Generation and amplification of primordial gravitational waves

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Plan of the talk

1. Vanilla, slow roll, inflation
2. Inflation with toppings
3. Summary

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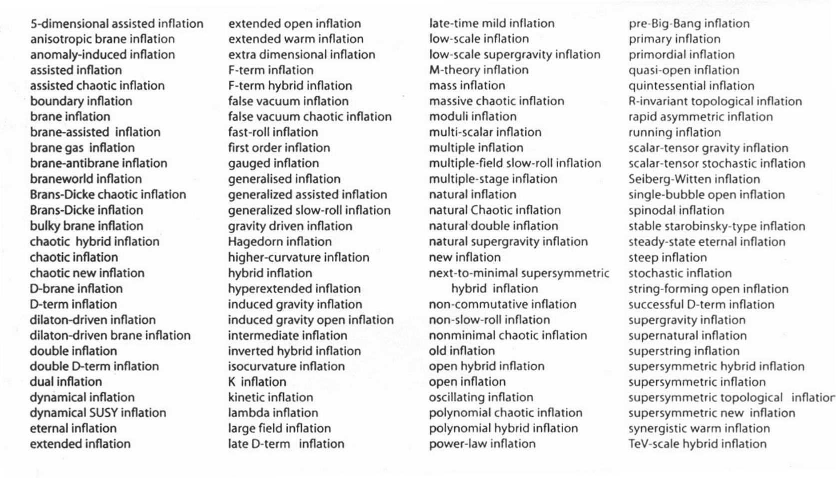
Proliferation of inflationary models


5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflation
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A partial list of inflationary models[1]. The goal is to rule out as many models as possible! Detecting the imprints of tensor perturbations can considerably help in achieving this goal.

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[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

The tensor-to-scalar ratio and the tensor spectral index

At the leading order in the slow roll approximation, the primordial tensor amplitude can be expressed in terms of the inflationary potential $V(\phi)$ as

$$\mathcal{P}_{_{\rm T}}(k) \simeq \frac{2}{3\,\pi^2}\,\left(\frac{V}{M_{_{\rm Pl}}^4}\right)_{k=a\,H},$$

while the spectral index can be estimated to be

$$n_{_{\rm T}} = \left(\frac{{\rm d}\ln \mathcal{P}_{_{\rm T}}}{{\rm d}\ln k}\right)_{k=a\,H} = -2\,\epsilon_1.$$

The tensor-to-scalar ratio is then given by

$$r = \frac{\mathcal{P}_{_{\rm T}}(k)}{\mathcal{P}_{_{\rm S}}(k)} \simeq 16\,\epsilon_1 = -8\,n_{_{\rm T}},$$

where $\mathcal{P}_{_{\rm S}}(k)$ denotes the scalar power spectrum, and the last equality is referred to as the consistency relation[2].

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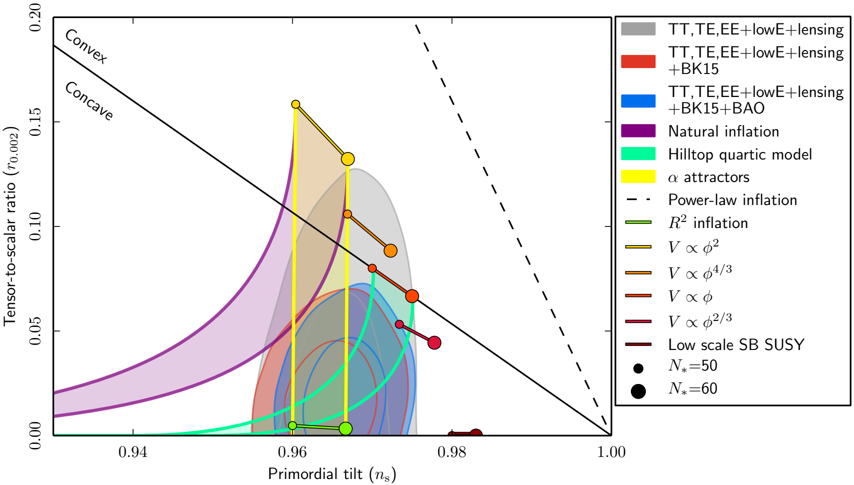
[2] See, for instance, B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).

