Decoding the cosmos: Tracing origins with rays and ripples

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Colloquium

Department of Physics, Ashoka University, Sonipat

March 5, 2025

Plan of the talk

- Standard model of cosmology
- Inflationary scenario
- 3 Constraints on inflation from the Planck CMB data
- 4 GWs provide a new window to the universe
- 5 Generation of GWs in the early universe
- 6 Observations by the PTAs and the stochastic GW background
- Outlook

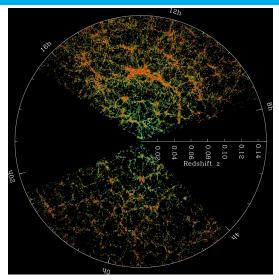


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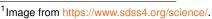
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Distribution of galaxies in the universe

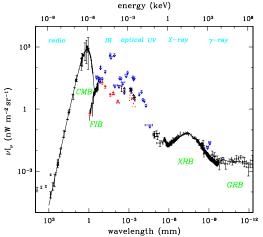


Distribution of galaxies as observed by the Sloan Digital Sky Survey¹.





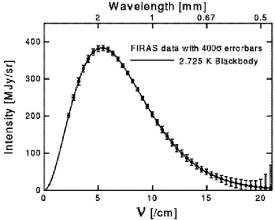
Spectrum of radiation in the universe



The spectrum of the cosmological background radiation has been plotted as a function of wavelength². Note that the cosmic microwave background (CMB) contributes the most to the background radiation.

²Figure from D. Scott, arXiv:astro-ph/9912038.

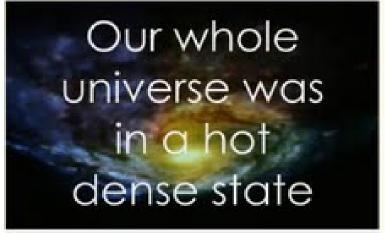
Spectrum of the CMB



The spectrum of the CMB as measured by the COBE satellite³. It is such a perfect Planck spectrum (corresponding to a temperature of 2.725° K) that it is unlikely to be bettered in the laboratory. The error bars in the graph above have been amplified 400 times so that they can be seen!

³Image from http://www.astro.ucla.edu/~wright/cosmo_01.htm.

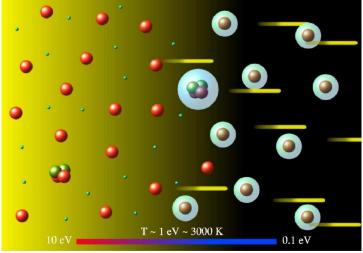
Big bang model seems popular!



The current view of the universe, encapsulated in the hot big bang model, seems popular. The above image is a screen grab from the theme song of the recent American sitcom 'The Big Bang Theory'⁴!

⁴See http://www.cbs.com/shows/big_bang_theory/.

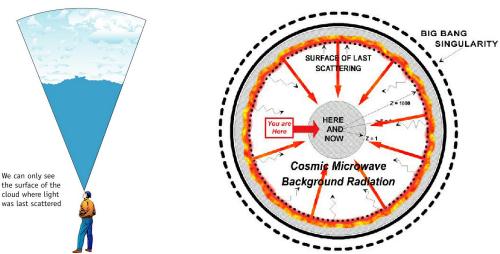
Decoupling of matter and radiation⁵



Matter and radiation cease to interact at a temperature of about $T \simeq 3000^\circ$ K, which corresponds to a redshift of about $z \simeq 1000$.

⁵Image from W. H. Kinney, arXiv:astro-ph/0301448v2.

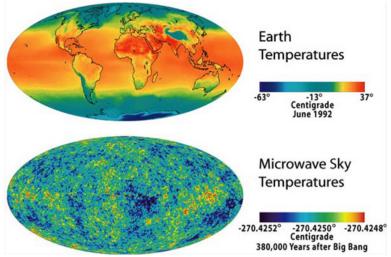
Surface of last scattering and the free streaming of CMB photons



CMB photons stream to us freely from the surface of last scattering when radiation decoupled from matter⁶.

⁶Image from http://planck.caltech.edu/epo/epo-cmbDiscovery4.html.

