Gravitational waves from the early universe

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Plan of the talk

1. The need for inflation
2. Constraints on inflation from the CMB data
3. GWs provide a new window to the universe
4. Reheating can boost the strengths of primary GWs
5. Generation of GWs by enhanced scalar perturbations on small scales
6. The NANOGrav 15-year data and its implications
7. Outlook

This talk is based on. . .

- H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Suppression of scalar power on large scales and associated bispectra, Phys. Rev. D **106**, 043535 (2022) [arXiv:2003.01099 [astro-ph.CO]].
- M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, *Generating primordial features at large scales in two field models of inflation*, JCAP **08**, 025 (2020) [arXiv:2004.00672 [astro-ph.CO]].
- M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation*, JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Primordial black holes and secondary gravitational waves from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, *Decoding the phases of early and late time reheating through imprints on primordial gravitational waves*, Phys. Rev. D **104**, 063513 (2021) [arXiv:2105.09242 [astro-ph.CO]].

This talk is based on. . .

- H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, Galaxies **11**, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].
- S. Maiti, D. Maity and L. Sriramkumar, Constraining inflationary magnetogenesis and reheating via GWs in light of PTA data, arXiv:2401.01864 [astro-ph.CO].
- S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, Constraining the history of reheating with the NANOGrav 15-year data, in preparation.

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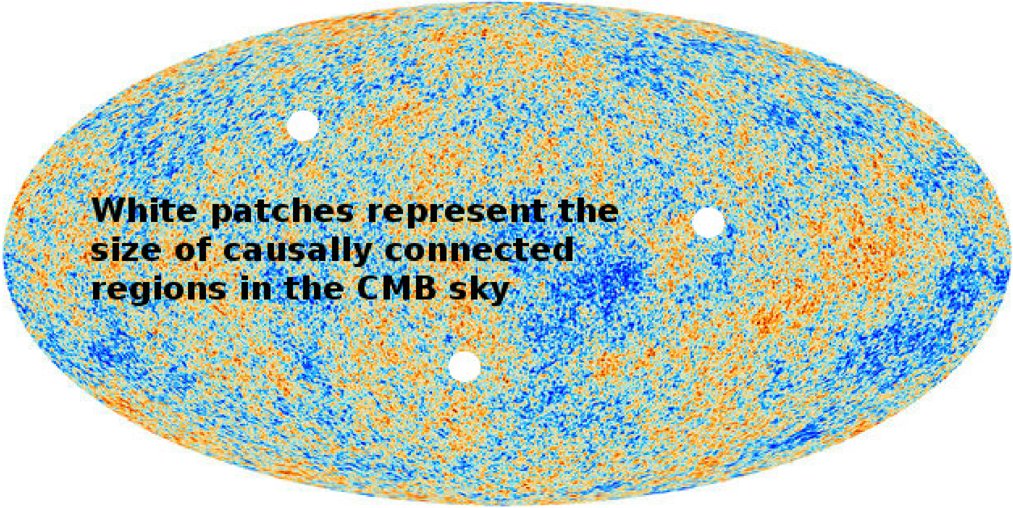
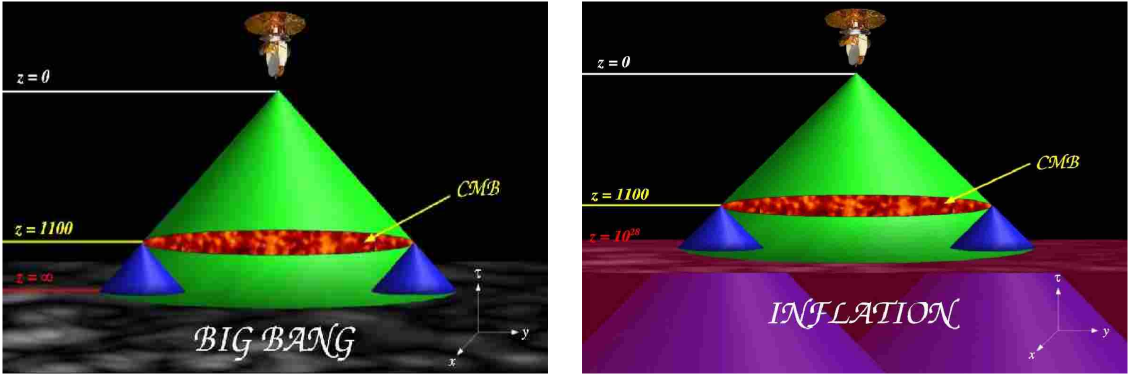
The horizon problem



The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the surface of last scattering, which subtends an angle of about 1° today, could not have interacted before decoupling.

The resolution of the horizon problem in the inflationary scenario



Another illustration of the horizon problem (on the left), and an illustration of its resolution (on the right) through an early and sufficiently long epoch of inflation[1].

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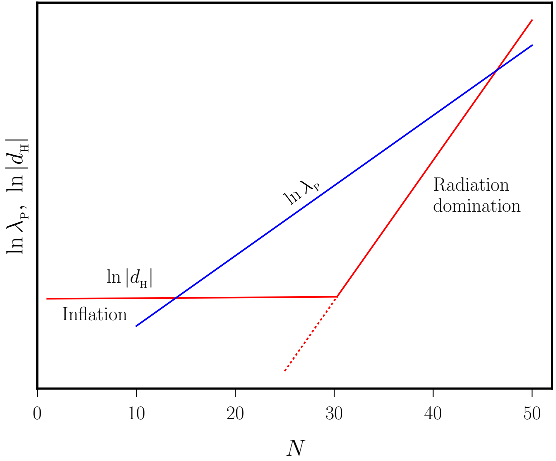
[1] Images from W. Kinney, astro-ph/0301448.

Bringing the modes inside the Hubble radius



Evolution of comoving lengths

The physical wavelength $\lambda_{\mathrm{P}} \propto a$ (in blue) and the Hubble radius $d_{\mathrm{H}} = H^{-1}$ (in red) in the inflationary scenario[2]. The scale factor is expressed in terms of e-folds N as $a(N) \propto \mathrm{e}^{N}$.

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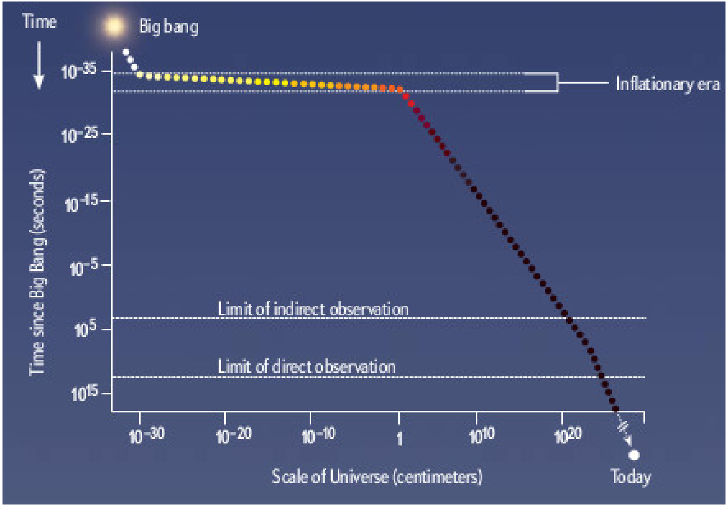
[2] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Time and duration of inflation



Inflation—a brief period of accelerated expansion—is expected to have taken place during the very early stages of the universe[3].

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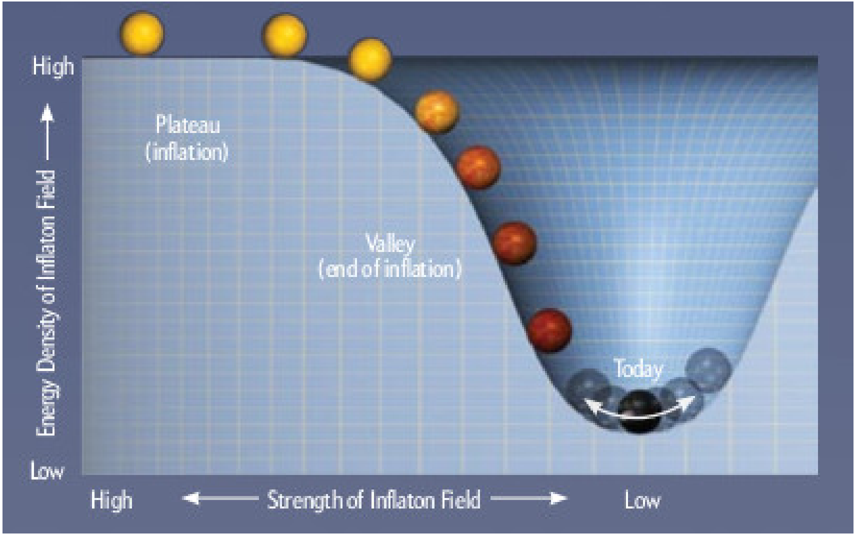
[3] Image from P. J. Steinhardt, Sci. Am. **304**, 18 (2011).

