

Can inflation be falsified?

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Colloquium

National Centre for Radio Astrophysics, Pune

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Plan of the talk

- 1 The inflationary paradigm
- 2 The scalar and tensor power spectra generated during inflation
- 3 Constraints from Planck
- 4 The Maldacena formalism to evaluate the inflationary scalar bispectrum
- 5 Constraints on non-Gaussianities
- 6 BINGO: An efficient code to numerically compute the bispectrum
- 7 Are bouncing models viable?
- 8 Summary and outlook

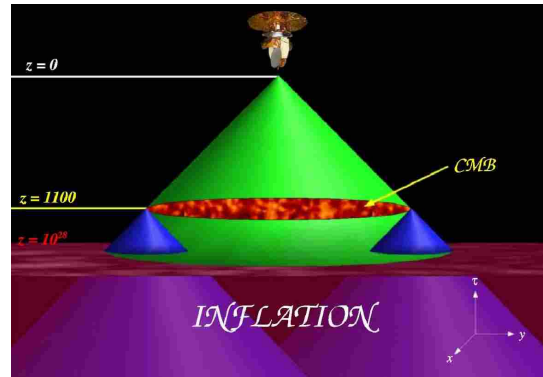
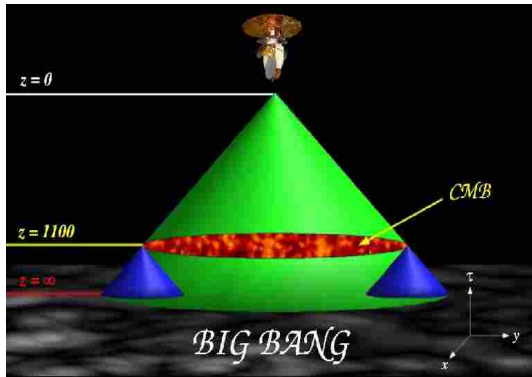


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The resolution of the horizon problem in inflation

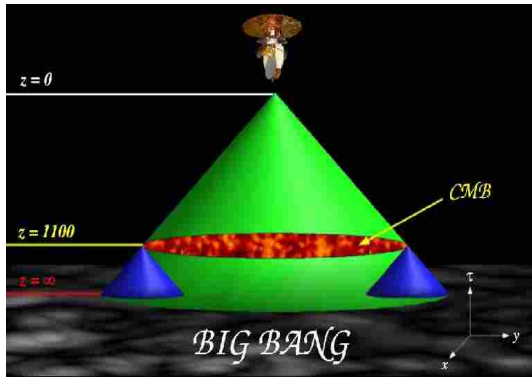


Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

¹ Images from [W. Kinney, astro-ph/0301448](#).

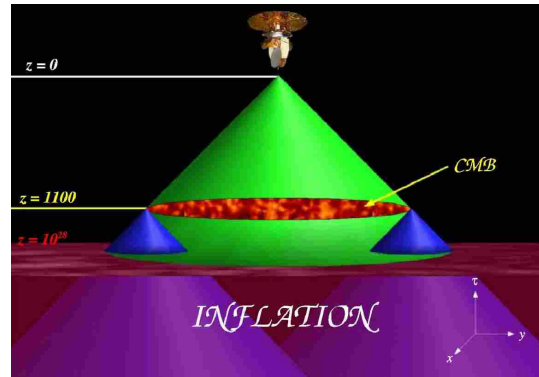


The resolution of the horizon problem in inflation



Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

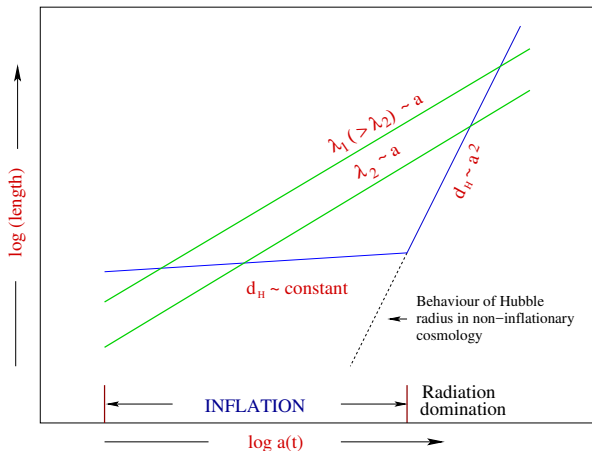
Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem¹.



¹Images from [W. Kinney, astro-ph/0301448](#).



Bringing the modes inside the Hubble radius



The behavior of the physical wavelength $\lambda_P \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs².

[Back to bounce](#)

²See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.



Driving inflation with scalar fields

If we require that $\lambda_P < d_H$ at a sufficiently early time, then we need to have an epoch wherein λ_P decreases faster than the Hubble scale *as we go back in time*, i.e. a regime during which

$$-\frac{d}{dt} \left(\frac{\lambda_P}{d_H} \right) < 0 \quad \Rightarrow \quad \ddot{a} > 0.$$

³See, for instance, B. A. Bassett, S. Tsujikawa and D. Wands, *Rev. Mod. Phys.* **78**, 537 (2006).



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From the Friedmann equations, we then require that, during this epoch,

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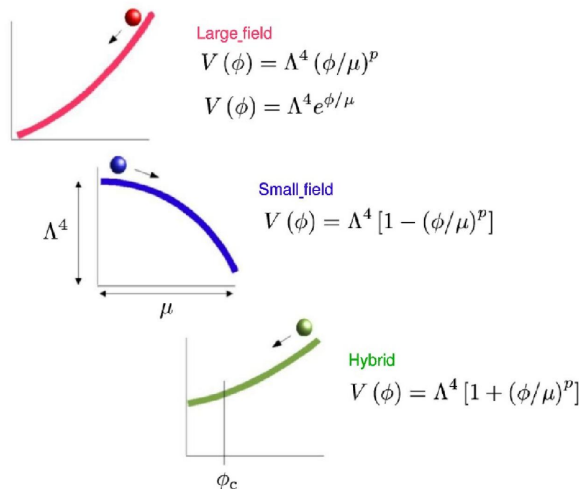
$$\dot{\phi}^2 < V(\phi).$$

This condition can be achieved if the scalar field ϕ is initially displaced from a minima of the potential, and inflation will end when the field approaches a minima with zero or negligible potential energy³.

³See, for instance, B. A. Bassett, S. Tsujikawa and D. Wands, *Rev. Mod. Phys.* **78**, 537 (2006).



A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation⁴. Often, these potentials are classified as small field, large field and hybrid models.

⁴Image from [W. Kinney, astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448).



Proliferation of inflationary models

5-dimensional assisted inflation	extended open inflation	late-time mild inflation	pre-Big-Bang inflation
anisotropic brane inflation	extended warm inflation	low-scale inflation	primary inflation
anomaly-induced inflation	extra dimensional inflation	low-scale supergravity inflation	primordial inflation
assisted inflation	F-term inflation	M-theory inflation	quasi-open inflation
assisted chaotic inflation	F-term hybrid inflation	mass inflation	quintessential inflation
boundary inflation	false vacuum inflation	massive chaotic inflation	R-invariant topological inflation
brane inflation	false vacuum chaotic inflation	moduli inflation	rapid asymmetric inflation
brane-assisted inflation	fast-roll inflation	multi-scalar inflation	running inflation
brane gas inflation	first order inflation	multiple inflation	scalar-tensor gravity inflation
brane-antibrane inflation	gauged inflation	multiple-field slow-roll inflation	scalar-tensor stochastic inflation
braneworld inflation	generalised inflation	multiple-stage inflation	Seiberg-Witten inflation
Brans-Dicke chaotic inflation	generalized assisted inflation	natural inflation	single-bubble open inflation
Brans-Dicke inflation	generalized slow-roll inflation	natural Chaotic inflation	spinodal inflation
bulky brane inflation	gravity driven inflation	natural double inflation	stable starobinsky-type inflation
chaotic hybrid inflation	Hagedorn inflation	natural supergravity inflation	steady-state eternal inflation
chaotic inflation	higher-curvature inflation	new inflation	steep inflation
chaotic new inflation	hybrid inflation	next-to-minimal supersymmetric hybrid inflation	stochastic inflation
D-brane inflation	hyperextended inflation	non-commutative inflation	string-forming open inflation
D-term inflation	induced gravity inflation	non-slow-roll inflation	successful D-term inflation
dilaton-driven inflation	induced gravity open inflation	nonminimal chaotic inflation	supergravity inflation
dilaton-driven brane inflation	intermediate inflation	old inflation	supernatural inflation
double inflation	inverted hybrid inflation	open hybrid inflation	superstring inflation
double D-term inflation	isocurvature inflation	open inflation	supersymmetric hybrid inflation
dual inflation	K inflation	oscillating inflation	supersymmetric inflation
dynamical inflation	kinetic inflation	polynomial chaotic inflation	supersymmetric topological inflator
dynamical SUSY inflation	lambda inflation	polynomial hybrid inflation	supersymmetric new inflation
eternal inflation	large field inflation	power-law inflation	synergistic warm inflation
extended inflation	late D-term inflation		TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models⁵. Actually, it may not even be possible to rule out some of these models!

