The inflationary scalar bispectrum
– Status and possibilities –

L. Sriramkumar

Department of Physics, Indian Institute of Technology Madras, Chennai

Chennai Mathematical Institute, Chennai

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Proliferation of inflationary models[1]

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflation
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A partial list of ever-increasing number of inflationary models!

[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Non-Gaussianities – Pre-Planck status

- If one assumes the bispectrum to be, say, of the so-called local form, the WMAP 9-year data constrains the non-Gaussianity parameter $f_{_{\rm NL}}$ to be 37.2 ± 19.9, at 68% confidence level[2].

[2] C. L. Bennett *et al.*, arXiv:1212.5225v1 [astro-ph.CO].

[3] J. Maldacena, JHEP **05**, 013 (2003).

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- If missions such as Planck indeed detect a large level of non-Gaussianity as suggested by the above mean value of $f_{_{\rm NL}}$, then it can result in a substantial tightening in the constraints on the various inflationary models. For example, canonical scalar field models that lead to nearly scale invariant primordial spectra contain only a small amount of non-Gaussianity and, hence, will cease to be viable[3].

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- If missions such as Planck indeed detect a large level of non-Gaussianity as suggested by the above mean value of $f_{\rm NL}$, then it can result in a substantial tightening in the constraints on the various inflationary models. For example, canonical scalar field models that lead to nearly scale invariant primordial spectra contain only a small amount of non-Gaussianity and, hence, will cease to be viable[3].
- However, it is known that primordial spectra with features can lead to reasonably large non-Gaussianities[4]. Therefore, if the non-Gaussianity parameter $f_{\rm NL}$ actually proves to be large, then either one has to reconcile with the fact that the primordial spectrum contains features or we have to turn our attention to non-canonical scalar field models such as, say, D brane inflation models[5].

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Constraints on non-Gaussianities from Planck[6]

- The constraints from Planck on the local form of the non-Gaussianity parameter $f_{\rm NL}$ proves to be 2.7 ± 5.8.

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Constraints on non-Gaussianities from Planck[6]

- The constraints from Planck on the local form of the non-Gaussianity parameter $f_{\rm NL}$ proves to be 2.7 ± 5.8.
- In other words, preliminary investigations seem to suggest that inflationary models that lead to rather large non-Gaussianities are likely to be ruled out by the data.

[6] P. A. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

Plan of the talk

1. The inflationary paradigm
2. Confronting inflationary power spectra with the CMB data
3. The scalar bispectrum and the non-Gaussianity parameter – Definitions
4. The Maldacena formalism for evaluating the bispectrum
5. BINGO: An efficient code to numerically compute the bispectrum
6. Constraints from Planck on non-Gaussianities
7. Are features consistent with small non-Gaussianities?
8. Outlook

A few words on the conventions and notations

- We shall work in units such that $c = \hbar = 1$, and define the Planck mass to be $\mathrm{M}_{_{\mathrm{Pl}}}^2 = (8\,\pi\,G)^{-1}$.

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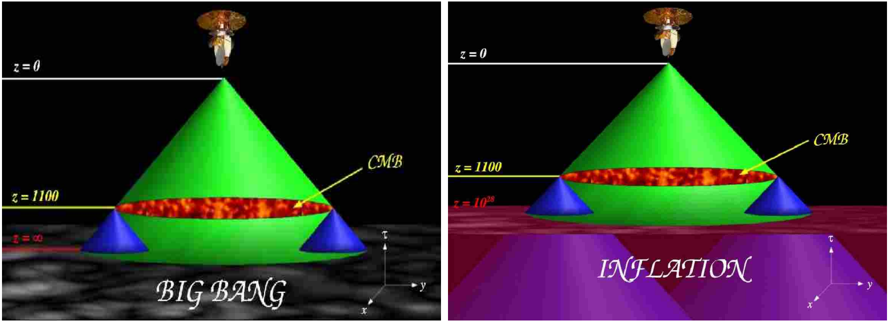
- Further, N shall denote the number of e-folds.

Inflation resolves the horizon problem



Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

[7] Images from W. Kinney, astro-ph/0301448.

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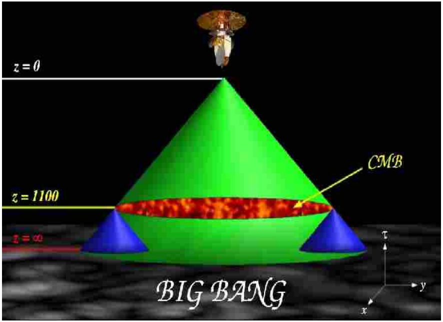
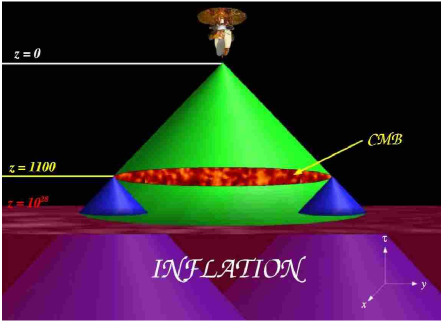
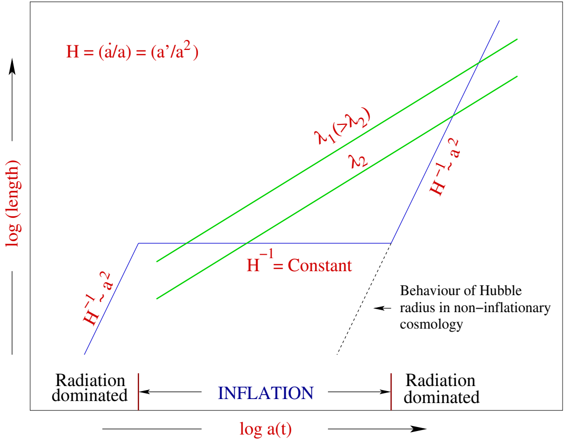




Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.
Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[7].

[7] Images from W. Kinney, astro-ph/0301448.

Bringing the modes inside the Hubble radius



A schematic diagram illustrating the behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[8].

