Did the universe bang or bounce?

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Plan of the talk

This talk is based on. . .

- D. Chowdhury, V. Sreenath and L. Sriramkumar, *The tensor bispectrum in a matter bounce*, JCAP **1511**, 002 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- R. N. Raveendran, D. Chowdhury and L. Sriramkumar, *Viable tensor-to-scalar ratio in a symmetric matter bounce*, arXiv:1703.10061v1 [gr-qc].

A few words on the conventions and notations

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$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\boldsymbol{x}^2 = a^2(\eta)\,\left(-\mathrm{d}\eta^2 + \mathrm{d}\boldsymbol{x}^2\right),$$

  where t is the cosmic time, $a(t)$ is the scale factor and $\eta = \int \mathrm{d}t/a(t)$ denotes the conformal time coordinate.

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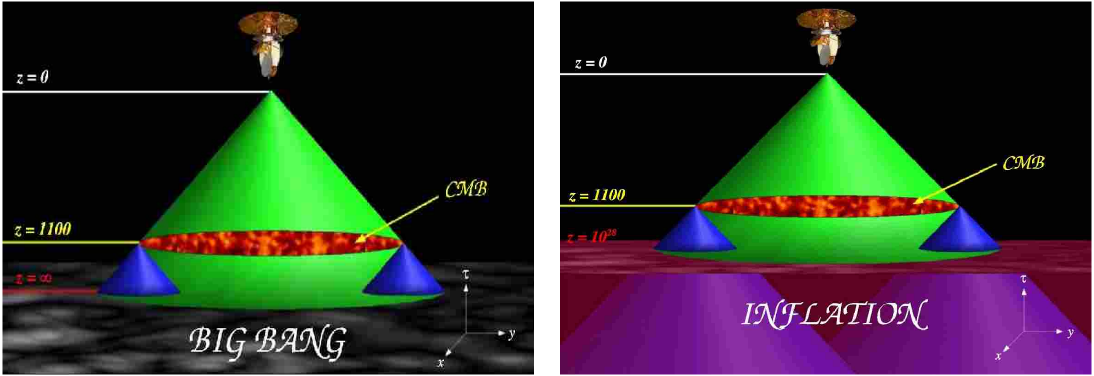
- We shall denote differentiation with respect to the cosmic and the conformal times t and η by an overdot and an overprime, respectively.
- Moreover, N shall denote the number of e-folds and, as usual, $H = \dot{a}/a$ shall denote the Hubble parameter associated with the Friedmann universe.

Plan of the talk

The resolution of the horizon problem in inflation



Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

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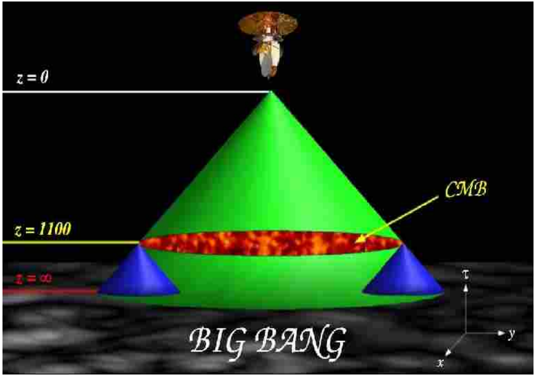
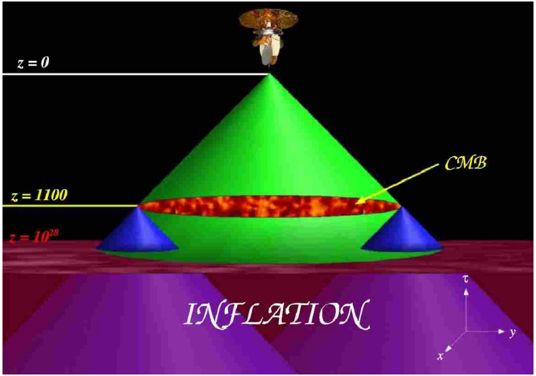


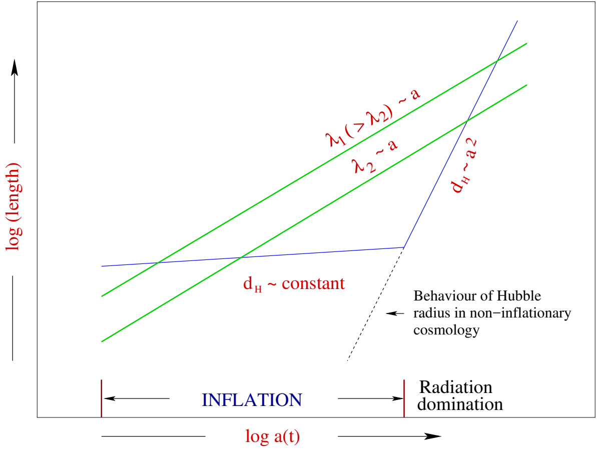


Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[1].

[1] Images from W. Kinney, astro-ph/0301448.

Bringing the modes inside the Hubble radius



The behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[2].

Back to bounce

[2] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Driving inflation with scalar fields

If we require that $\lambda_{\rm P} < d_{\rm H}$ at a sufficiently early time, then we need to have an epoch wherein $\lambda_{\rm P}$ decreases faster than the Hubble scale *as we go back in time*, i.e. a regime during which

$$-\frac{{\rm d}}{{\rm d}\,t}\left(\frac{\lambda_{\rm P}}{d_{\rm H}}\right) < 0 \quad \Rightarrow \quad \ddot{a} > 0.$$

[3]See, for instance, B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).

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From the Friedmann equations, we then require that, during this epoch,

$$(\rho + 3\,p) < 0.$$

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If we require that $\lambda_{\mathrm{P}} < d_{\mathrm{H}}$ at a sufficiently early time, then we need to have an epoch wherein λ_{P} decreases faster than the Hubble scale *as we go back in time*, i.e. a regime during which

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In the case of canonical scalar fields, this condition simplifies to

$$\dot{\phi}^2 < V(\phi).$$

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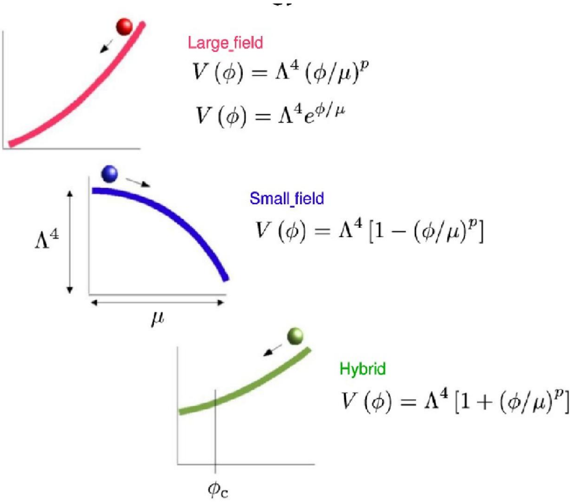
$$\dot{\phi}^2 < V(\phi).$$

This condition can be achieved if the scalar field ϕ is initially displaced from a minima of the potential, and inflation will end when the field approaches a minima with zero or negligible potential energy[3].

[3]See, for instance, B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).

A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[4]. Often, these potentials are classified as small field, large field and hybrid models.

[4] Image from W. Kinney, astro-ph/0301448.

Proliferation of inflationary models

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflation
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models[5]. Actually, it may not even be possible to rule out some of these models!

[5] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Plan of the talk

The character of the perturbations

In a Friedmann universe, the perturbations in the metric and the matter can be classified according to their behavior with respect to local rotation of the spatial coordinates on hypersurfaces of constant time as follows[6]:

- Scalar perturbations – Density and pressure perturbations
- Vector perturbations – Rotational velocity fields
- Tensor perturbations – Gravitational waves

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It is the fluctuations in the inflaton field ϕ that act as the seeds for the scalar perturbations that are primarily responsible for the anisotropies in the CMB and, eventually, the present day inhomogeneities.

[6] See, for instance, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

The quadratic action governing the perturbations

One can show that, at the quadratic order, the action governing the curvature perturbation $\mathcal{R}$ and the tensor perturbation γ_{ij} are given by[7]

$$\begin{aligned} \mathcal{S}_2[\mathcal{R}] &= \frac{1}{2}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; z^2\, \left[\mathcal{R}'^2 - (\partial\mathcal{R})^2\right], \\ \mathcal{S}_2[\gamma_{ij}] &= \frac{M_{_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; a^2\, \left[\gamma_{ij}'^{\,2} - (\partial\gamma_{ij})^2\right]. \end{aligned}$$

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These actions lead to the following equations of motion governing the scalar and tensor modes, say, f_k and h_k:

$$\begin{aligned} f_k'' + 2\,\frac{z'}{z}\, f_k' + k^2\, f_k &= 0, \\ h_k'' + 2\,\frac{a'}{a}\, h_k' + k^2\, h_k &= 0, \end{aligned}$$

where $z = a\, M_{_{\mathrm{Pl}}}\,\sqrt{2\,\epsilon_1}$, with $\epsilon_1 = -\,\mathrm{d}\ln H/\mathrm{d}N$ being the first slow roll parameter.

