Computation and characteristics of inflationary three-point functions

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Plan of the talk

1. Some remarks on the computation of the power spectra during inflation
2. Evaluation of the scalar bispectrum generated during inflation
3. BINGO: An efficient code to numerically compute the bispectrum
4. Constraints from Planck on the scalar bispectrum
5. Evaluating the other three-point functions
6. The squeezed limit and the consistency relations
7. Summary

This talk is based on. . .

- D. K. Hazra, L. Sriramkumar and J. Martin, *BINGO: A code for the efficient computation of the scalar bispectrum*, JCAP **1305**, 026 (2013).
- V. Sreenath, R. Tibrewala and L. Sriramkumar, *Numerical evaluation of the three-point scalar-tensor cross-correlations and the tensor bispectrum*, JCAP **1312**, 037 (2013).
- V. Sreenath and L. Sriramkumar, *Examining the consistency relations describing the three-point functions involving tensors*, JCAP 1410, 021 (2014).
- V. Sreenath, D. K. Hazra and L. Sriramkumar, *On the scalar consistency relation away from slow roll*, arXiv:1410.0252 [astro-ph.CO].

A few words on the conventions and notations

- We shall work in units such that $c = \hbar = 1$, and define the Planck mass to be $M_{_{\rm Pl}} = (8\,\pi\,G)^{-1/2}$.

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- Moreover, N shall denote the number of e-folds.
- Further, as usual, a and $H = \dot{a}/a$ shall denote the scale factor and the Hubble parameter associated with the Friedmann universe.

Arriving at the action governing the perturbations I

Recall that, in the ADM formalism, the spacetime metric is expressed in terms of the lapse function N, the shift vector N^i and the spatial metric h_{ij} as

$$\mathrm{d}s^2 = -N^2\,\mathrm{d}t^2 + h_{ij}\,\left(N^i\,\mathrm{d}t + \mathrm{d}x^i\right)\,\left(N^j\,\mathrm{d}t + \mathrm{d}x^j\right),$$

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The case of our interest is Einsteinian gravity which is sourced by a canonical and minimally coupled scalar field, say, the inflaton ϕ that is governed by the potential $V(\phi)$.

We shall work in the co-moving gauge wherein the perturbations in the scalar field are assumed to be absent. Also, we shall assume that the spatial metric h_{ij} is given by

$$h_{ij} = a^2(t)\,\mathrm{e}^{2\,\mathcal{R}(t,\boldsymbol{x})}\,\left[\mathrm{e}^{\gamma(t,\boldsymbol{x})}\right]_{ij},$$

where $\mathcal{R}$ denotes the curvature perturbation describing the scalars, while γ_{ij} represents the transverse and traceless (*i.e.* $\partial_j\gamma_{ij} = \gamma_{ii} = 0$) tensor perturbations.

Arriving at the action governing the perturbations II

The action describing such a system can be written in terms of the metric variables N, N^i and h_{ij} and the homogeneous scalar field ϕ as follows:

$$S[N, N^i, h_{ij}, \phi] = \int \mathrm{d}t \int \mathrm{d}^3\boldsymbol{x}\, N\,\sqrt{h}\,\left\{\frac{M_{_{\mathrm{Pl}}}^2}{2}\left[\frac{1}{N^2}\,\left(E_{ij}E^{ij} - E^2\right) + {}^{(3)}R\right] + \left[\frac{\dot{\phi}^2}{2\,N^2} - V(\phi)\right]\right\},$$

where $h \equiv \det\,(h_{ij})$ and ${}^{(3)}R$ is the spatial curvature associated with the metric h_{ij}.

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where $h \equiv \det\,(h_{ij})$ and ${}^{(3)}R$ is the spatial curvature associated with the metric h_{ij}.

The quantity E_{ij} is given by

$$E_{ij} = \frac{1}{2}\, \left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right),$$

with $E = h_{ij}\, E^{ij}$.

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Solving the constraint equations and substituting the solutions back in the above action permits one to arrive at the actions describing the dynamical variables of our interest, *viz.* $\mathcal{R}$ and γ_{ij}.

The quadratic action governing the perturbations

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[1] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

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One can show that, at the quadratic order, the actions governing the curvature perturbation $\mathcal{R}$ and the tensor perturbation γ_{ij} are given by[1]

$$\mathcal{S}^2_{\mathcal{R}\mathcal{R}}[\mathcal{R}] = \frac{1}{2}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\ z^2\left[\mathcal{R}'^2 - (\partial\mathcal{R})^2\right],$$

where $z = \sqrt{2\,\epsilon_1}\,M_{_{\mathrm{Pl}}}\,a$, with $\epsilon_1 = \dot{H}/H^2$ being the first slow roll parameter, and

$$\mathcal{S}^2_{\gamma\gamma}[\gamma_{ij}] = \frac{M_{_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\ a^2\left[\gamma_{ij}'^{\,2} - (\partial\gamma_{ij})^2\right].$$

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These actions lead to the following equations of motion governing the Fourier modes, say, f_k and g_k, of the scalar and the tensor perturbations:

$$f_k'' + 2\,(z'/z)\, f_k' + k^2\, f_k = 0,$$
$$g_k'' + 2\,(a'/a)\, g_k' + k^2\, g_k = 0.$$

[1] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

Quantization of the scalar and tensor perturbations

On quantization, the operators $\hat{\mathcal{R}}(\eta, \boldsymbol{x})$ and $\hat{\gamma}_{ij}(\eta, \boldsymbol{x})$ representing the scalar and the tensor perturbations can be expressed in terms of the corresponding Fourier modes f_k and g_k as[2]

$$\begin{aligned}
\hat{\mathcal{R}}(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \\
&= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \left[\hat{a}_{\boldsymbol{k}}\, f_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^{\dagger}_{\boldsymbol{k}}\, f^{*}_k(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right], \\
\hat{\gamma}_{ij}(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\gamma}^{\boldsymbol{k}}_{ij}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \\
&= \sum_s \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \left[\hat{b}^{s}_{\boldsymbol{k}}\, \varepsilon^{s}_{ij}(\boldsymbol{k})\, g_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^{s\dagger}_{\boldsymbol{k}}\, \varepsilon^{s*}_{ij}(\boldsymbol{k})\, g^{*}_k(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right].
\end{aligned}$$

In these decompositions, the operators $(\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}})$ and $(\hat{b}^{s}_{\boldsymbol{k}}, \hat{b}^{s\dagger}_{\boldsymbol{k}})$ satisfy the standard commutation relations, while the quantity $\varepsilon^{s}_{ij}(\boldsymbol{k})$ represents the transverse and traceless polarization tensor describing the gravitational waves.