[8] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Scalar fields can drive inflation

If we require that $\lambda_{\rm P} < d_{\rm H}$ at a sufficiently early time, then we need to have an epoch wherein $\lambda_{\rm P}$ decreases faster than the Hubble scale *as we go back in time*, i.e. a regime during which[9]

$$-\frac{{\rm d}}{{\rm d}\,t}\left(\frac{\lambda_{\rm P}}{d_{\rm H}}\right) < 0 \quad \rightarrow \quad \ddot{a} > 0.$$

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In the case of a canonical scalar field, say, ϕ, this condition simplifies to

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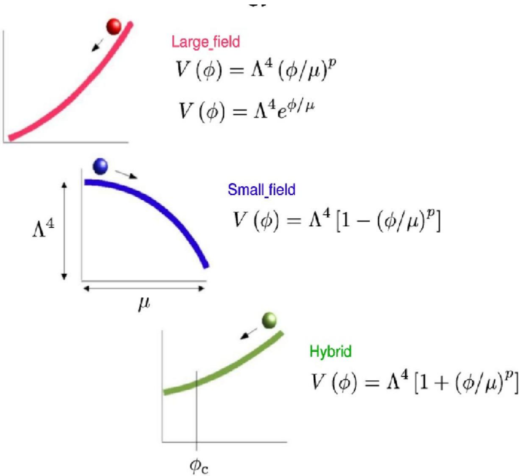
This condition can be achieved if the scalar field is initially displaced from a minima of the potential, and inflation will end when the field approaches the minima with zero or negligible potential energy.

[9] For a review, see L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[10]. Often, these potentials are classified as small field, large field and hybrid models.

[10] Image from W. Kinney, astro-ph/0301448.

The character of the perturbations

In a Friedmann universe, the perturbations in the metric and the matter can be classified according to their behavior with respect to a local rotation of the spatial coordinates on hypersurfaces of constant time as follows[11]:

- ✦ Scalar perturbations – Density and pressure perturbations
- ✦ Vector perturbations – Rotational velocity fields
- ✦ Tensor perturbations – Gravitational waves

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It is the fluctuations in the inflaton field ϕ that act as the seeds for the scalar perturbations that are primarily responsible for the anisotropies in the CMB and, eventually, the present day inhomogeneities.

[11] See, for instance, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

The curvature perturbation and the governing equation

On quantization, the operator corresponding to the curvature perturbation $\mathcal{R}(\eta, \boldsymbol{x})$ can be expressed as

$$\begin{aligned}\hat{\mathcal{R}}(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \\ &= \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\,\pi)^{3/2}} \left[\hat{a}_{\boldsymbol{k}}\, f_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^{\dagger}\, f_{\boldsymbol{k}}^{*}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],\end{aligned}$$

where $\hat{a}_{\boldsymbol{k}}$ and $\hat{a}_{\boldsymbol{k}}^{\dagger}$ are the usual creation and annihilation operators that satisfy the standard commutation relations.

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$$= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \left[\hat{a}_{\boldsymbol{k}}\, f_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^{\dagger}_{\boldsymbol{k}}\, f^{*}_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where $\hat{a}_{\boldsymbol{k}}$ and $\hat{a}^{\dagger}_{\boldsymbol{k}}$ are the usual creation and annihilation operators that satisfy the standard commutation relations.

The modes $f_{\boldsymbol{k}}$ are governed by the differential equation

$$f''_{\boldsymbol{k}} + 2\,\frac{z'}{z}\, f'_{\boldsymbol{k}} + k^2\, f_{\boldsymbol{k}} = 0,$$

where $z = a\,\mathrm{M}_{_{\mathrm{Pl}}}\,\sqrt{2\,\epsilon_1}$, with $\epsilon_1 = -\mathrm{d}\ln H/\mathrm{d}N$ being the first slow roll parameter.

The scalar and the tensor perturbation spectra

The dimensionless scalar power spectrum $\mathcal{P}_{_{\rm S}}(k)$ is defined in terms of the correlation function of the Fourier modes of the curvature perturbation $\hat{\mathcal{R}}_{\boldsymbol{k}}$ as follows:

$$\langle 0|\hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\,\hat{\mathcal{R}}_{\boldsymbol{p}}(\eta)|0\rangle = \frac{(2\,\pi)^2}{2\,k^3}\,\mathcal{P}_{_{\rm S}}(k)\;\delta^{(3)}\,(\boldsymbol{k}+\boldsymbol{p})\,,$$

where $|0\rangle$ is the Bunch-Davies vacuum, defined as $\hat{a}_{\boldsymbol{k}}|0\rangle = 0\;\forall\;\boldsymbol{k}$ and, in terms of the quantity $f_{\boldsymbol{k}}$, the power spectrum is given by

$$\mathcal{P}_{_{\rm S}}(k) = \frac{k^3}{2\,\pi^2}\,|f_{\boldsymbol{k}}|^2.$$

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The tensor modes, say, $h_{\boldsymbol{k}}$, satisfy the differential equation

$$h_{\boldsymbol{k}}'' + 2\,\mathcal{H}\; h_{\boldsymbol{k}}' + k^2\; h_{\boldsymbol{k}} = 0,$$

where $\mathcal{H} = (a'/a)$ is the conformal Hubble parameter and, the tensor power spectrum, viz. $\mathcal{P}_{\rm T}(k)$, is given by[12]

$$\mathcal{P}_{\rm T}(k) = \frac{8}{{\rm M}_{\rm Pl}^2}\, \frac{k^3}{2\pi^2}\, |h_{\boldsymbol{k}}|^2.$$

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Evaluation of the inflationary power spectra

As is well known[13], analytically, the so-called Bunch-Davies initial conditions[14] are imposed on the modes in the sub-Hubble limit, *viz.* when $k/(a\,H) \to \infty$, and the scalar as well as the tensor power spectra are evaluated in the super-Hubble limit, *i.e.* when $k/(a\,H) \to 0$.

[13] See, for example, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

[14] T. Bunch and P. C. W. Davies, Proc. Roy. Soc. Lond. A **360**, 117 (1978).

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While comparing specific inflationary models with the data, the power spectra often need to be evaluated numerically. In such situations, the Bunch-Davies initial conditions are imposed on the modes when they are *well inside the Hubble radius*, and the power spectra are evaluated at suitably late times when the modes are *sufficiently outside*[15].

[13] See, for example, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

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The tensor-to-scalar ratio and the spectral indices

The tensor-to-scalar ratio r is given by

$$r(k) \equiv \frac{\mathcal{P}_{_{\rm T}}(k)}{\mathcal{P}_{_{\rm S}}(k)},$$

while the scalar and the tensor spectral indices are defined as follows:

$$n_{_{\rm S}} \equiv 1 + \frac{{\rm d}\ln \mathcal{P}_{_{\rm S}}}{{\rm d}\ln k} \quad \text{and} \quad n_{_{\rm T}} \equiv \frac{{\rm d}\ln \mathcal{P}_{_{\rm T}}}{{\rm d}\ln k}.$$

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While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_{\mathrm{S}}(k) = \mathcal{A}_{\mathrm{S}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} \quad \text{and} \quad \mathcal{P}_{\mathrm{T}}(k) = \mathcal{A}_{\mathrm{T}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{T}}},$$

wherein the spectral indices n_{S} and n_{T} are assumed to be constant. The quantity k_* denotes a specific scale at which the scalar and the tensor amplitudes $\mathcal{A}_{\mathrm{S}}$ and $\mathcal{A}_{\mathrm{T}}$ are quoted.