⁵From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).



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The character of the perturbations

In a Friedmann universe, the perturbations in the metric and the matter can be classified according to their behavior with respect to local rotation of the spatial coordinates on hypersurfaces of constant time as follows⁶:

- ◆ Scalar perturbations – Density and pressure perturbations
- ◆ Vector perturbations – Rotational velocity fields
- ◆ Tensor perturbations – Gravitational waves

⁶See, for instance, [L. Sriramkumar, Curr. Sci. 97, 868 \(2009\)](#).



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It is the fluctuations in the inflaton field ϕ that act as the seeds for the scalar perturbations that are primarily responsible for the anisotropies in the CMB and, eventually, the present day inhomogeneities.

⁶See, for instance, [L. Sriramkumar, Curr. Sci. 97, 868 \(2009\)](#).



The quadratic action governing the perturbations

One can show that, at the quadratic order, the action governing the curvature perturbation \mathcal{R} and the tensor perturbation γ_{ij} are given by⁷

$$\mathcal{S}_2[\mathcal{R}] = \frac{1}{2} \int d\eta \int d^3\mathbf{x} z^2 \left[\mathcal{R}'^2 - (\partial\mathcal{R})^2 \right],$$

$$\mathcal{S}_2[\gamma_{ij}] = \frac{M_{\text{Pl}}^2}{8} \int d\eta \int d^3\mathbf{x} a^2 \left[\gamma'_{ij}{}^2 - (\partial\gamma_{ij})^2 \right].$$

▶ [Back to the cubic action](#)

⁷V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).



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[Back to the cubic action](#)

These actions lead to the following equations of motion governing the scalar and tensor modes, say, f_k and h_k :

$$f_k'' + 2 \frac{z'}{z} f_k' + k^2 f_k = 0,$$

$$h_k'' + 2 \frac{a'}{a} h_k' + k^2 h_k = 0,$$

where $z = a M_{\text{Pl}} \sqrt{2\epsilon_1}$, with $\epsilon_1 = -d \ln H / dN$ being the first slow roll parameter.

⁷V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).



Quantization of the scalar and tensor perturbations

On quantization, the operators $\hat{\mathcal{R}}(\eta, \mathbf{x})$ and $\hat{\gamma}_{ij}(\eta, \mathbf{x})$ representing the scalar and the tensor perturbations can be expressed in terms of the corresponding Fourier modes $f_{\mathbf{k}}$ and $g_{\mathbf{k}}$ as⁸

$$\begin{aligned}\hat{\mathcal{R}}(\eta, \mathbf{x}) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \hat{\mathcal{R}}_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{a}_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \\ \hat{\gamma}_{ij}(\eta, \mathbf{x}) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \hat{\gamma}_{ij}^{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \\ &= \sum_s \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{b}_{\mathbf{k}}^s \varepsilon_{ij}^s(\mathbf{k}) g_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{b}_{\mathbf{k}}^{s\dagger} \varepsilon_{ij}^{s*}(\mathbf{k}) g_{\mathbf{k}}^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right].\end{aligned}$$

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In these decompositions, the operators $(\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}}^\dagger)$ and $(\hat{b}_{\mathbf{k}}^s, \hat{b}_{\mathbf{k}}^{s\dagger})$ satisfy the standard commutation relations, while the quantity $\varepsilon_{ij}^s(\mathbf{k})$ represents the transverse and traceless polarization tensor describing the gravitational waves.

⁸See, for instance, L. Sriramkumar, *Curr. Sci.* **97**, 868 (2009).



The scalar and tensor power spectra

The dimensionless scalar and tensor power spectra $\mathcal{P}_S(k)$ and $\mathcal{P}_T(k)$ are defined in terms of the correlation functions of the Fourier modes $\hat{\mathcal{R}}_{\mathbf{k}}$ and $\hat{\gamma}_{mn}^{\mathbf{k}}$ as follows:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}}(\eta) \hat{\mathcal{R}}_{\mathbf{k}'}(\eta) \rangle = \frac{(2\pi)^2}{2k^3} \mathcal{P}_S(k) \delta^{(3)}(\mathbf{k} + \mathbf{k}'),$$

$$\langle \hat{\gamma}_{m_1 n_1}^{\mathbf{k}}(\eta) \hat{\gamma}_{m_2 n_2}^{\mathbf{k}'}(\eta) \rangle = \frac{(2\pi)^2}{8k^3} \Pi_{m_1 n_1, m_2 n_2}^{\mathbf{k}} \mathcal{P}_T(k) \delta^3(\mathbf{k} + \mathbf{k}'),$$

where $\Pi_{m_1 n_1, m_2 n_2}^{\mathbf{k}} = \sum_s \varepsilon_{m_1 n_1}^s(\mathbf{k}) \varepsilon_{m_2 n_2}^{s*}(\mathbf{k})$.



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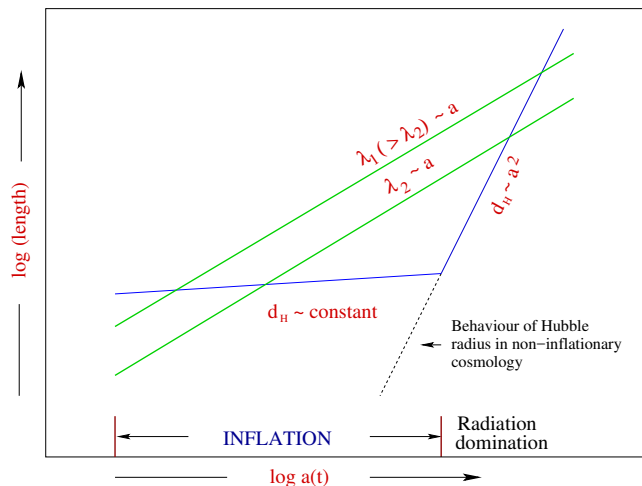
In the Bunch-Davies vacuum, say, $|0\rangle$, which is defined as $\hat{a}_{\mathbf{k}}|0\rangle = 0$ and $\hat{b}_{\mathbf{k}}^s|0\rangle = 0 \forall \mathbf{k}$ and s , we can express the power spectra in terms of the quantities $f_{\mathbf{k}}$ and $g_{\mathbf{k}}$ as

$$\mathcal{P}_S(k) = \frac{k^3}{2\pi^2} |f_{\mathbf{k}}|^2, \quad \mathcal{P}_T(k) = 4 \frac{k^3}{2\pi^2} |h_{\mathbf{k}}|^2.$$

With the initial conditions imposed in the sub-Hubble domain, *viz.* when $k/(aH) \gg 1$, these spectra are to be evaluated on super-Hubble scales, *i.e.* as $k/(aH) \ll 1$.