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Quantization of the scalar and tensor perturbations

On quantization, the operators $\hat{\mathcal{R}}(\eta, \boldsymbol{x})$ and $\hat{\gamma}_{ij}(\eta, \boldsymbol{x})$ representing the scalar and the tensor perturbations can be expressed in terms of the corresponding Fourier modes f_k and h_k as[8]

$$\hat{\mathcal{R}}(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \left[\hat{a}_{\boldsymbol{k}}\, f_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}^{\dagger}_{\boldsymbol{k}}\, f^{*}_k(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

$$\hat{\gamma}_{ij}(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\gamma}^{\boldsymbol{k}}_{ij}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$= \sum_{s} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \left[\hat{b}^{s}_{\boldsymbol{k}}\, \varepsilon^{s}_{ij}(\boldsymbol{k})\, h_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^{s\dagger}_{\boldsymbol{k}}\, \varepsilon^{s*}_{ij}(\boldsymbol{k})\, h^{*}_k(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right].$$

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In these decompositions, the operators $(\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}})$ and $(\hat{b}^{s}_{\boldsymbol{k}}, \hat{b}^{s\dagger}_{\boldsymbol{k}})$ satisfy the standard commutation relations, while the quantity $\varepsilon^{s}_{ij}(\boldsymbol{k})$ represents the transverse and traceless polarization tensor describing the gravitational waves.

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The scalar and tensor power spectra

The dimensionless scalar and tensor power spectra $\mathcal{P}_{_{\rm S}}(k)$ and $\mathcal{P}_{_{\rm T}}(k)$ are defined in terms of the correlation functions of the Fourier modes $\hat{\mathcal{R}}_{\boldsymbol{k}}$ and $\hat{\gamma}_{mn}^{\boldsymbol{k}}$ as follows:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}(\eta)\, \hat{\mathcal{R}}_{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{2\,k^3}\, \mathcal{P}_{_{\rm S}}(k)\; \delta^{(3)}\left(\boldsymbol{k}+\boldsymbol{k}'\right),$$

$$\langle \hat{\gamma}_{m_1 n_1}^{\boldsymbol{k}}(\eta)\, \hat{\gamma}_{m_2 n_2}^{\boldsymbol{k}'}(\eta)\rangle = \frac{(2\,\pi)^2}{8\,k^3}\, \Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}}\, \mathcal{P}_{_{\rm T}}(k)\; \delta^{3}\left(\boldsymbol{k}+\boldsymbol{k}'\right),$$

where $\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}} = \sum_s\, \varepsilon_{m_1 n_1}^{s}(\boldsymbol{k})\, \varepsilon_{m_2 n_2}^{s*}(\boldsymbol{k})$.

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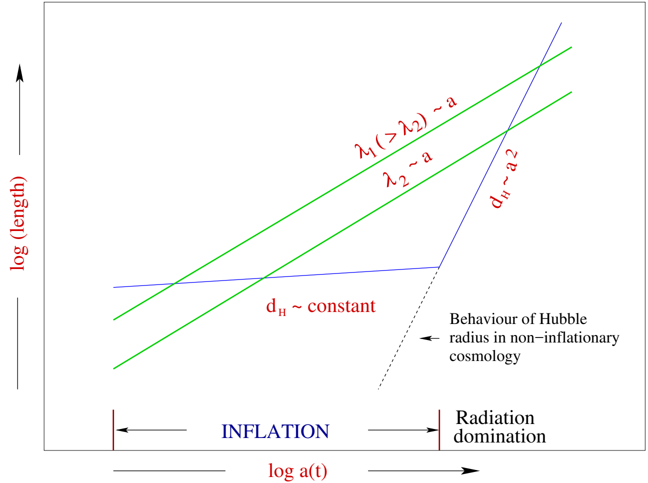
In the Bunch-Davies vacuum, say, $|0\rangle$, which is defined as $\hat{a}_{\boldsymbol{k}}|0\rangle = 0$ and $\hat{b}_{\boldsymbol{k}}^{s}|0\rangle = 0\ \forall\ \boldsymbol{k}$ and s, we can express the power spectra in terms of the quantities f_k and g_k as

$$\mathcal{P}_{_{\rm S}}(k) = \frac{k^3}{2\,\pi^2}\,|f_k|^2,\quad \mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,|h_k|^2.$$

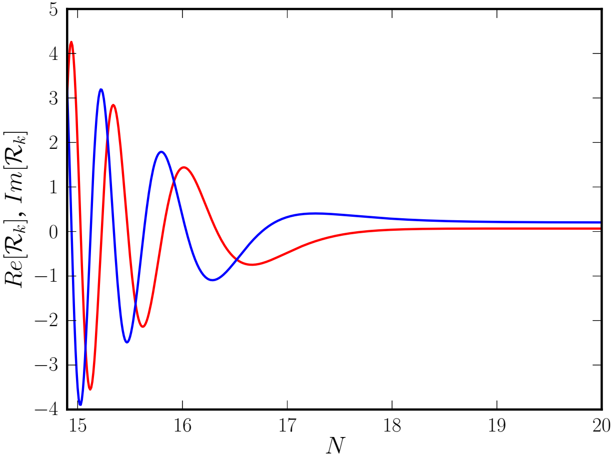
With the initial conditions imposed in the sub-Hubble domain, *viz.* when $k/(a\,H) \gg 1$, these spectra are to be evaluated on super-Hubble scales, *i.e.* as $k/(a\,H) \ll 1$.

From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the modes are well inside the Hubble radius (*viz.* when $k/(a\,H) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(a\,H) \ll 1$).

Typical evolution of the scalar modes



Typical evolution of the real and the imaginary parts of the scalar modes during slow roll inflation. The mode considered leaves the Hubble radius at about 18 e-folds[9]

[9] Figure from V. Sreenath, *Computation and characteristics of inflationary three-point functions*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2015).

Spectral indices and the tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_{_{\rm S}}(k) = \mathcal{A}_{_{\rm S}}\,\left(\frac{k}{k_\ast}\right)^{n_{_{\rm S}}-1}, \qquad \mathcal{P}_{_{\rm T}}(k) = \mathcal{A}_{_{\rm T}}\,\left(\frac{k}{k_\ast}\right)^{n_{_{\rm T}}},$$

with the spectral indices $n_{_{\rm S}}$ and $n_{_{\rm T}}$ assumed to be constant.

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The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_{_{\rm T}}(k)}{\mathcal{P}_{_{\rm S}}(k)}$$

and it is usual to further set $r = -8\, n_{_{\rm T}}$, *viz.* the so-called consistency relation, which is valid during slow roll inflation.

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In general, the scalar and tensor spectral indices are defined as[10]

$$n_{_{\rm S}} = 1 + \frac{{\rm d}\ln \mathcal{P}_{_{\rm S}}}{{\rm d}\ln k}, \qquad n_{_{\rm T}} = \frac{{\rm d}\ln \mathcal{P}_{_{\rm T}}}{{\rm d}\ln k}.$$

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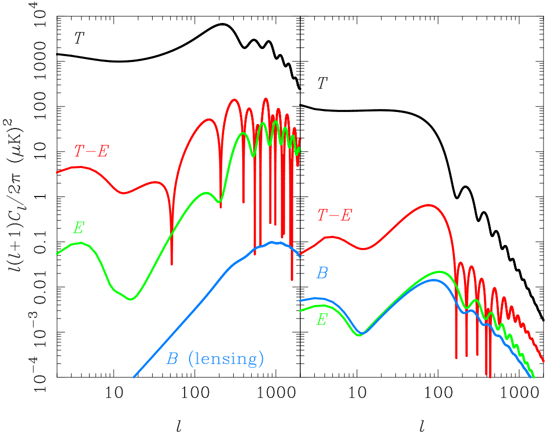
Plan of the talk

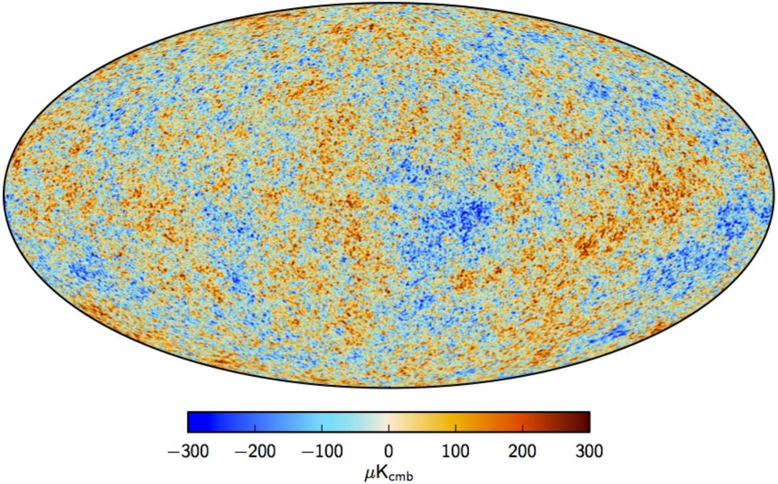
Theoretical angular power spectra[11]



The *theoretically* computed, CMB angular power and cross-correlation spectra – temperature (T, in black), E (in green), B (in blue), and T-E (in red) – arising due to scalars (on the left) and tensors (on the right) corresponding to a tensor-to-scalar ratio of $r = 0.24$. The B-mode spectrum induced by weak gravitational lensing has also been shown (in blue) in the panel on the left.