[2] See, for instance, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

The scalar and tensor power spectra

The dimensionless scalar and tensor power spectra $\mathcal{P}_{_{\mathrm{S}}}(k)$ and $\mathcal{P}_{_{\mathrm{T}}}(k)$ are defined in terms of the correlation functions of the Fourier modes $\hat{\mathcal{R}}_{\boldsymbol{k}}$ and $\hat{\gamma}_{mn}^{\boldsymbol{k}}$ as follows:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\, \hat{\mathcal{R}}_{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{2\,k^3}\, \mathcal{P}_{_{\mathrm{S}}}(k)\, \delta^{(3)}\,(\boldsymbol{k}+\boldsymbol{k}'),$$

$$\langle \hat{\gamma}_{m_1n_1}^{\boldsymbol{k}}(\eta)\, \hat{\gamma}_{m_2n_2}^{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{8\,k^3}\, \Pi_{m_1n_1,m_2n_2}^{\boldsymbol{k}}\, \mathcal{P}_{_{\mathrm{T}}}(k)\, \delta^{3}\,(\boldsymbol{k}+\boldsymbol{k}'),$$

where

$$\Pi_{m_1n_1,m_2n_2}^{\boldsymbol{k}} = \sum_s \varepsilon_{m_1n_1}^{s}(\boldsymbol{k})\, \varepsilon_{m_2n_2}^{s*}(\boldsymbol{k}).$$

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The dimensionless scalar and tensor power spectra $\mathcal{P}_{\rm S}(k)$ and $\mathcal{P}_{\rm T}(k)$ are defined in terms of the correlation functions of the Fourier modes $\hat{\mathcal{R}}_{\boldsymbol{k}}$ and $\hat{\gamma}_{mn}^{\boldsymbol{k}}$ as follows:

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\,\hat{\mathcal{R}}_{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{2\,k^3}\,\mathcal{P}_{\rm S}(k)\;\delta^{(3)}\left(\boldsymbol{k}+\boldsymbol{k}'\right),$$

$$\langle\hat{\gamma}_{m_1 n_1}^{\boldsymbol{k}}(\eta)\,\hat{\gamma}_{m_2 n_2}^{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{8\,k^3}\,\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}}\;\mathcal{P}_{\rm T}(k)\;\delta^{3}\left(\boldsymbol{k}+\boldsymbol{k}'\right),$$

where

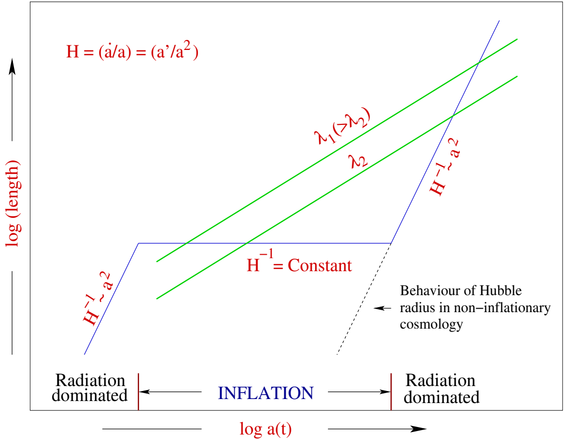
$$\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}} = \sum_{s}\,\varepsilon_{m_1 n_1}^{s}(\boldsymbol{k})\;\varepsilon_{m_2 n_2}^{s*}(\boldsymbol{k}).$$

In the Bunch-Davies vacuum, say, $\vert 0\rangle$, which is defined as $\hat{a}_{\boldsymbol{k}}\vert 0\rangle = 0$ and $\hat{b}_{\boldsymbol{k}}^{s}\vert 0\rangle = 0\;\forall\;\boldsymbol{k}$ and s, we can express the power spectra in terms of the quantities f_k and g_k as

$$\mathcal{P}_{\rm S}(k) = \frac{k^3}{2\,\pi^2}\,\vert f_k\vert^2 \quad \text{and} \quad \mathcal{P}_{\rm T}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\vert g_k\vert^2.$$

With the initial conditions imposed in the sub-Hubble domain, *viz.* when $k/(a\,H) \gg 1$, these spectra are to be evaluated on super-Hubble scales, *i.e.* as $k/(a\,H) \ll 1$.

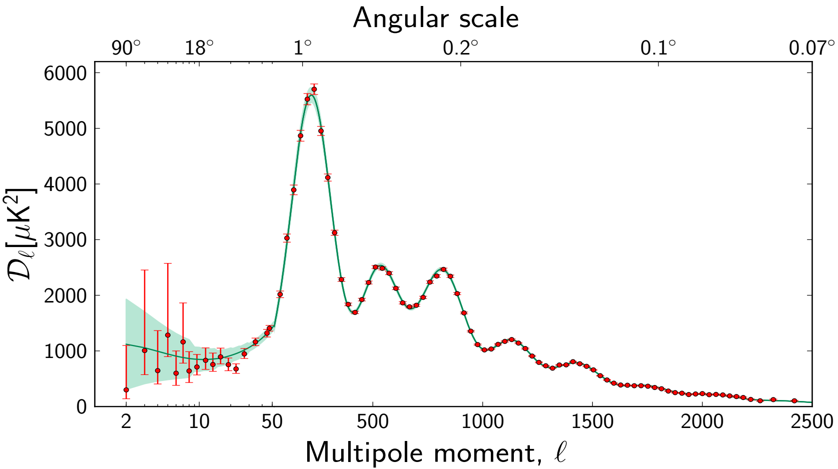
From inside the Hubble radius to super-Hubble scales



A schematic diagram illustrating the behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[3].