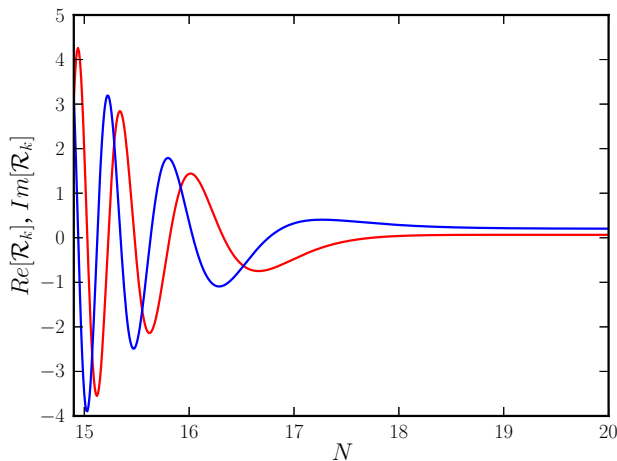
From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the modes are well inside the Hubble radius (*viz.* when $k/(aH) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(aH) \ll 1$).



Typical evolution of the scalar modes



Typical evolution of the real and the imaginary parts of the scalar modes during slow roll inflation. The mode considered leaves the Hubble radius at about 18 e-folds⁹.

⁹Figure from V. Sreenath, *Computation and characteristics of inflationary three-point functions*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2015).



Spectral indices and the tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_S(k) = \mathcal{A}_S \left(\frac{k}{k_*} \right)^{n_S - 1}, \quad \mathcal{P}_T(k) = \mathcal{A}_T \left(\frac{k}{k_*} \right)^{n_T},$$

with the spectral indices n_S and n_T assumed to be constant.



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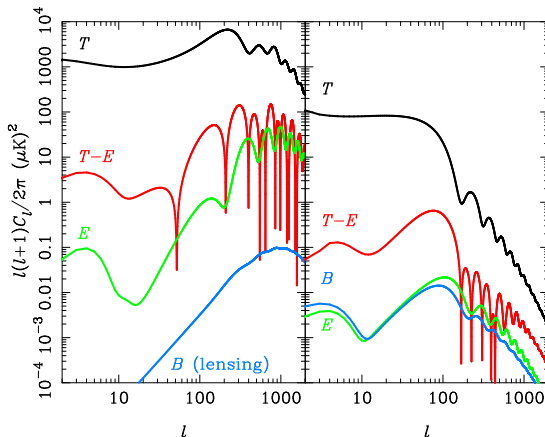
The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)}$$

and it is usual to further set $r = -8n_T$, viz. the so-called consistency relation, which is valid during slow roll inflation.



Theoretical angular power spectra¹⁰

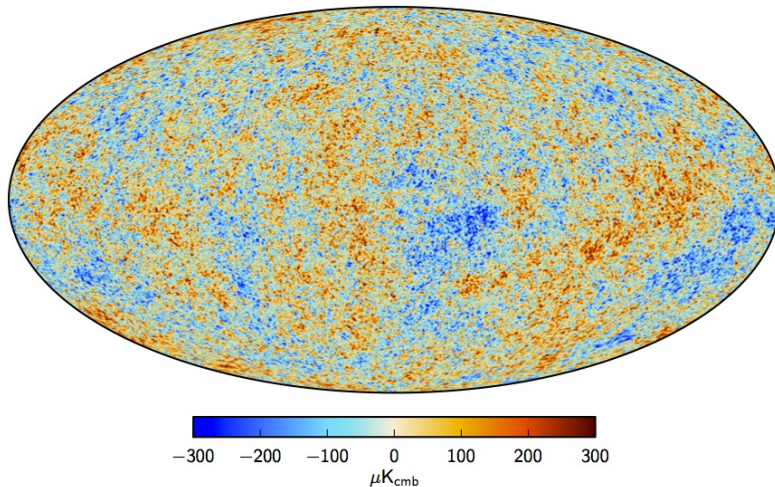


The *theoretically* computed, CMB angular power and cross-correlation spectra – temperature (T , in black), E (in green), B (in blue), and $T-E$ (in red) – arising due to scalars (on the left) and tensors (on the right) corresponding to a tensor-to-scalar ratio of $r = 0.24$. The B -mode spectrum induced by weak gravitational lensing has also been shown (in blue) in the panel on the left.

¹⁰Figure from A. Challinor, arXiv:1210.6008 [astro-ph.CO].



CMB anisotropies as seen by Planck

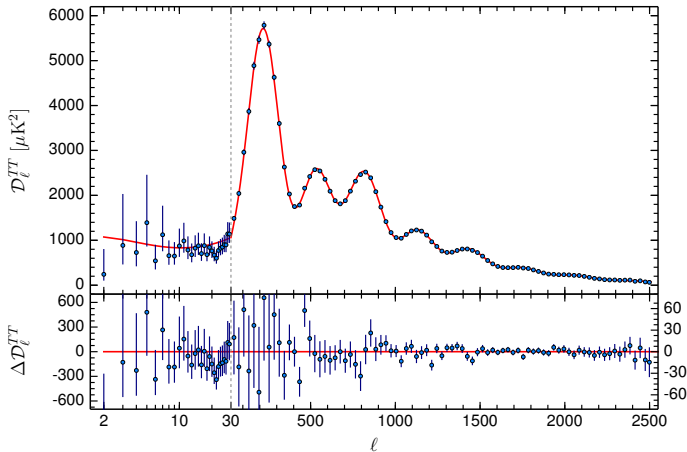


CMB intensity map at $5'$ resolution derived from the joint analysis of Planck, WMAP, and 408 MHz observations¹¹.

¹¹Planck Collaboration (R. Adam *et al.*), *Astron. Astrophys.* **594**, A1 (2016).



CMB TT angular power spectrum from Planck

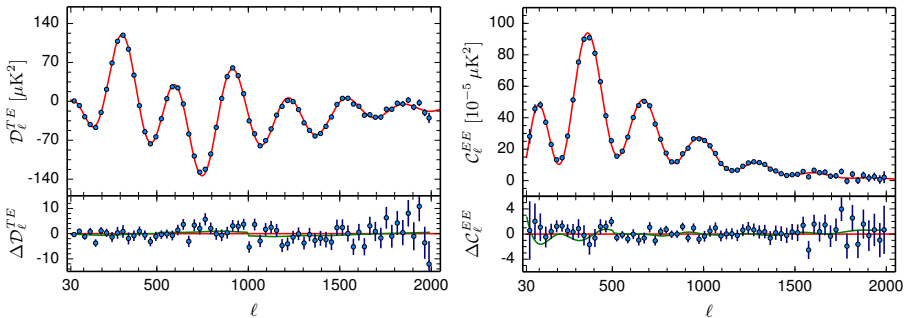


The CMB TT angular power spectrum from the Planck 2015 data (the blue dots with error bars) and the theoretical, best fit Λ CDM model with a power law primordial spectrum (the solid red curve)¹².

¹²Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).



CMB TE and EE angular power spectra from Planck



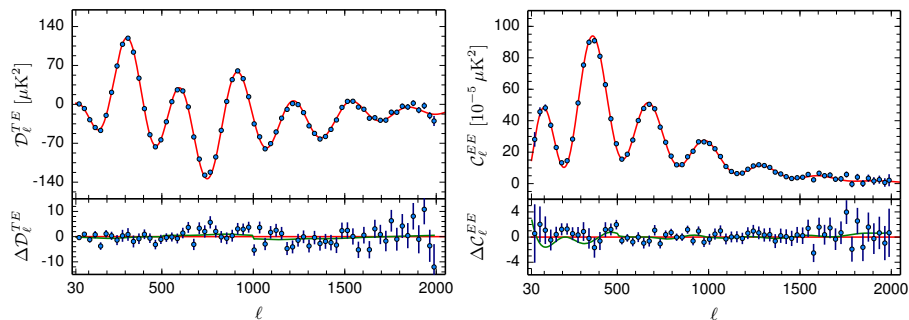
The CMB TE (on the left) and EE (on the right) angular power spectra from the Planck 2015 data (the blue dots with error bars) and the theoretical, best fit Λ CDM model with a power law primordial spectrum (the solid red curves)¹³.