Driving inflation with scalar fields



Inflation can be achieved with scalar fields encountered in high energy physics[4].

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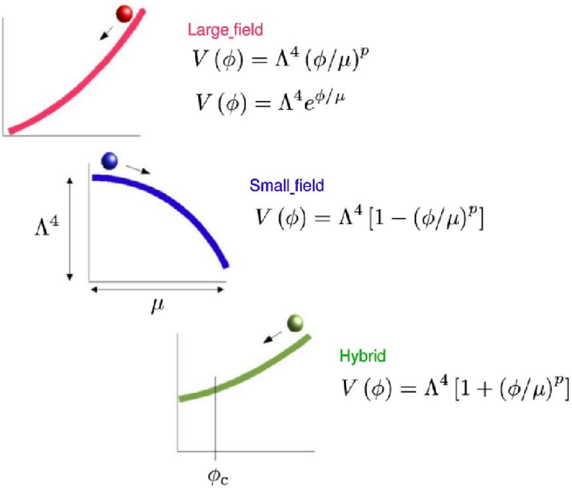
[4] Image from P. J. Steinhardt, Sci. Am. **304**, 34 (2011).

A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[5]. Often, these potentials are classified as small field, large field and hybrid models.

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[5] Image from W. Kinney, astro-ph/0301448.

Proliferation of inflationary models

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric
hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflatior
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models[6]. Actually, it may not even be possible to rule out some of these models!

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[6] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

The quadratic action governing the perturbations

One can show that, at the quadratic order, the action governing the curvature perturbation $\mathcal{R}$ and the tensor perturbation γ_{ij} are given by[7]

$$\begin{aligned}
\mathcal{S}_2[\mathcal{R}(\eta, \boldsymbol{x})] &= \frac{1}{2}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; z^2\, \left[\mathcal{R}'^2 - (\partial\mathcal{R})^2\right],\\
\mathcal{S}_2[\gamma_{ij}(\eta, \boldsymbol{x})] &= \frac{M_{_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; a^2\, \left[\gamma_{ij}'^{\,2} - (\partial\gamma_{ij})^2\right].
\end{aligned}$$

These actions lead to the following equations of motion governing the scalar and tensor modes, say, f_k and h_k:

$$\begin{aligned}
f_k'' + 2\,\frac{z'}{z}\, f_k' + k^2\, f_k &= 0,\\
g_k'' + 2\,\frac{a'}{a}\, g_k' + k^2\, g_k &= 0,
\end{aligned}$$

where $z = a\, M_{_{\mathrm{Pl}}}\, \sqrt{2\,\epsilon_1}$, with $\epsilon_1 = -\,\mathrm{d}\ln H/\mathrm{d}N$ being the first slow roll parameter.

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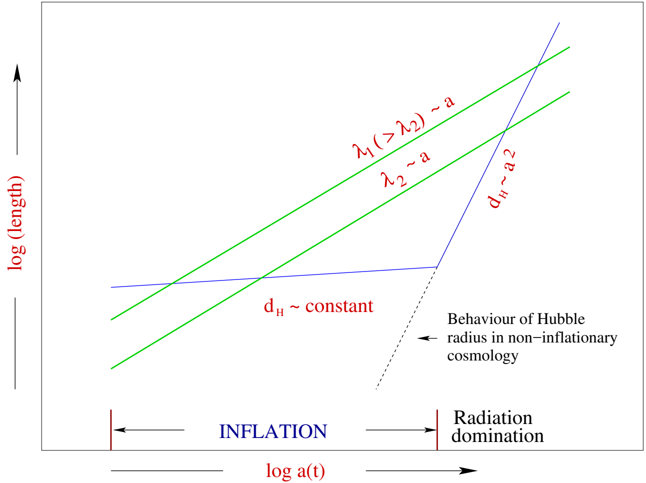
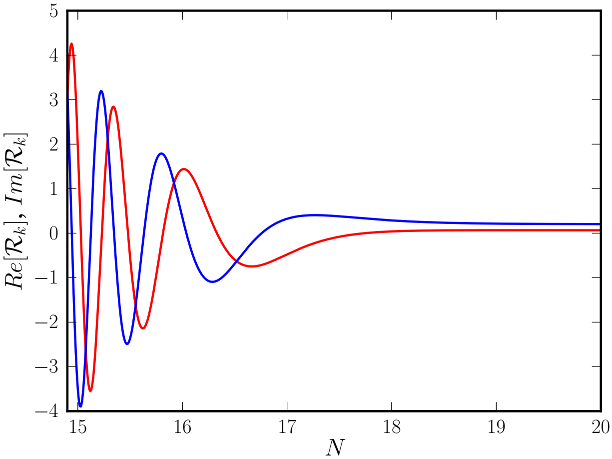
[7] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the modes are well inside the Hubble radius (*viz.* when $k/(a\,H) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(a\,H) \ll 1$).

Typical evolution of the perturbations



Typical evolution of the real and the imaginary parts of the scalar modes during slow roll inflation. The mode considered here leaves the Hubble radius at about 18 e-folds[8].

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[8] Figure from V. Sreenath, *Computation and characteristics of inflationary three-point functions*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2015).

Spectral indices and the tensor-to-scalar ratio

The scalar and tensor power spectra, viz. $\mathcal{P}_{_{\rm S}}(k)$ and $\mathcal{P}_{_{\rm T}}(k)$, can be expressed in terms of the Fourier modes f_k and g_k as follows:

$$\begin{aligned}\mathcal{P}_{_{\rm S}}(k) &= \frac{k^3}{2\,\pi^2}\,\vert f_k(\eta_{\rm e})\vert^2,\\ \mathcal{P}_{_{\rm T}}(k) &= 8\,\frac{k^3}{2\,\pi^2}\,\vert g_k(\eta_{\rm e})\vert^2,\end{aligned}$$

with $\eta_{\rm e}$ corresponding to suitably late times during inflation.

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_{_{\rm S}}(k) = A_{_{\rm S}}\,\left(\frac{k}{k_\ast}\right)^{n_{\rm S}-1}, \qquad \mathcal{P}_{_{\rm T}}(k) = A_{_{\rm T}}\,\left(\frac{k}{k_\ast}\right)^{n_{\rm T}},$$

with the spectral indices $n_{_{\rm S}}$ and $n_{_{\rm T}}$ assumed to be constant. The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_{_{\rm T}}(k)}{\mathcal{P}_{_{\rm S}}(k)}.$$

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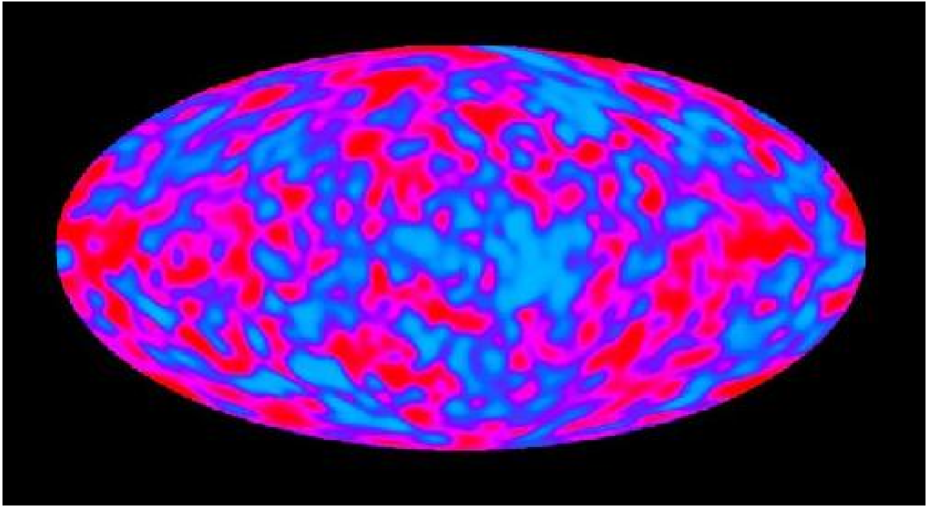
Anisotropies in the CMB



The fluctuations in the temperature of the CMB as seen by COBE[9]. The CMB turns out to be isotropic to one part in 10^5.