[3] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Angular power spectrum from the Planck data[4]



The CMB TT angular power spectrum from the Planck data (the red dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid green curve).

[4] P. A. R. Ade *et al.*, arXiv:1303.5075 [astro-ph.CO].

The scalar bispectrum

The scalar bispectrum $\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time $\eta_{\rm e}$, as follows[5]:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e}) \rangle = (2\,\pi)^3\; \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\; \delta^{(3)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right).$$

For convenience, we shall set

$$\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\; G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

[5] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

The non-Gaussianity parameter $f_{_{\rm NL}}$

The observationally relevant non-Gaussianity parameter $f_{_{\rm NL}}$ is basically introduced through the relation[6]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}_{_{\rm G}}(\eta, \boldsymbol{x}) - \frac{3\, f_{_{\rm NL}}}{5}\, \left[\mathcal{R}_{_{\rm G}}^2(\eta, \boldsymbol{x}) - \left\langle \mathcal{R}_{_{\rm G}}^2(\eta, \boldsymbol{x})\right\rangle\right],$$

where $\mathcal{R}_{_{\rm G}}$ denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

Utilizing the above relation and Wick's theorem, one can arrive at the three-point correlation function of the curvature perturbation in Fourier space in terms of the parameter $f_{_{\rm NL}}$. It is found to be

$$\begin{aligned}\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}\, \hat{\mathcal{R}}_{\boldsymbol{k}_3}\rangle &= -\frac{3\, f_{_{\rm NL}}}{10}\, \frac{(2\,\pi)^{5/2}}{k_1^3\, k_2^3\, k_3^3}\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\\ &\quad \times \left[k_1^3\, \mathcal{P}_{_{\rm S}}(k_2)\, \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right].\end{aligned}$$

[6] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

The relation between $f_{_{\rm NL}}$ and the scalar bispectrum

Upon making use of the above expression for the three-point function of the curvature perturbation and the definition of the scalar bispectrum, we can, in turn, arrive at the following relation[7]:

$$
\begin{aligned}
f_{_{\rm NL}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= -\frac{10}{3}\,(2\,\pi)^{1/2}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad\times\,\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)\,\mathcal{P}_{_{\rm S}}(k_3)+\text{two permutations}\right]^{-1}\\
&= -\frac{10}{3}\,\frac{1}{(2\,\pi)^4}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad\times\,\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)\,\mathcal{P}_{_{\rm S}}(k_3)+\text{two permutations}\right]^{-1}.
\end{aligned}
$$

[7] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

The action at the cubic order

It can be shown that, the third order term in the action describing the curvature perturbation is given by[8]

$$\mathcal{S}^3_{\mathcal{R}\mathcal{R}\mathcal{R}}[\mathcal{R}] = M_{_{\mathrm{Pl}}}^2 \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial^i\mathcal{R})\,(\partial_i\chi) + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial^i\mathcal{R})\,(\partial_i\chi)\,(\partial^2\chi) + \frac{\epsilon_1}{4}\,(\partial^2\mathcal{R})\,(\partial\chi)^2 + \mathcal{F}_1\left(\frac{\delta\mathcal{L}^2_{\mathcal{R}\mathcal{R}}}{\delta\mathcal{R}}\right)\biggr],$$

where $\epsilon_2 = \mathrm{d}\ln\epsilon_1/\mathrm{d}N$ denotes the second slow roll parameter, the quantity $\mathcal{F}_1(\delta\mathcal{L}^2_{\mathcal{R}\mathcal{R}}/\delta\mathcal{R})$ represents terms involving the variation of the second order Lagrangian density with respect to $\mathcal{R}$, and χ is related to the curvature perturbation $\mathcal{R}$ through the relation: $\partial^2\chi = a\,\epsilon_1\,\mathcal{R}'$.

[8] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

Evaluating the scalar bispectrum

At the leading order in the perturbations, one then finds that the scalar three-point correlation function in Fourier space is described by the integral[9]

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e})\rangle = -i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a(\eta)\;\left\langle\left[\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e}), \hat{H}_{\rm I}(\eta)\right]\right\rangle,$$

where $\hat{H}_{\rm I}$ is the Hamiltonian corresponding to the above third order action, while $\eta_{\rm i}$ denotes a sufficiently early time when the initial conditions are imposed on the modes, and $\eta_{\rm e}$ denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are to be evaluated in the perturbative vacuum.

[9] See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).

The resulting bispectrum

The quantity $G_{\mathcal{RRR}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ evaluated towards the end of inflation at the conformal time $\eta = \eta_{\rm e}$ can be written as[10]

$$
\begin{aligned}
G_{\mathcal{RRR}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &\equiv \sum_{C=1}^{7} G_{\mathcal{RRR}}^{(C)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&\equiv M_{\rm Pl}^2 \sum_{C=1}^{6} \Big\{ \left[f_{k_1}(\eta_{\rm e})\, f_{k_2}(\eta_{\rm e})\, f_{k_3}(\eta_{\rm e})\right]\, \mathcal{G}_{\mathcal{RRR}}^{(C)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&\qquad + \left[f_{k_1}^{\ast}(\eta_{\rm e})\, f_{k_2}^{\ast}(\eta_{\rm e})\, f_{k_3}^{\ast}(\eta_{\rm e})\right]\, \mathcal{G}_{\mathcal{RRR}}^{(C)\ast}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \Big\} \\
&\qquad + G_{\mathcal{RRR}}^{(7)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3),
\end{aligned}
$$

where the quantities $\mathcal{G}_{\mathcal{RRR}}^{(C)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ with $C = (1, 6)$ correspond to the six terms in the interaction Hamiltonian.