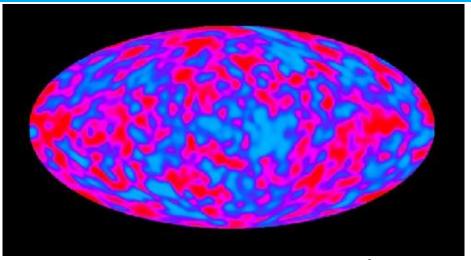
Projecting the surface of last scattering



As the surface of the Earth is often illustrated, the temperature of the CMB on the surface of last scattering can be projected on to a plane using the Mollweide projection⁷.

⁷Image from http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/planckcmb.html.

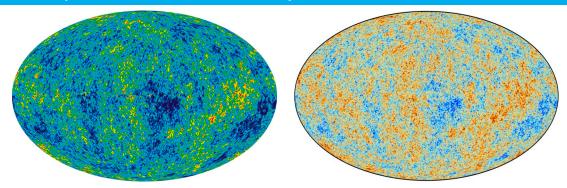
Anisotropies in the CMB



The fluctuations in the temperature of the CMB as seen by COBE⁸. The CMB turns out to be isotropic to one part in 10^5 .

⁸Image from http://aether.lbl.gov/www/projects/cobe/COBE Home/DMR Images.html.

Anisotropies in the CMB as seen by WMAP and Planck



Left: All-sky map of the anisotropies in the CMB created from nine years of Wilkinson Microwave Anisotropy Probe (WMAP) data⁹.

Right: CMB intensity map derived from the joint analysis of Planck, WMAP, and $408\,\mathrm{MHz}$ observations ¹⁰. The above images show temperature variations (as color differences) of the order of $200\,\mu\mathrm{K}$.



⁹Image from http://wmap.gsfc.nasa.gov/media/121238/index.html.

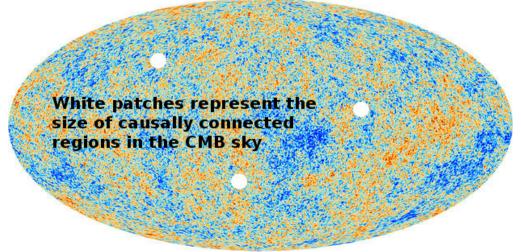
¹⁰P. A. R. Ade *et al.*, arXiv:1502.01582 [astro-ph.CO].

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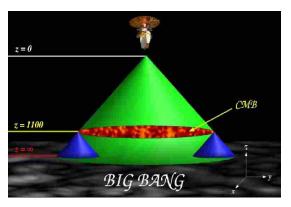


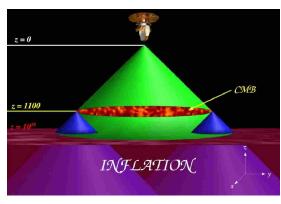
Horizon problem



The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the surface of last scattering, which subtends an angle of about 1° today, count have interacted before decoupling.

Resolution of the horizon problem in the inflationary scenario



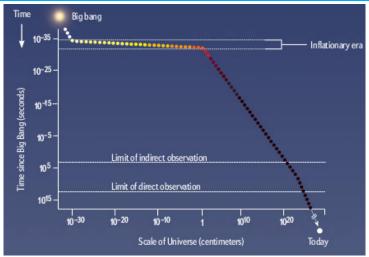


Another illustration of the horizon problem (on the left), and an illustration of its resolution (on the right) through an early and sufficiently long epoch of inflation¹¹.



¹¹Images from W. Kinney, astro-ph/0301448.

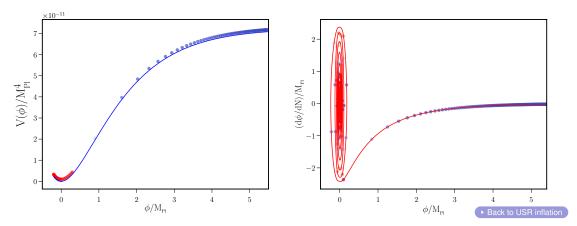
Time and duration of inflation



Inflation—a brief period of accelerated expansion—is expected to have taken place during the very early stages of the universe¹².

¹²Image from P. J. Steinhardt, Sci. Am. **304**, 18 (2011).

