm e}^{i\,\alpha\,k/k_0}.
\end{aligned}$$

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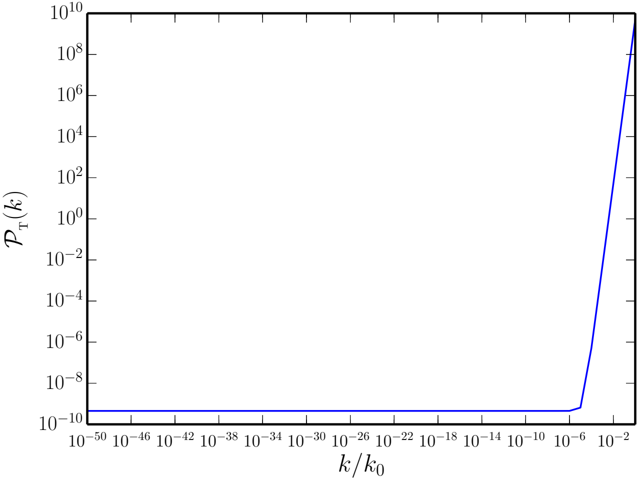
$$B_k = \frac{\sqrt{2}}{M_{\rm Pl}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{2\,a_0\,\alpha^2}\,(1+\alpha^2)^2\,\left(\frac{3\,i\,k_0}{\alpha^2\,k}+\frac{3}{\alpha}-\frac{i\,k}{k_0}\right)\,{\rm e}^{i\,\alpha\,k/k_0}.$$

If we evaluate the tensor power spectrum after the bounce at $\eta = \beta\,\eta_0$, we find that it can be expressed as

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\vert A_k + B_k\,f(\beta)\vert^2.$$

The tensor power spectrum



The behavior of the tensor power spectrum has been plotted as a function of k/k_0 for a wide range of wavenumbers. In plotting this figure, we have set $k_0/M_{_{\rm Pl}} = 1$, $a_0 = 10^5$, $\alpha = 10^5$ and $\beta = 10^2$. Note that the power spectrum is scale invariant for $k \ll k_0/\alpha$.

Tensor bi-spectrum and non-Gaussianity parameter

The tensor bi-spectrum, evaluated at the conformal time, say, η_{e}, is defined as

$$\langle\hat{\gamma}^{\boldsymbol{k}_1}_{m_1n_1}(\eta_{\mathrm{e}})\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2n_2}(\eta_{\mathrm{e}})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3n_3}(\eta_{\mathrm{e}})\rangle = (2\,\pi)^3\,\mathcal{B}^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times\,\delta^{(3)}\,(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$$

and, for convenience, we shall set

$$\mathcal{B}^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = (2\,\pi)^{-9/2}\;G^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).$$

[16] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

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As in the scalar case, one can define a dimensionless non-Gaussianity parameter to characterize the amplitude of the tensor bi-spectrum as follows[16]:

$$h_{_{\mathrm{NL}}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\left(\frac{4}{2\,\pi^2}\right)^2\,\left[k_1^3\,k_2^3\,k_3^3\;G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right] \times\,\left[\Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2}\,\Pi^{\boldsymbol{k}_2}_{m_3 n_3, \bar{m}\bar{n}}\;k_3^3\;\mathcal{P}_{_{\mathrm{T}}}(k_1)\;\mathcal{P}_{_{\mathrm{T}}}(k_2) + \text{five permutations}\right]^{-1}.$$

[16] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

The third order action and the tensor bi-spectrum

The third order action that leads to the tensor bi-spectrum is given by[17]

$$S^3_{\gamma\gamma\gamma}[\gamma_{ij}] = \frac{M_{_{\mathrm{Pl}}}^2}{2} \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x} \left[\frac{a^2}{2}\, \gamma_{lj}\, \gamma_{im}\, \partial_l \partial_m \gamma_{ij} - \frac{a^2}{4}\, \gamma_{ij}\, \gamma_{lm}\, \partial_l \partial_m \gamma_{ij}\right].$$

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The tensor bi-spectrum calculated in the perturbative vacuum using the Maldacena formalism, can be written in terms of the modes h_k as follows:

$$\begin{aligned} G^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= M_{\rm Pl}^2\,\Big[\Big(\Pi^{\boldsymbol{k}_1}_{m_1n_1,ij}\,\Pi^{\boldsymbol{k}_2}_{m_2n_2,im}\,\Pi^{\boldsymbol{k}_3}_{m_3n_3,lj} - \frac{1}{2}\,\Pi^{\boldsymbol{k}_1}_{m_1n_1,ij}\,\Pi^{\boldsymbol{k}_2}_{m_2n_2,ml}\,\Pi^{\boldsymbol{k}_3}_{m_3n_3,ij}\Big)\,k_{1m}\,k_{1l} \\ &\quad + \text{five permutations}\Big] \\ &\quad \times\,\big[h_{k_1}(\eta_{\rm e})\,h_{k_2}(\eta_{\rm e})\,h_{k_3}(\eta_{\rm e})\,\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \text{complex conjugate}\big], \end{aligned}$$