The additional, seventh term $G_{\mathcal{RRR}}^{(7)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ arises due to a field redefinition, and its contribution to $G_{\mathcal{RRR}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is given by

$$
G_{\mathcal{RRR}}^{(7)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{\epsilon_2(\eta_{\rm e})}{2} \left(|f_{k_2}(\eta_{\rm e})|^2\, |f_{k_3}(\eta_{\rm e})|^2 + \text{two permutations}\right).
$$

[10] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

The integrals involved

The quantities $\mathcal{G}^{(C)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ with $C=(1,6)$ are described by the integrals

$$\mathcal{G}^{(1)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2\,i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^2\,\left(f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right),$$

$$\mathcal{G}^{(2)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\,2\,i\,\left(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + \text{two permutations}\right)\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^2\,f_{k_1}^{\ast}\,f_{k_2}^{\ast}\,f_{k_3}^{\ast},$$

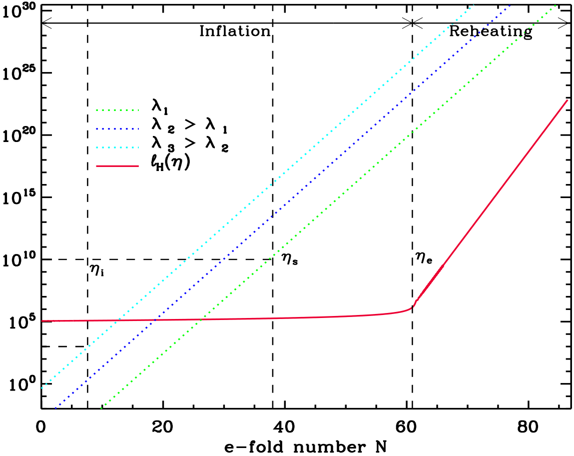
$$\mathcal{G}^{(3)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\,2\,i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^2\,\left[\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\right) f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{five permutations}\right],$$

$$\mathcal{G}^{(4)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1\,\epsilon_2^{\prime}\,\left(f_{k_1}^{\ast}\,f_{k_2}^{\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right),$$

$$\mathcal{G}^{(5)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^3\,\left[\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\right) f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{five permutations}\right],$$

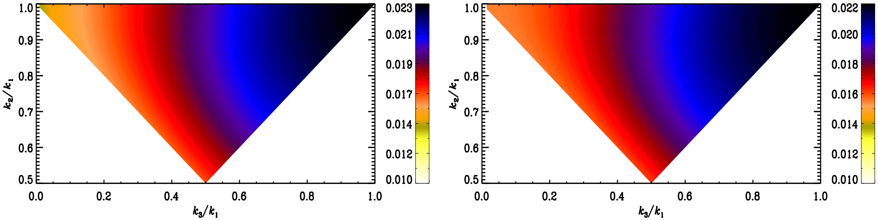
$$\mathcal{G}^{(6)}_{\mathcal{RRR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{i}{2}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\, a^2\,\epsilon_1^3\,\left\{\left[\frac{k_1^2\,\left(\boldsymbol{k}_2\cdot\boldsymbol{k}_3\right)}{k_2^2\,k_3^2}\right] f_{k_1}^{\ast}\,f_{k_2}^{\prime\ast}\,f_{k_3}^{\prime\ast} + \text{two permutations}\right\}.$$

The various times of interest



The exact behavior of the physical wavelengths and the Hubble radius plotted as a function of the number of e-folds in the case of the archetypical quadratic potential, which allows us to illustrate the various times of our interest, *viz.* $\eta_{\rm i}$, $\eta_{\rm s}$ and $\eta_{\rm e}$.

Results from BINGO[11]



A comparison of the analytical results (on the left) for the non-Gaussianity parameter $f_{\rm NL}$ with the numerical results from the code BIspectra and Non-Gaussianity Operator or, simply, BINGO (on the right) for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential. The maximum difference between the numerical and the analytic results is found to be about 5%.

[11] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026, (2013).

Inflationary models permitting deviations from slow roll

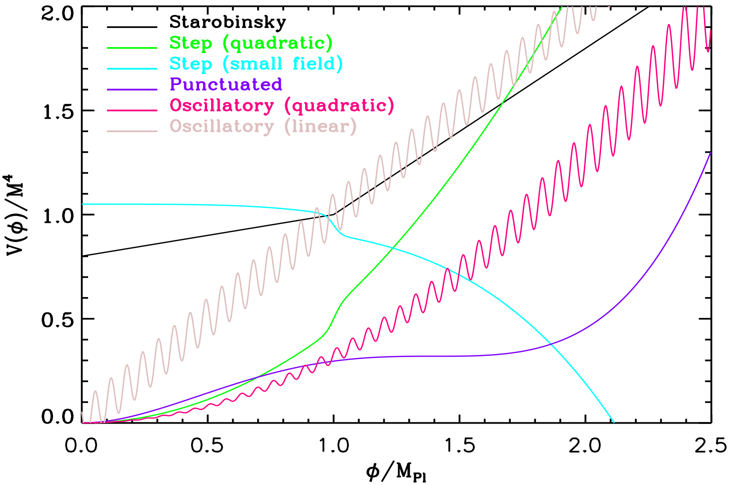
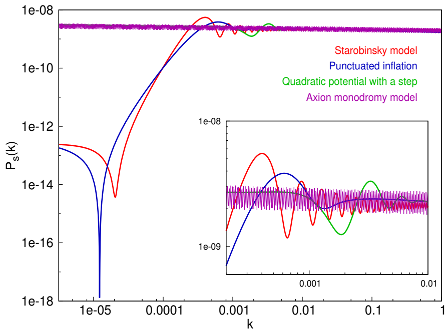
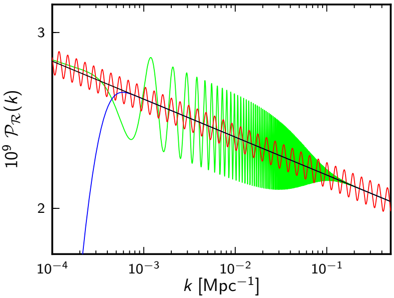


Illustration of potentials that admit departures from slow roll.