Inflationary attractor



Evolution of the scalar field in the popular Starobinsky model, which leads to slow roll inflation, is indicated (as circles, in blue and red) at regular intervals of time (on the left). Illustration of the behavior of the scalar field in phase space (on the right)¹³.

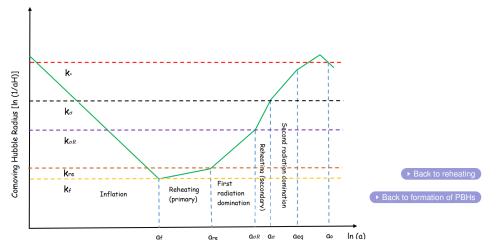
¹³ Figure from H. V. Ragavendra, *Observational imprints of non-trivial inflationary dynamics over large and small, scales*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2022).

Origin of the primordial perturbations

- ◆ The vacuum fluctuations in the scalar fields that drive inflation lead to perturbations in the metric. The perturbations in the metric and matter are related through the Einstein's equations.
- ◆ The scalar perturbations leave the largest imprints on the CMB, and are primarily responsible for the inhomogeneities in the distribution of matter in the universe.
- Whereas, the tensor perturbations, i.e. the gravitational waves (GWs), can be generated even in the absence of sources.



Behavior of the comoving wave number and Hubble radius



Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_{\rm H}/a = (a\,H)^{-1}$ (in green) across different epochs¹⁴.

¹⁴Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Describing the primordial perturbations

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra¹⁵:

$$\mathcal{P}_{\scriptscriptstyle \mathrm{S}}(k) = A_{\scriptscriptstyle \mathrm{S}} \, \left(rac{k}{k_*}
ight)^{n_{\scriptscriptstyle \mathrm{S}}-1}, \qquad \mathcal{P}_{\scriptscriptstyle \mathrm{T}}(k) = A_{\scriptscriptstyle \mathrm{T}} \, \left(rac{k}{k_*}
ight)^{n_{\scriptscriptstyle \mathrm{T}}},$$

where $A_{\rm S}$ and $A_{\rm T}$ denote the scalar and tensor amplitudes, k_{*} represents the so-called pivot scale at which the amplitudes are quoted, while the spectral indices $n_{\rm S}$ and $n_{\rm T}$ are assumed to be constant.

The tensor-to-scalar ratio r is defined as

$$r(k) = rac{\mathcal{P}_{_{\mathrm{T}}}(k)}{\mathcal{P}_{_{\mathrm{S}}}(k)}.$$



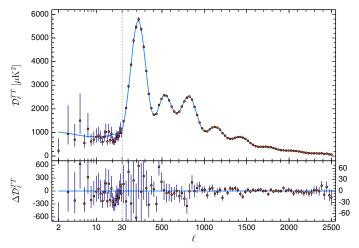
¹⁵See, for instance, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

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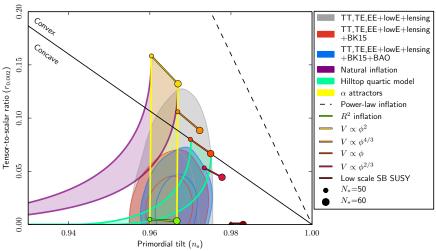
CMB angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit Λ CDM model with a power law primordial spectrum (solid blue curve)¹⁶

¹⁶Planck Collaboration (N. Aghanim et al.), Astron. Astrophys. **641**, A6 (2020).

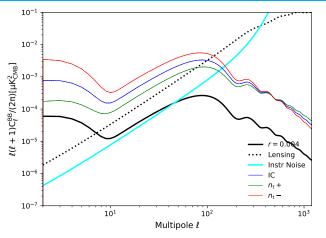
Performance of inflationary models in the $n_{\rm s}$ -r plane



Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models¹⁷.

¹⁷Planck Collaboration (Y. Akrami et al.), Astron. Astrophys. 641, A10 (2020).