Performance of specific inflationary models in the n_{s}-r plane



Joint constraints on n_{s} and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[3].

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[3] Planck Collaboration (Y. Akrami *et al.*), arXiv:1807.06211 [astro-ph.CO].

Primordial tensors determine the scale of inflation

In slow roll inflation, the tensor-to-scalar ratio at the pivot scale $k_\ast$ can be expressed as

$$r_\ast \simeq \frac{2\,V}{3\,\pi^2\,M_{_{\rm Pl}}^4}\,\frac{1}{A_{_{\rm S}}},$$

where $A_{_{\rm S}}$ denotes the amplitude of the scalar perturbations.

According to the recent Planck data[4], $A_{_{\rm S}} \simeq 2.1 \times 10^{-9}$ and $r_\ast < 0.06$ at $k_\ast = 0.002\,{\rm Mpc}^{-1}$, which implies that we have

$$V^{1/4} \lesssim 3.3 \times 10^{16}\, r_\ast^{1/4}\,{\rm GeV} \lesssim 1.6 \times 10^{16}\,{\rm GeV}.$$

Note that, if $H_{_{\rm I}}$ represents the Hubble scale during the inflationary epoch, as $H_{_{\rm I}}^2 \simeq V/(3\,M_{_{\rm Pl}}^2)$ during slow roll, the above inequality also leads to

$$\frac{H_{_{\rm I}}}{M_{_{\rm Pl}}} \lesssim 2.4 \times 10^{-5}.$$

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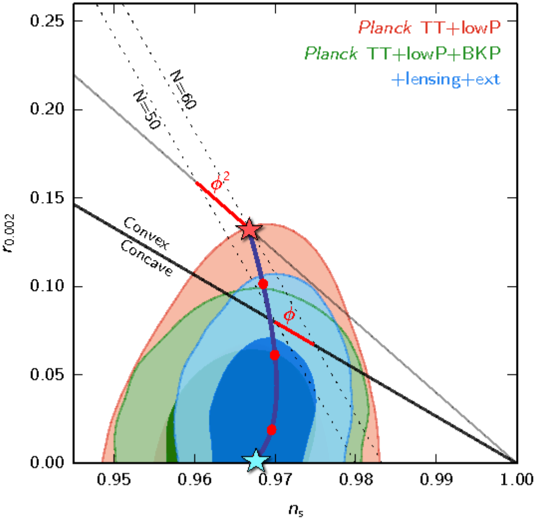
[4] Planck Collaboration (Y. Akrami *et al.*), arXiv:1807.06211 [astro-ph.CO].

Models with small tensor-to-scalar ratio



The α-attractor models can lead to rather small values of the tensor-to-scalar ratio[5].

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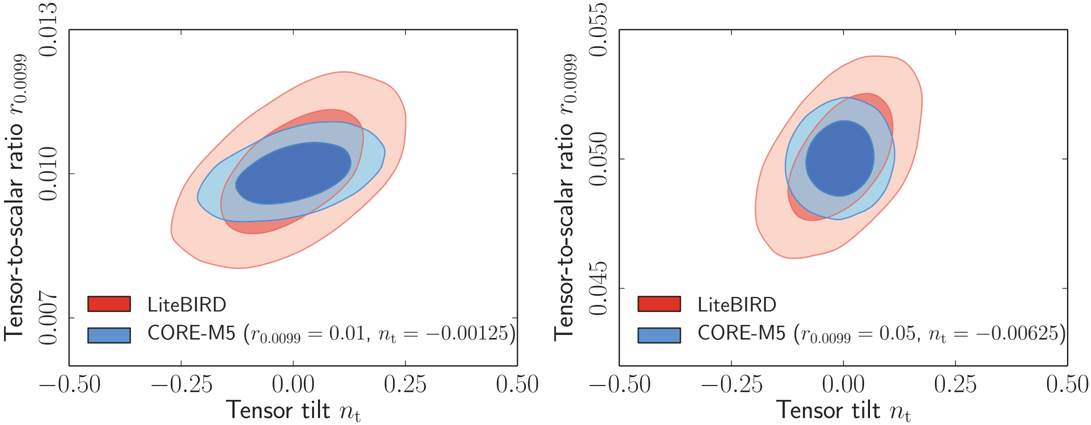
[5] R. Kallosh and A. Linde, Phys. Rev. D **91**, 083528 (2015).