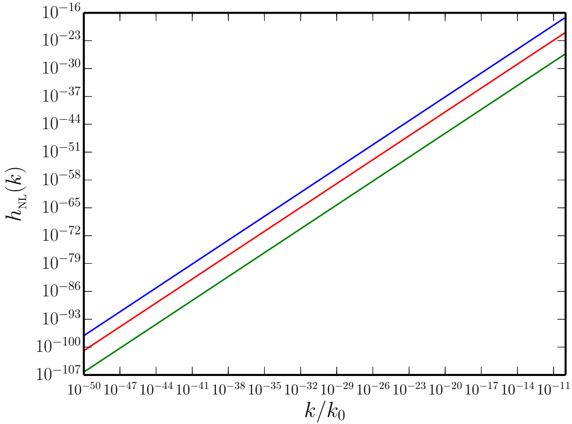
where $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ is described by the integral

$$\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\frac{i}{4}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\,h^{\ast}_{k_1}\,h^{\ast}_{k_2}\,h^{\ast}_{k_3},$$

with $\eta_{\rm i}$ denoting the time when the initial conditions are imposed on the perturbations.

[17] J. Maldacena, JHEP **0305**, 013 (2003).

The contributions due to the three domains



The contributions to the non-Gaussianity parameter $h_{\rm NL}$ in the equilateral limit from the first (in green), the second (in red) and the third (in blue) domains have been plotted as a function of k/k_0 for $k \ll k_0/\alpha$. Clearly, the third domain gives rise to the maximum contribution to $h_{\rm NL}$[18].

[18] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The effect of the long wavelength tensor modes

Since the amplitude of a long wavelength mode freezes on super-Hubble scales during inflation, such modes can be treated as a background as far as the smaller wavelength modes are concerned. Let us denote the constant amplitude of the long wavelength tensor mode as γ_{ij}^{B}.

[19] S. Kundu, JCAP **1404**, 016 (2014).

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In the presence of such a long wavelength mode, the background FLRW metric can be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,[\mathrm{e}^{\gamma^{\mathrm{B}}}]_{ij}\,\mathrm{d}\boldsymbol{x}^i\,\mathrm{d}\boldsymbol{x}^j,$$

i.e. the spatial coordinates are modified according to a spatial transformation of the form $\boldsymbol{x}' = \Lambda\,\boldsymbol{x}$, where $\Lambda_{ij} = [\mathrm{e}^{\gamma^{\mathrm{B}}/2}]_{ij}$.

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i.e. the spatial coordinates are modified according to a spatial transformation of the form $\boldsymbol{x}' = \Lambda\, \boldsymbol{x}$, where $\Lambda_{ij} = [\mathrm{e}^{\gamma^{\mathrm{B}}/2}]_{ij}$.

Under such a spatial transformation, the small wavelength tensor perturbation transforms as[19]

$$\gamma_{ij}^{\boldsymbol{k}} \to \det\left(\Lambda^{-1}\right) \gamma_{ij}^{\Lambda^{-1}\boldsymbol{k}},$$

where $\det\left(\Lambda^{-1}\right) = 1$.

[19] S. Kundu, JCAP **1404**, 016 (2014).

The behavior of the two and three-point functions

On using the above results, one finds that the tensor two-point function in the presence of a long wavelength mode denoted by, say, the wavenumber k, can be written as

$$\langle \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1}\, \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2} \rangle_k = \frac{(2\,\pi)^2}{2\,k_1^3}\, \frac{\Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2}}{4}\, \mathcal{P}_{_{\rm T}}(k_1)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \times \left[1 - \left(\frac{n_{_{\rm T}} - 3}{2}\right)\, \gamma^{\rm B}_{ij}\, \hat{n}_{1i}\, \hat{n}_{1j}\right],$$

where $\hat{n}_{1i} = k_{1i}/k_1$.