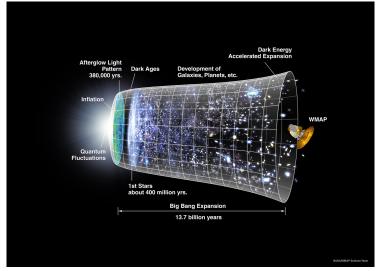
Prospects of observing the imprints of the tensor perturbations



The B-mode angular power spectra of the CMB resulting from the primordial tensor perturbations for three models with $r_{0.05} = 0.05$ have been plotted, along with the CMB lensing signal and the instrumental noise of a LiteBIRD-like configuration¹⁸.

¹⁸D. Paoletti, F. Finelli, J. Valiviita and M. Hazumi, Phys. Rev. D **106**, 083528 (2022).

Timeline of the universe



► Observations of GWs

A pictorial timeline of the universe¹⁹.



¹⁹See http://wmap.gsfc.nasa.gov/media/060915/060915_CMB_Timeline150.jpg.

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Laser Interferometer Gravitational-Wave Observatory (LIGO)



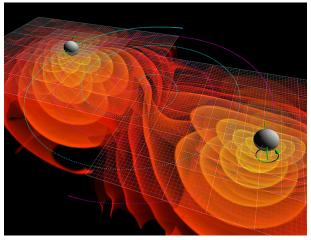


Views of LIGO at Hanford (on the left) and at Livingston (on the right). These observatories are essentially Michelson-Morley interferometers with rather long arms (of length about 4 km) that are extremely sensitive to the smallest disturbances of the mirrors²⁰.



²⁰Images from https://www.advancedligo.mit.edu/summary.html.

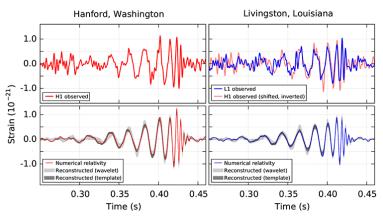
GWs from inspiralling black holes (BHs)



Numerical simulations of the GWs emitted by the coalescence of two black holes. The orange contours represent the amplitude of the GWs and the blue lines represent the orbits of the black holes²¹.

²¹Image from E. Berti, Physics 9, 17 (2016).

First observation of the merger of binary BHs



On September 14, 2015, similar signals were observed in both of LIGO's interferometers. The top panels show the measured signal in the Hanford (top left) and Livingston (top right) detectors. The bottom panels show the expected signal produced by the merger of two BHs, based on numerical simulations²².

²²Figure from B. P. Abbott et al., Phys. Rev. Lett. **116**, 061102 (2016).

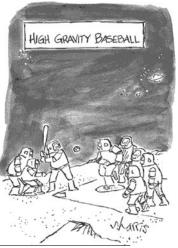
Some essential properties of GWs

- The GWs are small disturbances in a given spacetime (very much like ripples in water), which travel at the speed of light.
- They satisfy the wave equation in the given background spacetime.
- ◆ The GWs are transverse in nature and are characterized by two degrees of polarization²³.



Sources of GWs²⁴

In order to generate GWs of detectable amplitude, the gravitational fields of the sources ought to be very strong.



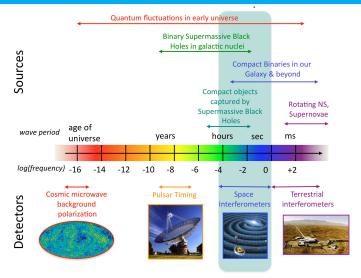
Strong sources of GWs include

- Rotating neutron stars
- Exploding supernovae
- Coalescing binary neutron stars or black holes
- Supermassive binary black holes at the centre of galaxies
- Fluctuations in the early universe



²⁴Cartoon from http://www.sciencecartoonsplus.com/gallery/physics/galphys2b.php.

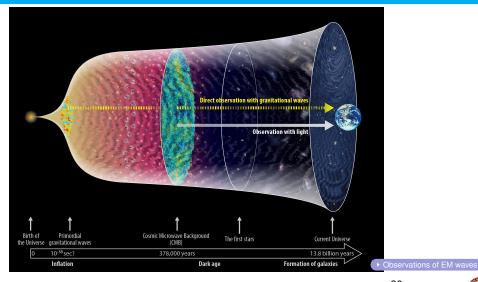
Spectral range of GWs



Different sources of GWs and corresponding detectors²⁵.



Probing the primordial universe through GWs



GWs provide a unique window to probe the primordial universe²⁶.