CORE forecasts for the slow roll consistency relation



Marginalized forecasts for the slow roll consistency relation corresponding to two different values of the tensor-to-scalar ratio[6]. Deviations from the consistency relation would imply inflation driven by multiple fields.

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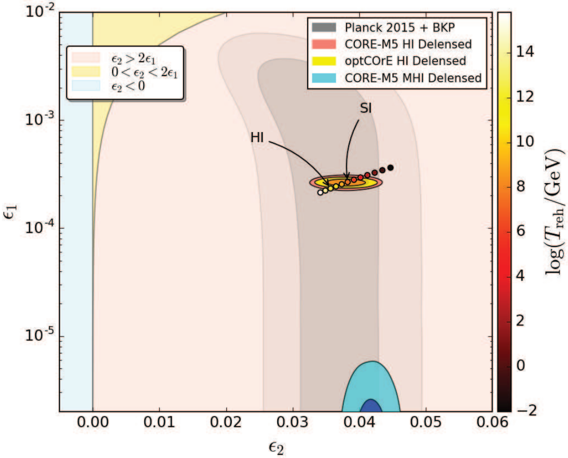
[6] F. Finelli *et al.*, arXiv:1612.08270 [astro-ph.CO].

Discriminating between different post-inflationary dynamics



CORE can, in principle, distinguish between Higgs inflation (HI) and Starobinsky inflation (SI), which share the same inflationary potential[7], but have different reheating temperatures (around $10^{12}\,\mathrm{GeV}$ for HI and $10^{8}\,\mathrm{GeV}$ for SI).

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[7] F. Finelli *et al.*, arXiv:1612.08270 [astro-ph.CO].

The gravitational wave energy density spectrum

The gravitational wave background at a given conformal time, say, η, can be characterized by the spectrum of energy density $\Omega_{_{\mathrm{GW}}}(k,\eta)$, which is defined as[8]

$$\Omega_{_{\mathrm{GW}}}(k,\eta) = \frac{1}{\rho_{_{\mathrm{C}}}(\eta)}\,\frac{\mathrm{d}\langle\hat{\rho}_{_{\mathrm{GW}}}(\eta)\rangle}{\mathrm{d}\ln k}.$$

The quantity $\rho_{_{\mathrm{GW}}}$ denotes the energy density associated with the gravitational waves, while $\rho_{_{\mathrm{C}}}(\eta) = 3\,H^2\,M_{_{\mathrm{Pl}}}^2$ represents the critical density.

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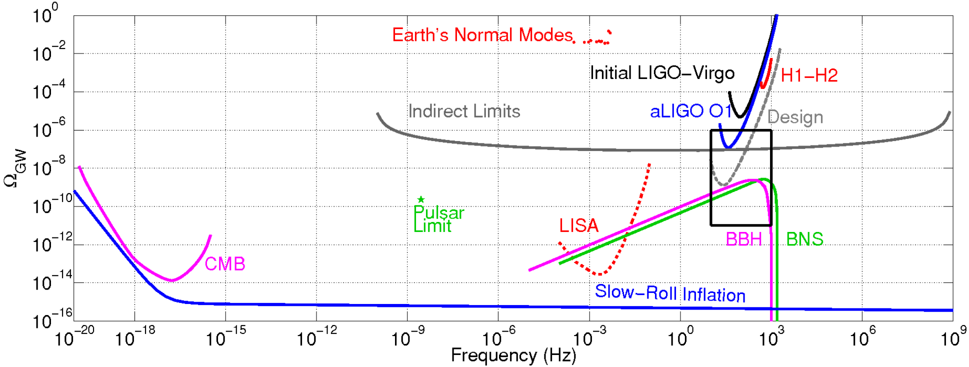
[8] See, for instance, L. A. Boyle and P. J. Steinhardt, Phys. Rev. D **77**, 063504 (2008).