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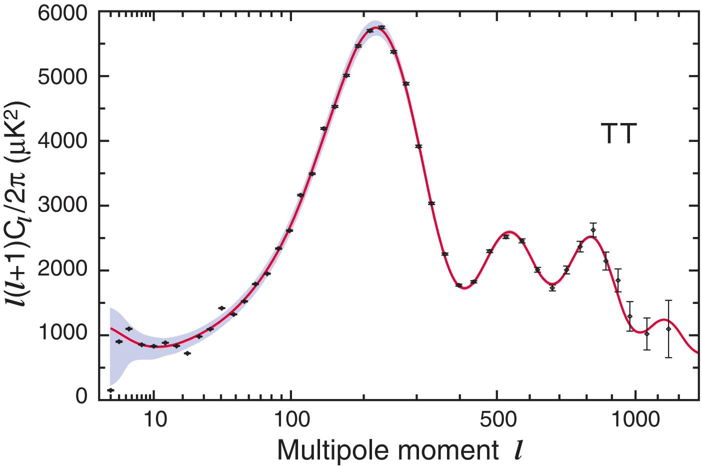
wherein the spectral indices $n_{_{\rm S}}$ and $n_{_{\rm T}}$ are assumed to be constant. The quantity $k_\ast$ denotes a specific scale at which the scalar and the tensor amplitudes $\mathcal{A}_{_{\rm S}}$ and $\mathcal{A}_{_{\rm T}}$ are quoted.

Moreover, it is usual to further set $r = -8\, n_{_{\rm T}}$, viz. the so-called consistency relation, which is valid during slow roll inflation.

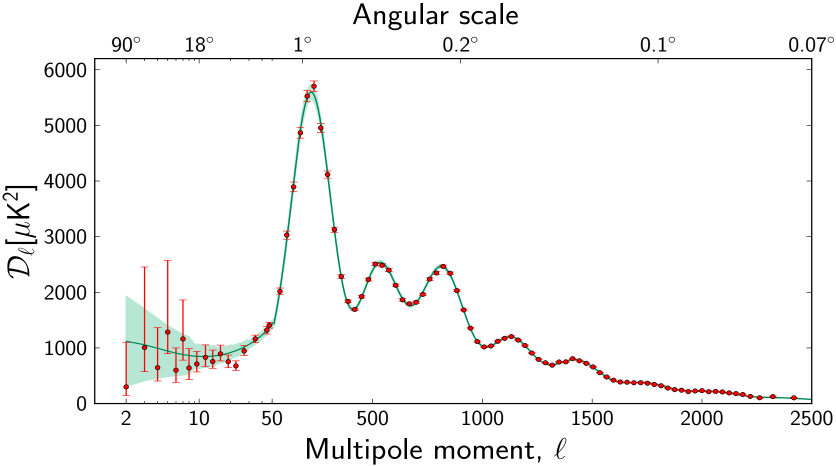
Angular power spectrum from the WMAP 9-year data[16]



The WMAP 9-year data for the CMB TT angular power spectrum (the black dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid red curve).

[16] C. L. Bennett *et al.*, arXiv:1212.5225v1 [astro-ph.CO].

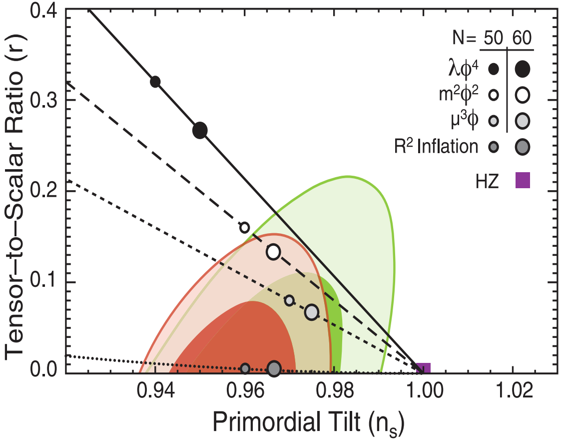
Angular power spectrum from the Planck data[17]



The CMB TT angular power spectrum from the Planck data (the red dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid green curve).

[17] P. A. R. Ade *et al.*, arXiv:1303.5075 [astro-ph.CO].

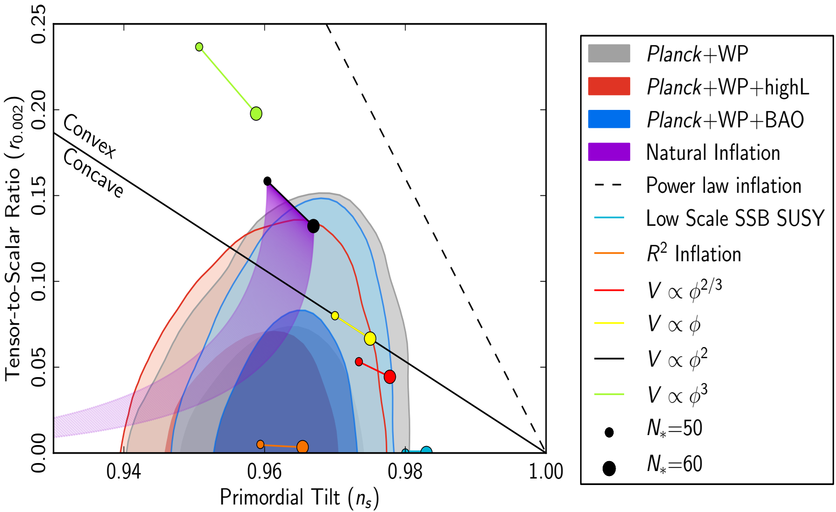
Constraints from the WMAP data[18]



Joint constraints from the recent WMAP 9-year and other cosmological data on the inflationary parameters $n_{\rm s}$ and r for large field models with potentials of the form $V(\phi) \propto \phi^n$.

[18] G. Hinshaw *et al.*, arXiv:1212.5226v1 [astro-ph.CO].

Constraints from Planck[19]



The corresponding constraints from the Planck data for various models.

[19] P. A. R. Ade *et al.*, arXiv:1303.5082 [astro-ph.CO].

The scalar bispectrum

The scalar bispectrum $\mathcal{B}_{\mathrm{S}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time η_{e}, as follows[20]:

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\mathrm{e}})\rangle = (2\,\pi)^3\;\mathcal{B}_{\mathrm{S}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\;\delta^{(3)}\,(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\,.$$

[20] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011);
C. L. Bennett *et al.*, arXiv:1212.5225v1 [astro-ph.CO].

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In our discussion below, for the sake of convenience, we shall set

$$\mathcal{B}_{\mathrm{S}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\; G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

[20] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011);
C. L. Bennett *et al.*, arXiv:1212.5225v1 [astro-ph.CO].