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¹⁴D. N. Spergel and M. Zaldarriaga, *Phys. Rev. Lett.* **79**, 2180 (1997).



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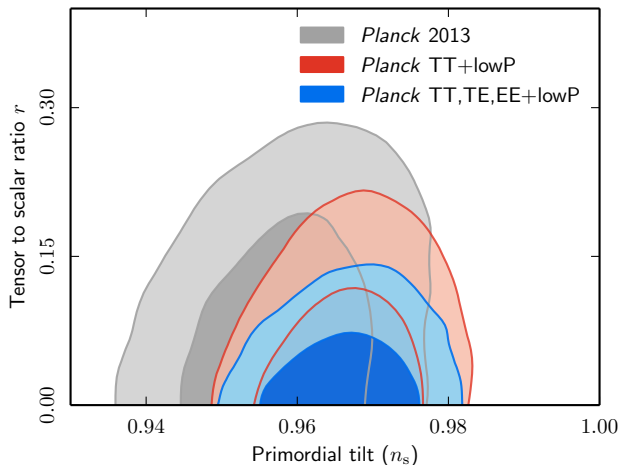
The large angle ($50 < \ell < 150$) TE anti-correlation detected by Planck (and earlier by WMAP) is a distinctive signature of primordial, super-Hubble, adiabatic perturbations¹⁴.

¹³ Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).

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Joint constraints on r and n_s



Marginalized joint confidence contours for (n_s, r) , at the 68% and 95% CL, in the presence of running of the spectral indices, and for the same combinations of data as in the previous figure¹⁵.

¹⁵Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).



Specific inflationary models of interest I

Power law potentials: In power law potentials of the following form¹⁶:

$$V(\phi) = \lambda M_{\text{Pl}}^4 (\phi/M_{\text{Pl}})^n,$$

inflation occurs for large values of the field, *i.e.* $\phi > M_{\text{Pl}}$.

¹⁶A. D. Linde, Phys. Lett. B **129**, 177 (1983).

¹⁷L. Boubekeur and D. Lyth, JCAP **0507**, 010 (2005).

¹⁸K. Freese, J. .A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).



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Hilltop models: The hilltop models are described by the potential¹⁷:

$$V(\phi) \simeq \Lambda^4 [1 - (\phi/\mu)^p + \dots]$$

and, in these models, the inflaton rolls away from an unstable equilibrium. The ellipsis indicates higher order terms that are considered to be negligible during inflation, but ensure positiveness of the potential.

¹⁶ A. D. Linde, Phys. Lett. B **129**, 177 (1983).

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¹⁸ K. Freese, J. .A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).



Specific inflationary models of interest I

Power law potentials: In power law potentials of the following form¹⁶:

$$V(\phi) = \lambda M_{\text{Pl}}^4 (\phi/M_{\text{Pl}})^n,$$

inflation occurs for large values of the field, *i.e.* $\phi > M_{\text{Pl}}$.

Hilltop models: The hilltop models are described by the potential¹⁷:

$$V(\phi) \simeq \Lambda^4 [1 - (\phi/\mu)^p + \dots]$$

and, in these models, the inflaton rolls away from an unstable equilibrium. The ellipsis indicates higher order terms that are considered to be negligible during inflation, but ensure positiveness of the potential.

Natural inflation: In natural inflation, a non-perturbative shift symmetry is invoked to suppress radiative corrections, leading to the periodic potential¹⁸

$$V(\phi) = \Lambda^4 [1 + \cos(\phi/f)],$$

where f is the scale which determines the curvature of the potential.

¹⁶A. D. Linde, Phys. Lett. B **129**, 177 (1983).

¹⁷L. Boubekeur and D. Lyth, JCAP **0507**, 010 (2005).

¹⁸K. Freese, J. .A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).



Specific inflationary models of interest II

R^2 inflation: The first inflationary model proposed with the action¹⁹

$$S[g_{\mu\nu}] = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{R^2}{6 M^2} \right),$$

corresponds to the following potential in the Einstein frame:

$$V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{2/3} \phi/M_{\text{Pl}}} \right)^2.$$

¹⁹A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

²⁰See, for example, R. Kallosh, A. D. Linde and D. Roest, JHEP **1311**, 198 (2013).



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α -attractors: A class of inflationary models motivated by recent developments in conformal symmetry and supergravity correspond to the potential²⁰

$$V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{2/3} \phi/(\sqrt{\alpha} M_{\text{Pl}})} \right]^2.$$

A second class of models, called the super-conformal α -attractors, are described by the following potential:

$$V(\phi) = \Lambda^4 \tanh^{2m} \left(\phi/\sqrt{6 \alpha M_{\text{Pl}}} \right).$$

¹⁹A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

²⁰See, for example, R. Kallosh, A. D. Linde and D. Roest, JHEP **1311**, 198 (2013).



Accounting for post-inflationary evolution

The uncertainties post-inflation are modeled by two parameters: the energy scale ρ_{th} by which the universe has thermalized, and the parameter w_{int} which characterizes the effective equation of state between the end of inflation and the energy scale specified by ρ_{th} .

²¹ A. R. Liddle and S. M. Leach, Phys. Rev. D **68**, 103503, (2003);
J. Martin and C. Ringeval, Phys. Rev. D **82**, 023511 (2010).



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The number of e-folds to the end of inflation is described by the expression²¹

$$N_* \approx 67 - \ln \left(\frac{k_*}{a_0 H_0} \right) + \frac{1}{4} \ln \left(\frac{V_*^2}{M_{\text{Pl}}^4 \rho_{\text{end}}} \right) + \frac{1 - 3w_{\text{int}}}{12(1 + w_{\text{int}})} \ln \left(\frac{\rho_{\text{th}}}{\rho_{\text{end}}} \right) - \frac{1}{12} \ln(g_{\text{th}}),$$

where ρ_{end} is the energy density at the end of inflation, $a_0 H_0$ is the present Hubble scale, V_* is the potential energy when k_* left the Hubble radius during inflation, and g_{th} is the number of effective bosonic degrees of freedom at the energy scale ρ_{th} .

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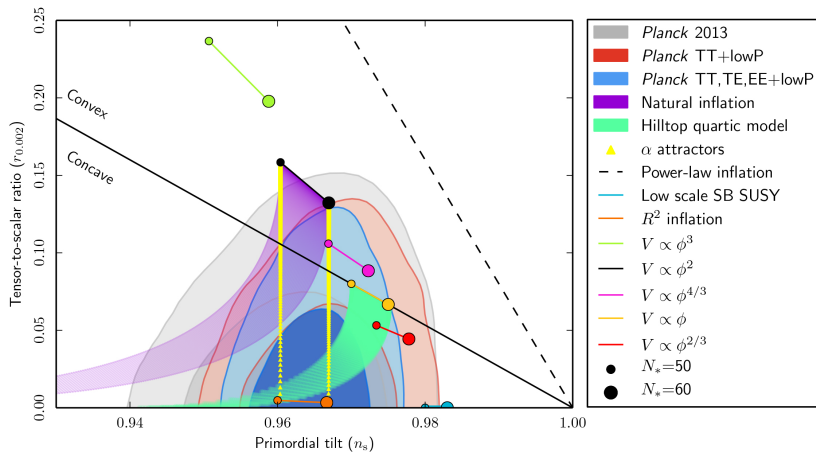
where ρ_{end} is the energy density at the end of inflation, $a_0 H_0$ is the present Hubble scale, V_* is the potential energy when k_* left the Hubble radius during inflation, and g_{th} is the number of effective bosonic degrees of freedom at the energy scale ρ_{th} .