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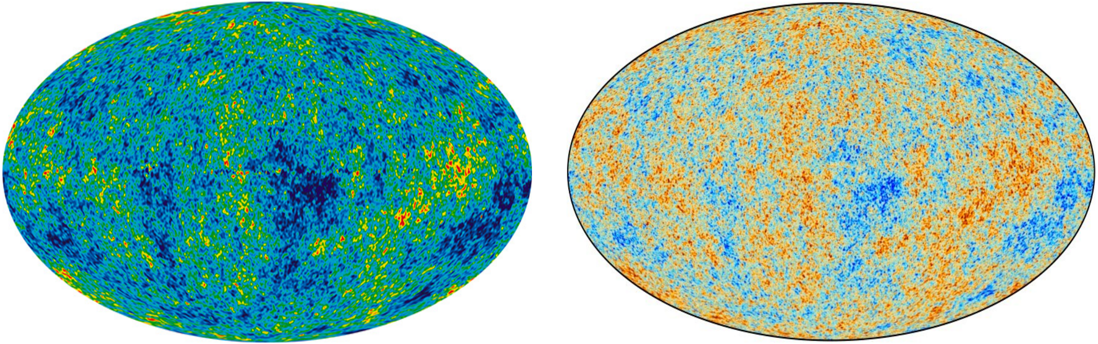
[9] Image from http://aether.lbl.gov/www/projects/cobe/COBE_Home/DMR_Images.html.

Anisotropies in the CMB as seen by WMAP and Planck



Left: All-sky map of the anisotropies in the CMB created from nine years of Wilkinson Microwave Anisotropy Probe (WMAP) data[10].

Right: CMB intensity map derived from the joint analysis of Planck, WMAP, and $408\,\mathrm{MHz}$ observations[11]. The above images show temperature variations (as color differences) of the order of $200\,\mu\mathrm{K}$.

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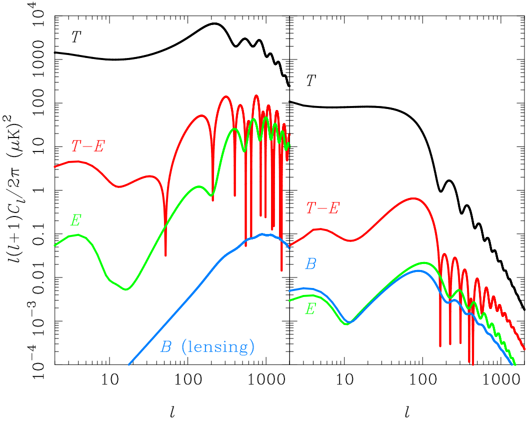
[10] Image from http://wmap.gsfc.nasa.gov/media/121238/index.html.

[11] P. A. R. Ade *et al.*, arXiv:1502.01582 [astro-ph.CO].

Theoretical angular power spectra[12]



The *theoretically* computed, angular power and cross-correlation spectra of the CMB arising due to scalars (on the left) and tensors (on the right) corresponding to a tensor-to-scalar ratio of $r = 0.24$. The B-mode spectrum induced by weak gravitational lensing has also been shown (in blue) in the panel on the left.

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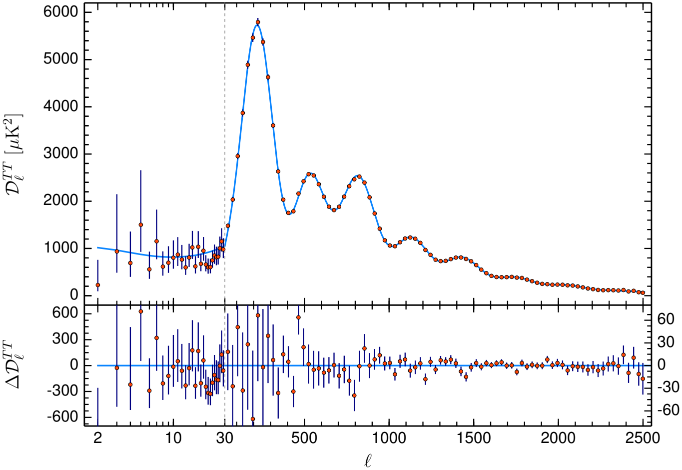
[12] Figure from, A. Challinor, arXiv:1210.6008 [astro-ph.CO].

CMB angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit ΛCDM model with a power law primordial spectrum (solid blue curve)[13]

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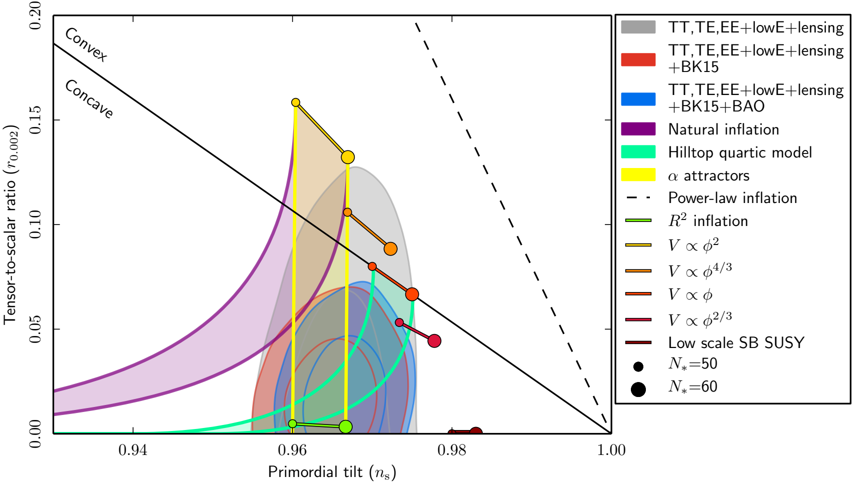
[13] Planck Collaboration (N. Aghanim *et al.*), Astron. Astrophys. **641**, A6 (2020).

Performance of inflationary models in the n_{s}-r plane



Joint constraints on n_{s} and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[14].

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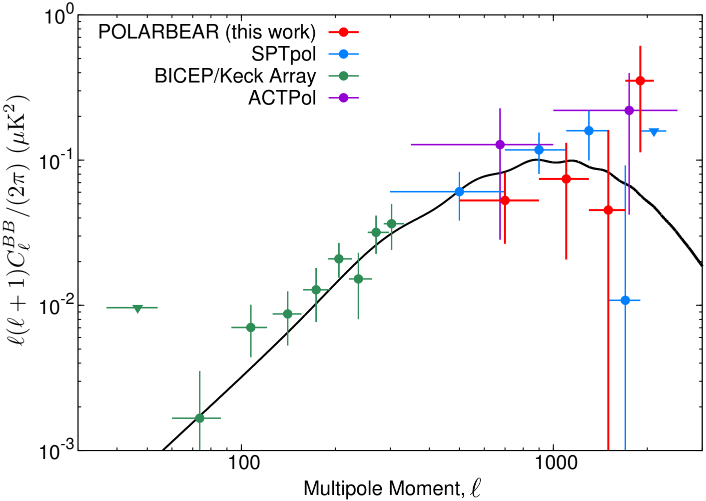
[14] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Constraints on the B-mode polarization of the CMB



Constraints on the B-mode, angular power spectrum of the CMB from two years of POLARBEAR data[15].

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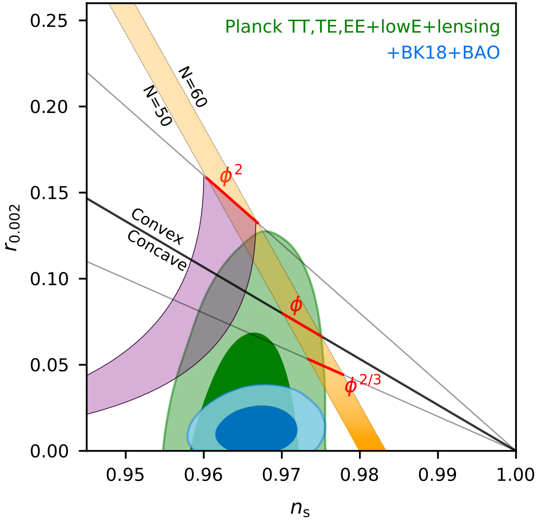
[15] POLARBEAR Collaboration (P. A. R. Ade *et al.*), Ap. J. **848**, 141, (2017).

Latest constraints on the tensor-to-scalar ratio r



Latest constraints on the tensor-to-scalar ratio r from the BICEP/Keck telescopes[16].