[20] V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

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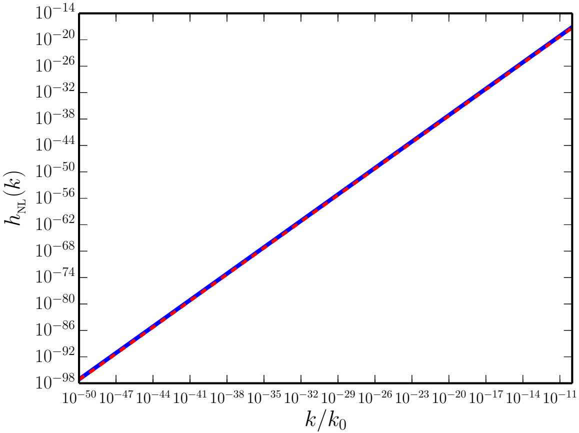
where $\hat{n}_{1i} = k_{1i}/k_1$.

One can also show that, in the presence of a long wavelength mode, the tensor bi-spectrum can be written as[20]

$$\langle\, \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1}\, \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2}\, \hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}\, \rangle_{k_3} = -\frac{(2\,\pi)^{5/2}}{4\,k_1^3\,k_3^3} \left(\frac{n_{_{\rm T}} - 3}{32}\right) \mathcal{P}_{_{\rm T}}(k_1)\, \mathcal{P}_{_{\rm T}}(k_3) \times \Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2}\, \Pi^{\boldsymbol{k}_3}_{m_3 n_3, ij}\, \hat{n}_{1i}\, \hat{n}_{1j}\, \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2).$$

[20] V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

The complete contribution to $h_{\rm NL}$



The behavior of $h_{\rm NL}$ in the equilateral (in blue) and the squeezed (in red) limits plotted as a function of k/k_0 for $k \ll k_0/\alpha$. The resulting $h_{\rm NL}$ is considerably small when compared to the values that arise in de Sitter inflation wherein $3/8 \lesssim h_{\rm NL} \lesssim 1/2$. Moreover, we find that $h_{\rm NL}$ behaves as k^2 in the equilateral and the squeezed limits, with similar amplitudes[21].

[21] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).

Modeling the matter bounce with scalar fields

As we had discussed, the matter bounce scenario described by the scale factor

$$a(\eta) = a_0\, \left(1 + \eta^2/\eta_0^2\right) = a_0\, \left(1 + k_0^2\, \eta^2\right)$$

can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

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We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ, that are governed by the following action[22]:

$$S[\phi,\chi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - V(\phi) - \alpha\,\left(-\frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi\right)^2\right],$$

where α is a dimensionless constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6\,M_{_{\rm Pl}}^2\,k_0^2/a_0^2}{\cosh^6(\sqrt{12}\,\phi/M_{_{\rm Pl}})}.$$

[22] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, Work in progress.

The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$\mathrm{d}s^2 = -(1+2\,A)\,\mathrm{d}t^2 + 2\,a(t)\,(\partial_i B)\,\mathrm{d}t\,\mathrm{d}x^i + a^2(t)\,\left[(1-2\,\psi)\,\delta_{ij} + 2\,(\partial_i\,\partial_j E)\right]\mathrm{d}x^i\,\mathrm{d}x^j,$$

where, evidently, the quantities A, ψ, B and E represent the metric perturbations.

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where, evidently, the quantities A, ψ, B and E represent the metric perturbations.

The gauge invariant curvature and isocurvature perturbations $\mathcal{R}$ and $\mathcal{S}$ can be defined as in terms of the above metric perturbations and the perturbations $\delta\phi$ and $\delta\chi$ in the scalar fields as follows:

$$\mathcal{R} = \frac{H}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\phi}\,\overline{\delta\phi} - \alpha\,\dot{\chi}^3\,\overline{\delta\chi}\right), \quad \mathcal{S} = \frac{H\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\chi}\,\overline{\delta\phi} - \dot{\phi}\,\overline{\delta\chi}\right).$$

The quantities $\overline{\delta\phi}$ and $\overline{\delta\chi}$ denote the gauge invariant versions of the perturbations in the scalar fields, and are given by

$$\overline{\delta\phi} = \delta\phi + \frac{\dot{\phi}\,\psi}{H}, \qquad \overline{\delta\chi} = \delta\chi + \frac{\dot{\chi}\,\psi}{H}.$$