The Planck team set $k_* = 0.002 \text{ Mpc}^{-1}$, $g_{\text{th}} = 10^3$ and a logarithmic prior was chosen on ρ_{th} (in the interval $[(10^3 \text{ GeV})^4, \rho_{\text{end}}]$).

²¹ A. R. Liddle and S. M. Leach, *Phys. Rev. D* **68**, 103503, (2003);
J. Martin and C. Ringeval, *Phys. Rev. D* **82**, 023511 (2010).



Performance of models in the n_s - r plane



Marginalized joint **68%** and **95%** CL regions for n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models²².

²²Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).



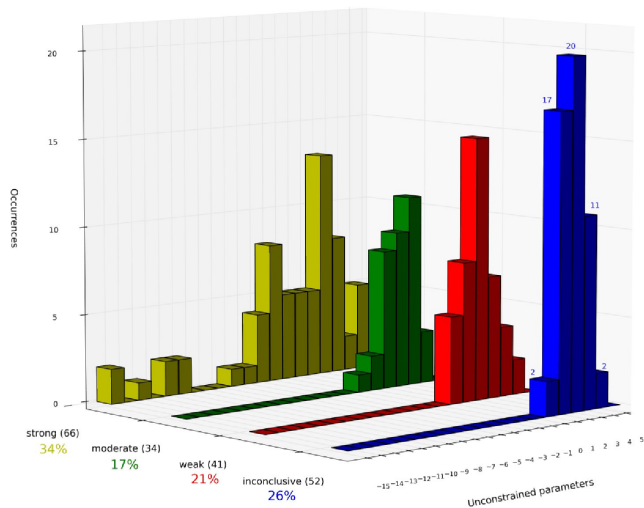
Bayesian evidence for specific inflationary models

Inflationary model	$\Delta\chi^2$		$\ln B_{0X}$	
	$w_{\text{int}} = 0$	$w_{\text{int}} \neq 0$	$w_{\text{int}} = 0$	$w_{\text{int}} \neq 0$
$R + R^2/(6M^2)$	+0.8	+0.3	...	+0.7
$n = 2/3$	+6.5	+3.5	-2.4	-2.3
$n = 1$	+6.2	+5.5	-2.1	-1.9
$n = 4/3$	+6.4	+5.5	-2.6	-2.4
$n = 2$	+8.6	+8.1	-4.7	-4.6
$n = 3$	+22.8	+21.7	-11.6	-11.4
$n = 4$	+43.3	+41.7	-23.3	-22.7
Natural	+7.2	+6.5	-2.4	-2.3
Hilltop ($p = 2$)	+4.4	+3.9	-2.6	-2.4
Hilltop ($p = 4$)	+3.7	+3.3	-2.8	-2.6
Double well	+5.5	+5.3	-3.1	-2.3
Brane inflation ($p = 2$)	+3.0	+2.3	-0.7	-0.9
Brane inflation ($p = 4$)	+2.8	+2.3	-0.4	-0.6
Exponential inflation	+0.8	+0.3	-0.7	-0.9
SB SUSY	+0.7	+0.4	-2.2	-1.7
Supersymmetric α -model	+0.7	+0.1	-1.8	-2.0
Superconformal ($m = 1$)	+0.9	+0.8	-2.3	-2.2
Superconformal ($m \neq 1$)	+0.7	+0.5	-2.4	-2.6

The improvement in the likelihood $\Delta\chi^2$ (with respect to the base Λ CDM model) and the Bayes factor (with respect to R^2 inflation) for a set of inflationary models.



Performance of inflationary models

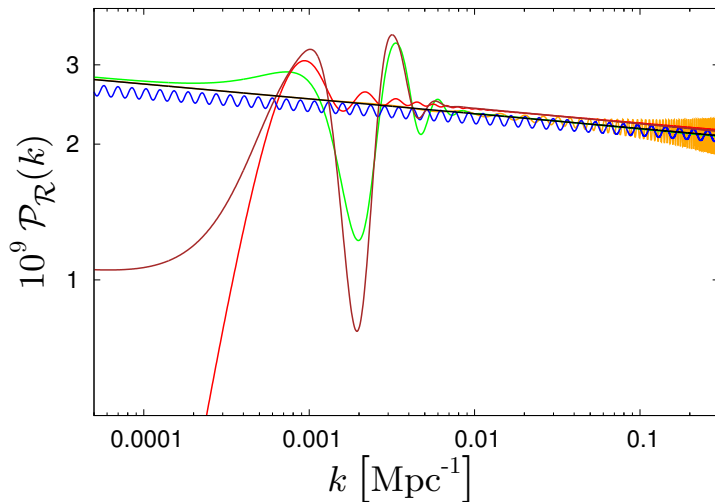


The efficiency of the inflationary paradigm leads to a situation wherein, despite the strong constraints, a variety of models continue to remain consistent with the data²³.

²³ J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP **1403**, 039 (2014).



Power spectra with features



Primordial power spectra with features that lead to an improved fit to the data than the conventional, nearly scale, invariant spectra²⁴.

²⁴Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).



Inflationary models permitting deviations from slow roll

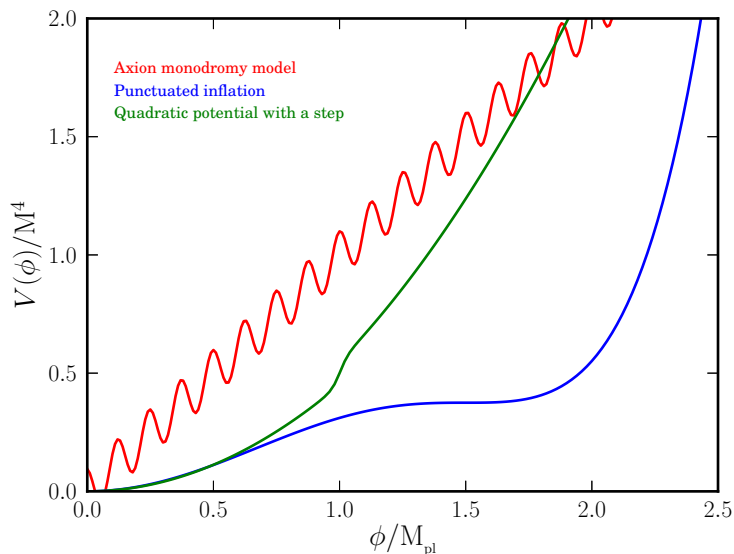
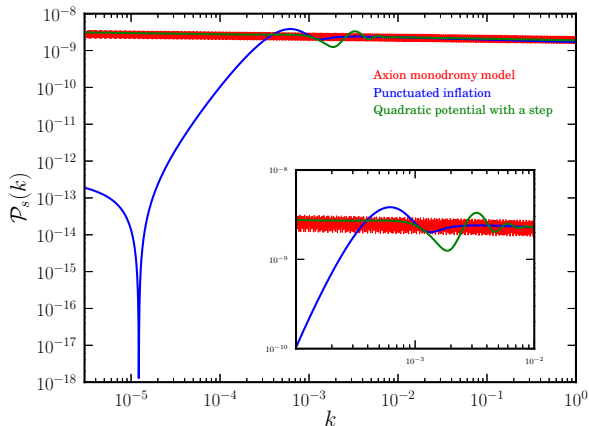


Illustration of potentials that admit departures from slow roll.



Spectra leading to an improved fit to the CMB data



The scalar power spectra in different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum²⁵.

²⁵ R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);
 D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010);
 M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013).



Plan of the talk

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The scalar bispectrum

The scalar bispectrum $\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time η_e , as follows²⁶:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e) \rangle = (2\pi)^3 \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3).$$

²⁶D. Larson *et al.*, *Astrophys. J. Suppl.* **192**, 16 (2011);
E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).



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Note that the delta function on the right hand side imposes the triangularity condition, *viz.* that the three wavevectors \mathbf{k}_1 , \mathbf{k}_2 and \mathbf{k}_3 have to form the edges of a triangle.