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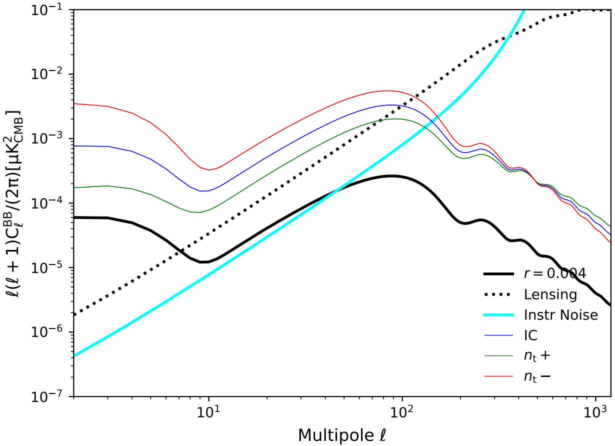
[16] BICEP/Keck Collaboration (P. A. R. Ade *et al.*), arXiv:2203.16556 [astro-ph.CO].

Prospects of observing the imprints of the tensor perturbations



The B-mode angular power spectra of the CMB resulting from the primordial tensor perturbations for three models with $r_{0.05} = 0.05$ have been plotted, along with the CMB lensing signal and the instrumental noise of a LiteBIRD-like configuration[17].

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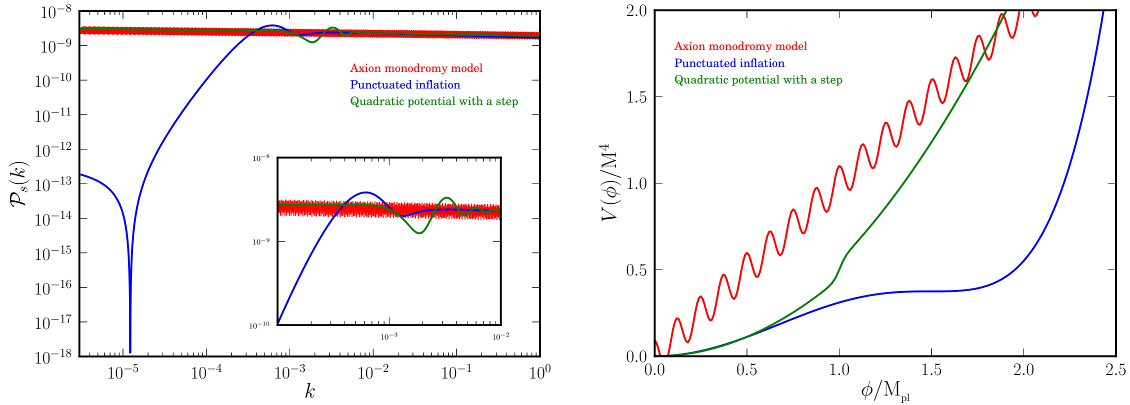
[17] D. Paoletti, F. Finelli, J. Valiviita and M. Hazumi, Phys. Rev. D **106**, 083528 (2022).

Spectra leading to an improved fit to the CMB data



The scalar power spectra (on the left) arising in different inflationary models (on the right) that lead to a better fit to the CMB data than the conventional power law spectrum[18].

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[18] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010);
M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013);
For a recent discussion, see H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D **106**, 043535 (2022).

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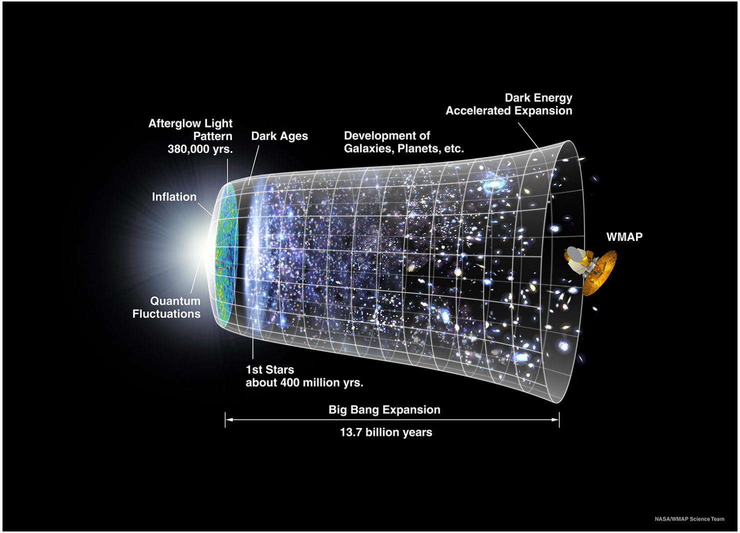
Timeline of the universe



A pictorial timeline of the universe[19].

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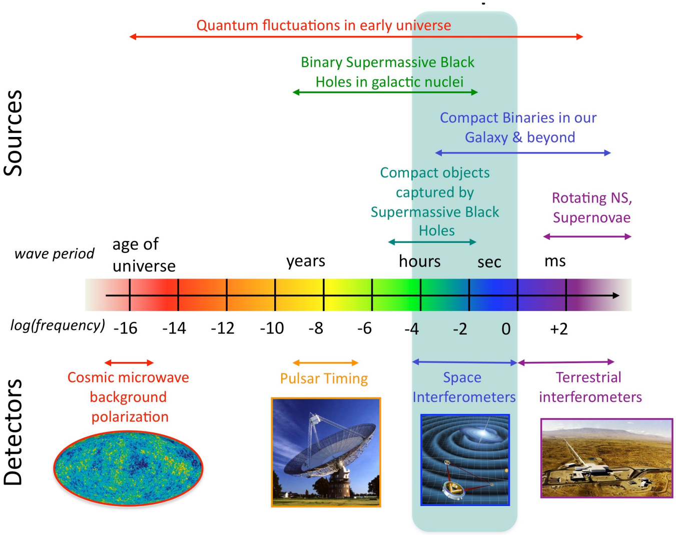
[19] See http://wmap.gsfc.nasa.gov/media/060915/060915_CMB_Timeline150.jpg.

The spectrum of GWs



Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

wave period
age of universe
years
hours
sec
ms

log(frequency)
-16
-14
-12
-10
-8
-6
-4
-2
0
+2

Detectors

Cosmic microwave background polarization

Pulsar Timing

Space Interferometers

Terrestrial interferometers

Different sources of GWs and corresponding detectors[20].

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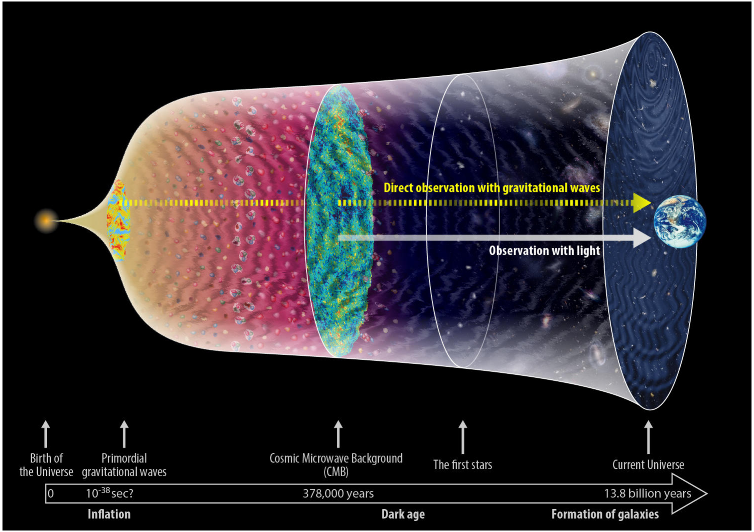
[20] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Probing the primordial universe through GWs



GWs provide a unique window to probe the primordial universe[21].

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[21] Image from https://gwpo.nao.ac.jp/en/gallery/000061.html.