Spectra leading to an improved fit to the CMB data



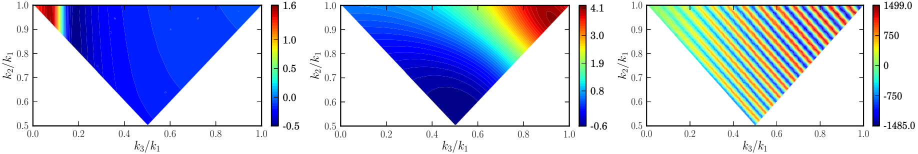


Left: The scalar power spectra in different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum[12].
Right: A set of spectra with features considered by the Planck team[13].

[12] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013).

[13] P. A. R. Ade *et al.*, arXiv:1303.5082 [astro-ph.CO].

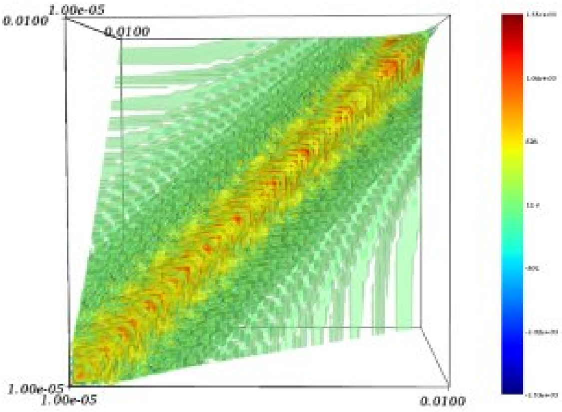
$f_{\rm NL}$ in models with features[14]



The scalar non-Gaussianity parameter $f_{\rm NL}$ in the punctuated inflationary scenario (on the left), quadratic potential with a step (in the middle) and the axion monodromy model (on the right).

[14] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026 (2013);
V. Sreenath, D. K. Hazra and L. Sriramkumar, arXiv:1410.0252 [astro-ph.CO].

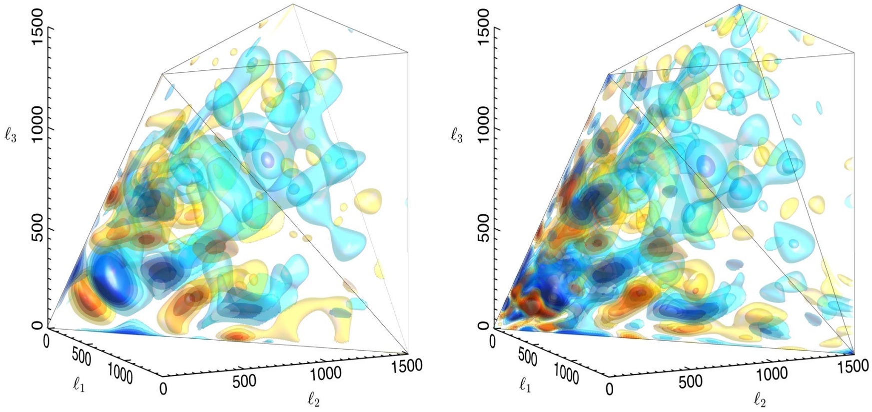
The inflationary scalar bispectrum



The shape of the inflationary scalar bispectrum (actually, the non-Gaussianity parameter $f_{\rm NL}$) in the case of the axion monodromy model[15].

[15] V. Sreenath, D. K. Hazra and L. Sriramkumar, arXiv:1410.0252 [astro-ph.CO].

The observed CMB TTT angular bispectrum



The CMB TTT angular bispectrum as observed by Planck, arrived at using two different methods[16].

[16] P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

Template bispectra

For comparison with the observations, the scalar bispectrum is often expressed as follows[17]:

$$G_{\mathcal{RRR}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = f_{\rm NL}^{\rm loc}\, G_{\mathcal{RRR}}^{\rm loc}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm eq}\, G_{\mathcal{RRR}}^{\rm eq}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm orth}\, G_{\mathcal{RRR}}^{\rm orth}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3),$$

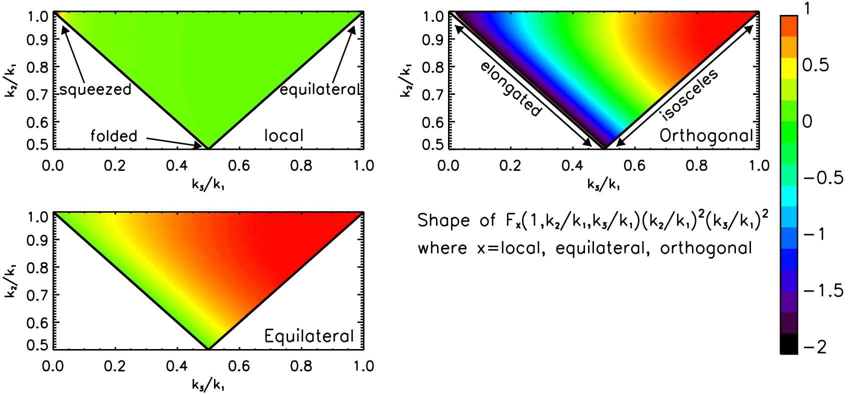
where $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm eq}$ and $f_{\rm NL}^{\rm orth}$ are free parameters that are to be estimated, and the local, the equilateral, and the orthogonal template bi-spectra are given by:

$$\begin{aligned}
G_{\mathcal{RRR}}^{\rm loc}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{6}{5}\, \frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\, \left(k_1^3\, \mathcal{P}_{_{\rm S}}(k_2)\, \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right),\\
G_{\mathcal{RRR}}^{\rm eq}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{3}{5}\, \frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\, \Bigl(6\, k_2\, k_3^2\, \mathcal{P}_{_{\rm S}}(k_1)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_2)\, \mathcal{P}_{_{\rm S}}^{1/3}(k_3) - 3\, k_3^3\, \mathcal{P}_{_{\rm S}}(k_1)\, \mathcal{P}_{_{\rm S}}(k_2)\\
&\quad -2\, k_1\, k_2\, k_3\, \mathcal{P}_{_{\rm S}}^{2/3}(k_1)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_2)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_3) + \text{five permutations}\Bigr),\\
G_{\mathcal{RRR}}^{\rm orth}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{3}{5}\, \frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\, \Bigl(18\, k_2\, k_3^2\, \mathcal{P}_{_{\rm S}}(k_1)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_2)\, \mathcal{P}_{_{\rm S}}^{1/3}(k_3) - 9\, k_3^3\, \mathcal{P}_{_{\rm S}}(k_1)\, \mathcal{P}_{_{\rm S}}(k_2)\\
&\quad -8\, k_1\, k_2\, k_3\, \mathcal{P}_{_{\rm S}}^{2/3}(k_1)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_2)\, \mathcal{P}_{_{\rm S}}^{2/3}(k_3) + \text{five permutations}\Bigr).
\end{aligned}$$

The basis $(f_{\rm NL}^{\rm loc}, f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth})$ for the scalar bispectrum is considered to be large enough to encompass a range of interesting models.