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Note that the delta function on the right hand side imposes the triangularity condition, *viz.* that the three wavevectors \mathbf{k}_1 , \mathbf{k}_2 and \mathbf{k}_3 have to form the edges of a triangle.

For convenience, we shall set

$$\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^{-9/2} G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

²⁶D. Larson *et al.*, *Astrophys. J. Suppl.* **192**, 16 (2011);
E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).



The cubic order action governing the perturbations

It can be shown that, the third order term in the action describing the curvature perturbation is given by²⁷

$$\begin{aligned} \mathcal{S}_{\mathcal{R}\mathcal{R}\mathcal{R}}^3[\mathcal{R}] = M_{\text{Pl}}^2 \int d\eta \int d^3\mathbf{x} \left[a^2 \epsilon_1^2 \mathcal{R} \mathcal{R}'^2 + a^2 \epsilon_1^2 \mathcal{R} (\partial\mathcal{R})^2 \right. \\ \left. - 2 a \epsilon_1 \mathcal{R}' (\partial^i \mathcal{R}) (\partial_i \chi) + \frac{a^2}{2} \epsilon_1 \epsilon_2' \mathcal{R}^2 \mathcal{R}' + \frac{\epsilon_1}{2} (\partial^i \mathcal{R}) (\partial_i \chi) (\partial^2 \chi) \right. \\ \left. + \frac{\epsilon_1}{4} (\partial^2 \mathcal{R}) (\partial \chi)^2 + \mathcal{F}_1 \left(\frac{\delta \mathcal{L}_{\mathcal{R}\mathcal{R}}^2}{\delta \mathcal{R}} \right) \right], \end{aligned}$$

where $\mathcal{F}_1(\delta \mathcal{L}_{\mathcal{R}\mathcal{R}}^2 / \delta \mathcal{R})$ denotes terms involving the variation of the second order action with respect to \mathcal{R} , while χ is related to the curvature perturbation \mathcal{R} through the relation

$$\partial^2 \chi = a \epsilon_1 \mathcal{R}'.$$

► The quadratic action

²⁷J. Maldacena, JHEP **0305**, 013 (2003);

D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);

X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).



Evaluating the scalar bispectrum

At the leading order in the perturbations, one then finds that the scalar three-point correlation function in Fourier space is described by the integral²⁸

$$\begin{aligned} & \langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e) \rangle \\ &= -i \int_{\eta_i}^{\eta_e} d\eta a(\eta) \left\langle \left[\hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e) \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e), \hat{H}_I(\eta) \right] \right\rangle, \end{aligned}$$

where \hat{H}_I is the Hamiltonian corresponding to the above third order action, while η_i denotes a sufficiently early time when the initial conditions are imposed on the modes, and η_e denotes a very late time, say, close to when inflation ends.

²⁸See, for example, D. Seery and J. E. Lidsey, *JCAP* **0506**, 003 (2005);
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where \hat{H}_I is the Hamiltonian corresponding to the above third order action, while η_i denotes a sufficiently early time when the initial conditions are imposed on the modes, and η_e denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are evaluated in the initial vacuum state (*viz.* the Bunch-Davies vacuum in the situation of our interest).

²⁸See, for example, D. Seery and J. E. Lidsey, *JCAP* **0506**, 003 (2005);
X. Chen, *Adv. Astron.* **2010**, 638979 (2010).



The non-Gaussianity parameter f_{NL}

The observationally relevant non-Gaussianity parameter f_{NL} is basically introduced through the relation²⁹

$$\mathcal{R}(\eta, \mathbf{x}) = \mathcal{R}_{\text{G}}(\eta, \mathbf{x}) - \frac{3 f_{\text{NL}}}{5} [\mathcal{R}_{\text{G}}^2(\eta, \mathbf{x}) - \langle \mathcal{R}_{\text{G}}^2(\eta, \mathbf{x}) \rangle],$$

where \mathcal{R}_{G} denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

²⁹E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).



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where \mathcal{R}_{G} denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

Utilizing the above relation and Wick's theorem, one can arrive at the three-point correlation function of the curvature perturbation in Fourier space in terms of the parameter f_{NL} . It is found to be

$$\begin{aligned} \langle \hat{\mathcal{R}}_{\mathbf{k}_1} \hat{\mathcal{R}}_{\mathbf{k}_2} \hat{\mathcal{R}}_{\mathbf{k}_3} \rangle &= -\frac{3 f_{\text{NL}}}{10} (2\pi)^{5/2} \left(\frac{1}{k_1^3 k_2^3 k_3^3} \right) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \\ &\times [k_1^3 \mathcal{P}_{\text{S}}(k_2) \mathcal{P}_{\text{S}}(k_3) + \text{two permutations}]. \end{aligned}$$

²⁹E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).



The relation between f_{NL} and the scalar bispectrum

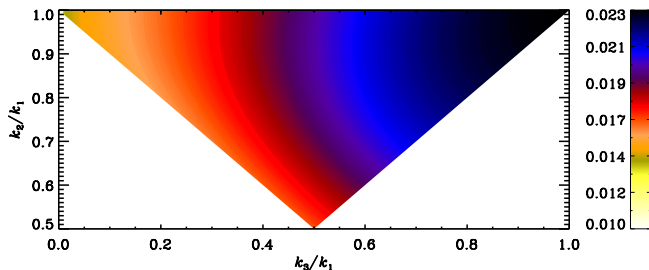
Upon making use of the above expression for the three-point function of the curvature perturbation and the definition of the scalar bispectrum, we can, in turn, arrive at the following relation³⁰:

$$\begin{aligned}
 f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= -\frac{10}{3} (2\pi)^{1/2} (k_1^3 k_2^3 k_3^3) \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\quad \times [k_1^3 \mathcal{P}_s(k_2) \mathcal{P}_s(k_3) + \text{two permutations}]^{-1} \\
 &= -\frac{10}{3} \frac{1}{(2\pi)^4} (k_1^3 k_2^3 k_3^3) G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
 &\quad \times [k_1^3 \mathcal{P}_s(k_2) \mathcal{P}_s(k_3) + \text{two permutations}]^{-1}.
 \end{aligned}$$

³⁰J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).



The shape of the slow roll bispectrum



The results for the non-Gaussianity parameter f_{NL} , evaluated analytically in the slow roll approximation, has been plotted as a function of k_3/k_1 and k_2/k_1 for the case of the popular quadratic potential. Note that the non-Gaussianity parameter peaks in the equilateral limit wherein $k_1 = k_2 = k_3$. In slow roll scenarios involving the canonical scalar field, the largest value of f_{NL} is found to be of the order of the first slow roll parameter ϵ_1 , while $f_{\text{NL}} \sim \epsilon_1/c_s^2$ in non-canonical models, where c_s denotes the speed of the scalar perturbations³¹. The most recent results imply that $c_s \geq 0.024$ ³².

³¹See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);

X. Chen, Adv. Astron. **2010**, 638979 (2010).

³²P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].



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Template bispectra

For comparison with the observations, the scalar bispectrum is often expressed in terms of the parameters f_{NL}^{loc} , f_{NL}^{eq} and f_{NL}^{orth} as follows:

$$G_{RRR}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = f_{NL}^{loc} G_{RRR}^{loc}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + f_{NL}^{eq} G_{RRR}^{eq}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + f_{NL}^{orth} G_{RRR}^{orth}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

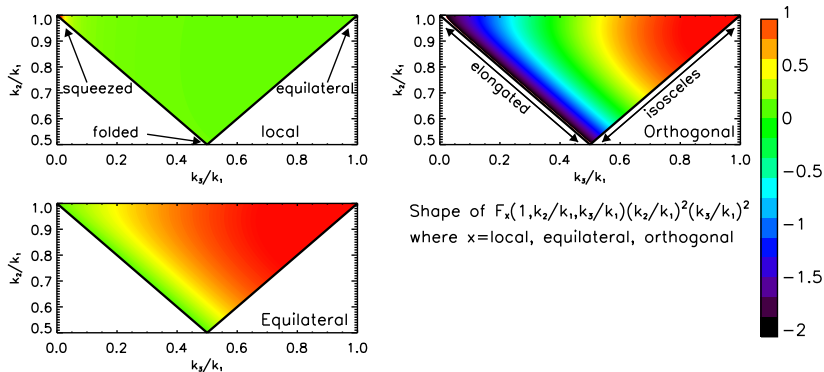


Illustration of the three template basis bispectra³³.