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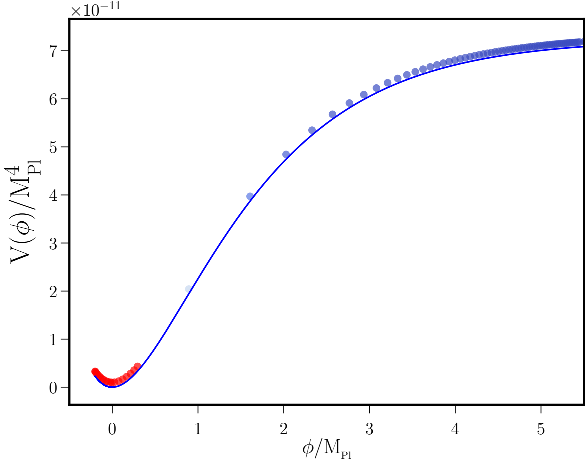
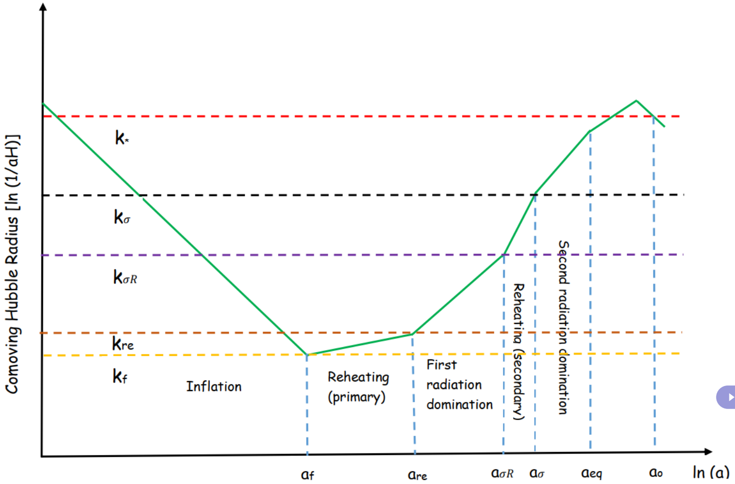
Evolution of the scalar field in an inflationary potential



The evolution of the scalar field in the so-called Starobinsky model has been indicated (as circles, in blue and red) at regular intervals of time. Inflation is terminated as the field approaches the bottom of the potential (near the light blue dot). Thereafter, the field oscillates at the bottom of the potential (indicated by the red dots).

Behavior of the comoving wave number and Hubble radius



▸ Evolution of physical lengths

▸ Back to secondary GWs

Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_{\mathrm{H}}/a = (a\,H)^{-1}$ (in green) across different epochs[22].

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[22] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Duration of reheating and the reheating temperature

The duration of the epoch of reheating N_{re} and the reheating temperature T_{re} can be expressed in terms of the equation of state parameter w_{re} during reheating and the inflationary parameters as follows[23]:

$$
\begin{aligned}
N_{\mathrm{re}} &= \frac{4}{(3\,w_{\mathrm{re}}-1)}\left[N_* + \frac{1}{4}\,\ln\left(\frac{30}{\pi^2\,g_{*,\mathrm{re}}}\right) + \frac{1}{3}\,\ln\left(\frac{11\,g_{s,\mathrm{re}}}{43}\right) + \ln\left(\frac{k_*}{a_0\,T_0}\right) + \ln\left(\frac{\rho_{\mathrm{e}}^{1/4}}{H_{_{\mathrm{I}}}}\right)\right],\\
T_{\mathrm{re}} &= \left(\frac{43}{11\,g_{s,\mathrm{re}}}\right)^{1/3}\left(\frac{a_0\,H_{_{\mathrm{I}}}}{k_*}\right)\,\mathrm{e}^{-(N_*+N_{\mathrm{re}})}\,T_0,
\end{aligned}
$$

where $H_{_{\mathrm{I}}}$ is the Hubble parameter during inflation, $T_0 = 2.725\,\mathrm{K}$ is the present temperature of the CMB, and H_0 denotes the current value of the Hubble parameter.

Note that $k_*/a_0 \simeq 0.05\,\mathrm{Mpc}^{-1}$ represents the CMB pivot scale and N_* denotes the number of e-folds *prior to the end of inflation* when the pivot scale leaves the Hubble radius.

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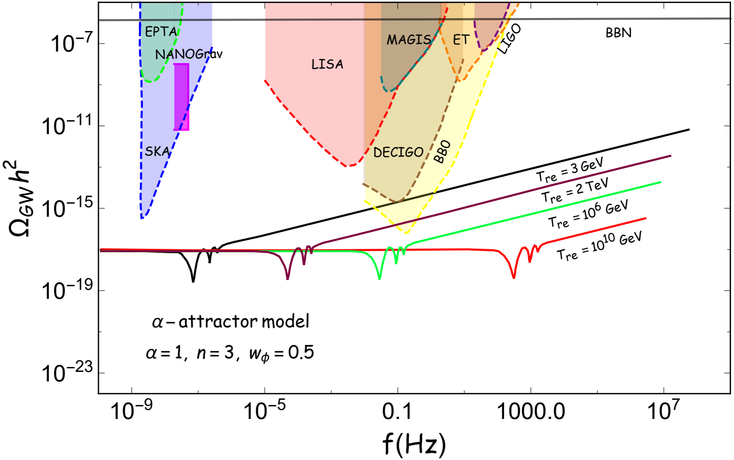
[23] J. Martin and C. Ringeval, Phys. Rev. D **82**, 023511 (2010);
L. Dai, M. Kamionkowski and J. Wang, Phys. Rev. Lett. **113**, 041302 (2014);
J. L. Cook, E. Dimastrogiovanni, D. A. Easson and L. M. Krauss, JCAP **04**, 047 (2015).

Effects on $\Omega_{\rm GW}(f)$ due to reheating



The behavior of the dimensionless spectral energy density of primary GWs today, viz. $\Omega_{\rm GW}$, has been plotted, over a wide range of frequency f, for different reheating temperatures (in red, green, brown and black)[24].

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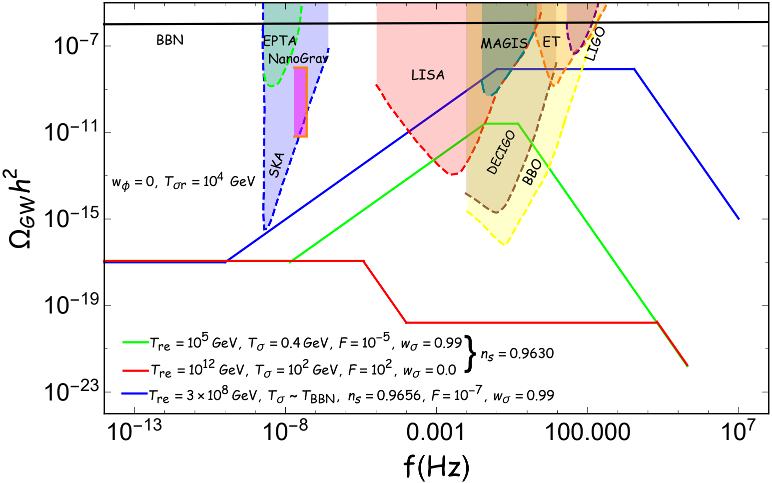
[24] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Effects on $\Omega_{\rm GW}(f)$ due to late time entropy production



The dimensionless spectral energy density of primary GWs observed today $\Omega_{\rm GW}(f)$ has been plotted in a scenario involving late time production of entropy[25].

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[25] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

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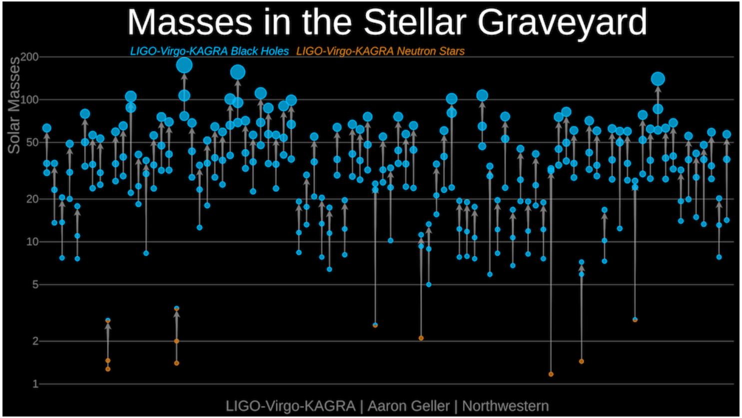
2. Constraints on inflation from the CMB data
3. GWs provide a new window to the universe
4. Reheating can boost the strengths of primary GWs
5. Generation of GWs by enhanced scalar perturbations on small scales
6. The NANOGrav 15-year data and its implications
7. Outlook

Coalescence of compact binaries observed by LIGO



In November 2021, the LIGO-Virgo-KAGRA Collaboration released the results of the second-half of their third observing run. This third GW Transient Catalog is the largest catalog of mergers involving black holes and neutron stars released thus far and includes events released in prior observing runs[26].