[11] Figure from A. Challinor, arXiv:1210.6008 [astro-ph.CO].

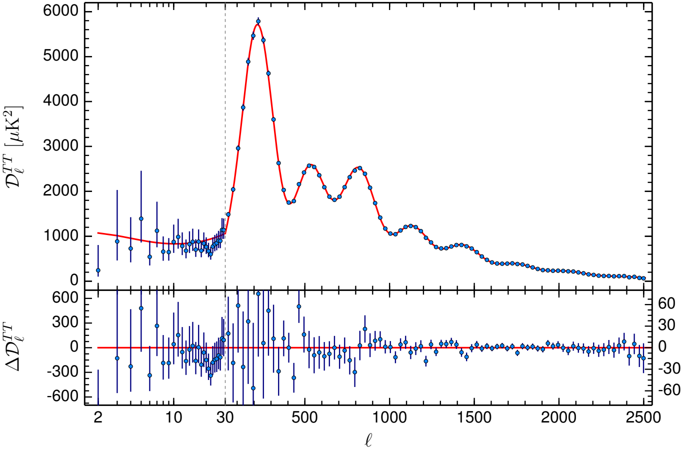
CMB anisotropies as seen by Planck



CMB intensity map at $5'$ resolution derived from the joint analysis of Planck, WMAP, and $408\,\mathrm{MHz}$ observations[12].

[12] Planck Collaboration (R. Adam *et al.*), Astron. Astrophys. **594**, A1 (2016).

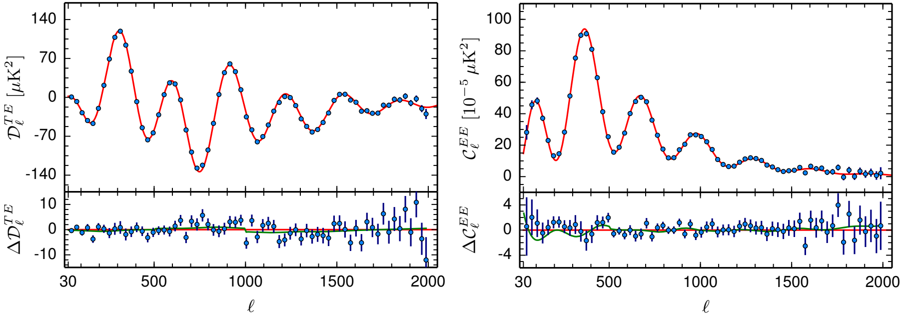
CMB TT angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2015 data (the blue dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid red curve)[13].

[13] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

CMB TE and EE angular power spectra from Planck



The CMB TE (on the left) and EE (on the right) angular power spectra from the Planck 2015 data (the blue dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid red curves)[14].

[14] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

[15] D. N. Spergel and M. Zaldarriaga, Phys. Rev. Lett. **79**, 2180 (1997).

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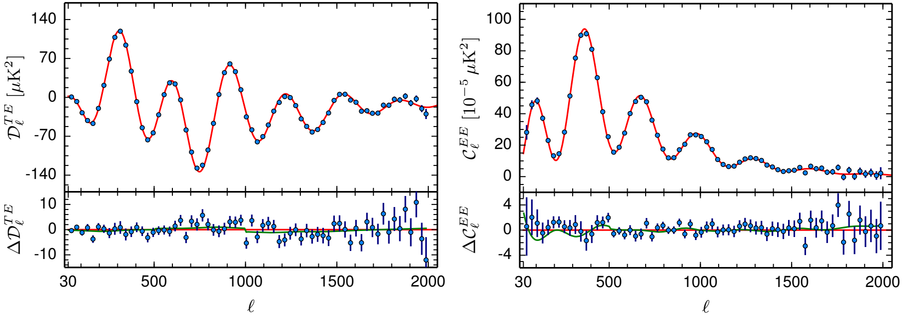

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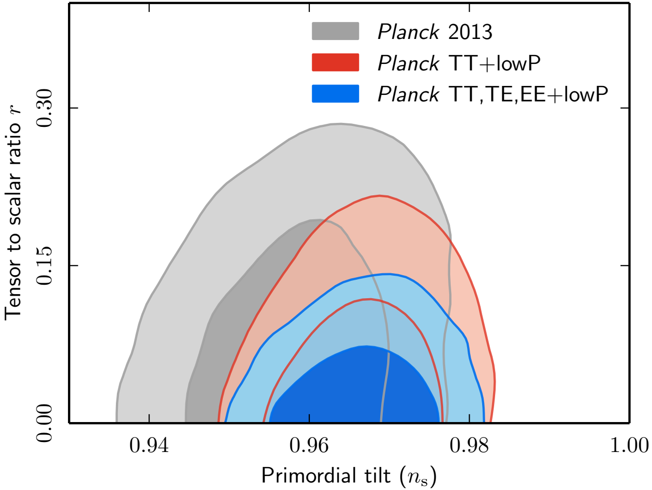
The large angle ($50 < \ell < 150$) TE anti-correlation detected by Planck (and earlier by WMAP) is a distinctive signature of primordial, super-Hubble, adiabatic perturbations[15].

[14] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

[15] D. N. Spergel and M. Zaldarriaga, Phys. Rev. Lett. **79**, 2180 (1997).

Joint constraints on r and $n_{\rm s}$



Marginalized joint confidence contours for $(n_{\rm s}, r)$, at the $68\,\%$ and $95\,\%$ CL, in the presence of running of the spectral indices[16].

[16] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

Specific inflationary models of interest I

Power law potentials: In power law potentials of the following form[17]:

$$V(\phi) = \lambda\, M_{_{\rm Pl}}^4\, \left(\phi/M_{_{\rm Pl}}\right)^n,$$

inflation occurs for large values of the field, *i.e.* $\phi > M_{_{\rm Pl}}$.

[17] A. D. Linde, Phys. Lett. B **129**, 177 (1983).

[18] L. Boubekeur and D. Lyth, JCAP **0507**, 010 (2005).

[19] K. Freese, J. .A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

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Hilltop models: The hilltop models are described by the potential[18]:

$$V(\phi) \simeq \Lambda^4\, \left[1 - (\phi/\mu)^p + \ldots\right]$$

and, in these models, the inflaton rolls away from an unstable equilibrium. The ellipsis indicates higher order terms that are considered to be negligible during inflation, but ensure positiveness of the potential.

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Natural inflation: In natural inflation, a non-perturbative shift symmetry is invoked to suppress radiative corrections, leading to the periodic potential[19]

$$V(\phi) = \Lambda^4\, \left[1 + \cos\left(\phi/f\right)\right],$$

where f is the scale which determines the curvature of the potential.

[17] A. D. Linde, Phys. Lett. B **129**, 177 (1983).

[18] L. Boubekeur and D. Lyth, JCAP **0507**, 010 (2005).

[19] K. Freese, J. .A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

Specific inflationary models of interest II

D-brane inflation: In brane world scenarios, if the standard model of particle physics is confined to our three dimensional brane, the distance between our brane and the anti-brane can drive inflation. In such a scenario, the effective potential driving inflation is given by[20]

$$V(\phi) = \Lambda^4 \left[1 - (\mu/\phi)^p + \ldots\right].$$

[20] S. Kachru, R. Kallosh, A. D. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, JCAP **0310**, 013 (2003); G. Dvali, Q. Shafi and S. Solganik, arXiv:hep-th/0105203.

[21] See, for instance, A. Goncharov and A. D. Linde, Sov. Phys. JETP **59**, 930 (1984); M. Cicoli, C. Burgess and F. Quevedo, JCAP **0903**, 013, (2009).