The non-Gaussianity parameter $f_{\rm NL}$

With the so-called local limit in mind, the observationally relevant non-Gaussianity parameter $f_{\rm NL}$ is introduced through the relation[21]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}_{\rm G}(\eta, \boldsymbol{x}) - \frac{3\, f_{\rm NL}}{5}\, \left[\mathcal{R}^2_{\rm G}(\eta, \boldsymbol{x}) - \left\langle \mathcal{R}^2_{\rm G}(\eta, \boldsymbol{x}) \right\rangle\right],$$

where $\mathcal{R}_{\rm G}$ denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

Utilizing the above relation and Wick's theorem, one can arrive at the three point correlation function of the curvature perturbation in Fourier space in terms of the parameter $f_{\rm NL}$. It is found to be

$$\begin{aligned} \langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}\, \hat{\mathcal{R}}_{\boldsymbol{k}_3} \rangle &= -\frac{3\, f_{\rm NL}}{10}\, (2\,\pi)^{5/2}\, \left(\frac{1}{k_1^3\, k_2^3\, k_3^3}\right)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \\ &\quad \times \left[k_1^3\, \mathcal{P}_{\rm S}(k_2)\, \mathcal{P}_{\rm S}(k_3) + \text{two permutations}\right]. \end{aligned}$$

[21] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

The relation between $f_{_{\rm NL}}$ and the bispectrum

Upon making use of the above expression for the three point function of the curvature perturbation and the definition of the bispectrum, we can, in turn, arrive at the following relation[22]:

$$\begin{aligned} f_{_{\rm NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\frac{10}{3}\,(2\,\pi)^{1/2}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,\mathcal{B}_{_{\rm S}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\\ &\quad\times\,\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)\,\mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1}\\ &= -\frac{10}{3}\,\frac{1}{(2\,\pi)^4}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\\ &\quad\times\,\left[k_1^3\,\mathcal{P}_{_{\rm S}}(k_2)\,\mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1}. \end{aligned}$$

[22] See, for instance, S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

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Note that, in the equilateral limit, *i.e.* when $k_1 = k_2 = k_3$, this expression for $f_{_{\rm NL}}$ simplifies to

$$f_{_{\rm NL}}^{\rm eq}(k) = -\frac{10}{9}\, \frac{1}{(2\,\pi)^4}\, \frac{k^6\, G(k)}{\mathcal{P}_{_{\rm S}}^2(k)}.$$

[22] See, for instance, S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

The action at the cubic order

It can be shown that, the third order term in the action describing the curvature perturbation is given by[23]

$$\mathcal{S}_3[\mathcal{R}] = \mathrm{M}_{_{\mathrm{Pl}}}^2 \int \mathrm{d}\eta \int \mathrm{d}^3\mathbf{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial^i\mathcal{R})\,(\partial_i\chi) + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial^i\mathcal{R})\,(\partial_i\chi)\,(\partial^2\chi) + \frac{\epsilon_1}{4}\,(\partial^2\mathcal{R})\,(\partial\chi)^2 + \mathcal{F}\left(\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\right)\biggr],$$

where $\mathcal{F}(\delta\mathcal{L}_2/\delta\mathcal{R})$ denotes terms involving the variation of the second order action with respect to $\mathcal{R}$, while the quantity χ is related to the curvature perturbation $\mathcal{R}$ through the relation

$$\partial^2\chi = a\,\epsilon_1\,\mathcal{R}'.$$

[23] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

Evaluating the bispectrum

At the leading order in the perturbations, one then finds that the three point correlation in Fourier space is described by the integral[24]

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e})\rangle = -\,i\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a(\eta)\;\left\langle\left[\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e}), \hat{H}_{\rm I}(\eta)\right]\right\rangle,$$

where $\hat{H}_{\rm I}$ is the Hamiltonian corresponding to the above third order action, while $\eta_{\rm i}$ denotes a sufficiently early time when the initial conditions are imposed on the modes, and $\eta_{\rm e}$ denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are evaluated in the initial vacuum state (*viz.* the Bunch-Davies vacuum in the situation of our interest).

[24] See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).

The resulting bispectrum

The quantity $G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ evaluated towards the end of inflation at the conformal time $\eta = \eta_{\rm e}$ can be written as[25]

$$
\begin{aligned}
G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &\equiv \sum_{C=1}^{7} G_{_C}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&\equiv {\rm M}_{_{\rm Pl}}^2 \sum_{C=1}^{6} \Big\{ \left[f_{\boldsymbol{k}_1}(\eta_{\rm e})\, f_{\boldsymbol{k}_2}(\eta_{\rm e})\, f_{\boldsymbol{k}_3}(\eta_{\rm e})\right] \, \mathcal{G}_{_C}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
&\qquad + \left[f_{\boldsymbol{k}_1}^{*}(\eta_{\rm e})\, f_{\boldsymbol{k}_2}^{*}(\eta_{\rm e})\, f_{\boldsymbol{k}_3}^{*}(\eta_{\rm e})\right] \, \mathcal{G}_{_C}^{*}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \Big\} + G_7(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3),
\end{aligned}
$$

where the quantities $\mathcal{G}_{_C}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ with $C = (1, 6)$ correspond to the six terms in the interaction Hamiltonian.

The additional, seventh term $G_7(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ arises due to a field redefinition, and its contribution to $G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is given by

$$
G_7(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{\epsilon_2(\eta_{\rm e})}{2} \, \left(|f_{\boldsymbol{k}_2}(\eta_{\rm e})|^2 \, |f_{\boldsymbol{k}_3}(\eta_{\rm e})|^2 + \text{two permutations}\right).
$$

[25] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

The integrals involved

The quantities $\mathcal{G}_C(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ with $C = (1, 6)$ are described by the integrals

$$\mathcal{G}_1(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = 2\, i \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1^2\, \left(f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{\prime *}\, f_{\boldsymbol{k}_3}^{\prime *} + \text{two permutations}\right),$$

$$\mathcal{G}_2(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -2\, i\, \left(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2 + \text{two permutations}\right) \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1^2\, f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{*}\, f_{\boldsymbol{k}_3}^{*},$$

$$\mathcal{G}_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -2\, i \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1^2 \left[\left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\right) f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{\prime *}\, f_{\boldsymbol{k}_3}^{\prime *} + \text{five permutations}\right],$$

$$\mathcal{G}_4(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1\, \epsilon_2^{\prime}\, \left(f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{*}\, f_{\boldsymbol{k}_3}^{\prime *} + \text{two permutations}\right),$$

$$\mathcal{G}_5(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{i}{2} \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1^3 \left[\left(\frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\right) f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{\prime *}\, f_{\boldsymbol{k}_3}^{\prime *} + \text{five permutations}\right],$$

$$\mathcal{G}_6(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{i}{2} \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\, a^2\, \epsilon_1^3 \left\{\left[\frac{k_1^2\, (\boldsymbol{k}_2 \cdot \boldsymbol{k}_3)}{k_2^2\, k_3^2}\right] f_{\boldsymbol{k}_1}^{*}\, f_{\boldsymbol{k}_2}^{\prime *}\, f_{\boldsymbol{k}_3}^{\prime *} + \text{two permutations}\right\},$$

where ϵ_2 is the second slow roll parameter that is defined with respect to the first as follows: $\epsilon_2 = \mathrm{d} \ln \epsilon_1 / \mathrm{d}N$.