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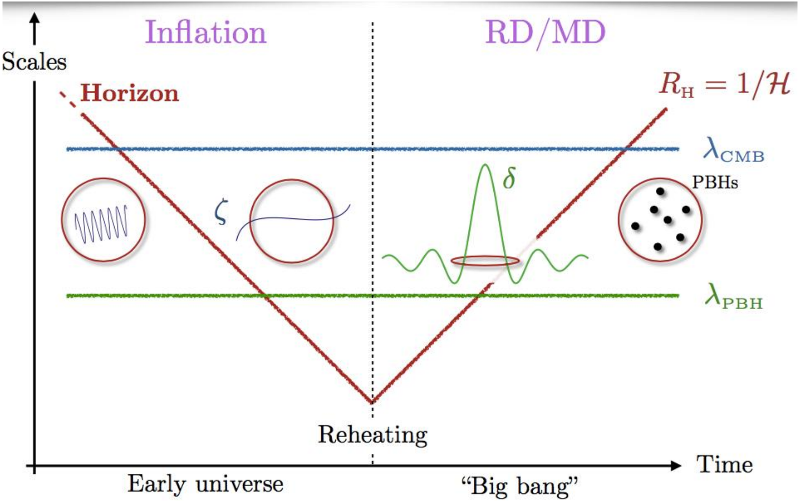
[26] Image from https://www.ligo.org/detections/O3bcatalog.php.

Formation of primordial black holes (PBHs)



BHs can form in the primordial universe when perturbations with significant amplitudes on small scales reenter the Hubble radius during the radiation dominated epoch[27].

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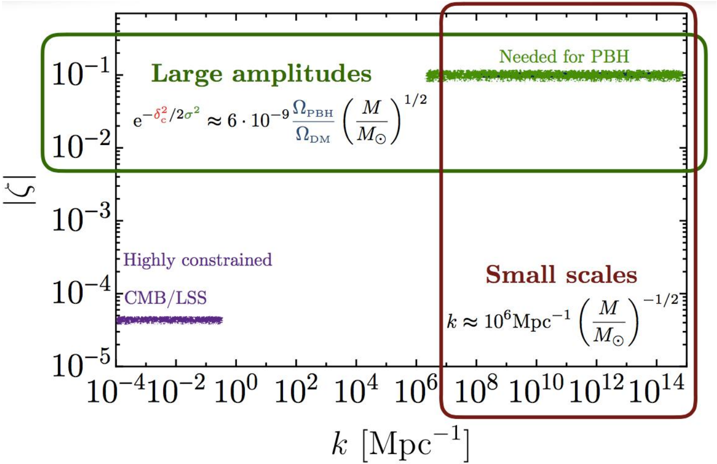
[27] Figure from G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

Amplitude required to form significant number of PBHs



In order to form significant number of black holes, the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity[28].

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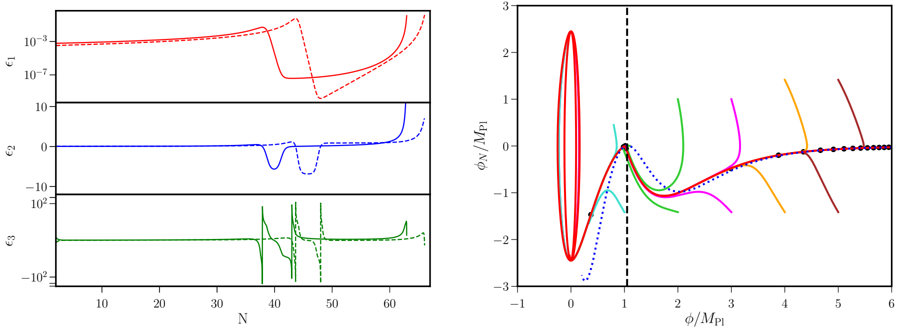
[28] Figure credit G. Franciolini.

Single-field models admitting ultra slow roll inflation



Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)[29]:

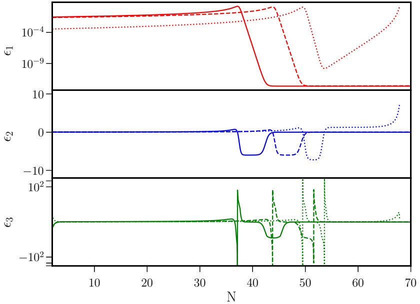
$$\mathrm{USR1}: V(\phi) = V_0\, \frac{6\,x^2 - 4\,\alpha\,x^3 + 3\,x^4}{(1+\beta\,x^2)^2},$$

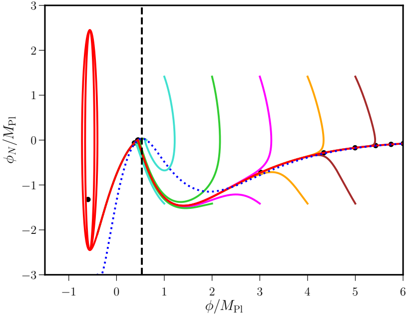
$$\mathrm{USR2}: V(\phi) = V_0\, \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

---

[29] J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Single-field models permitting punctuated inflation





Potentials admitting punctuated inflation[30]:

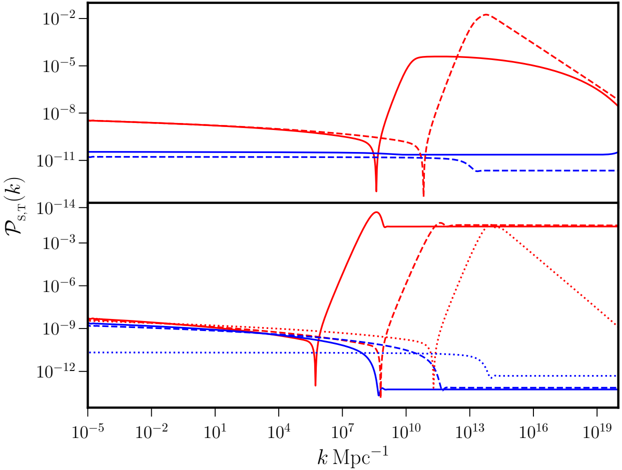
$$\mathrm{PI1}: V(\phi) = V_0\,\left(1+B\,\phi^4\right), \quad \mathrm{PI2}: V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4,$$

$$\mathrm{PI3}: V(\phi) = V_0\,\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right)\right]^2.$$

---

[30] D. Roberts, A. R. Liddle and D. H. Lyth, Phys. Rev. D **51**, 4122 (1995);
R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Power spectra in models permitting ultra slow roll inflation



The scalar (in red) and the tensor (in blue) power spectra arising in various single field models that permit a period of ultra slow roll inflation[31].

---

[31] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021); Also see H. V. Ragavendra and L. Sriramkumar, Galaxies **11**, 34 (2023).

The two field model of interest

It has been shown that two scalar fields ϕ and χ governed by the following action:

$$S[\phi,\chi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \frac{f(\phi)}{2}\,\partial^\mu\chi\,\partial_\mu\chi - V(\phi,\chi)\right]$$

described by the potential

$$V(\phi,\,\chi) = V_0\,\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{m_\chi^2}{2}\,\chi^2$$

and the non-canonical coupling functions

$$f_1(\phi) = \mathrm{e}^{2\,b_1\,\phi} \quad \text{or} \quad f_2(\phi) = \mathrm{e}^{2\,b_2\,\phi^2}$$

can lead to features in the scalar power spectrum[32].

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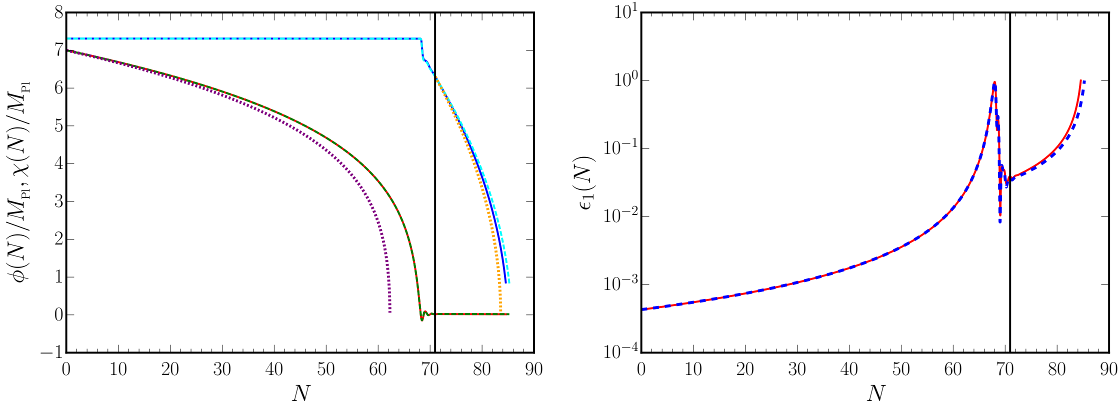
[32] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08** 025 (2020).