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Exponential potentials: Exponential potentials of the following form are ubiquitous in inflationary models motivated by supergravity and string theory[21]:

$$V(\phi) = \Lambda^4 \left(1 - \mathrm{e}^{-q\,\phi/M_{\mathrm{Pl}}} + \dots\right).$$

As in the case of the hilltop models, the ellipsis indicates possible higher order terms that are considered to be negligible during inflation, but ensure positiveness of the potential.

[20] S. Kachru, R. Kallosh, A. D. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, JCAP **0310**, 013 (2003); G. Dvali, Q. Shafi and S. Solganik, arXiv:hep-th/0105203.

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Specific inflationary models of interest III

Spontaneously broken SUSY: Hybrid inflationary models predicting $n_{\rm S} > 1$ are strongly disfavored by the Planck data. An example of a hybrid model that leads to $n_{\rm S} < 1$ is the case wherein slow roll inflation is driven by loop corrections in spontaneously broken supersymmetric (SUSY) grand unified theories described by the following potential[22]:

$$V(\phi) = \Lambda^4\,\left[1 + \alpha_h\,\log(\phi/M_{_{\rm Pl}})\right],$$

where $\alpha_h > 0$ is a dimensionless parameter.

[22] E. J. Copeland, A. R. Liddle, D. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D **49**, 6410 (1994); G. R. Dvali, Q. Shafi and R. K. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994).

Specific inflationary models of interest III

R^2 inflation: The first inflationary model proposed with the action[23]

$$S[g_{\mu\nu}] = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x\, \sqrt{-g}\, \left(R + \frac{R^2}{6\, M^2}\right),$$

corresponds to the following potential in the Einstein frame:

$$V(\phi) = \Lambda^4\, \left(1 - {\rm e}^{-\sqrt{2/3}\,\phi/M_{\rm Pl}}\right)^2.$$

[23] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

[24] See, for example, R. Kallosh, A. D. Linde and D. Roest, JHEP **1311**, 198 (2013).

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α-attractors: A class of inflationary models motivated by recent developments in conformal symmetry and supergravity correspond to the potential[24]

$$V(\phi) = \Lambda^4 \, \left[1 - {\rm e}^{-\sqrt{2/3}\, \phi/(\sqrt{\alpha}\, M_{\rm Pl})}\right]^2.$$

A second class of models, called the super-conformal α-attractors, are described by the following potential:

$$V(\phi) = \Lambda^4 \tanh^{2\, m} \left(\phi/\sqrt{6\, \alpha\, M_{\rm Pl}}\right).$$

[23] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

[24] See, for example, R. Kallosh, A. D. Linde and D. Roest, JHEP **1311**, 198 (2013).

Specific inflationary models of interest IV

Non-minimally coupled inflaton: Inflationary predictions are quite sensitive to the non-minimal coupling, $\xi\, R\, \phi^2$, of the inflaton to the Ricci scalar. An interesting aspect of non-minimal coupling would be to reconcile, say, the quartic potential $V(\phi) = \lambda\, \phi^4/4$, with the Planck observations for $\xi \ll 1$. It is found that this model lies within the 95% CL region for $\xi > 0.0019$[25].

[25] P. A. R. Ade *et al.*, Astron. Astrophys. **571**, A22 (2014).

[26] F. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).

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Higgs inflation model: The Higgs inflation model, in which inflation is driven by the potential[26]

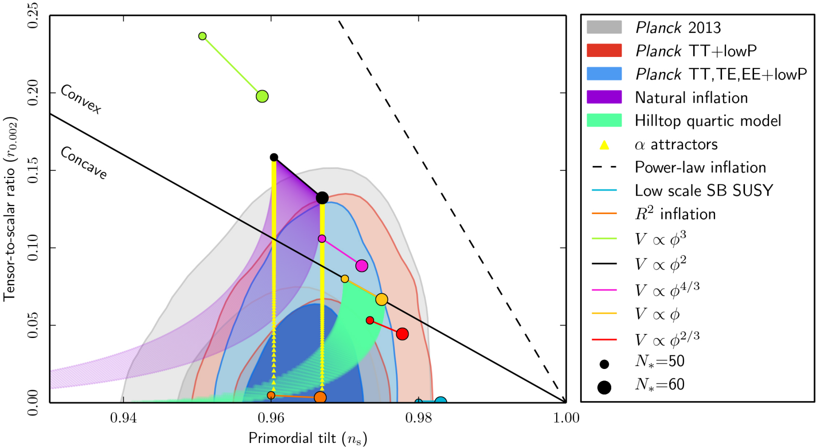
$$V(\phi) = \lambda\, (\phi^2 - \phi_0^2)^2/4$$

is found to lead to the same predictions as the R^2 model to the lowest order (in the slow roll approximation at tree level) for $\xi \gg 1$ and $\phi \gg \phi_0$.

[25] P. A. R. Ade *et al.*, Astron. Astrophys. **571**, A22 (2014).

[26] F. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).

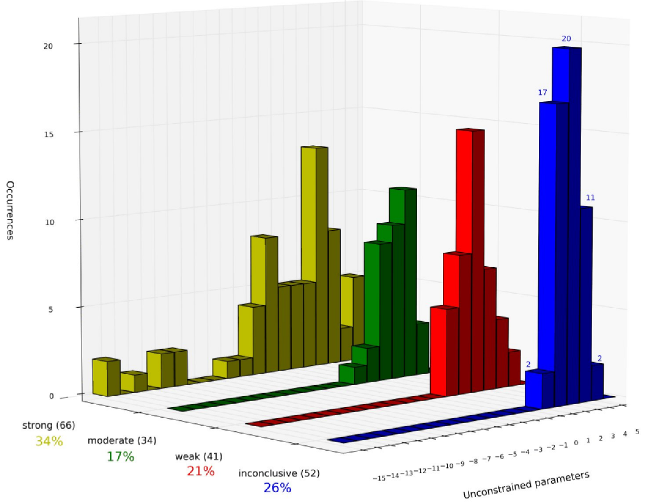
Performance of models in the n_s-r plane



Marginalized joint $68\,\%$ and $95\,\%$ CL regions for n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models[27].

[27] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

Performance of inflationary models



The efficiency of the inflationary paradigm leads to a situation wherein, despite the strong constraints, a variety of models continue to remain consistent with the data[28].

[28] J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP **1403**, 039 (2014).

Plan of the talk

The scalar bispectrum

The scalar bispectrum $\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time $\eta_{\rm e}$, as follows[29]:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e})\rangle = (2\,\pi)^3\;\; \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\; \delta^{(3)}\,(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\,.$$

[29] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

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Note that the delta function on the right hand side imposes the triangularity condition, *viz.* that the three wavevectors $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$ have to form the edges of a triangle.

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For convenience, we shall set

$$\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\; G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

[29] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

The cubic order action governing the perturbations

It can be shown that, the third order term in the action describing the curvature perturbation is given by[30]

$$\mathcal{S}^{3}_{\mathcal{R}\mathcal{R}\mathcal{R}}[\mathcal{R}] = M_{\rm Pl}^{2} \int {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial^i\mathcal{R})\,(\partial_i\chi) + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial^i\mathcal{R})\,(\partial_i\chi)\,(\partial^2\chi) + \frac{\epsilon_1}{4}\,(\partial^2\mathcal{R})\,(\partial\chi)^2 + \mathcal{F}_1\left(\frac{\delta\mathcal{L}^2_{\mathcal{R}\mathcal{R}}}{\delta\mathcal{R}}\right)\biggr],$$

where $\mathcal{F}_1(\delta\mathcal{L}^2_{\mathcal{R}\mathcal{R}}/\delta\mathcal{R})$ denotes terms involving the variation of the second order action with respect to $\mathcal{R}$, while χ is related to the curvature perturbation $\mathcal{R}$ through the relation

$$\partial^2\chi = a\,\epsilon_1\,\mathcal{R}'.$$

[30] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

Evaluating the scalar bispectrum

At the leading order in the perturbations, one then finds that the scalar three-point correlation function in Fourier space is described by the integral[31]

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\mathrm{e}})\rangle = -i\int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta\; a(\eta)\;\left\langle\left[\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\mathrm{e}}), \hat{H}_{\mathrm{I}}(\eta)\right]\right\rangle,$$

where $\hat{H}_{\mathrm{I}}$ is the Hamiltonian corresponding to the above third order action, while η_{i} denotes a sufficiently early time when the initial conditions are imposed on the modes, and η_{e} denotes a very late time, say, close to when inflation ends.

[31] See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).

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where $\hat{H}_{\rm I}$ is the Hamiltonian corresponding to the above third order action, while $\eta_{\rm i}$ denotes a sufficiently early time when the initial conditions are imposed on the modes, and $\eta_{\rm e}$ denotes a very late time, say, close to when inflation ends.

Note that, while the square brackets imply the commutation of the operators, the angular brackets denote the fact that the correlations are evaluated in the initial vacuum state (*viz.* the Bunch-Davies vacuum in the situation of our interest).