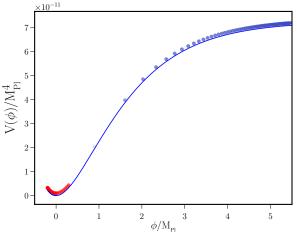


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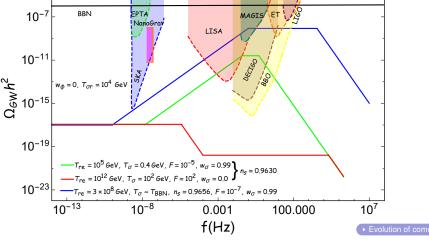


Evolution of the scalar field in an inflationary potential



The evolution of the scalar field in the so-called Starobinsky model has been indicated (as circles, in blue and red) at regular intervals of time. Inflation is terminated as the field approaches the bottom of the potential (near the light blue dot). Thereafter, the field oscillates at the bottom of the potential (indicated by the red dots).

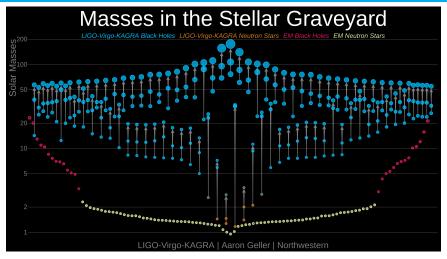
Effects on $\Omega_{\text{GW}}(f)$ due to late time entropy production



The dimensionless spectral energy density of primary GWs observed today $\Omega_{\rm GW}(f)$ has been plotted in a scenario involving late time production of entropy²⁷.

²⁷Md. R. Hague, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

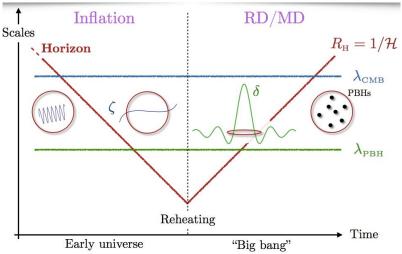
Coalescence of compact binaries observed by LIGO



The third GW Transient Catalog of mergers involving black holes and neutron stars observed by the LIGO-Virgo-KAGRA collaboration²⁸.

²⁸Image from https://www.ligo.caltech.edu/LA/image/ligo20211107a.

Formation of BHs in the early universe

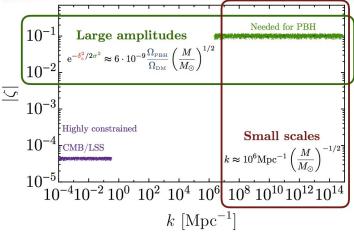


BHs can form when perturbations with significant amplitudes reenter the Hubble radius during the radiation dominated epoch²⁹.

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²⁹ Figure from G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

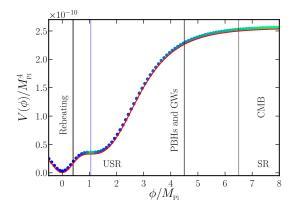
Amplitude required to form significant number of PBHs

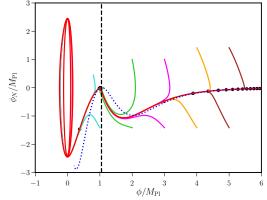


In order to form significant number of primordial black holes (PBHs), the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity³⁰.

³⁰Figure credit G. Franciolini.

Single-field models admitting ultra slow roll inflation





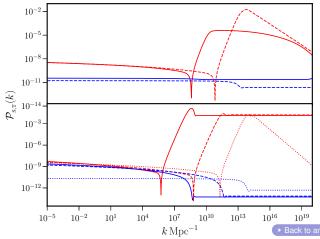
▶ Inflationary attractor

Potentials which contain a point of inflection lead to ultra slow roll (USR) inflation³¹.