Non-trivial inflationary dynamics in the two-field model

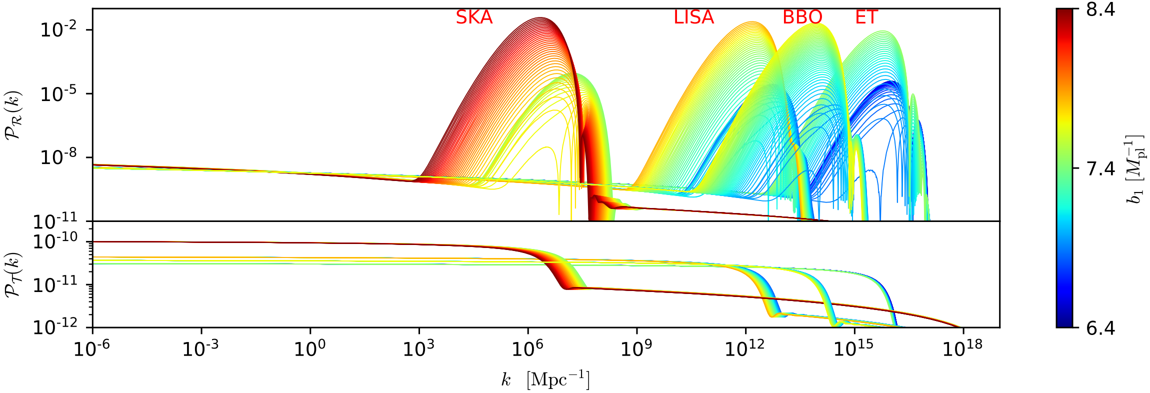


Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the two field model of our interest[33]. Note that there arises a turn in the field space around $N = 70$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

---

[33] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Enhanced power on small scales in the two-field model

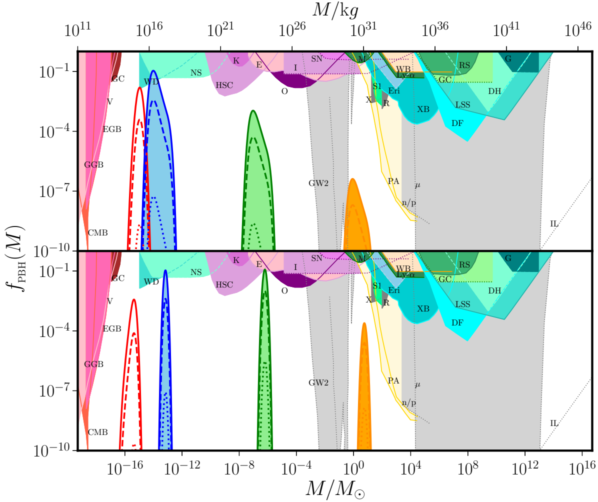


The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of a particular parameter[34].

---

[34] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

$f_{\rm PBH}(M)$ in ultra slow roll and punctuated inflation

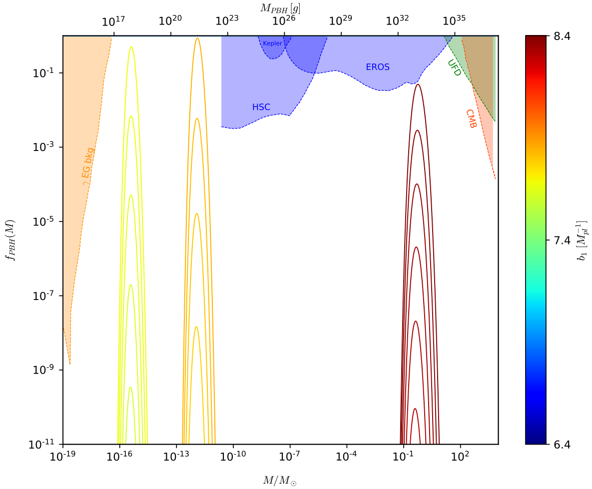


The fraction of PBHs contributing to the cold dark matter density today $f_{\rm PBH}(M)$ has been plotted for different models, viz. USR2 (on top, in red) and PI3 (at the bottom, in red)[35].

---

[35] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$f_{\rm PBH}(M)$ in the two-field model



The fraction of PBHs contributing to the dark matter density today $f_{\rm PBH}(M)$ in the two-field model of our interest[36].

---

[36] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Spectral density of secondary GWs today

The dimensionless spectral energy density of GWs $\Omega_{\mathrm{GW}}(k,\eta)$, when evaluated at late times during the radiation dominated epoch, can be expressed as[37]

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{\rho_{\mathrm{GW}}(k,\eta)}{\rho_{\mathrm{cr}}(\eta)} = \frac{1}{972}\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left[\frac{4\,v^2-(1+v^2-u^2)^2}{4\,u\,v}\right]^2 \mathcal{P}_{\mathrm{S}}(k\,v)\,\mathcal{P}_{\mathrm{S}}(k\,u) \times \left[\mathcal{I}_c^2(u,v)+\mathcal{I}_s^2(u,v)\right],$$

where the quantities $\mathcal{I}_c(u,v)$ and $\mathcal{I}_s(u,v)$ are determined by the transfer function $\mathcal{T}(k,\eta)$ for the scalar perturbations.

We can express $\Omega_{\mathrm{GW}}(k)$ today in terms of the above $\Omega_{\mathrm{GW}}(k,\eta)$ as follows:

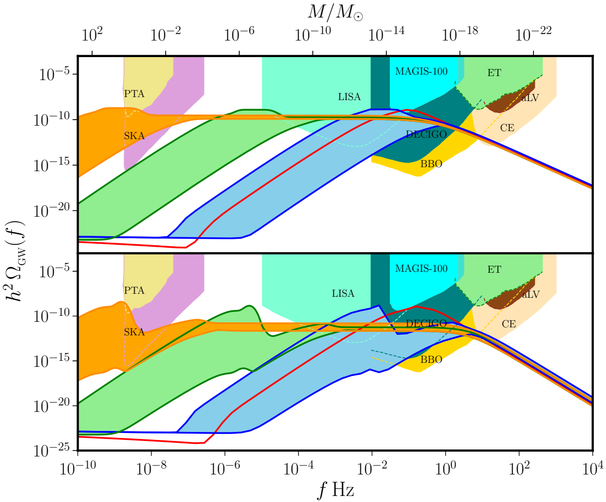
$$h^2\,\Omega_{\mathrm{GW}}(k) \simeq 1.38\times 10^{-5}\left(\frac{g_{*,k}}{106.75}\right)^{-1/3}\left(\frac{\Omega_r\,h^2}{4.16\times 10^{-5}}\right)\Omega_{\mathrm{GW}}(k,\eta),$$

where Ω_{r} denotes the dimensionless energy density of radiation today, while $g_{*,k}$ and $g_{*,0}$ represent the number of relativistic degrees of freedom at reentry and today, respectively.

---

[37] K. Kohri and T. Terada, Phys. Rev. D **97**, 123532 (2018);
J. R. Espinosa, D. Racco and A. Riotto, JCAP **09**, 012 (2018).

$\Omega_{\mathrm{GW}}(f)$ in ultra slow roll and punctuated inflation

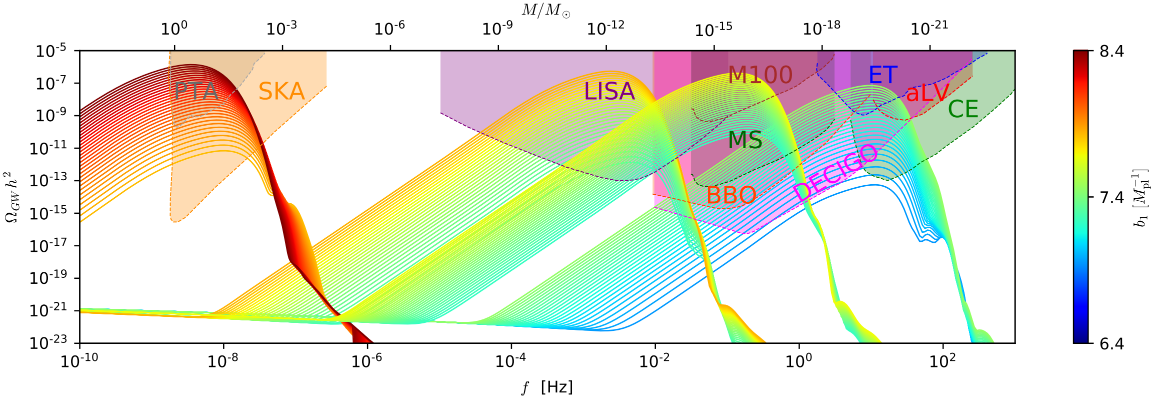


The dimensionless spectral density of GWs $\Omega_{\mathrm{GW}}(f)$ arising in the models of USR2 (in red, on top) as well as PI3 (in red, at the bottom)[38].