[31] See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).

The non-Gaussianity parameter $f_{_{\rm NL}}$

The observationally relevant non-Gaussianity parameter $f_{_{\rm NL}}$ is basically introduced through the relation[32]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}_{_{\rm G}}(\eta, \boldsymbol{x}) - \frac{3\, f_{_{\rm NL}}}{5}\, \left[\mathcal{R}_{_{\rm G}}^2(\eta, \boldsymbol{x}) - \langle \mathcal{R}_{_{\rm G}}^2(\eta, \boldsymbol{x})\rangle\right],$$

where $\mathcal{R}_{_{\rm G}}$ denotes the Gaussian quantity, and the factor of $3/5$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

[32] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

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Utilizing the above relation and Wick's theorem, one can arrive at the three-point correlation function of the curvature perturbation in Fourier space in terms of the parameter $f_{_{\rm NL}}$. It is found to be

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}\, \hat{\mathcal{R}}_{\boldsymbol{k}_3}\rangle = -\frac{3\, f_{_{\rm NL}}}{10}\, (2\,\pi)^{5/2}\, \left(\frac{1}{k_1^3\, k_2^3\, k_3^3}\right)\, \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$
$$\times \left[k_1^3\, \mathcal{P}_{_{\rm S}}(k_2)\, \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right].$$

[32] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

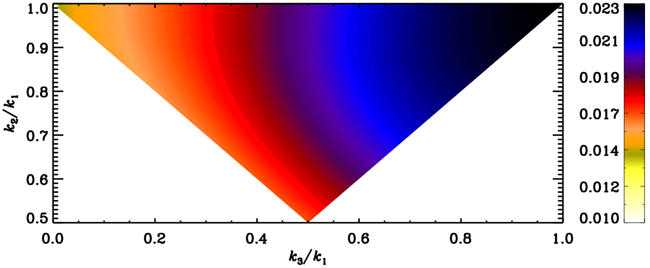
The relation between $f_{\rm NL}$ and the scalar bispectrum

Upon making use of the above expression for the three-point function of the curvature perturbation and the definition of the scalar bispectrum, we can, in turn, arrive at the following relation[33]:

$$\begin{aligned}
f_{\rm NL}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= -\frac{10}{3}\,(2\,\pi)^{1/2}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad\times\,\left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3)+\text{two permutations}\right]^{-1}\\
&= -\frac{10}{3}\,\frac{1}{(2\,\pi)^4}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\
&\quad\times\,\left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3)+\text{two permutations}\right]^{-1}.
\end{aligned}$$

[33] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

The shape of the slow roll bispectrum



The results for the non-Gaussianity parameter $f_{\rm NL}$, evaluated analytically in the slow roll approximation, has been plotted as a function of k_3/k_1 and k_2/k_1 for the case of the popular quadratic potential. Note that the non-Gaussianity parameter peaks in the equilateral limit wherein $k_1 = k_2 = k_3$. In slow roll scenarios involving the canonical scalar field, the largest value of $f_{\rm NL}$ is found to be of the order of the first slow roll parameter ϵ_1, while $f_{\rm NL} \sim \epsilon_1/c_{\rm S}^2$ in non-canonical models, where $c_{\rm S}$ denotes the speed of the scalar perturbations[34]. The most recent results imply that $c_{\rm S} \geq 0.024$[35].

[34] See, for example, D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005); X. Chen, Adv. Astron. **2010**, 638979 (2010).

[35] P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

Template bispectra

For comparison with the observations, the scalar bispectrum is often expressed in terms of the parameters $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm eq}$ and $f_{\rm NL}^{\rm orth}$ as follows:

$$G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = f_{\rm NL}^{\rm loc}\, G_{\mathcal{R}\mathcal{R}\mathcal{R}}^{\rm loc}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + f_{\rm NL}^{\rm eq}\, G_{\mathcal{R}\mathcal{R}\mathcal{R}}^{\rm eq}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + f_{\rm NL}^{\rm orth}\, G_{\mathcal{R}\mathcal{R}\mathcal{R}}^{\rm orth}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3).$$

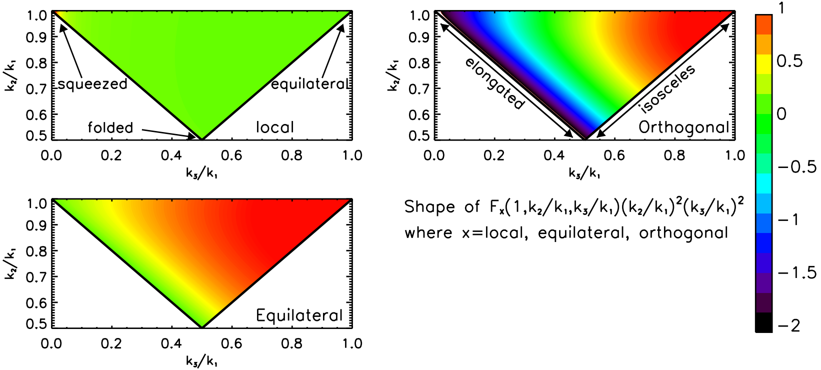


Illustration of the three template basis bispectra[36].

[36] E. Komatsu, Class. Quantum Grav. **27**, 124010 (2010).

Constraints on the scalar non-Gaussianity parameters

The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows[37]:

$$\begin{aligned} f_{\mathrm{NL}}^{\mathrm{loc}} &= 0.8 \pm 5.0, \\ f_{\mathrm{NL}}^{\mathrm{eq}} &= -4 \pm 43, \\ f_{\mathrm{NL}}^{\mathrm{orth}} &= -26 \pm 21. \end{aligned}$$

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These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

[37] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A17 (2016).

Can inflation be falsified I?

As the cosmological data continues to improve with its inevitable twists, it has become evident that whatever the observations turn out to be they will be lauded as 'proof of inflation'. This was poignantly brought to the fore when the BICEP2 data was released, in the wake of Planck's initial cosmological papers. Even though the two datasets taken at face-value contradicted each other, they were both advertised as proof of inflation. With the demise of the BICEP2 claim no one seems to have noted the flaw subjacent to this attitude: inflation can in fact predict practically anything. Interesting sociology will no doubt be reenacted when Planck's polarization data makes its mark, in the hopefully not too distant future[a].

[a] G. Gubitosi, M. Lagos, J. Magueijo and R. Allison, JCAP **06**, 002 (2016).

Can inflation be falsified II?

As the recent hiccups in CMB polarization observations demonstrated, whatever the data turns out to be it will be paraded as proof of inflation. This is because for any observation there is an inflationary model fitting it. Concomitantly, there is a trend in parroting the death of inflation's alternatives (such as cyclic models, string gas cosmology and varying speed of light models).

Putting aside sociology, these perceptions may derive from an important scientific issue. We argued here that the oft-used concept of Bayesian evidence fails to adequately capture the tenet that falsifiability is the hallmark of a scientific theory. Whilst the concept may be perfectly appropriate for comparing models within a paradigm, it fails to suitably penalize the whole paradigm for not making a prediction that could rule it out[a].

[a] G. Gubitosi, M. Lagos, J. Magueijo and R. Allison, JCAP **06**, 002 (2016).

Plan of the talk

Bouncing scenarios as an alternative paradigm[38]

- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

[38] See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008); D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

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- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.

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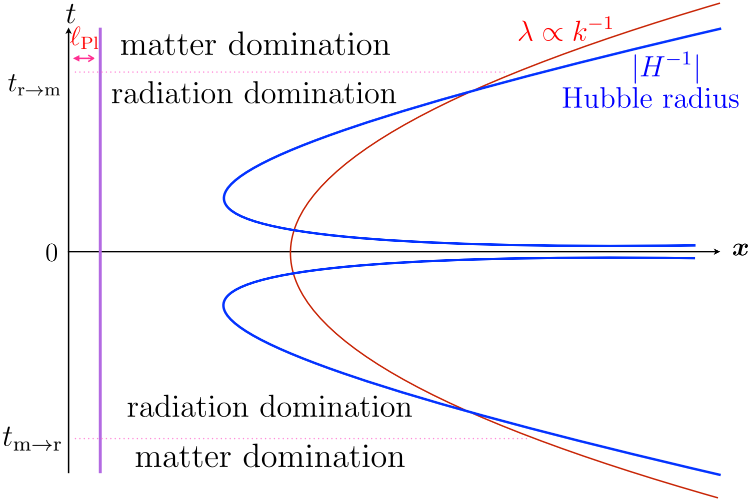
- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- However, matter fields *will* have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity can be supposed to fail and quantum gravitational effects are expected to take over.

[38] See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008); D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

Overcoming the horizon problem in bouncing models



Behavior in inflation

Evolution of the physical wavelength and the Hubble radius in a bouncing scenario[39].