³³E. Komatsu, *Class. Quantum Grav.* **27**, 124010 (2010).



Constraints on the scalar non-Gaussianity parameters

The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows³⁴:

$$\begin{aligned}f_{\text{NL}}^{\text{loc}} &= 0.8 \pm 5.0, \\f_{\text{NL}}^{\text{eq}} &= -4 \pm 43, \\f_{\text{NL}}^{\text{orth}} &= -26 \pm 21.\end{aligned}$$

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Also, the constraints on each of the f_{NL} parameters have been arrived at assuming that the other two parameters are zero.

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Also, the constraints on each of the f_{NL} parameters have been arrived at assuming that the other two parameters are zero.

These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

³⁴Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A17 (2016).



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The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows³⁴:

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It should be stressed that these are constraints on the primordial values.

Also, the constraints on each of the f_{NL} parameters have been arrived at assuming that the other two parameters are zero.

These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

We should also add that these constraints become less stringent if the primordial spectra are assumed to contain features.

³⁴Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A17 (2016).

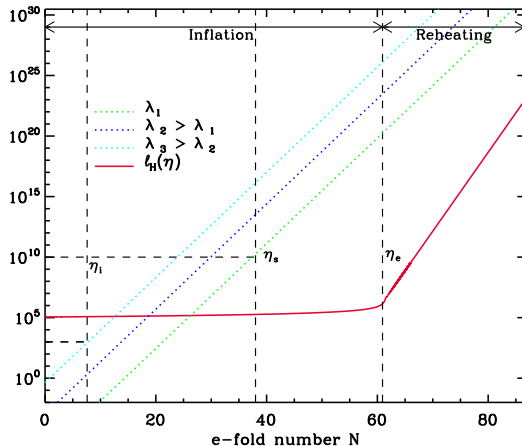


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The various times of interest³⁵

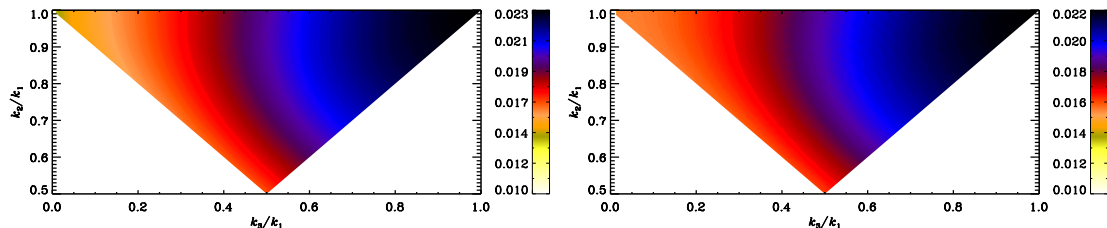


The exact behavior of the physical wavelengths and the Hubble radius plotted as a function of the number of e-folds in the case of the archetypical quadratic potential, which allows us to illustrate the various times of our interest, *viz.* η_i , η_s and η_e .

³⁵D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026 (2013).



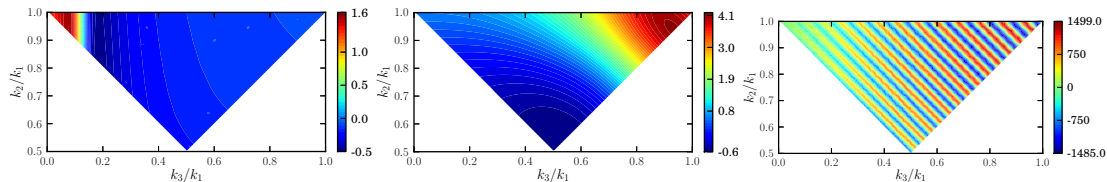
Results from BINGO



A comparison of the analytical results (on the left) for the non-Gaussianity parameter f_{NL} with the numerical results from the code Blspectra and Non-Gaussianity Operator or, simply, BINGO (on the right) for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential³⁶ The maximum difference between the numerical and the analytic results is found to be about 5%.

³⁶D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026, (2013).



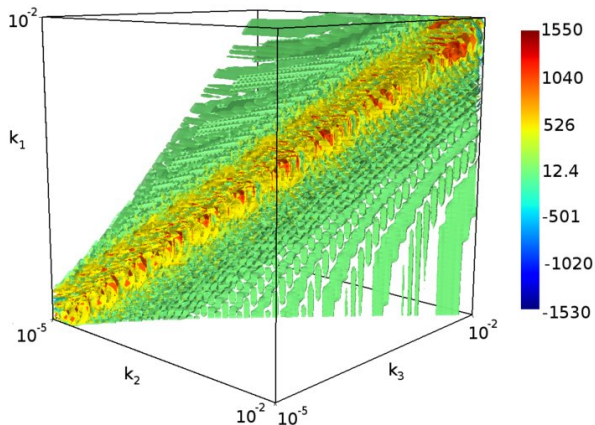
f_{NL} in models with features

The scalar non-Gaussianity parameter f_{NL} in the punctuated inflationary scenario (on the left), quadratic potential with a step (in the middle) and the axion monodromy model (on the right)³⁷.

³⁷ D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026 (2013);
V. Sreenath, D. K. Hazra and L. Sriramkumar, JCAP **1502**, 029 (2015).



The inflationary scalar bispectrum



The shape of the inflationary scalar bispectrum (actually, the non-Gaussianity parameter f_{NL}) in the case of the axion monodromy model³⁸.

³⁸V. Sreenath, D. K. Hazra and L. Sriramkumar, JCAP **1502**, 029 (2015).



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Bouncing scenarios as an alternative paradigm³⁹

- ◆ Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

³⁹See, for instance, [M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 \(2008\);](#)
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Bouncing scenarios as an alternative paradigm³⁹

- ◆ Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.
- ◆ They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.

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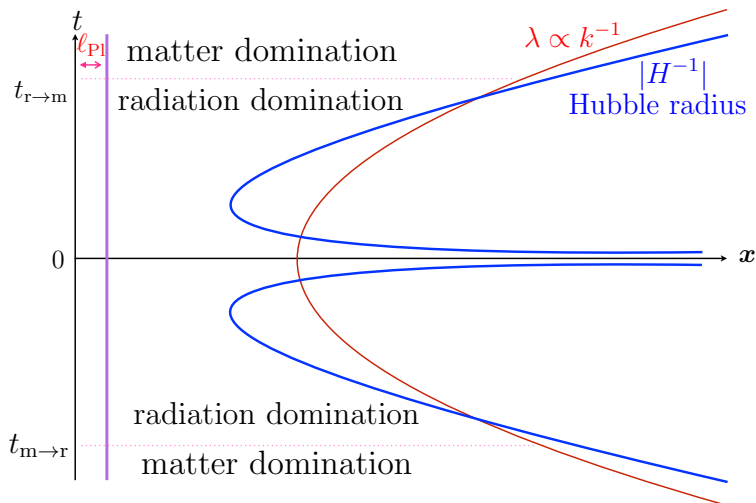
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- ◆ They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- ◆ However, matter fields *will* have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity can be supposed to fail and quantum gravitational effects are expected to take over.

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Overcoming the horizon problem in bouncing models



► Behavior in inflation

Evolution of the physical wavelength and the Hubble radius in a bouncing scenario⁴⁰

⁴⁰Figure from, D. Battefeld and P. Peter, *Phys. Rept.* **571**, 1 (2015).



Violation of the null energy condition near the bounce

Recall that, according to the Friedmann equations

$$\dot{H} = -4\pi G (\rho + p).$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.