[17] C. L. Bennett *et al.*, Astrophys. J. Suppl. **208**, 20 (2013).

Illustration of the template bi-spectra



An illustration of the three template basis bi-spectra, *viz.* the local (top left), the equilateral (bottom) and the orthogonal (top right) forms for a generic triangular configuration of the wavevectors[18].

[18] E. Komatsu, Class. Quantum Grav. **27**, 124010 (2010).

Constraints on $f_{_{\rm NL}}$

The constraints on the non-Gaussianity parameters from the recent Planck data are as follows[19]:

$$
\begin{aligned}
f_{_{\rm NL}}^{\rm loc} &= 2.7 \pm 5.8, \\
f_{_{\rm NL}}^{\rm eq} &= -42 \pm 75, \\
f_{_{\rm NL}}^{\rm orth} &= -25 \pm 39.
\end{aligned}
$$

It should be stressed here that these are constraints on the primordial values.

Also, the constraints on each of the $f_{_{\rm NL}}$ parameters have been arrived at assuming that the other two parameters are zero.

We should also add that these constraints become less stringent if the primordial spectra are assumed to contain features.

[19] P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

The cross-correlations and the tensor bispectrum

The cross-correlations involving two scalars and a tensor and a scalar and two tensors are defined as

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3n_3}(\eta_{\rm e})\,\rangle = (2\,\pi)^3\,\mathcal{B}^{m_3n_3}_{\mathcal{R}\mathcal{R}\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\,\delta^{(3)}\,(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,,$$

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2n_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3n_3}(\eta_{\rm e})\rangle = (2\,\pi)^3\,\mathcal{B}^{m_2n_2m_3n_3}_{\mathcal{R}\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\times\delta^{(3)}\,(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,,$$

while the tensor bispectrum is given by

$$\langle\hat{\gamma}^{\boldsymbol{k}_1}_{m_1n_1}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2n_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3n_3}(\eta_{\rm e})\rangle = (2\,\pi)^3\,\mathcal{B}^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\times\delta^{(3)}\,(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,.$$

As in the pure scalar case, we shall set

$$\mathcal{B}_{\rm ABC}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = (2\,\pi)^{-9/2}\,G_{\rm ABC}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3),$$

where each of $(\mathrm{A},\mathrm{B},\mathrm{C})$ can be either a $\mathcal{R}$ or a γ.

The corresponding non-Gaussianity parameters

As in the scalar case, one can define dimensionless non-Gaussianity parameters to characterize the scalar-scalar-tensor and the scalar-tensor-tensor cross-correlations and the tensor bispectrum as follows[20]:

$$
\begin{aligned}
C_{\rm NL}^{\mathcal{R}} &= -\frac{4}{(2\,\pi^2)^2}\,\left[k_1^3\,k_2^3\,k_3^3\,G_{\mathcal{R}\mathcal{R}\gamma}^{m_3n_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right]\\
&\quad\times\left(\Pi_{m_3n_3,\bar{m}\bar{n}}^{\boldsymbol{k}_3}\right)^{-1}\,\left\{\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)+k_2^3\,\mathcal{P}_{_{\rm S}}(k_1)\right]\,\mathcal{P}_{_{\rm T}}(k_3)\right\}^{-1},\\
C_{\rm NL}^{\gamma} &= -\frac{4}{(2\,\pi^2)^2}\,\left[k_1^3\,k_2^3\,k_3^3\,G_{\mathcal{R}\gamma\gamma}^{m_2n_2m_3n_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right]\\
&\quad\times\left\{\mathcal{P}_{_{\rm S}}(k_1)\,\left[\Pi_{m_2n_2,m_3n_3}^{\boldsymbol{k}_2}\,k_3^3\,\mathcal{P}_{_{\rm T}}(k_2)+\Pi_{m_3n_3,m_2n_2}^{\boldsymbol{k}_3}\,k_2^3\,\mathcal{P}_{_{\rm T}}(k_3)\right]\right\}^{-1},\\
h_{\rm NL} &= -\left(\frac{4}{2\,\pi^2}\right)^2\,\left[k_1^3\,k_2^3\,k_3^3\,G_{\gamma\gamma\gamma}^{m_1n_1m_2n_2m_3n_3}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\right]\\
&\quad\times\left[\Pi_{m_1n_1,m_2n_2}^{\boldsymbol{k}_1}\,\Pi_{m_3n_3,\bar{m}\bar{n}}^{\boldsymbol{k}_2}\,k_3^3\,\mathcal{P}_{_{\rm T}}(k_1)\,\mathcal{P}_{_{\rm T}}(k_2)+\text{five permutations}\right]^{-1}.
\end{aligned}
$$

[20] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

The actions governing the other three point functions

The actions that lead to the correlations involving two scalars and one tensor, one scalar and two tensors and three tensors are given by