[39]Figure from, D. Battefeld and P. Peter, Phys. Rept. **571**, 1 (2015).

Violation of the null energy condition near the bounce

Recall that, according to the Friedmann equations

$$\dot{H} = -4\,\pi\, G\, (\rho + p)\,.$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

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In such cases, one finds that $\dot{H}$ *will* be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve such bounces.

Classical bounces and sources

Consider for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0 \left(1 + k_0^2\, \eta^2\right)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale $M_{\rm Pl}$.

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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3\,q)$ and $w_2 = (2-q)/(3\,q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2\,q+1)/q}$ and $\rho_2 = M_2/a^{2\,(1+q)/q}$, where $M_1 = 12\, k_0^2\, M_{_{\rm Pl}}^2\, a_0^{1/q}$ and $M_2 = -M_1\, a_0^{1/q}$.

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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0/a^3$ and $\rho_2 = -12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0^2/a^4$.

E-$\mathcal{N}$-folds

The conventional e-fold N is defined $N = \log(a/a_{\rm i})$ so that $a(N) = a_{\rm i} \exp N$. However, the function e^N is a monotonically increasing function of N.

[40] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

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In completely symmetric bouncing scenarios, an obvious choice for the scale factor seems to be[40]

$$a(\mathcal{N}) = a_0\,\exp\left(\mathcal{N}^2/2\right),$$

with $\mathcal{N}$ being the new time variable that we shall consider for integrating the differential equation governing the background as well as the perturbations.

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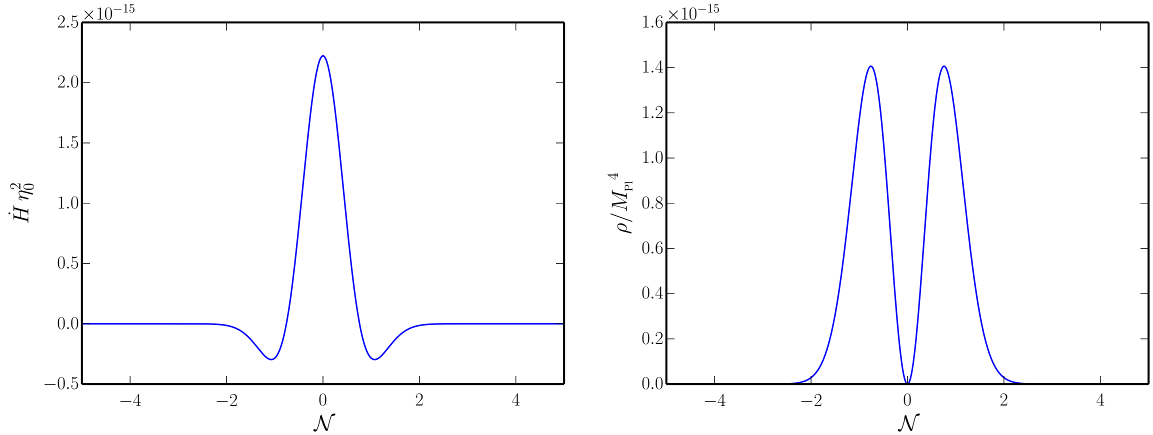
We shall refer to the variable $\mathcal{N}$ as e-$\mathcal{N}$-fold since the scale factor grows roughly by the amount ${\rm e}^{\mathcal{N}}$ between $\mathcal{N}$ and $(\mathcal{N}+1)$.

[40] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

Behavior of $\dot{H}$ and ρ in a matter bounce



The behavior of $\dot{H}$ (on the left) and the total energy density ρ (on the right) in a symmetric matter bounce scenario has been plotted as a function of $\mathcal{N}$. Note that the maximum value of ρ is much smaller than $M_{\rm Pl}^4$, which suggests that the bounce can be treated completely classically.

Back to scalar perturbations

Duality between de Sitter inflation and matter bounce

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation[41].

[41] D. Wands, Phys. Rev. D **60**, 023507 (1999).

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Given a scale factor a, the corresponding dual, say, $\tilde{a}$, which leads to the same equation for the variable u_k is given by

$$a(\eta) \to \tilde{a}(\eta) = C\, a(\eta) \int_{\eta_*}^{\eta} \frac{{\rm d}\bar{\eta}}{a^2(\bar{\eta})},$$

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It is straightforward to show that the dual solution to de Sitter inflation corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

[41] D. Wands, Phys. Rev. D **60**, 023507 (1999).

Plan of the talk

The matter bounce

We shall assume that the scale factor describing the bouncing scenario is given in terms of the conformal time coordinate η by the relation

$$a(\eta) = a_0\,\left(1+\eta^2/\eta_0^2\right) = a_0\,\left(1+k_0^2\,\eta^2\right).$$

As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch[42].

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The quantity a''/a corresponding to the above scale factor is given by

$$\frac{a''}{a} = \frac{2\, k_0^2}{1 + k_0^2\, \eta^2},$$

which is essentially a Lorentzian profile.

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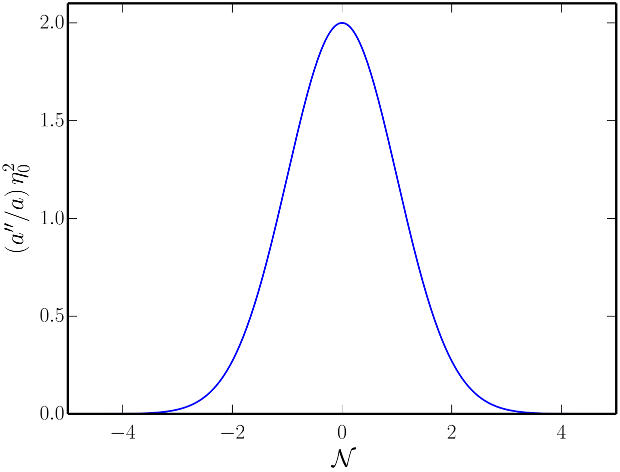
The behavior of a''/a



The behavior of the quantity a''/a has been plotted as a function of $\mathcal{N}$ for the matter bounce scenario of interest. Note that the maximum value of a''/a is of the order of k_0^2.

The tensor modes in the first domain

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0\, k_0^2\, \eta^2$, so that $a''/a \simeq 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.

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The modes h_k can be easily obtained in such a case and the positive frequency modes that correspond to the vacuum state at early times are given by

$$h_k(\eta) = \frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{a_0\,k_0^2\,\eta^2}\,\left(1 - \frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta}.$$

The modes in the second domain

Let us now consider the behavior of the modes in the domain $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, where, say, $\beta \simeq 10^2$. Since we are interested in scales much smaller than k_0, we shall assume that $\eta_k \ll -\alpha\,\eta_0$, which corresponds to $k \ll k_0/\alpha$.

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In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h_k'(\eta_*)\, a^2(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^2(\tilde{\eta})},$$

where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

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where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

If we choose $\eta_* = -\alpha\,\eta_0$, we can make use of the solution in the first domain to determine the constants and express the solution in the second domain as follows:

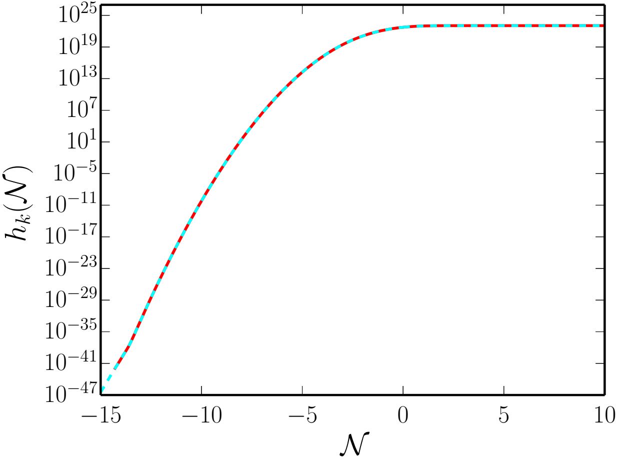
$$h_k = \mathcal{A}_k + \mathcal{B}_k\, f(k_0\,\eta),$$

where the function $f(k_0\,\eta)$ is given by

Back to scalar perturbations

$$f(k_0\,\eta) = \frac{k_0\,\eta}{1 + k_0^2\,\eta^2} + \tan^{-1}(k_0\,\eta)\,.$$

Evolution of the tensor modes across the bounce



A comparison of the numerical results (in solid red) with the analytical results (in dashed cyan) for the amplitude of the tensor mode $|h_k|$ corresponding to $k/k_0 = 10^{-20}$. We have set $k_0 = M_{\rm Pl}$, $a_0 = 3\times10^7$, and we have chosen $\alpha = 10^5$ for plotting the analytical results[43].

[43] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).