Splitting the integrals

To begin with, we shall divide each of the integrals $\mathcal{G}_{C}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$, where $C = (1, 6)$, into two parts as follows[26]:

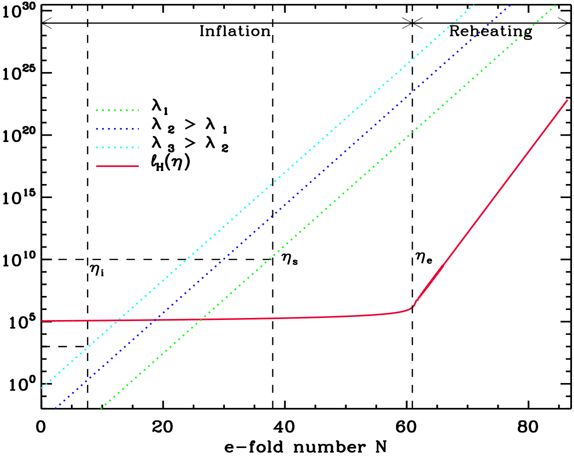
$$\mathcal{G}_{C}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \mathcal{G}_{C}^{\rm is}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \mathcal{G}_{C}^{\rm se}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

The integrals in the first term $\mathcal{G}_{C}^{\rm is}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ run from the earliest time (*i.e.* $\eta_{\rm i}$) when the smallest of the three wavenumbers k_1, k_2 and k_3 is sufficiently inside the Hubble radius [typically corresponding to $k/(a\, H) \simeq 100$] to the time (say, $\eta_{\rm s}$) when the largest of the three wavenumbers is well outside the Hubble radius [say, when $k/(a\, H) \simeq 10^{-5}$].

Then, evidently, the second term $\mathcal{G}_{C}^{\rm se}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ will involve integrals which run from the latter time $\eta_{\rm s}$ to the end of inflation at $\eta_{\rm e}$.

[26] D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].

The various times of interest



The exact behavior of the physical wavelengths and the Hubble radius plotted as a function of the number of e-folds in the case of the archetypical quadratic potential, which allows us to illustrate the various times of our interest, *viz.* $\eta_{\rm i}$, $\eta_{\rm s}$ and $\eta_{\rm e}$.

An estimate of the super-Hubble contribution to $f_{\rm NL}^{\rm eq}$

Consider power law inflation of the form $a(\eta) = a_1\,(\eta/\eta_1)^{\gamma+1}$, where a_1 and η_1 are constants, while γ is a free index. For such an expansion, the first slow roll parameter is a constant, and is given by $\epsilon_1 = (\gamma+2)/(\gamma+1)$.

In such a case, one can easily obtain that

$$f_{\rm NL}^{\rm eq\,(se)}(k) = \frac{5}{72\,\pi}\,\left[12 - \frac{9\,(\gamma+2)}{\gamma+1}\right]\,\Gamma^2\left(\gamma+\frac{1}{2}\right)\,2^{2\,\gamma+1}\,(2\,\gamma+1)\,(\gamma+2)$$
$$\times\,(\gamma+1)^{-2\,(\gamma+1)}\,\sin\,(2\,\pi\,\gamma)\,\left[1 - \frac{H_{\rm s}}{H_{\rm e}}\,{\rm e}^{-3\,(N_{\rm e}-N_{\rm s})}\right]\,\left(\frac{k}{a_{\rm s}\,H_{\rm s}}\right)^{-(2\,\gamma+1)}.$$

and, in arriving at this expression, for convenience, we have set η_1 to be η_s.

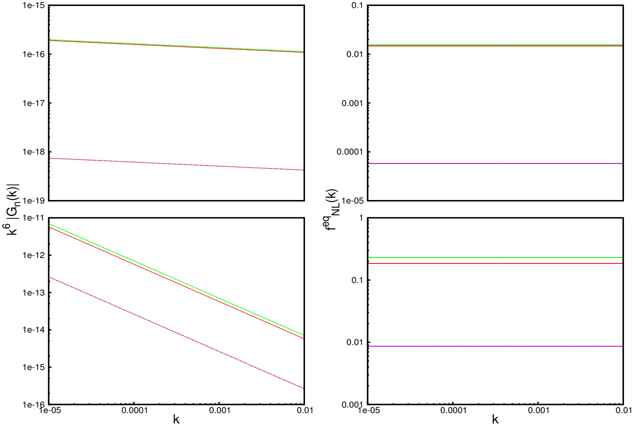
For $\gamma = -(2+\varepsilon)$, where $\varepsilon \simeq 10^{-2}$, the above estimate for $f_{_{\rm NL}}$ reduces to[27]

$$f_{\rm NL}^{\rm eq\,(se)}(k) \lesssim -\frac{5\,\varepsilon^2}{9}\,\left(\frac{k_{\rm s}}{a_{\rm s}\,H_{\rm s}}\right)^3 \simeq -10^{-19},$$

where, in obtaining the final value, we have set $k_{\rm s}/(a_{\rm s}\,H_{\rm s}) = 10^{-5}$.

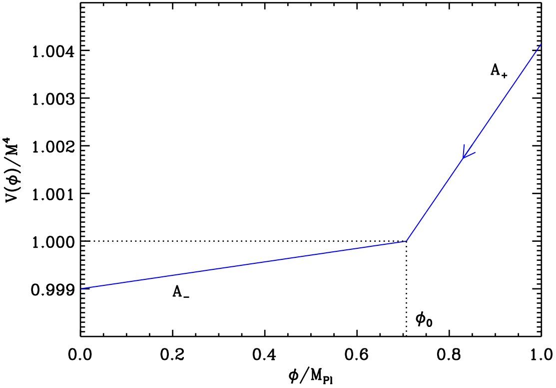
[27] D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].

The spectral dependence in power law inflation



The different non-zero contributions to the bispectrum, viz. the quantities k^6 times the absolute values of $G_1 + G_3$ (in green), G_2 (in red) and $G_5 + G_6$ (in purple), in power law inflation (on the left) and the corresponding contributions to the non-Gaussianity parameter $f_{\rm NL}^{\rm eq}$ (on the right), arrived at using BINGO (BIspectrum and Non-Gaussianity Operator), have been plotted as solid lines for two different values of γ (above and below). The dots on the lines represent the analytical results.