Constraints on $\Omega_{\mathrm{GW}}^{0}(f)$ from LIGO and Virgo on small scales



Assuming a scale-invariant primordial tensor spectrum, LIGO and Virgo set an *upper bound* on the gravitational wave density parameter over the frequency range 20–$86\,\mathrm{Hz}$ (corresponding to the wavenumbers $k = 2\,\pi\,f = (1.3–5.5) \times 10^{16}\,\mathrm{Mpc}^{-1}$) today to be[9]

$$\Omega_{\mathrm{GW}}^{0}(f) \leq 1.7 \times 10^{-7}.$$

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[9] The LIGO Scientific Collaboration, the Virgo Collaboration (B. P. Abbott, *et al.*), Phys. Rev. Lett. **118**, 121101 (2017).

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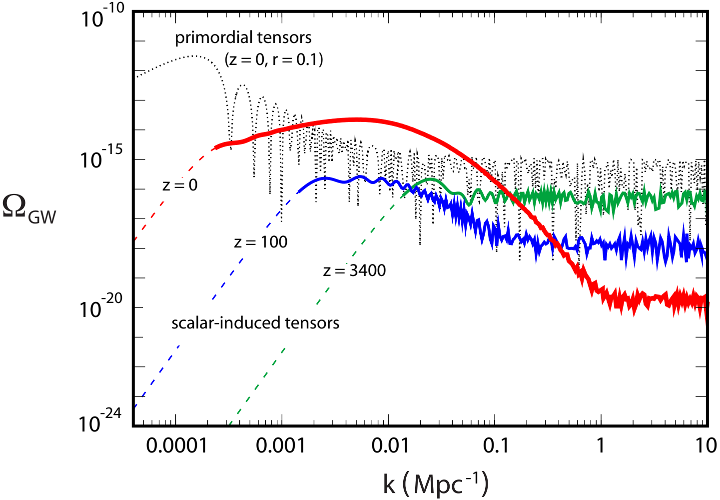
The primordial and induced gravitational wave spectrum today



The primordial as well as the induced (by the second order scalar perturbations) gravitational wave energy density spectrum[10].

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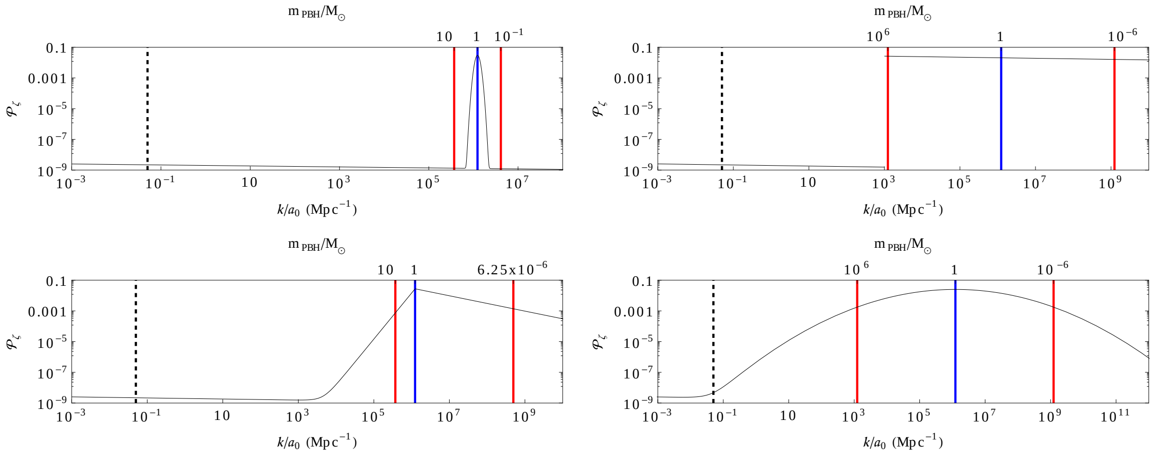
[10] D. Baumann, P. J. Steinhardt, K. Takahashi and K. Ichiki, Phys. Rev. D **76**, 084019 (2007).