The tensor power spectrum after the bounce

The quantities $\mathcal{A}_k$ and $\mathcal{B}_k$ are given by

$$\mathcal{A}_k = \frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{a_0\,\alpha^2}\,\left(1+\frac{i\,k_0}{\alpha\,k}\right)\,{\rm e}^{i\,\alpha\,k/k_0}+\mathcal{B}_k\,f(\alpha),$$

$$\mathcal{B}_k = \frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{2\,a_0\,\alpha^2}\,\left(1+\alpha^2\right)^2\,\left(\frac{3\,i\,k_0}{\alpha^2\,k}+\frac{3}{\alpha}-\frac{i\,k}{k_0}\right)\,{\rm e}^{i\,\alpha\,k/k_0}.$$

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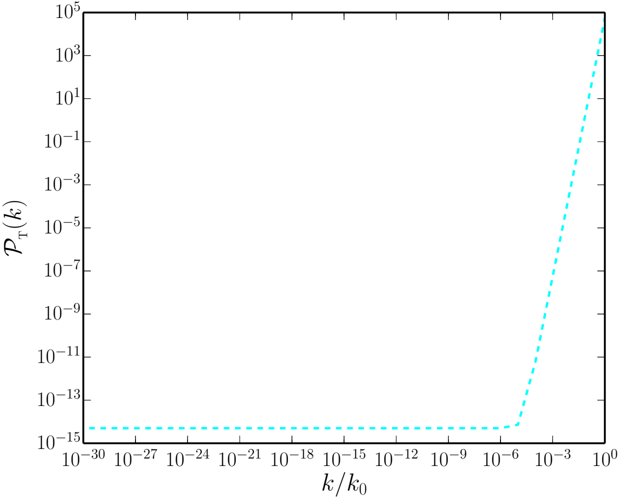
If we evaluate the tensor power spectrum after the bounce at $\eta = \beta\,\eta_0$, we find that it can be expressed as

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\vert\mathcal{A}_k + \mathcal{B}_k\,f(\beta)\vert^2.$$

The tensor power spectrum



The behavior of the tensor power spectrum has been plotted as a function of k/k_0 for a wide range of wavenumbers. In plotting this figure, we have set $k_0 = M_{_{\rm Pl}}$, $a_0 = 3 \times 10^7$, $\alpha = 10^5$ and $\beta = 10^2$. Note that the power spectrum is scale invariant for $k \ll k_0/\alpha$.

Plan of the talk

A new model for the completely symmetric matter bounce

As we had discussed, the matter bounce scenario described by the scale factor

$$a(\eta) = a_0\,\left(1 + \eta^2/\eta_0^2\right) = a_0\,\left(1 + k_0^2\,\eta^2\right)$$

can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

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We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ, that are governed by the following action[44]:

$$S[\phi,\chi] = -\int \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + V(\phi) + U_0\,\left(-\frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi\right)^2\right],$$

where U_0 is a constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6\,M_{_{\rm Pl}}^2\,(k_0^2/a_0^2)}{\cosh^6[\phi/(\sqrt{12}\,M_{_{\rm Pl}})]}.$$

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The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$\mathrm{d}s^2 = -(1+2\,A)\,\mathrm{d}t^2 + 2\,a(t)\,(\partial_i B)\,\mathrm{d}t\,\mathrm{d}x^i + a^2(t)\,\left[(1-2\,\psi)\,\delta_{ij} + 2\,(\partial_i\,\partial_j E)\right]\mathrm{d}x^i\,\mathrm{d}x^j,$$

where, evidently, the quantities A, ψ, B and E represent the metric perturbations.

[45] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

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The gauge invariant curvature and isocurvature perturbations $\mathcal{R}$ and $\mathcal{S}$ can be defined in terms of the above metric perturbations and the perturbations $\delta\phi$ and $\delta\chi$ in the scalar fields as follows[45]:

$$\mathcal{R} = \frac{H}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\,\left(\dot{\phi}\,\overline{\delta\phi} - U_0\,\dot{\chi}^3\,\overline{\delta\chi}\right), \quad \mathcal{S} = \frac{H\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\,\left(\dot{\chi}\,\overline{\delta\phi} - \dot{\phi}\,\overline{\delta\chi}\right).$$

[45] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$\mathrm{d}s^2 = -(1 + 2\,A)\,\mathrm{d}t^2 + 2\,a(t)\,(\partial_i B)\,\mathrm{d}t\,\mathrm{d}x^i + a^2(t)\,\left[(1 - 2\,\psi)\,\delta_{ij} + 2\,(\partial_i\,\partial_j E)\right]\mathrm{d}x^i\,\mathrm{d}x^j,$$

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The quantities $\overline{\delta\phi}$ and $\overline{\delta\chi}$ denote the gauge invariant versions of the perturbations in the scalar fields, and are given by

$$\overline{\delta\phi} = \delta\phi + \frac{\dot{\phi}\,\psi}{H}, \qquad \overline{\delta\chi} = \delta\chi + \frac{\dot{\chi}\,\psi}{H}.$$

[45] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

Equations governing the curvature and isocurvature perturbations

We obtain the equations of motion describing the gauge invariant perturbations $\mathcal{R}_k$ and $\mathcal{S}_k$ in our model to be

$$\mathcal{R}_k'' + \frac{2\,\left(7+9\,k_0^2\,\eta^2-6\,k_0^4\,\eta^4\right)}{\eta\,\left(1-3\,k_0^2\,\eta^2\right)^2\,\left(1+k_0^2\,\eta^2\right)}\,\mathcal{R}_k' - \frac{k^2\,\left(5+9\,k_0^2\,\eta^2\right)}{3\,\left(1-3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k$$

$$= \frac{4\,\left(5+12\,k_0^2\,\eta^2\right)}{\sqrt{3}\,\eta\,\left(1-3\,k_0^2\,\eta^2\right)\sqrt{1+k_0^2\,\eta^2}}\,\mathcal{S}_k' - \frac{4\,\left[5-22\,k_0^2\,\eta^2-24\,k_0^4\,\eta^4+k^2\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^2\right]}{\sqrt{3}\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^{3/2}\,\left(1-3\,k_0^2\,\eta^2\right)}\,\mathcal{S}_k,$$

$$\mathcal{S}_k'' - \frac{2\,\left(9+7\,k_0^2\,\eta^2+6\,k_0^4\,\eta^4\right)}{\eta\,\left(1-3\,k_0^2\,\eta^2\right)\,\left(1+k_0^2\,\eta^2\right)}\,\mathcal{S}_k'$$

$$- \frac{18-85\,k_0^2\,\eta^2-25\,k_0^4\,\eta^4-6\,k_0^6\,\eta^6+k^2\,\eta^2\,\left(3-k_0^2\,\eta^2\right)\,\left(1+k_0^2\,\eta^2\right)^2}{\eta^2\,\left(1-3\,k_0^2\,\eta^2\right)\,\left(1+k_0^2\,\eta^2\right)^2}\,\mathcal{S}_k$$

$$= \frac{4\sqrt{3}\,\left(3-2\,k_0^2\,\eta^2\right)}{\eta\,\sqrt{1+k_0^2\,\eta^2}\,\left(1-3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k' + \frac{4\,k^2\,\sqrt{1+k_0^2\,\eta^2}}{\sqrt{3}\,\left(1-3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k.$$

Behavior of $\dot{H}$

However, some of the coefficients diverge when $\dot{H}$ and/or H vanish.

The uniform-χ gauge

The issue of diverging coefficients can be avoided by working in a gauge wherein $\delta\chi = 0$[46].

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In this gauge, the equations of motion for the metric perturbations A_k and ψ_k can be obtained to be

$$\begin{aligned} A_k'' + 4\,\mathcal{H}\,A_k' + \left(\frac{k^2}{3} - \frac{20\,a_0^2\,k_0^2}{a^2}\right) A_k &= -3\,\mathcal{H}\,\psi_k' + \frac{4\,k^2}{3}\,\psi_k, \\ \psi_k'' - 2\,\mathcal{H}\,\psi_k' + k^2\,\psi_k &= 2\,\mathcal{H}\,A_k' - \frac{20\,a_0^2\,k_0^2}{a^2}\,A_k, \end{aligned}$$

where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

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where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

Also, in the uniform-χ gauge, the curvature and isocurvature perturbations simplify to be

$$\mathcal{R}_k = \psi_k + \frac{2\,H\,M_{\rm Pl}^2}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\,\left(\dot{\psi}_k + H\,A_k\right), \quad \mathcal{S}_k = \frac{2\,H\,M_{\rm Pl}^2\,\sqrt{U_0\,\dot{\chi}^4}}{\left(\dot{\phi}^2 - U_0\,\dot{\chi}^4\right)\,\dot{\phi}}\,\left(\dot{\psi}_k + H\,A_k\right).$$

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Solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain

As in the case of tensors, we shall be interested in evaluating the power spectrum after the bounce at $\eta = \beta\,\eta_0$. Also, to arrive at the analytical approximations, as earlier, we shall divide period of interest into two domains, *viz*. $-\infty < \eta < -\alpha\,\eta_0$ and $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$.