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It can be shown that, if the modes of cosmological interest have to be inside the Hubble radius at early times during the contracting phase, *the universe needs to undergo non-accelerated contraction*.



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In such cases, one finds that \dot{H} will be positive near the bounce, which implies that $\rho + p < 0$ in this domain. In other words, the null energy condition needs to be violated in order to achieve such bounces.

► Behavior during inflation



A new model for the completely symmetric matter bounce

The so-called matter bounce scenario is described by the scale factor

$$a(\eta) = a_0 \left(1 + \eta^2/\eta_0^2\right) = a_0 \left(1 + k_0^2 \eta^2\right).$$

It can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

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We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ , that are governed by the following action⁴¹:

$$S[\phi, \chi] = - \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + U_0 \left(-\frac{1}{2} \partial_\mu \chi \partial^\mu \chi \right)^2 \right],$$

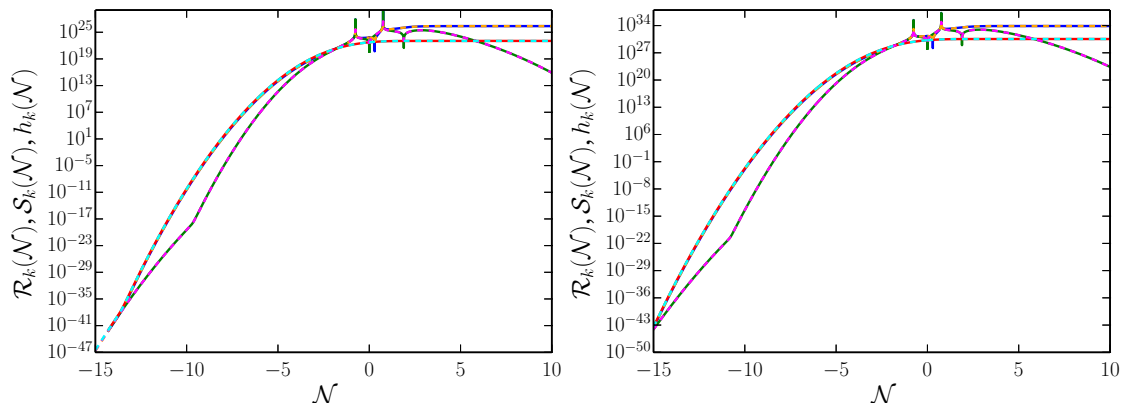
where U_0 is a constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6 M_{\text{Pl}}^2 (k_0^2/a_0^2)}{\cosh^6[\phi/(\sqrt{12} M_{\text{Pl}})]}.$$

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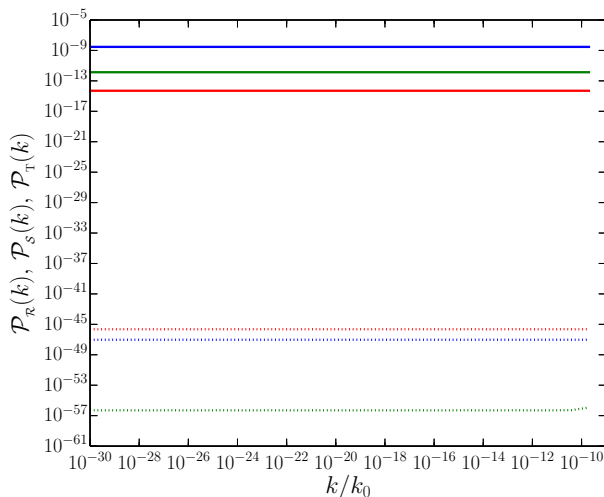
Evolution of \mathcal{R}_k , \mathcal{S}_k and h_k



The evolution of the curvature, isocurvature and tensor perturbations, viz. \mathcal{R}_k (in blue and orange), \mathcal{S}_k (in green and magenta) and h_k (in red and cyan) across the bounce for the modes $k/k_0 = 10^{-20}$ (on the left) and $k/k_0 = 10^{-25}$ (on the right). We have set $k_0 = M_{\text{Pl}}$, $a_0 = 3 \times 10^7$, $\alpha = 10^5$ and $\beta = 10^2$. The solid lines denote the results obtained numerically while the dashed lines represent the analytical approximations.



The scalar and tensor power spectra in the matter bounce

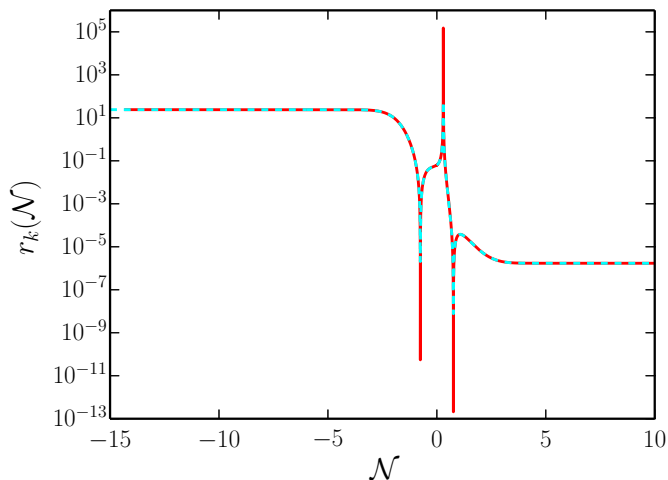


The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra have been plotted before (as dotted lines) as well as after (as solid lines) the bounce⁴²

⁴²R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].



The evolution of the tensor-to-scalar ratio



The evolution of the tensor-to-scalar ratio r across the symmetric matter bounce for a typical mode of cosmological interest. The solid (in red) and dashed (in cyan) lines represent the numerical and analytical results, respectively.



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Summary: The inflationary scenario

- ◆ A nearly scale invariant primordial spectrum as is generated in slow roll inflation is remarkably consistent with the cosmological data.

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- ◆ While there exist some anomalies (features in the power spectra, hemispherical asymmetry, etc.), they do not seem to have substantial statistical significance.
- ◆ One may need to carry out a systematic search involving the scalar and the tensor power spectra⁴³, the scalar and the tensor bispectra and the cross correlations to arrive at a smaller subset of viable inflationary models⁴⁴.

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Can inflation be falsified?

The difficulty with the inflationary paradigm

A theory that predicts everything predicts nothing^a.

^aP. J. Steinhardt, *Sci. Am.* **304**, 36 (2011).

Bayesian evidence for models across paradigms

As the recent hiccups in CMB polarization observations demonstrated, whatever the data turns out to be it will be paraded as proof of inflation. This is because for any observation there is an inflationary model fitting it. Concomitantly, there is a trend in parroting the death of inflation's alternatives (such as cyclic models, string gas cosmology and varying speed of light models).

Putting aside sociology, these perceptions may derive from an important scientific issue. We argued here that the oft-used concept of Bayesian evidence fails to adequately capture the tenet that falsifiability is the hallmark of a scientific theory. Whilst the concept may be perfectly appropriate for comparing models within a paradigm, it fails to suitably penalize the whole paradigm for not making a prediction that could rule it out^a.

^aG. Gubitosi, M. Lagos, J. Magueijo and R. Allison, *JCAP* **06**, 002 (2016).

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- ◆ We are currently working on constructing symmetric bouncing models that lead to scalar power spectra with a tilt as suggested by the cosmological data.
- ◆ It is also important to examine if the non-Gaussianities generated in such models are in agreement with the recent constraints from Planck.



Issues confronting bouncing models

- ◆ In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.

⁴⁵L. E. Allen and D. Wands, *Phys. Rev.* **70**, 063515 (2004).

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Collaborators I



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Debika Chowdhury



Rathul Nath Raveendran



Collaborators II



Pravabati Chingambam



Jinn-Ouk Gong



Rakesh Tibrewala



Tarun Souradeep



Jérôme Martin



Thank you for your attention