<sup>See, for example, C. Germani and T. Prokopec, Phys. Dark Univ. 18, 6 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP 01, 037 (2019).
Figures credits, H. V. Ragavendra and S. Maity.</sup>

Power spectra in models permitting USR inflation

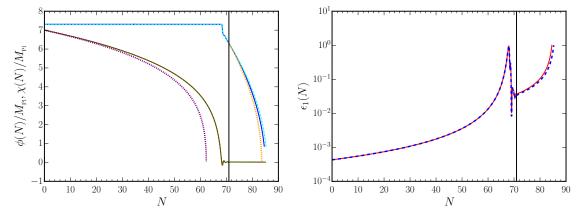


Scalar (in red) and the tensor (in blue) power spectra arising in different single-field models that permit a period of USR inflation³².



³²H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D 103, 083510 (2021); Also see H. V. Ragavendra and L. Sriramkumar, Galaxies 11, 34 (2023).

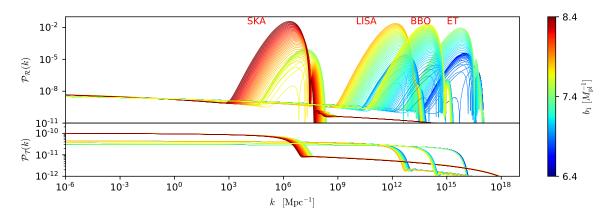
Non-trivial inflationary dynamics in a two-field model



Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the two field model of our interest³³. Note that there arises a turn in the field space around N=70, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

³³M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

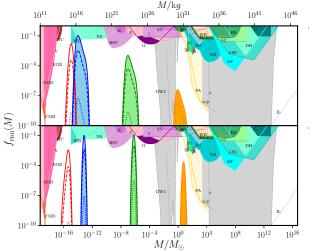
Enhanced power on small scales in the two-field model



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of a particular parameter³⁴.

³⁴M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

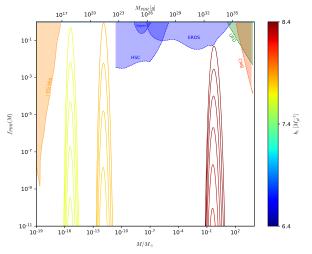
$f_{\scriptscriptstyle \mathrm{PBH}}(M)$ in models of ultra slow roll inflation



The fraction of PBHs contributing to the dark matter density today $f_{PBH}(M)$ has been plotted for different models and scenarios of interest³⁵.

³⁵H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

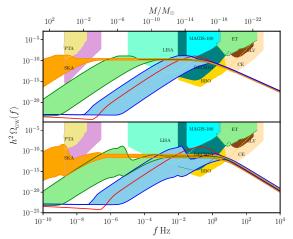
$f_{\scriptscriptstyle ext{PBH}}(M)$ in the two-field model



The fraction of PBHs contributing to the dark matter density today $f_{PBH}(M)$ in the two-field model of our interest³⁶.

³⁶M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

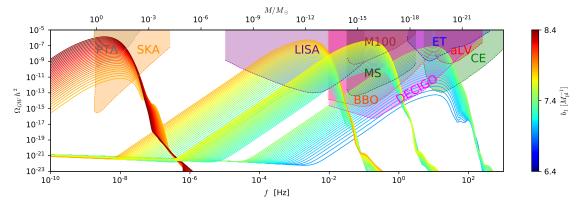
$\Omega_{\scriptscriptstyle \mathrm{GW}}(f)$ in ultra slow roll inflation



The dimensionless density parameter $\Omega_{\rm GW}$ arising in the models and reconstructed scenarios leading to an epoch of ultra slow roll inflation has been plotted as a function of the frequency f^{37} .

³⁷H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$\Omega_{\scriptscriptstyle \mathrm{GW}}(f)$ in the two-field model



The dimensionless density parameter $\Omega_{\rm GW}(f)$ arising in the two-field model has been plotted for a set of initial conditions for the background fields as well as a range of values of a parameter describing the model³⁸.



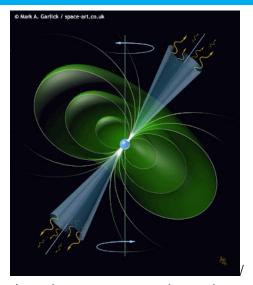
³⁸M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

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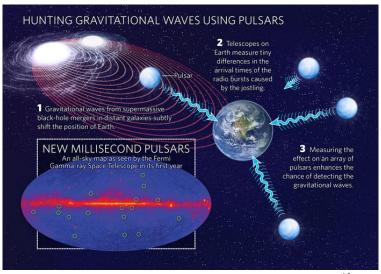
Pulsars



Pulsars are dense and rotating neutron stars that emit regular beams of light³⁹.