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In the first domain, we find that the solution to the curvature perturbation can be arrived at as in the case of tensors and is given by

$$\mathcal{R}_k(\eta) \simeq \frac{1}{\sqrt{6\,k}\,M_{_{\mathrm{Pl}}}\,a_0\,k_0^2\,\eta^2}\,\left(1-\frac{i}{k\,\eta}\right)\,\mathrm{e}^{-i\,k\,\eta}.$$

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Using this solution, it is straightforward to obtain the following solution for the isocurvature perturbation at early times:

$$\begin{aligned}\mathcal{S}_k(\eta) \;\simeq\; & \frac{1}{9\,\sqrt{2\,k^3}\,a_0\,k_0^3\,M_{_{\rm Pl}}\eta^4}\biggl(-12\,i\,\left(1+i\,k\,\eta\right)\,{\rm e}^{-i\,k\,\eta}+\frac{9}{3^{1/4}}\,k\,k_0\,\eta^2\,{\rm e}^{-i\,k\,\eta/\sqrt{3}}\\ & +4\,k^2\,\eta^2\,{\rm e}^{-i\,k\,\eta/\sqrt{3}}\,\left\{\pi+i\,{\rm Ei}\left[{\rm e}^{-i\,(3-\sqrt{3})\,k\,\eta/3}\right]\right\}\biggr).\end{aligned}$$

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In the second domain, upon ignoring the k^2 dependent terms, one finds that the combination $A_k + \psi_k$ satisfies the same equation of motion as the tensor modes.

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This feature helps us obtain the solutions for A_k and ψ_k, and they are given by

$$\begin{aligned} A_k(\eta) + \psi_k(\eta) &\simeq \frac{\mathcal{C}_k}{2\, a_0^2}\, f(k_0\, \eta) + \mathcal{D}_k, \\ A_k(\eta) &\simeq \frac{\mathcal{C}_k\, k_0\, \eta}{4\, a_0^2\, (1 + k_0^2\, \eta^2)} + \mathcal{E}_k\, \mathrm{e}^{-2\sqrt{5}\, \tan^{-1}(k_0\, \eta)} + \mathcal{F}_k\, \mathrm{e}^{2\sqrt{5}\, \tan^{-1}(k_0\, \eta)}, \end{aligned}$$

where $f(k_0\, \eta)$ is the same function that we had encountered earlier in the case of tensors, and $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$ and $\mathcal{F}_k$ are four constants of integration.

Function $f(k_0\, \eta)$

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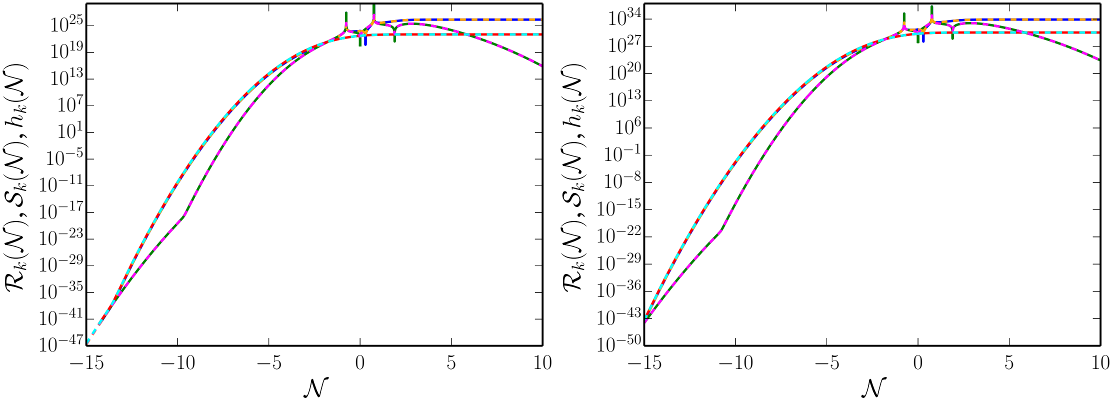
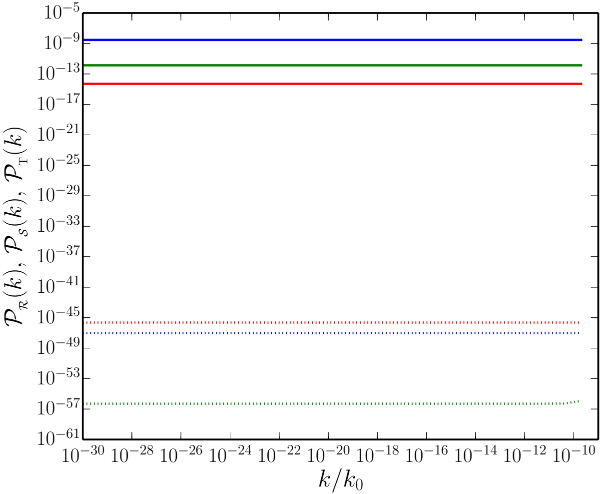
The four constants, *viz.* $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$ and $\mathcal{F}_k$, are determined by matching the above solutions with the solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain at $\eta = -\alpha\,\eta_0$.

Evolution of $\mathcal{R}_k$, $\mathcal{S}_k$ and h_k



The evolution of the curvature, isocurvature and tensor perturbations, *viz.* $\mathcal{R}_k$ (in blue and orange), $\mathcal{S}_k$ (in green and magenta) and h_k (in red and cyan) across the bounce for the modes $k/k_0 = 10^{-20}$ (on the left) and $k/k_0 = 10^{-25}$ (on the right). We have set $k_0 = M_{\rm Pl}$, $a_0 = 3 \times 10^7$, $\alpha = 10^5$ and $\beta = 10^2$. The solid lines denote the results obtained numerically, while the dashed lines represent the analytical approximations.

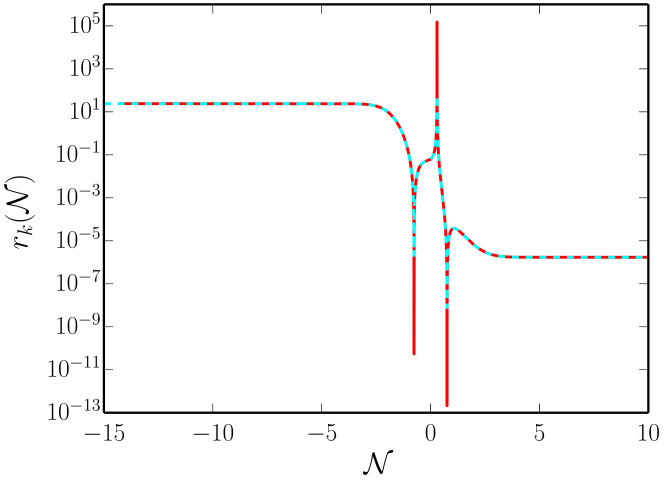
The scalar and tensor power spectra in the matter bounce



The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra have been plotted before (as dotted lines) as well as after (as solid lines) the bounce[47].

[47] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].

The evolution of the tensor-to-scalar ratio



The evolution of the tensor-to-scalar ratio r across the symmetric matter bounce for a typical mode of cosmological interest. The solid (in red) and dashed (in cyan) lines represent the numerical and analytical results, respectively.

Plan of the talk

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- We are currently working on constructing symmetric bouncing models that lead to scalar power spectra with a tilt as suggested by the cosmological data.
- It is also important to examine if the non-Gaussianities generated in such models are in agreement with the recent constraints from Planck.

Issues confronting bouncing models

- In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.

[48] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

[49] Y-F. Cai, R. Brandenberger and X. Zhang, Phys. Letts. B **703**, 25 (2011).

[50] J. Quintin, Z. Sherkatghanad, Y-F. Cai and R. Brandenberger, Phys. Rev. D **92**, 062532 (2015).

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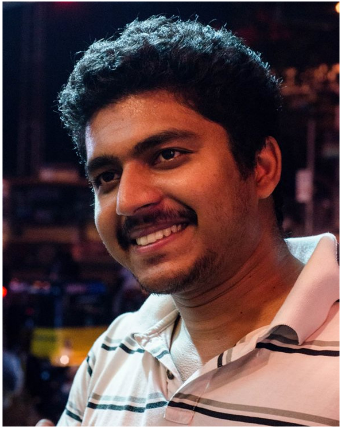
Collaborators: current and former students

Debika Chowdhury



Rathul Nath Raveendran

V. Sreenath

Thank you for your attention