The Starobinsky model

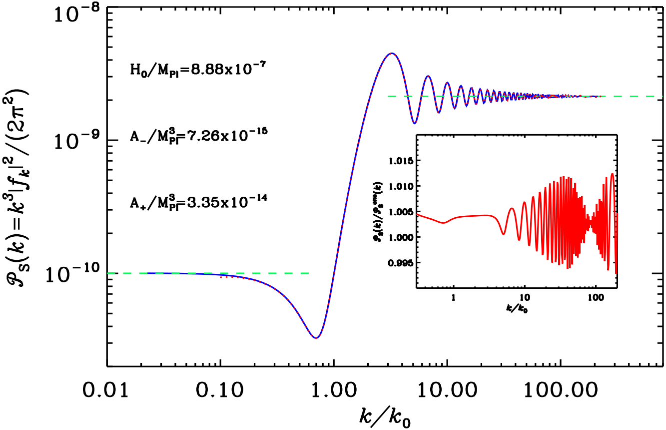


The Starobinsky model involves the canonical scalar field which is described by the potential[28]

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0) & \text{for } \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0) & \text{for } \phi < \phi_0. \end{cases}$$

[28] A. A. Starobinsky, Sov. Phys. JETP Lett. **55**, 489 (1992).

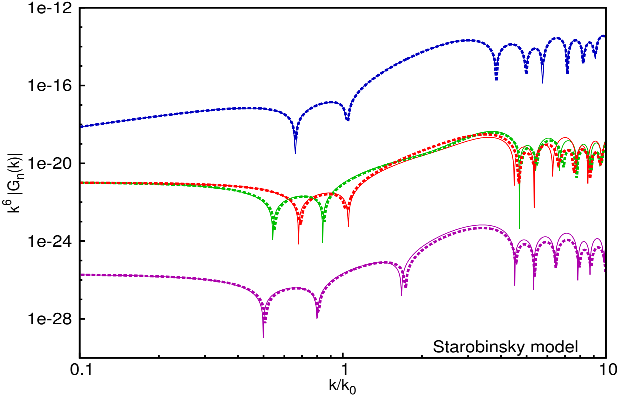
The scalar power spectrum in the Starobinsky model



The scalar power spectrum in the Starobinsky model[29]. While the solid blue curve denotes the analytic result, the red dots represent the scalar power spectrum that has been obtained through a complete numerical integration of the background as well as the perturbations.

[29] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

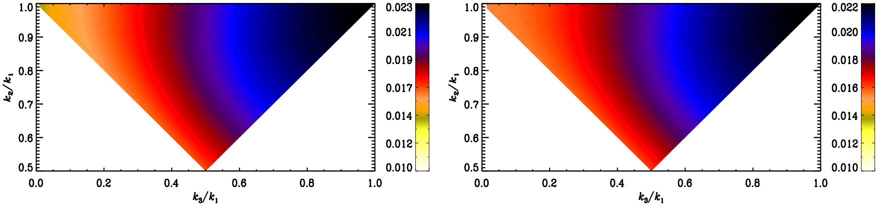
Comparison in the case of the Starobinsky model



A comparison of the analytical expressions (the solid curves) with the corresponding results from BINGO (the dashed curves) in the case of the Starobinsky model. While the contribution due to the term $G_4 + G_7$ appears in blue, we have chosen the same colors to denote the other contributions to the bispectrum as in the previous figure[30].

[30] See, J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012);
In this context, also see, F. Arroja, A. E. Romano and M. Sasaki, Phys. Rev. D **84**, 123503 (2011); F. Arroja and M. Sasaki, JCAP **1208**, 012 (2012).

Comparison for an arbitrary triangular configuration



A comparison of the analytical results (on the left) for the non-Gaussianity parameter $f_{\rm NL}$ with the results from BINGO (on the right) for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential. It should be mentioned that the contributions due to the first, the second, the third and the seventh terms (i.e. G_1, G_2, G_3 and G_7) have been taken into account in arriving at these results. The maximum difference between the numerical and the analytic results is found to be about 5%.

Template bispectra

For comparison with the observations, the bispectrum is often expressed as follows[31]:

$$G(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = f_{\rm NL}^{\rm loc}\, G_{\rm loc}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm eq}\, G_{\rm eq}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm orth}\, G_{\rm orth}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3),$$

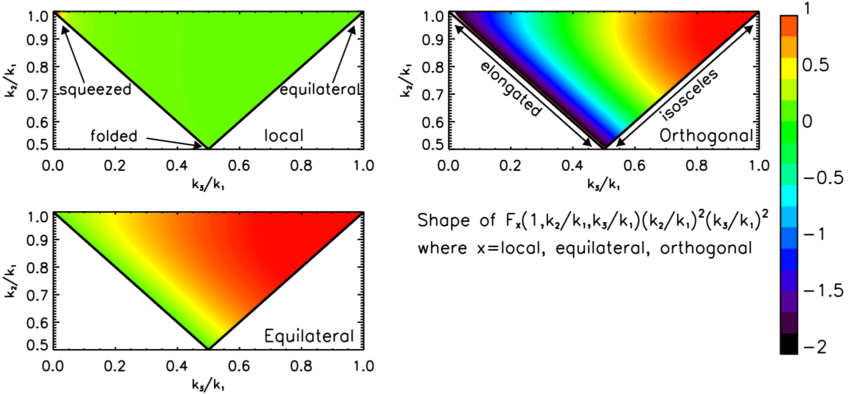
where $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm eq}$ and $f_{\rm NL}^{\rm orth}$ are free parameters that are to be estimated, and the local, the equilateral, and the orthogonal template bispectra are given by:

$$\begin{aligned}
G_{\rm loc}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{6}{5}\left[\frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\right]\left(k_1^3\, \mathcal{P}_{\rm S}(k_2)\, \mathcal{P}_{\rm S}(k_3) + \text{two permutations}\right),\\
G_{\rm eq}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{3}{5}\left[\frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\right)\left(6\, k_2\, k_3^2\, \mathcal{P}_{\rm S}(k_1)\, \mathcal{P}_{\rm S}^{2/3}(k_2)\, \mathcal{P}_{\rm S}^{1/3}(k_3) - 3\, k_3^3\, \mathcal{P}_{\rm S}(k_1)\, \mathcal{P}_{\rm S}(k_2)\right.\\
&\qquad \left. -2\, k_1\, k_2\, k_3\, \mathcal{P}_{\rm S}^{2/3}(k_1)\, \mathcal{P}_{\rm S}^{2/3}(k_2)\, \mathcal{P}_{\rm S}^{2/3}(k_3) + \text{five permutations}\right),\\
G_{\rm orth}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= \frac{3}{5}\left[\frac{(2\,\pi^2)^2}{k_1^3\, k_2^3\, k_3^3}\right]\left(18\, k_2\, k_3^2\, \mathcal{P}_{\rm S}(k_1)\, \mathcal{P}_{\rm S}^{2/3}(k_2)\, \mathcal{P}_{\rm S}^{1/3}(k_3) - 9\, k_3^3\, \mathcal{P}_{\rm S}(k_1)\, \mathcal{P}_{\rm S}(k_2)\right.\\
&\qquad \left. -8\, k_1\, k_2\, k_3\, \mathcal{P}_{\rm S}^{2/3}(k_1)\, \mathcal{P}_{\rm S}^{2/3}(k_2)\, \mathcal{P}_{\rm S}^{2/3}(k_3) + \text{five permutations}\right).
\end{aligned}$$

The basis $(f_{\rm NL}^{\rm loc}, f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth})$ for the three-point function is considered to be large enough to encompass a range of interesting models.