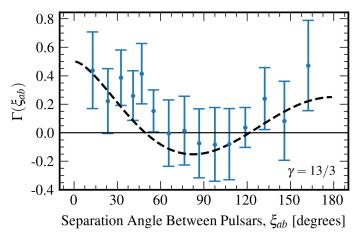
Pulsar timing arrays (PTAs)



The PTAs monitor an array of millisecond pulsars⁴⁰.



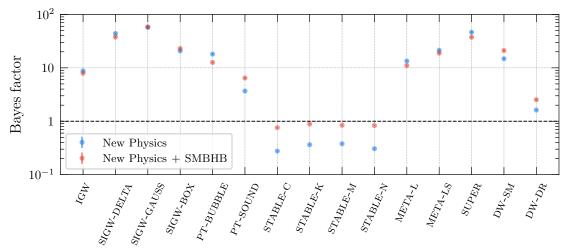
Hellings-Downs curve



The inter-pulsar correlations measured from 2,211 distinct pairings in the 67-pulsar array of the NANOGrav 15-year data. The dashed black line shows the Hellings-Downs correlation pattern⁴¹.

⁴¹NANOGrav Collaboration (G. Agazie et al.), Astrophys. J. Lett. **951**, 1 (2023).

Stochastic GW background observed by pulsar timing arrays (PTAs)



The Bayesian evidence for a variety of astrophysical and cosmological sources for the stochastic GW background suggested by the observations of the PTAs⁴².

⁴²NANOGrav Collaboration (G. Agazie et al.), Astrophys. J. Lett. **951**, 1 (2023).

Shape of the inflationary scalar power spectrum

We assume that the inflationary scalar power spectrum is given by⁴³

$$\mathcal{P}_{_{\mathrm{S}}}(k) = A_{_{\mathrm{S}}} \left(\frac{k}{k_{*}}\right)^{n_{_{\mathrm{S}}}-1} + A_{0} \left\{ egin{array}{l} \left(rac{k}{k_{\mathrm{peak}}}
ight)^{4} & k \leq k_{\mathrm{peak}}, \\ \left(rac{k}{k_{\mathrm{peak}}}
ight)^{n_{0}} & k \geq k_{\mathrm{peak}}, \end{array}
ight.$$

where $A_{\rm S}$ and $n_{\rm S}$ are the amplitude and spectral index of the power spectrum at the CMB pivot scale of $k_* = 0.05\,{\rm Mpc}^{-1}$.

We set the reheating temperature to the rather low value of $T_{\rm re} = 50 \, {\rm MeV}$.

We shall assume that the threshold value of the density contrast for the formation of PBHs is given by⁴⁴:

$$\delta_{\rm c}^{\rm an} = \frac{3(1+w_{\rm re})}{5+3w_{\rm re}} \sin^2\left(\frac{\pi\sqrt{w_{\rm re}}}{1+3w_{\rm re}}\right).$$

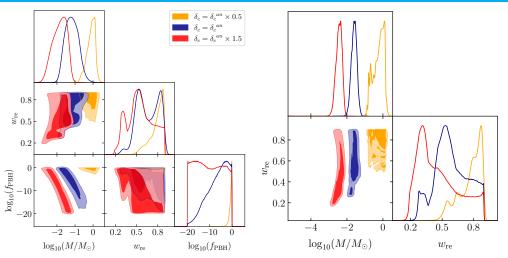


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⁴³For other forms of spectra, see G. Domènech, S. Pi, A. Wang and J. Wang, arXiv:2402.18965 [astro-ph.CO].

⁴⁴In this context, see T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. D **88**, 084051 (2013).

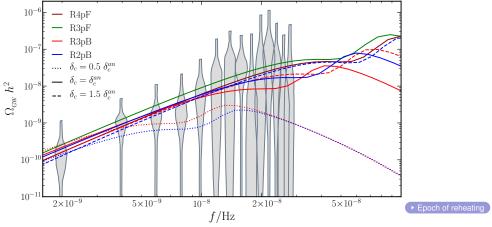
Constraints on the epoch of reheating



Constraints on the parameters in the models R3pB (on the left) and R2pB (on the right), arrived at upon comparison with the NANOGrav 15-year data⁴⁵.