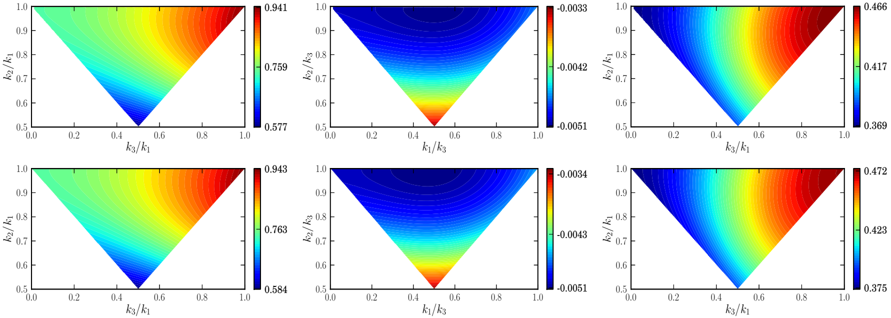
$$S^3_{\mathcal{R}\mathcal{R}\gamma}[\mathcal{R},\gamma_{ij}] = M_{_{\rm Pl}}^2 \int {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1\,\gamma_{ij}\,\partial_i\mathcal{R}\,\partial_j\mathcal{R} + \frac{1}{4}\,\partial^2\gamma_{ij}\,\partial_i\chi\,\partial_j\chi + \frac{a\,\epsilon_1}{2}\,\gamma_{ij}'\,\partial_i\mathcal{R}\,\partial_j\chi + \mathcal{F}^2_{ij}(\mathcal{R})\,\frac{\delta\mathcal{L}^2_{\gamma\gamma}}{\delta\gamma_{ij}} + \mathcal{F}^3(\mathcal{R},\gamma_{ij})\,\frac{\delta\mathcal{L}^2_{\mathcal{R}\mathcal{R}}}{\delta\mathcal{R}}\biggr],$$

$$S^3_{\mathcal{R}\gamma\gamma}[\mathcal{R},\gamma_{ij}] = \frac{M_{_{\rm Pl}}^2}{4} \int {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[\frac{a^2\,\epsilon_1}{2}\,\mathcal{R}\,\gamma_{ij}'\,\gamma_{ij}' + \frac{a^2\,\epsilon_1}{2}\,\mathcal{R}\,\partial_l\gamma_{ij}\,\partial_l\gamma_{ij} - a\,\gamma_{ij}'\,\partial_l\gamma_{ij}\,\partial_l\chi + \mathcal{F}^4_{ij}(\mathcal{R},\gamma_{mn})\,\frac{\delta\mathcal{L}^2_{\gamma\gamma}}{\delta\gamma_{ij}}\biggr],$$

$$S^3_{\gamma\gamma\gamma}[\gamma_{ij}] = \frac{M_{_{\rm Pl}}^2}{2} \int {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[\frac{a^2}{2}\,\gamma_{lj}\,\gamma_{im}\,\partial_l\partial_m\gamma_{ij} - \frac{a^2}{4}\,\gamma_{ij}\,\gamma_{lm}\,\partial_l\partial_m\gamma_{ij}\biggr].$$

The quantities $\mathcal{L}^2_{\mathcal{R}\mathcal{R}}$ and $\mathcal{L}^2_{\gamma\gamma}$ are the second order Lagrangian densities comprising of two scalars and tensors which lead to the equations of motion.

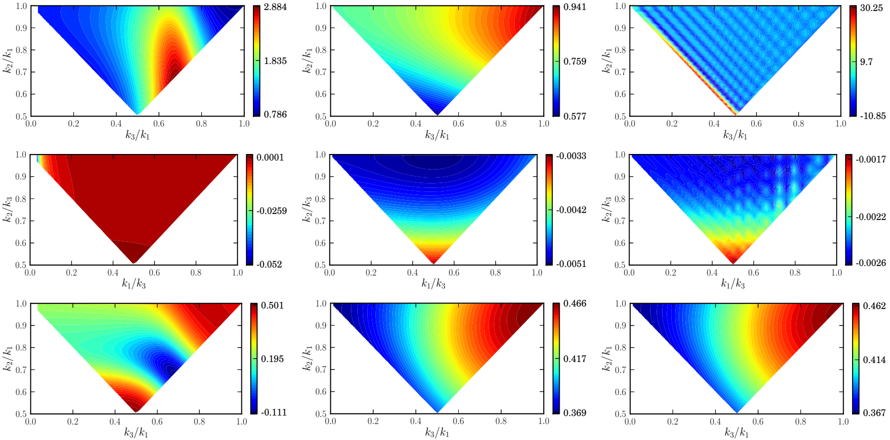
Comparison for an arbitrary triangular configuration



A comparison of the analytical results (at the bottom) for the non-Gaussianity parameters $C_{_{\rm NL}}^{\mathcal{R}}$ (on the left), $C_{_{\rm NL}}^{\gamma}$ (in the middle) and $h_{_{\rm NL}}$ (on the right) with the numerical results (on top) for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential[21]. As in the case of the scalar bispectrum, the maximum difference between the numerical and the analytic results is about 5%.

[21] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

Three point functions for models with features



Density plots of the non-Gaussianity parameters $C_{\rm NL}^{\mathcal{R}}$ (on top), $C_{\rm NL}^{\gamma}$ (in the middle) and $h_{\rm NL}$ (at the bottom) evaluated numerically for an arbitrary triangular configuration of the wavevectors for the case of the punctuated inflationary scenario (on the left), the quadratic potential with the step (in the middle) and the axion monodromy model (on the right).

The consistency relation for scalars

In the so-called squeezed limit of the scalar bispectrum, *i.e.* when $\boldsymbol{k}_1 = -\boldsymbol{k}_2$ and $\boldsymbol{k}_3 \to 0$, it can be shown that the non-Gaussianity parameter $f_{_{\rm NL}}$ can be expressed as[22]

$$f_{_{\rm NL}}(k) = \frac{5}{12}\,\left[n_{_{\rm S}}(k) - 1\right],$$

where $n_{_{\rm S}}$ is the scalar spectral index defined as

$$n_{_{\rm S}}(k) = 1 + \frac{{\rm d}\ln \mathcal{P}_{_{\rm S}}(k)}{{\rm d}\ln k}.$$

The above expression is often referred to as the consistency relation[23].

[22] J. Maldacena, JHEP **0305**, 013 (2003).

[23] P. Creminelli and M. Zaldarriaga, JCAP **0410**, 006 (2004).