[31] C. L. Bennett *et al.*, arXiv:1212.5225v1 [astro-ph.CO].

Illustration of the template bispectra



An illustration of the three template basis bispectra, viz. the local (top left), the equilateral (bottom) and the orthogonal (top right) forms for a generic triangular configuration of the wavevectors[32].

[32] E. Komatsu, Class. Quantum Grav. **27**, 124010 (2010).

Constraints on $f_{\rm NL}$

The constraints on the non-Gaussianity parameters from the recent Planck data are as follows[33]:

$$\begin{aligned} f_{\rm NL}^{\rm loc} &= 2.7 \pm 5.8, \\ f_{\rm NL}^{\rm eq} &= -42 \pm 75, \\ f_{\rm NL}^{\rm orth} &= -25 \pm 39. \end{aligned}$$

It should be stressed here that these are constraints are on the primordial values.

Also, the constraints on each of the $f_{\rm NL}$ parameters have been arrived at assuming that the other two parameters are zero.

[33] P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

Post-inflationary dynamics and non-linearities

- Post-inflationary dynamics, such as the curvaton and the modulated reheating scenarios can also lead to non-Gaussianities[34]. The strong constraints on $f_{\rm NL}^{\rm loc}$ from Planck suggests that the primordial non-Gaussianities are unlikely to have been generated post-inflation.
- Also, non-linear evolution, leading to and immediately after the epoch of decoupling, have been to shown to result in non-Gaussianities at the level of $\mathcal{O}(f_{\rm NL}) \sim 1-5$[35].

Clearly, these contributions need to be understood satisfactorily before the observational limits can be used to arrive at constraints on inflationary models.

[34] See, for instance, D. Langlois and T. Takahashi, arXiv:1301.3319v1 [astro-ph.CO].

[35] C. Pitrou, J.-P. Uzan and F. Bernardeau, JCAP **1007**, 003 (2010);
S.-C. Su, E. A. Lim and E. P. S. Shellard, arXiv:1212.6968v1 [astro-ph.CO].

Punctuated inflation

Punctuated inflation is a scenario wherein a brief period of rapid roll inflation or even a departure from inflation is sandwiched between two epochs of slow roll inflation[36].

[36] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010).

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Such a scenario can be achieved in inflaton potentials such as[37]

$$V(\phi) = (m^2/2)\ \phi^2 - \left(\sqrt{2\,\lambda\,(n-1)}\,m/n\right)\ \phi^n + (\lambda/4)\ \phi^{2(n-1)},$$

where $n > 2$ is an integer. This potential contains a point of inflection located at

$$\phi_0 = \left[\frac{2\,m^2}{(n-1)\,\lambda}\right]^{\frac{1}{2\,(n-2)}},$$

and it is the presence of this inflection point that admits punctuated inflation.

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These scenarios can lead to a sharp drop in power on large scales and result in an improved fit to the data at the low multipoles.

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[36] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010).

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Inflaton potentials with a step

Given a potential $V(\phi)$, one can introduce the step in the following fashion[38]:

$$V_{\rm step}(\phi) = V(\phi)\, \left[1 + \alpha\, \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right],$$

where, evidently, α, ϕ_0 and $\Delta\phi$ denote the height, the location, and the width of the step, respectively.

[38] J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64**, 123514 (2001).

[39] L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, Phys. Rev. D **74**, 083509 (2006);
M. J. Mortonson, C. Dvorkin, H. V. Peiris and W. Hu, Phys. Rev. D **79**, 103519 (2009);
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Such a step in potentials $V(\phi)$ which otherwise only result in slow roll lead to oscillatory features in the scalar power spectrum that provide a better fit to the outliers near $\ell = 20$ and $\ell = 44$[39].

[38] J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64**, 123514 (2001).

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Oscillating inflation potentials

Potentials containing oscillatory terms are encountered in string theory. A popular example is the axion monodromy model, which is described by the potential[40]

$$V(\phi) = \lambda \left[\phi + \alpha \cos\left(\frac{\phi}{\beta} + \delta\right)\right].$$

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Interestingly, such a potential leads to non-local features – *i.e.* a certain characteristic and repeated pattern that extends over a wide range of scales – in the primordial spectrum which result in an improved fit to the data[41].

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Another potential that has been considered in this context is the conventional quadratic potential which is superposed by sinusoidal oscillations as follows[42]:

$$V(\phi) = \frac{1}{2}\, m^2\, \phi^2 \left[1 + \alpha\, \sin\left(\frac{\phi}{\beta} + \delta\right)\right].$$

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The various models of interest

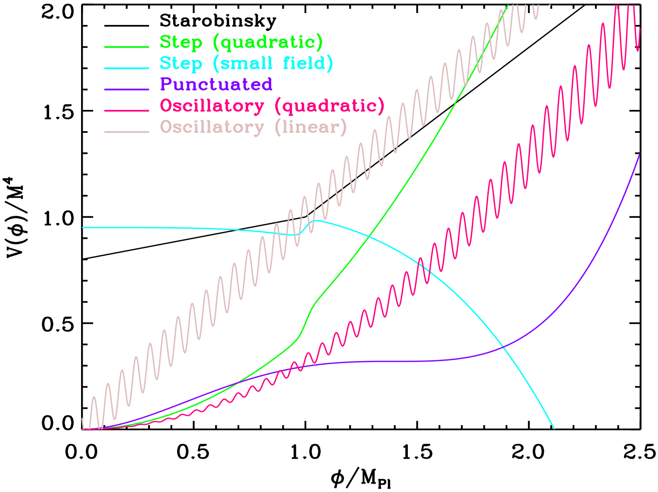
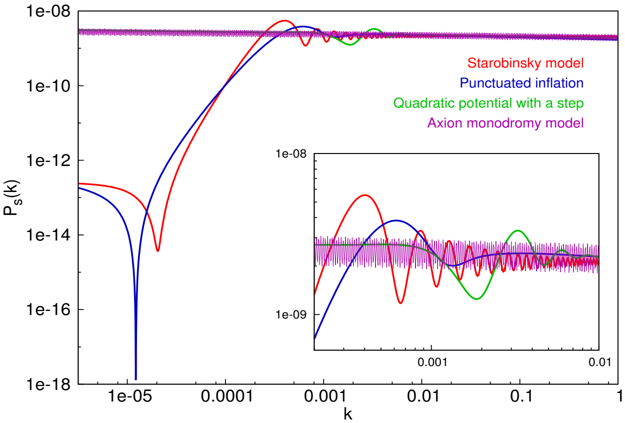


Illustration of the potentials in the different inflationary models of our interest.