Equations governing the curvature and isocurvature perturbations

We obtain the equations of motion describing the gauge invariant perturbations $\mathcal{R}$ and $\mathcal{S}$ to be

$$\mathcal{R}'' + \frac{2\,\left(7+9\,k_0^2\,\eta^2-6\,k_0^4\,\eta^4\right)}{\eta\,\left(1-2\,k_0^2\,\eta^{-}3\,k_0^4\,\eta^4\right)}\,\mathcal{R}' + \frac{k^2\,\left(5+9\,k_0^2\,\eta^2\right)}{\left(-3+9\,k_0^2\,\eta^2\right)}\,\mathcal{R}$$

$$= -\frac{4\,\left(5+12\,k_0^2\,\eta^2\right)}{\eta\,\left(-1+3\,k_0^2\,\eta^2\right)\,\sqrt{3+3\,k_0^2\,\eta^2}}\,\mathcal{S}' - \frac{4\,\left[5-22\,k_0^2\,\eta^2-24\,k_0^4\,\eta^4+k^2\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^2\right]}{\sqrt{3}\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^{3/2}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{S},$$

$$\mathcal{S}'' + \frac{2\,\left(9+7\,k_0^2\,\eta^2+6\,k_0^4\,\eta^4\right)}{\eta\,\left(-1+2\,k_0^2\,\eta^2+3\,k_0^4\,\eta^4\right)}\,\mathcal{S}'$$

$$+\frac{-18+85\,k_0^2\,\eta^2+25\,k_0^4\,\eta^4+6\,k_0^6\,\eta^6+k^2\,\left(-3+k_0^2\,\eta^2\right)\,\left(\eta+k_0^2\,\eta^3\right)^2}{\left(-1+3\,k_0^2\,\eta^2\right)\,\left(\eta+k_0^2\,\eta^3\right)^2}\,\mathcal{S}$$

$$= -\frac{4\,\sqrt{3}\,\left(-3+2\,k_0^2\,\eta^2\right)}{\eta\,\sqrt{1+k_0^2\,\eta^2}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{R}' - \frac{4\,k^2\,\sqrt{1+k_0^2\,\eta^2}}{\sqrt{3}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{R}.$$

However, note that some of the coefficients diverge at the bounce.

The uniform-χ gauge

The above issue can be avoided by working in a gauge wherein $\delta\chi = 0$[23]. In this gauge, the curvature and isocurvature perturbations simplify to be

$$\mathcal{R} = \psi + \frac{2\,H\,M_{_{\mathrm{Pl}}}^2}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\psi} + H\,A\right), \quad \mathcal{S} = \frac{2\,H\,M_{_{\mathrm{Pl}}}^2\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\frac{\dot{\chi}}{\dot{\phi}}\right)\,\left(\dot{\psi} + H\,A\right).$$

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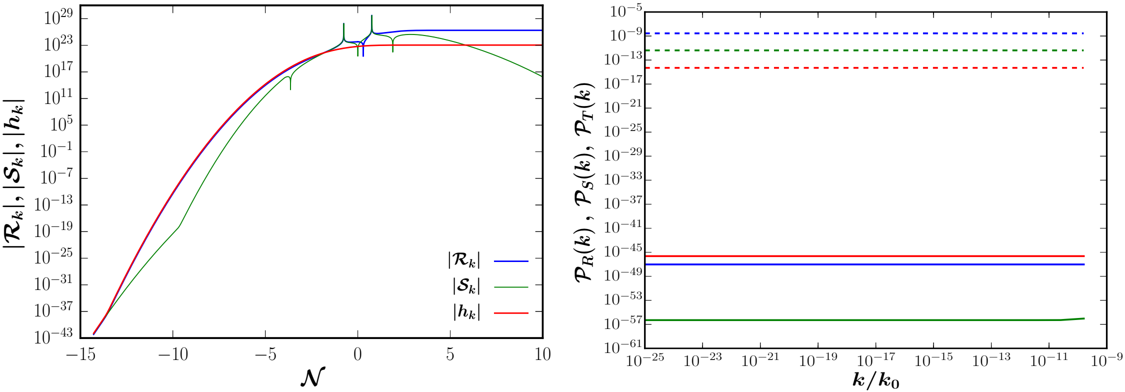
The equations of motion for $\mathcal{R}$ and $\mathcal{S}$ then lead to the following equations for the metric perturbations A and ψ:

$$\begin{aligned} A'' + 4\,\mathcal{H}\,A' + \left(\frac{k^2}{3} - \frac{5}{4}\,\frac{\alpha\,\chi'^4}{a^2\,M_{_{\rm Pl}}^2}\right)\,A &= -3\,\mathcal{H}\,\psi' + \frac{4\,k^2}{3}\,\psi, \\ \psi'' - 2\,\mathcal{H}\,\psi' + k^2\,\psi &= -2\,\mathcal{H}\,A' - \frac{5\,\alpha\chi'^4}{4\,M_{_{\rm Pl}}^2\,a^2}\,A, \end{aligned}$$

where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

[23] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

The scalar and tensor power spectra[24]



Left: The evolution of the scalar (curvature $\mathcal{R}_k$ and isocurvature $\mathcal{S}_k$) and tensor (h_k) perturbations across the bounce for the mode $k/k_0 = 10^{-20}$. We have set $k_0/M_{\rm Pl} = 1$, $a_0 = 3 \times 10^7$ and $\alpha\, M_{\rm Pl}^4 = 1$.