Consistency relations involving scalars and tensors

In the squeezed limit, it can be shown that one can arrive at the following consistency relations for the non-Gaussainity parameters describing the other three-point functions[24]:

$$\begin{aligned} C_{_{\rm NL}}^{\mathcal{R}}(k) &= \frac{1}{4}\,\left[n_{_{\rm S}}(k)-4\right], \\ C_{_{\rm NL}}^{\gamma}(k) &= \frac{n_{_{\rm T}}}{2}, \\ h_{_{\rm NL}}(k) &= \frac{1}{8}\,\left[n_{_{\rm T}}(k)-3\right], \end{aligned}$$

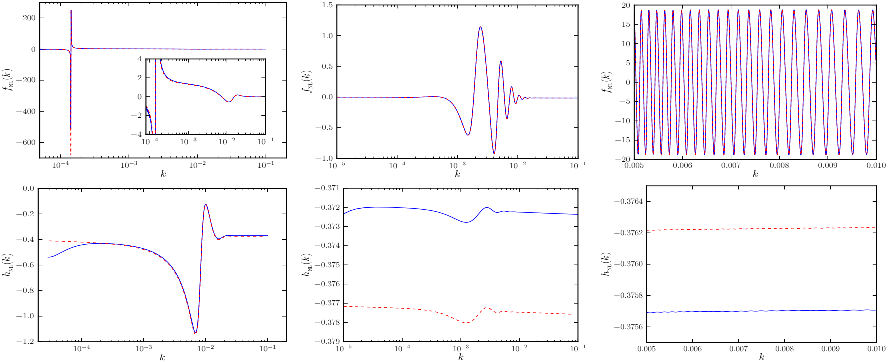
where, for simplicity, we have ignored quantities involving $\Pi^{\boldsymbol{k}}_{m_1n_1,m_2n_2}$, while $n_{_{\rm T}}$ is the tensor spectral index defined as

$$n_{_{\rm T}}(k) = \frac{{\rm d}\ln \mathcal{P}_{_{\rm T}}(k)}{{\rm d}\ln k}.$$

Note that, while writing down the consistency relation for $C_{_{\rm NL}}^{\mathcal{R}}$, we have taken the tensor mode to be the squeezed mode. Similarly, in the case of $C_{_{\rm NL}}^{\gamma}$, we have considered the scalar mode to be the squeezed mode.

[24] D. Jeong and M. Kamionkowski, Phys. Rev. Lett. **108**, 251301 (2012);
S. Kundu, JCAP **1404**, 016 (2014);
V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

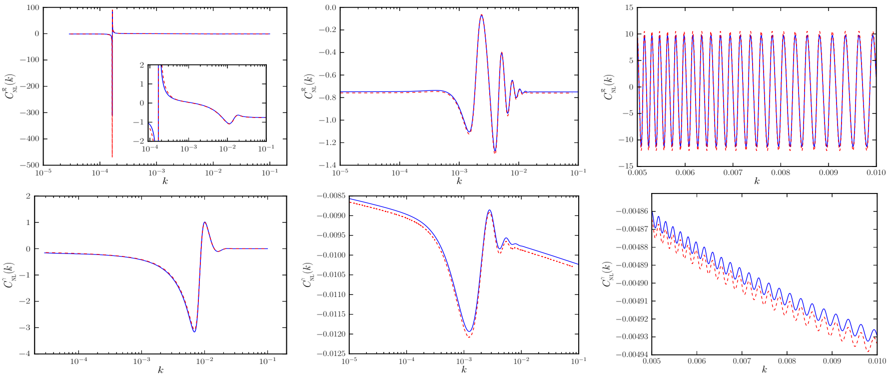
Consistency relations away from slow roll I



The behavior of the quantities $f_{\rm NL}$ (on top) and $h_{\rm NL}$ (at the bottom) in the squeezed limit has been plotted as a function of the wavenumber in the case of the punctuated inflationary scenario (on the left), the quadratic potential with a step (in the middle) and the axion monodromy model (on the right). The solid blue curves represent the numerical results obtained from the three-point functions, while the red dashed curves denote those arrived at using the consistency relations[25]. We find the match between the two to be better than 7%.

[25] V. Sreenath and L. Sriramkumar, JCAP 1410, 021 (2014).

Consistency relations away from slow roll II



The behavior of the quantities $C_{\rm NL}^{\mathcal{R}}$ (on top) and $C_{\rm NL}^{\gamma}$ (at the bottom) in the squeezed limit has been plotted as a function of the wavenumber for the three models of interest as in the previous figure. Evidently, the good match between the solid blue curves and the red dashed ones indicate the validity of the consistency relations even in situations involving strong departures from slow roll as in punctuated inflation.

Summary

- The strong constraints on the non-Gaussianity parameter $f_{\rm NL}$ from Planck suggests that inflationary and post-inflationary scenarios that lead to rather large non-Gaussianities are very likely to be ruled out by the data.

[26] P. A. R. Ade *et al.*, arXiv:1303.5082 [astro-ph.CO].

[27] In this context, see J. Martin, C. Ringeval and V. Vennin, arXiv:1303.3787 [astro-ph.CO].

Summary

- The strong constraints on the non-Gaussianity parameter $f_{\rm NL}$ from Planck suggests that inflationary and post-inflationary scenarios that lead to rather large non-Gaussianities are very likely to be ruled out by the data.
- In contrast, various analyses seem to point to the fact that the scalar power spectrum may contain features[26]. The possibility of such features can provide a strong handle on constraining inflationary models. Else, one may need to carry out a systematic search involving the scalar and the tensor power spectra[27], the scalar and the tensor bi-spectra and the cross correlations to arrive at a small subset of viable inflationary models.
- Interestingly, we find that, in single field inflationary models, the consistency conditions governing the three-point functions remain valid even in situations involving sharp departures from slow roll. Observational evidence of deviations from the consistency conditions can provide a powerful constraint, possibly ruling out all single field inflationary models.

[26] P. A. R. Ade *et al.*, arXiv:1303.5082 [astro-ph.CO].

[27] In this context, see J. Martin, C. Ringeval and V. Vennin, arXiv:1303.3787 [astro-ph.CO].

Thank you for your attention