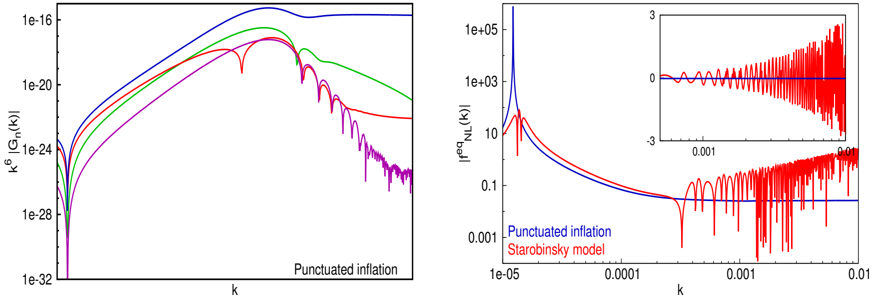
Inflationary models leading to features



The scalar power spectra in the different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum[43].

[43] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009); D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v2 [astro-ph.CO].

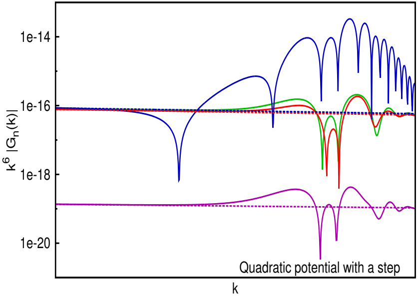
$f_{\rm NL}^{\rm eq}$ in punctuated inflation

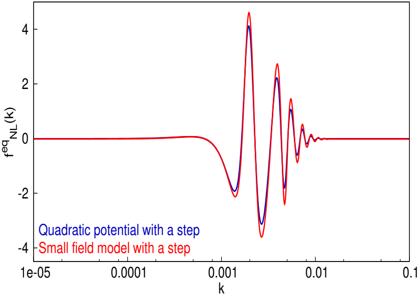


The contributions to the bispectrum due to the various terms (on the left), and the absolute value of $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right), in the punctuated inflationary scenario[44]. The absolute value of $f_{\rm NL}^{\rm eq}$ in a Starobinsky model that closely resembles the power spectrum in punctuated inflation has also been displayed. The large difference in $f_{\rm NL}^{\rm eq}$ between punctuated inflation and the Starobinsky model can be attributed to the considerable difference in the background dynamics.

[44] D. K. Hazra, L. Sriramkumar and J. Martin, arXiv:1201.0926v1 [astro-ph.CO].

$f_{\rm NL}^{\rm eq}$ in models with a step





The contributions due to the various terms (on the left) and $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right) when a step has been introduced in the popular chaotic inflationary model involving the quadratic potential[45]. The $f_{\rm NL}^{\rm eq}$ that arises in a small field model with a step has also been illustrated[46]. The background dynamics in these two models are very similar, and hence they lead to almost the same $f_{\rm NL}^{\rm eq}$.

[45] X. Chen, R. Easther and E. A. Lim, JCAP **0706**, 023 (2007); JCAP **0804**, 010 (2008); P. Adshead, W. Hu, C. Dvorkin and H. V. Peiris, Phys. Rev. D **84**, 043519 (2011); P. Adshead, C. Dvorkin, W. Hu and E. A. Lim, Phys. Rev. D **85**, 023531 (2012).

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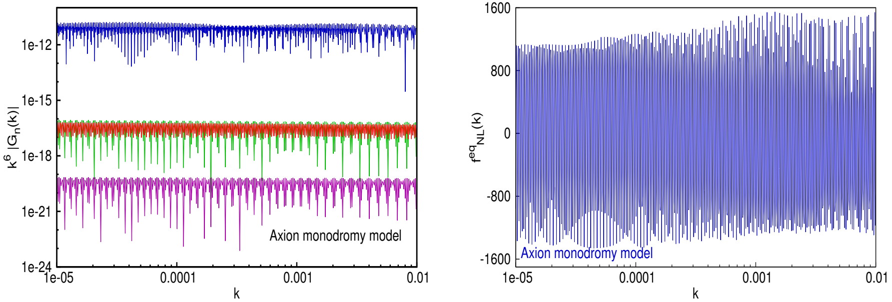
$f_{\rm NL}^{\rm eq}$ in the axion monodromy model



The contributions due to the various terms (on the left) and $f_{\rm NL}^{\rm eq}$ due to the dominant contribution (on the right) in the axion monodromy model[47]. The modulations in the potential give rise to a certain resonant behavior, leading to a large $f_{\rm NL}^{\rm eq}$[48].

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[48] S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010); R. Flauger and E. Pajer, JCAP **1101**, 017 (2011).

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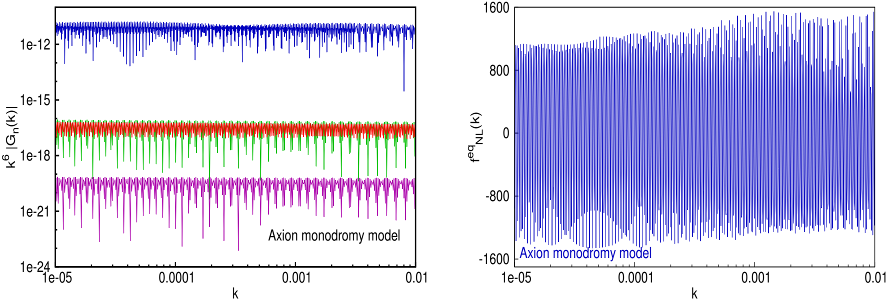

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Outlook

- The strong constraints on the non-Gaussianity parameter $f_{\rm NL}$ from Planck suggests that inflationary and post-inflationary scenarios that lead to rather large non-Gaussianities are very likely to be ruled out by the data.

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- The possibility of such features can provide a strong handle on constraining inflationary models.

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- The possibility of such features can provide a strong handle on constraining inflationary models.
- Else, one may need to carry out a systematic search involving the scalar and the tensor power spectra[50], the scalar and the tensor bispectra and the cross correlations.

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Thank you for your attention