⁴⁵S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

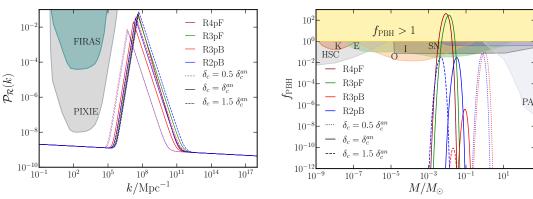
Generation of secondary GWs during the epoch of reheating



The dimensionless spectral energy density of the secondary GWs today $\Omega_{\rm GW}(f)$ is plotted for a given reheating temperature and the best-fit values of the parameters in the different models⁴⁶.

⁴⁶S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

Power spectra and the extent of PBHs formed



Scalar power spectra (on the left) and the extent of PBHs formed (on the right). We have assumed a specific reheating temperature and have plotted the fraction of PBHs that constitute the dark matter density today, viz. $f_{\rm PBH}(M)$, for the best-fit values of the parameters in the different models⁴⁷.

⁴⁷S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

Bayesian evidence

Model X	Model Y	$\mathrm{BF}_{Y,X}$		
		$\delta_{\rm c} = 0.5 \delta_{\rm c}^{\rm an}$	$\delta_{\mathrm{c}} = \delta_{\mathrm{c}}^{\mathrm{an}}$	$\delta_{\rm c} = 1.5 \delta_{\rm c}^{\rm an}$
SMBHB	R2pB	$1.7 \pm .06$	260.04 ± 19.21	350.61 ± 27.36

The Bayesian factors $BF_{Y,X}$ for the model R2pB that invokes primordial physics as the source of the stochastic GW background observed by the NANOGrav 15-year data, when compared to the astrophysical scenario that involves mergers of supermassive black hole binaries (SMBHB).

Bayesian factors $BF_{Y,X}$ that far exceed unity indicate strong evidence for the model Y with respect to the model X.

Clearly, when $\delta_c = \delta_c^{an}$ and $\delta_c = 1.5 \, \delta_c^{an}$, the NANOGrav 15-year data strongly favors the model R2pB when compared to the SMBHB model.

Plan of the talk

- Standard model of cosmology
- Inflationary scenario
- 3 Constraints on inflation from the Planck CMB data
- 4 GWs provide a new window to the universe
- Generation of GWs in the early universe
- 6 Observations by the PTAs and the stochastic GW background
- Outlook



Outlook

- ◆ The increasingly precise observations of the CMB by future missions such as Lite-BIRD (Light satellite for the studies of B-mode polarization and Inflation from cosmic background Radiation Detection), Primordial Inflation Explorer (PIXIE) and Exploring Cosmic History and Origin (ECHO, a proposed Indian effort) can be expected to help us improve the current constraints on the primordial correlations.
- ◆ The observations by LIGO are a culmination of almost fifty years of effort to detect GWs. They have opened up a new window to observe the universe.
- The observations by the PTAs and their possible implications for the stochastic GW background offer a wonderful opportunity to understand the physics operating in the early universe.
- Over the coming decades, GW observatories such as the Laser Interferometer Space Antenna, Einstein Telescope and Cosmic Explorer, can be expected to provide us with an unhindered view of the primordial universe.



This talk was based on...

- M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, Generating PBHs and small-scale GWs in two-field models of inflation, JCAP 08, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Primordial black holes and secondary gravitational waves from ultra slow roll and punctuated inflation*, Phys. Rev. D 103, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Decoding the phases of early and late time reheating through imprints on primordial gravitational waves, Phys. Rev. D 104, 063513 (2021) [arXiv:2105.09242 [astro-ph.CO]].
- ✦ H. V. Ragavendra and L. Sriramkumar, Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities, Galaxies 11, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].
- ◆ S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, Constraining the history of reheating with the NANOGrav 15-year data, JCAP 01, 118 (2025) [arXiv:2403.16963 [astro-ph.CO]].

Collaborators I



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Thank you for your attention