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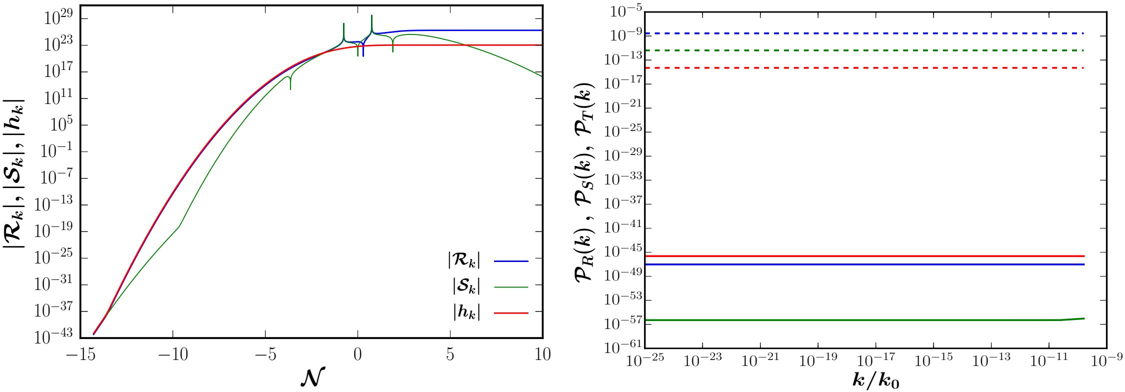

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Right: The corresponding power spectra have been plotted before (as dashed lines) as well as after (as solid lines) the bounce.

[24] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, Work in progress.

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- In a matter bounce which leads to a scale invariant tensor power spectrum as de Sitter inflation does, the amplitude of the tensor bi-spectrum proves to be considerably smaller. Moreover, due to the rapid growth of the amplitude of the tensor modes as one approaches the bounce, the consistency relation governing the tensor bi-spectrum is violated in these scenarios.

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- In a matter bounce which leads to a scale invariant tensor power spectrum as de Sitter inflation does, the amplitude of the tensor bi-spectrum proves to be considerably smaller. Moreover, due to the rapid growth of the amplitude of the tensor modes as one approaches the bounce, the consistency relation governing the tensor bi-spectrum is violated in these scenarios.
- It seems possible to construct matter bounce scenarios wherein the generated tensor-to-scalar ratios are consistent with the observations.

Issues confronting bouncing models

- The growth of the perturbations as one approaches the bounce during the contracting phase causes serious concerns about the validity of linear perturbation theory near the bounce. Is it, for instance, sufficient if the perturbations remain small in specific gauges? Is a divergent curvature perturbation acceptable? These are issues of considerable importance and they need to be addressed satisfactorily.

[25] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

[26] Y-F. Cai, R. Brandenberger and X. Zhang, Phys. Letts. B **703**, 25 (2011).

[27] J. Quintin, Z. Sherkatghanad, Y-F. Cai and R. Brandenberger, Phys. Rev. D **92**, 062532 (2015).

Issues confronting bouncing models

- The growth of the perturbations as one approaches the bounce during the contracting phase causes serious concerns about the validity of linear perturbation theory near the bounce. Is it, for instance, sufficient if the perturbations remain small in specific gauges? Is a divergent curvature perturbation acceptable? These are issues of considerable importance and they need to be addressed satisfactorily.
- Analysis in the cases of a few specific examples seem to suggest that bouncing models lead to a large tensor-to-scalar ratio that is inconsistent with the observations[25]. But it seems possible to construct models with lower tensor amplitudes. This aspect needs to be investigated in a wider set of models.

[25] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

[26] Y-F. Cai, R. Brandenberger and X. Zhang, Phys. Letts. B **703**, 25 (2011).

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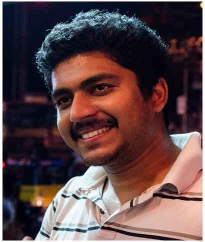
Collaborators: current and former students

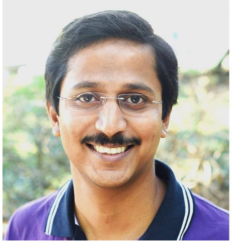
Debika Chowdhury



Rathul Nath Raveendran

Rajeev Kumar Jain

Kumar Atmjeet



V. Sreenath

Thank you for your attention