---

[38] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$\Omega_{\mathrm{GW}}(f)$ in the two-field model



The dimensionless spectral density of GWs $\Omega_{\mathrm{GW}}(f)$ arising in the two-field model has been plotted for a set of initial conditions for the background fields as well as a range of values of the parameter b_1[39].

---

[39] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

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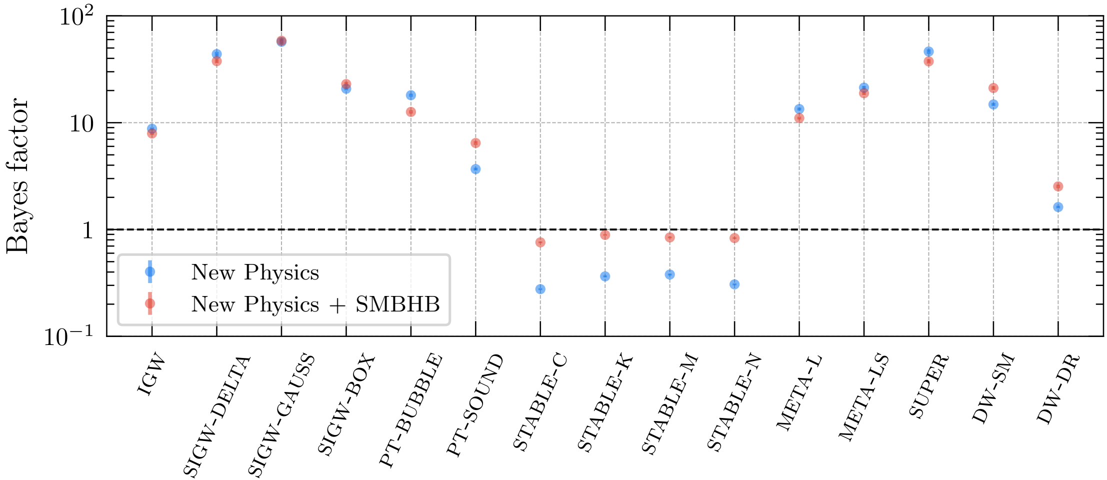
Stochastic GW background observed by pulsar timing arrays (PTAs)



The Bayesian evidence for a variety of astrophysical and cosmological sources for the stochastic GW background suggested by the observations of the PTAs [40].

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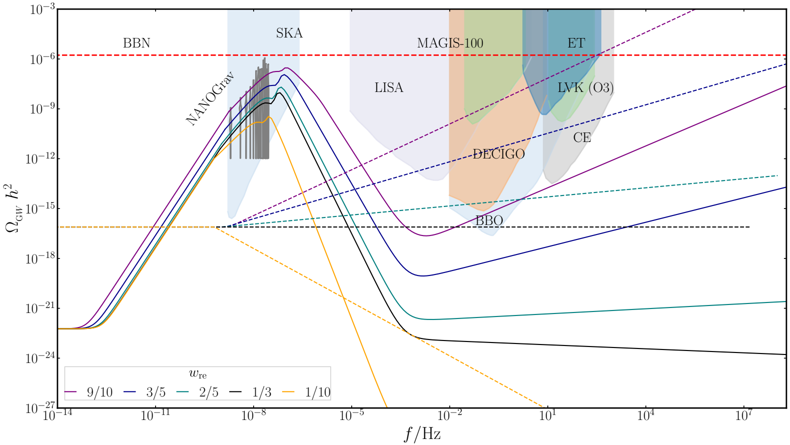
[40] NANOGrav Collaboration, Astrophys. J. Lett. **951**, L11 (2023).

Generation of secondary GWs during the epoch of reheating



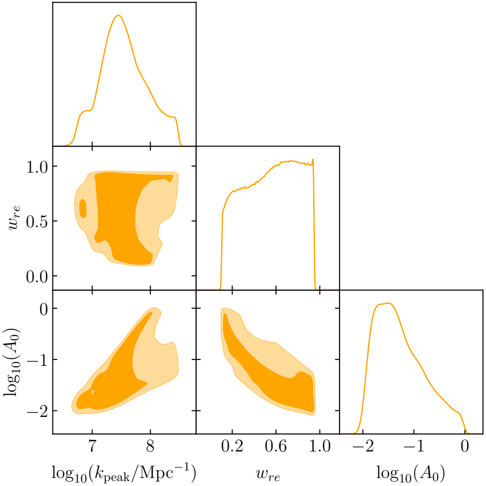
Epoch of reheating

The dimensionless spectral energy density of primary and secondary GWs today $\Omega_{\rm GW}(f)$ have been plotted for a given reheating temperature and different values of the parameter describing the equation of state during reheating[41].

---

[41] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, in preparation.

Constraints on the epoch of reheating

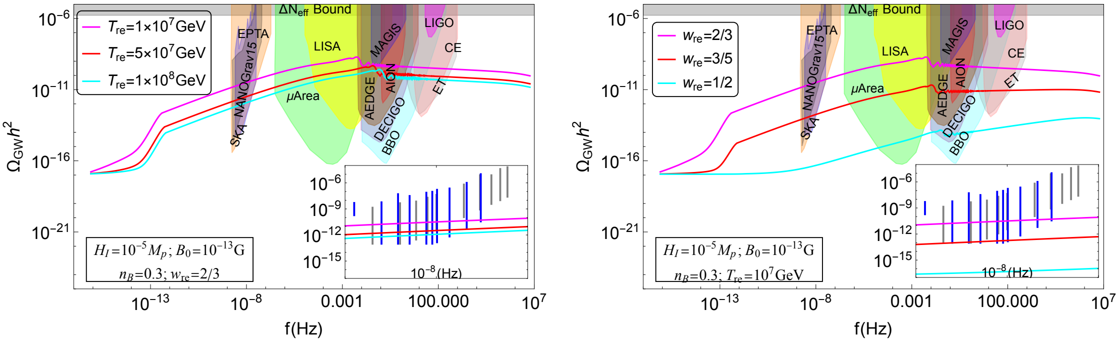


Constraints on the parameters describing the primordial scalar power spectrum and the epoch of reheating, arrived at upon comparison with the NANOGrav 15-year data[42].

---

[42] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, in preparation.

Secondary GWs induced by primordial magnetic fields (PMFs)



The dimensionless spectral energy density of secondary GWs observed today $\Omega_{\rm GW}(f)$, induced by the PMFs, have been plotted for different reheating temperatures (on the left) and different values of the parameter describing the equation of state during reheating (on the right)[43].

---

[43] S. Maiti, D. Maity and L. Sriramkumar, arXiv:2401.01864 [astro-ph.CO].

Plan of the talk

Outlook

- ✦ If one of the future CMB missions—such as LiteBIRD (Lite, Light satellite for the studies of B-mode polarization and Inflation from cosmicbackground Radiation Detection), Primordial Inflation Explorer (PIXIE) or Exploring Cosmic History and Origin (ECHO, a proposed Indian effort)—detect the signatures of the primordial GWs, it will help us arrive at strong constraints on the dynamics during inflation and reheating.
- ✦ The observations by LIGO are a culmination of almost fifty years of effort to detect GWs. They have opened up a new window to observe the universe.
- ✦ The observations by the PTAs and their possible implications for the stochastic GW background offer a wonderful opportunity to understand the physics operating over a wider range of scales in the early universe.
- ✦ During the coming decades, GW observatories such as the Laser Interferometer Space Antenna (LISA), Einstein Telescope and Cosmic Explorer, can be expected to provide us with an unhindered view of the primordial universe.

Collaborators I

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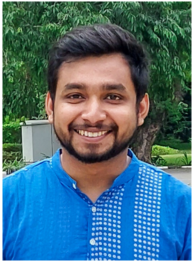
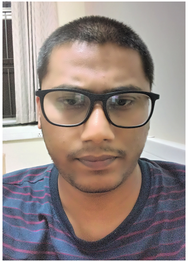
Subhasis Maiti



Suvashis Maity



Nilanjandev Bhaumik

Thank you for your attention