Scalar power spectra with features at small scales



If the scalar power spectrum contains features as in these spectra, then the second order effect can dominate the tensor perturbations generated at the first order[11].

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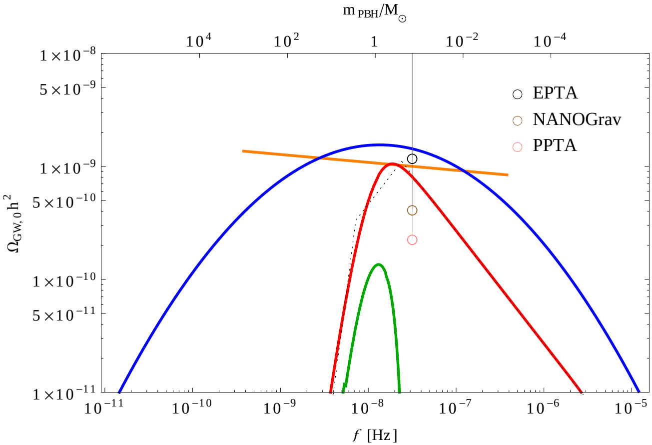
[11] S. Clesse, J. García-Bellido and S. Orani, arXiv:1812.11011 [astro-ph.CO].

The resulting gravitational wave energy density spectrum today



The gravitational wave energy density spectra today resulting from the different inflationary scalar power spectra displayed in the previous slide[12]. The circles indicate the best current limits from the different pulsar timing arrays.

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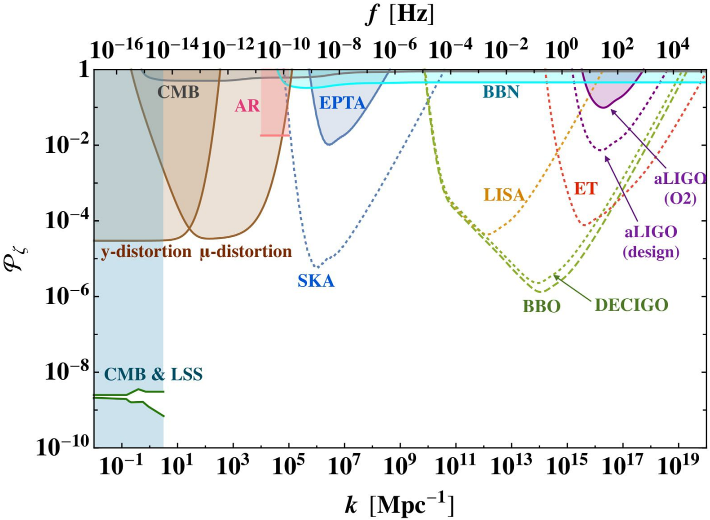
[12] S. Clesse, J. García-Bellido and S. Orani, arXiv:1812.11011 [astro-ph.CO].

Constraints on the scalar power spectrum



The current and expected limits on the scalar power spectrum over small scales[13].

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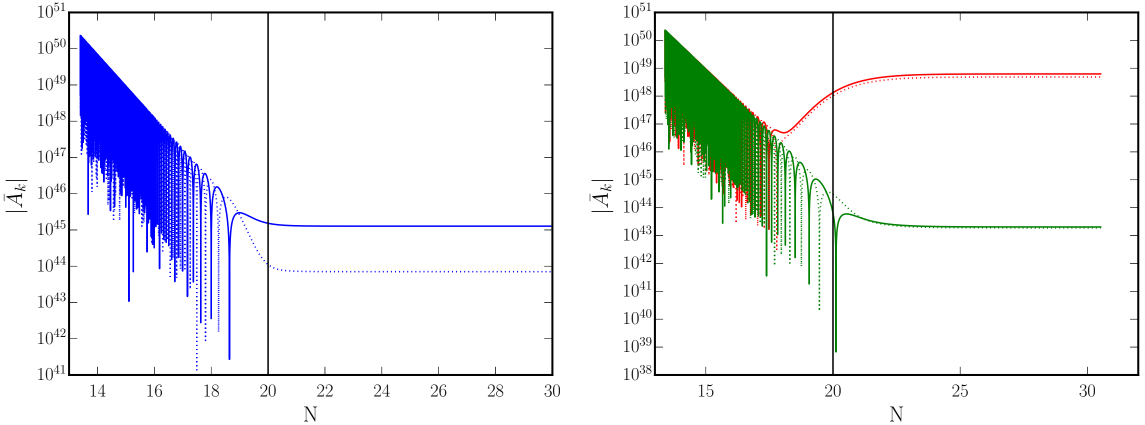
[13] K. Inomata and T. Nakama, Phys. Rev. D **99**, 043511 (2019).

Evolution of non-helical and helical electromagnetic modes



Evolution of the non-helical (on the left) and helical (on the right) electromagnetic modes during inflation[14]. The absolute values of the real (solid lines) and imaginary (dashed lines) parts of the vector potential have been plotted so that the oscillations are visible.

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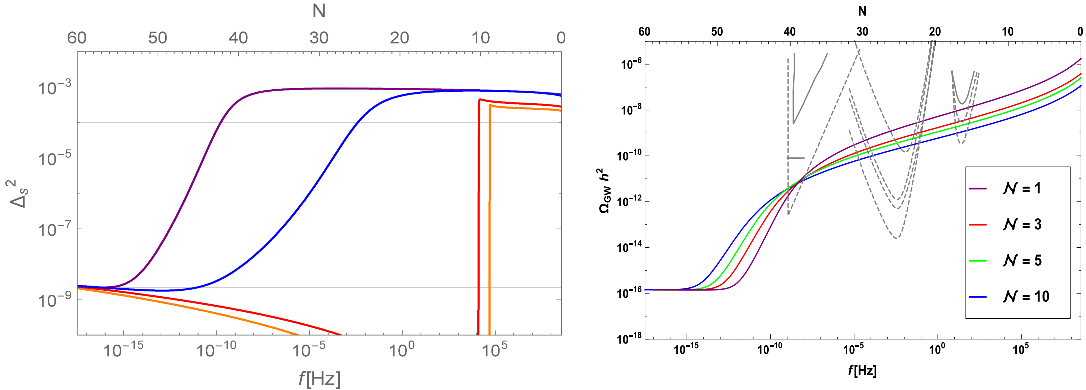
[14] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **1901**, 048 (2019).

Effects of helical fields on scalar and tensor power spectra



Left: The scalar power spectrum when the contribution due to the helical electromagnetic fields have been taken into account. While the upper horizontal line is an estimation of the bounds from primordial black holes, the lower one indicates COBE normalization.

Right: The corresponding spectrum of gravitational waves, along with the sensitivity curves for (from left to right): milli-second pulsar timing, e-LISA and advanced LIGO[15].

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[15] V. Domcke, M. Pieroni and P. Binétruy, JCAP **1606**, 031 (2016).

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Summary

Detection of the imprints of the primordial tensor perturbations at large as well as small scales seem essential to unravel the physics operating at the inflationary energy scales.

Thank you for your attention